Journal of Physics Special Topics
An undergraduate physics journal

A2_2 What If It's Always Sunny In Philadelphia?

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November 21, 2016

Abstract

We calculate the solar flux that would reach Philadelphia if it were always sunny in Philadelphia. We model how the intensity of the flux varies depending on the Earth's orientation with respect to the Sun finding that a sunny Philadelphia would receive enough solar flux to run 2 million TVs at once.

Introduction

In this paper we have decided to take the title of a popular sitcom - It's Always Sunny In Philadelphia, and interpret it literally. What if it were always sunny in Philadelphia? We will develop a model that allows us to calculate the solar flux that reaches Philadelphia at any time of the year, through considering daily and seasonal variations in terms of the Earth's orientation with respect to the Sun. This will then allow us to find the total energy transferred to philadelphia in a year.

Theory

First we assume the sky is clear. This means that 48% of the solar flux is transmitted through the atmosphere and reaches Philadelphia. [1]. Hence the solar flux that reaches Philadelphia is given by Eq. (1).

$$F = \frac{\alpha L}{4\pi r^2} \tag{1}$$

where F is the flux that reaches Philadelphia, α is the fraction of flux that is transmitted through the atmosphere (0.48), L is the luminosity of the Sun (3.9×10^{26} W) and r is the mean Earth-Sun distance (1 AU). Next we consider the Earth's orientation relative to the Sun.

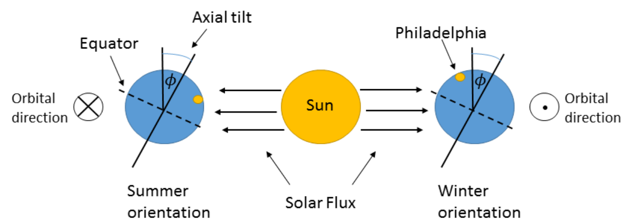

Figure 1: Philadelphias orientation relative to the sun as the seasons change. Philadelphia faces the solar flux at a shallower angle in the summer than in the winter.

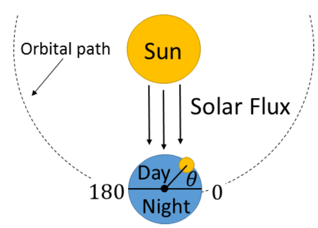

Figure 2: Philadelphias orientation relative to the sun as it changes throughout the day

Eq. (1) assumes that the unit normal to the surface of Philadelphia is parallel to the flux from the Sun. This is not the case as can be seen in

Fig. (1). We take Philadelphia at a latitude of 40 degrees North [2] and the Earth's axial tilt as 23.5 degrees. This means that Philadelphia will receive the most flux during the summer, when the Earth is tilted towards the Sun. We also assume that the solar flux is highest at noon, and that no solar flux reaches Philadelphia during the night. This is shown in Fig. (2). Considering the relative components of the solar flux that reaches Philadelphia dictated by ϕ (Fig. (1)) and θ (Fig. (2)) gives Eq. (2)

$$F = \sin(365\theta)\cos(\phi)\frac{\alpha L}{4\pi r^2} \tag{2}$$

where $0 < \theta < 180$, so that the sin term represents the component of flux that depends on the time of day ($\sin(365\theta) = 0$ for $180 < \theta < 360$ is imposed as a constraint), ϕ ranges from 16 to 64 degrees (40 ± 23.5) so that the cos term represents the component of flux that depends on the time of year. The sin term contains a constant because the period for the Earth's rotation must be 365 times smaller than the period of the Earth's orbit around the sun. The angular dependence of Eq. (2) is plotted in Fig. (3) which shows how the solar flux varies on daily and yearly scales.

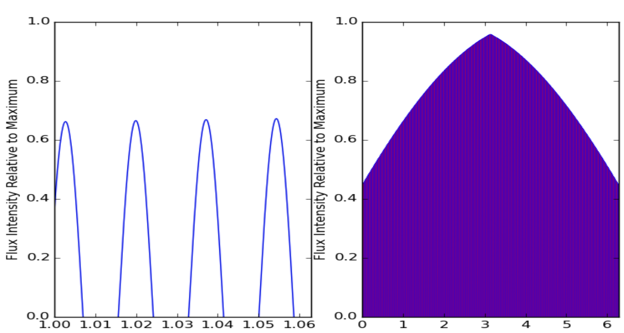

Figure 3: The daily fluctuation in solar flux reaching Philadelphia (left) and the change in daily fluctuations as the seasons pass (right).

The total energy for 1 year is given by Eq. (3). This is equivalent to the total area under the curve in Fig. (3) (right).

$$E = \int_0^{1yr}\int_{\frac{4\pi}{45}}^{\frac{16\pi}{45}}\int_0^{\pi} AF\,d\theta\,d\phi\,dt \tag{3}$$

A is the area of Philadelphia (349.9 km^2 [3]) and the limits are in time and then radians.

Errors

The errors associated with each of the variables are: $\Delta r = \pm 0.0167$ AU due to the Earth's elliptical orbit [4], $\Delta L = \pm 0.1\%$ due to solar variations [5], $\Delta\phi = \pm 0.05$ degrees and $\Delta\alpha = \pm 0.005\%$, both due to limitations in precision. Using standard error propagation we found the uncertainty associated with the flux to be $\Delta F = \pm 10.6\%$.

Conclusion

Using Eq. (2) we find a peak flux to be 640 ± 70 Wm^{-2} ($\theta = 90$ and $\phi = 16$), and Eq. (3) gives the total energy in 1 year as $2.48 \pm 0.26 \times 10^{16}$ J. This translates to a power of 786 MW and would be enough to run 2 million TV's playing It's Always Sunny In Philadelphia at once [6].

References

[1] http://earthobservatory.nasa.gov/Features/EnergyBalance/page4.php accessed on 10/11/2016

[2] https://www.distancesto.com/coordinates/us/philadelphia-latitude-longitude/history/726.html accessed on 10/11/2016

[3] http://www.bluebulbprojects.com/measureofthings/results.php?p=1&comp=area&unit=mi2&amt=134&sort=pr accessed on 10/11/2016

[4] https://en.wikipedia.org/wiki/Earth\%27s_orbit accessed on 26/10/16

[5] https://en.wikipedia.org/wiki/Solar_constant accessed on 26/10/16

[6] http://michaelbluejay.com/electricity/tv.html accessed on 05/11/16