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A2_3 Deep Space 2: Journey to another star

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Abstract

In designing a "Generation Ship", this paper discusses the relativistic effects on the velocity and travel time of a journey at an initial 1g acceleration to the star system Alpha Centauri. It was found that such a journey would take 3.8 years for the crew, 6.5 years for an observer on Earth and would have a peak velocity of 0.89c.

Introduction

In Deep Space 1: Staying Cool [1] we briefly touched upon the power required to keep a generation ship of density similar to the Space Shuttle and with the shape of a cylinder 50m long and 10m in radius constantly accelerating at 1g with advanced ion propulsion. However this did not take into account travel time, as a PWR lifecycle is around 18 months [2], if the journey could not be made in this time additional fuel would be needed. In order to correctly calculate the travel time it is necessary to take into account Special Relativity. This places an upper limit on velocity but also causes time dilation. Indeed a ship capable of just 1g acceleration would be capable of traversing the universe in a single human lifespan (relative to the passengers on board, not external observers) [3].

Travel Time

So, with such effects, just how long would it take to get to Alpha Centauri (which is $\sim 4 \times 10^{16}$m from the sun) with acceleration, a, of 1g? We can use the Lorenz transformations to show the velocity, v, of the ship to be:

$$v(t) = \frac{at + \gamma_0 v_0}{\sqrt{1 + \frac{(at+\gamma_0 v_0)^2}{c^2}}}, \tag{1}$$

at any time t in the rest frame of the ship after beginning its journey- assuming it starts at rest its initial velocity v_0 will be 0 and therefore its initial Lorentz factor γ_0 will be 1. However the ship will not always be accelerating in the direction of travel (assuming the ship is to stop gradually before reaching Alpha Centauri) so halfway through the journey the craft will need to reverse thrust and start accelerating against the direction of travel. This requires the journey to be split into 2 stages of 2×10^{16}m, the first with $a = g$ and the second with $a = -g$. In order to work out the time taken for a stage, first the position at a given time must be calculated:

$$r(t) = \int_{t_0}^{t_1} v(t)dt = \frac{c^2}{a}\sqrt{1 + \frac{(at + \gamma_0 v_0)^2}{c^2}}, \tag{2}$$

given that we know $r = 2 \times 10^{16}$ for a stage, we can solve for t for the first stage (where $v_0\gamma_0$ is 0) to find the time taken for the accelerating part of the journey: $t_1 = 9.23 \times 10^7$s or just less than 3 years. Obviously time dilation has had an effect (as Alpha Centauri is over 4 light years away)and later the proper time will be calculated. Substituting this back into 1 we find the velocity at the end of the first stage $v_1 = 2.84 \times 10^8$m/s or $\sim 0.95c$. We can just use symmetry to know that the second stage time would be the same as the first- assuming the ship comes to rest in the Alpha Centauri system (this is obviously not what would happen in reality) giving a total time taken t_{total} of 1.85×10^8s or $\sim$ 5.8years.

Earth Time

In order to discover how long the journey would take in the inertial frame of Earth, we must start with the equation for proper time, given constant acceleration [4]:

$$\tau = \frac{c}{a}\text{arcsinh}\left(\frac{at}{c}\right), \tag{3}$$

by setting the Proper time to t_1 and solving for t, a result of 3.12×10^8s was found. As above, by considering this half of the total time (given half the total distance) a total travel time from Earth is found to be ~ 19.7 years.

Conclusion

Setting aside the concerns of the required power output raised in Deep Space 1, the journey to Alpha Centauri could not be made within the 18 months of a single refuelling cycle. This adds to the problem with using a PWR to provide power to the generation ship. Additionally the crew would have had to be specifically trained to deal with long periods of time in a confined space with each other- 20 years for a one-way trip is a long time to spend with a small group of people. Of course, the reason this is a generation ship is that a long term mission would more than likely last longer than a human lifespan- if you are going to travel for 20 years, it would be a good idea to stay for quite a long time.

References

[1] M. Bryan, J. Forster, A. Stone *A2_1 Deep Space 1: Staying Cool*, PST 11, (2012).
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[4] L. Iorio *An analytical treatment of the Clock Paradox in the framework of the Special and General Theories of Relativity*, Foundations of Physics Letters, vol. 18, issue 1, pp. 1-19 (2005)