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P2_10 Shotgun!

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Abstract
This paper looks at the stopping power of shotguns as a function of distance from the target and number of shot in the shell. This relationship is determined, and it is found that the stopping power falls off quickly for larger numbers of pellets. Avenues for a more thorough investigation are also suggested.

P2_1 Firearms

Introduction

A shotgun is a type of firearm that generally uses a fixed shell size to fire a number of spherical pellets, known as shot. The number of pellets inside is often varied depending upon the application. Bird shot (used for bird hunting) uses a large number of shots in each shell, while buck shot (used for hunting deer) uses fewer shots in each shell.

The stopping power of a shotgun is not universally defined. However for the purpose of this article we will refer to it as the total kinetic energy of the pellets.

The article aims to determine the stopping power of a shotgun as a function of range for different types of ammunition.

Method

Modelling the shells to find the change in velocity with varying number of shot inside will provide evidence for the difference in the stopping power that the different types of ammunition can impart upon their target. We have a variable number of projectiles ranging from 2 to 64, with the total mass of the shot remaining the same.

The drag force on the spherical shots can be approximated using [1]

$$F = -\frac{1}{2}\rho_{air} A C_d v^2 \quad (1)$$

where F is the drag force, ρ_{air} is the density of the air 1.2kgm^{-3} [1], A is the cross sectional area of the shot, v is the velocity of the shot and C_d is the drag coefficient, taken to be 0.5 for spherical shot [4]. Acceleration therefore is given from Newton's second law as

$$a = -\frac{1}{2}\rho_{air}\frac{A}{m}C_d v^2 \quad (2)$$

where m is the mass of the pellet and a is the acceleration.

The number of shots in a shell can be expressed as

$$n = \frac{M}{\frac{4}{3}\pi r^3 \rho_{steel}} \quad (3)$$

where n is the number of shots, r is the radius of the spherical shot, M is the total mass of the shell and ρ_{steel} is the density of the shot, modelled here as steel (the standard material for shot) taken to be 7850kgm^{-3}[1].

Rearranging equation (3) for r gives

$$r = \sqrt[3]{\frac{3M}{4\pi n \rho_{steel}}} \quad (4)$$

Substituting this into equation (2) we can find the acceleration

$$a = -v^2 \frac{\rho_{air} C_d}{2m} \pi \left(\frac{3M}{4\pi n \rho_{steel}}\right)^{\frac{2}{3}} \quad (5)$$

With specifications for the total mass of ammunition used, the only unknowns in this equation will be a and v the rest can be considered a constant C

$$\frac{dv}{dt} = -v^2 C \quad (6)$$

Integrating equation (6) and using separation of variables gives

$$\int \frac{1}{v^2} dv = -C\int dt$$

$$\frac{-1}{v} = -Ct + c$$

where *c* is an integration constant. Rearranging for *v* and imposing the boundary condition $v=v_0$ at $t=0$ (where v_0 is the initial velocity of the pellets as they leave the muzzle) gives the integration constant *c* as $\frac{1}{v_0}$

$$v = \frac{v_0}{1+Ctv_0} \quad (7)$$

We can find the change in velocity with respect to distance travelled in the direction of motion.

$$d = \int v\, dt$$

Substituting in the velocity for equation (7), and rearranging for *t* gives

$$d = \int \frac{v_0}{1+Ctv_0} dt$$

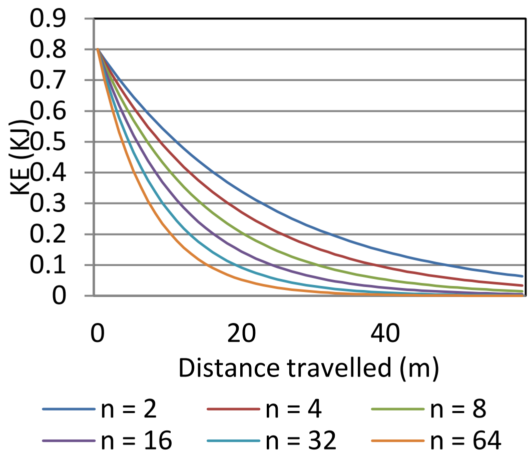
$$d = \frac{log(1+Ctv_0)}{C}$$

$$t = \frac{e^{Cd}-1}{Cv_0} \quad (8)$$

Substituting (8) into equation (7) we can find the change in velocity with respect to the distance travelled.

$$v = v_0 e^{-Cd} \quad (9)$$

using this equation (8) and the fact that kinetic is given by $\frac{1}{2}mv^2$ we can created graph 1 below, showing how the total kinetic energy varies with distance for different values of *n* the values used for the total mass is 0.01kg and the muzzle velocity is $400ms^{-1}$ [5].



Graph 1, shows how the total kinetic energy of the shell changes with distance for a variety of different number of projectiles.

Discussion

We assume that all the shot from a shotgun will be spherical; there are however different types of shotgun shells available other than spherical shot and these would require an alteration to the method shown in this article.

In any real situation there would be many other variables to take into account more than just the air resistance used in this model including wind, trajectory, obstacles etc.

The graph indicates that there would always be greater stopping power with fewer pellets. However, there is an issue regarding the stopping power of the shells that has not been addressed in this article. That is how the shots spread as they travel through the air. Although the stopping power falls off more steeply for larger number of pellets inside the shell, the chance of hitting the target is increased. The magnitude and form of this effect would be an area of further study.

Conclusion

The different number of shots in a shell will have an effect on the stopping power that the shotgun will have. Larger numbers of small pellets clearly lose kinetic energy over a shorter distance than smaller numbers of larger pellets. This is because the drag force has a greater total effect on the smaller pellets. For a more thorough conclusion, data on the amount of damage on a target, as a function of shot kinetic energy would be required. This would allow us to draw conclusions regarding the maximum effective range for different values of *n* against small animals as a hunting weapon, or against humans as a self-defence weapon.

References

[1] Stokes equation, *http://www.engineeringtoolbox.com/drag-coefficient-d_627.html*
[2]Density of steel, http://www.engineeringtoolbox.com/metal-alloys-densities-d_50.html
[3]Density of air, http://www.engineeringtoolbox.com/air-properties-d_156.html
[4] Drag coefficient for a sphere, *http://www.grc.nasa.gov/WWW/K-12/airplane/shaped.html*
[5] Muzzle velocity, total mass of the shell, *http://library.med.utah.edu/WebPath/TUTORIAL/GUNS/GUNBLST.html (shotgun ballistics)*