Journal of Physics Special Topics
An undergraduate physics journal

A5_1 A Drop of Rain

L. Carvalho, S. Hughes, S. Taper, K. Crutcher

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

November 8, 2019

Abstract

How much energy could we transfer from rainfall to power a 60 W lightbulb by fitting a hydroturbine into the base of a drainpipe attached to the Physics building at the University of Leicester (UoL)? We discovered that the maximum energy you could produce is 142.00 MJ which would be enough to power a 60 W lightbulb for 662.03 hours over the course of a year.

Introduction

Falling objects convert their gravitational potential energy (GPE) into kinetic energy (KE). If a small hydroelectric turbine was installed in a drainpipe how much energy would be produced by falling rain? To tackle this problem, we decided to calculate the amount of rainfall over the course of a year on a single building. We assumed that the total amount of water fallen could be stored and released with no losses. The system works via dispensing the water down a drainpipe with a hydroelectric turbine at the bottom, this is turned by the falling water generating electricity, as shown in Figure 1. It is assumed there is a full energy transfer from GPE to KE.

To calculate the energy generated over a year we first looked at how much energy was converted in a single release of the tank, E_r. We assumed the tank and pipe diameter match and that air resistance is negligible in order to model the water release as freefall. The energy transferred from GPE into electricity from water in freefall through a turbine is (1).

$$E_r = mgh\eta \tag{1}$$

Where η is the global efficiency ratio of a hydroturbine, m is mass of the water, g is gravitational acceleration and h is the height the water falls from. The mass of water in the tank can be put in terms of the water density, ρ, and cross-sectional area of the pipe or tank, A_p, giving (2).

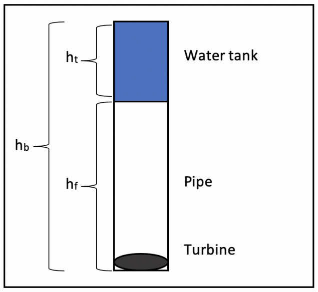

Figure 1: Shows the relation between h_b, the height of the building, h_t, the height of the tank and h_f, the height of the fall from the bottom of the tank to the turbine.

$$m = \int_{h_f}^{h_b} \rho A_p dh \tag{2}$$

After substituting in the new expression for mass and integrating we get (3).

$$E_r = \eta g \rho A_p \int_{h_f}^{h_b} h dh = \frac{1}{2}\eta g \rho A_p ({h_b}^2 - {h_f}^2) \tag{3}$$

To get the total energy released in a year, E_t, we multiply E_r (3) by the number of times water is released from the tank in a year, n (4), giving us (5). We calculated n by dividing the volume of rainfall in a year by the volume of the tank, assuming that n is large enough that non-integer values of n would have a linear relation to energy for simplicity.

$$n = \frac{V_{rain}}{V_{tank}} = \frac{A_b h_r}{A_p h_t} \tag{4}$$

Where A_b is the area of the building's roof and h_r is the height of the rainfall in a year.

$$E_t = \frac{\eta g \rho A_b h_r ({h_b}^2 - {h_f}^2)}{2h_t} \tag{5}$$

As $h_b = h_f + h_t$, (5) can be simplified to (6).

$$E_t = \frac{1}{2}\eta g \rho A_b h_r (h_t + 2h_f) \tag{6}$$

Discussion

Using the Physics building at UoL as our catchment area for the rainfall, we calculated A_b to be 2990.50 ± 11.33 m^2 using satellite maps [5] which have inbuilt measurement tools. We took 5 measurements of the area to find the mean and standard deviation. We estimated the height of the building to be 3 m per floor with 0.5 m thick floors for a total height, h_b of 11 m. The average rainfall in Leicester is 620 mm per year [1]. A standard drainpipe is 6 inches in diameter [2] giving it a cross-sectional area, A_p of 0.0182 m^2. η is usually between 0.7 and 0.9 [3] with a secondary source claiming that the efficiency of a small turbine is 75.1% [4]. The density of water is 997 kg m^{-3}.

According to the values stated above and (6) we could produce 74.903 MJ by having the height of the tank, h_t, equal to the height of the building, h_b and 142.00 MJ by having the tank be 1 m tall. Decreasing the height of the tank would increase this further but 1 m seemed a realistic tank size to us. This amount of energy would be enough to power our 60 W light bulb for a total of 662.03 hours over the course of a year.

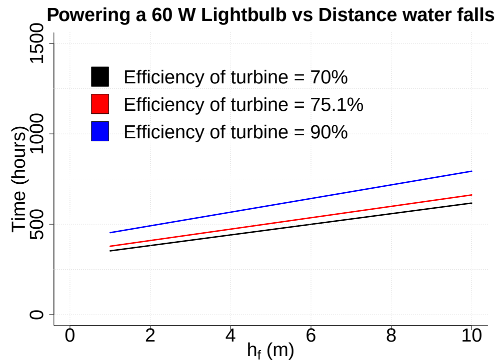

Figure 2: The duration that a 60 W light bulb can be powered is linearly related to height that the water falls from above the turbine. Each line shows different efficiency levels.[3]. The time at a height of 0 m would be 346.77 hours for 75.1% efficiency due to h_t being maximised and h_f being minimised in (6).

Conclusion

The final result shows that by optimising the height that the water falls from, if we were able to store water up until it filled 1 m of the drainpipe and then released it at once with no losses (not unrealistic), we would be able to power a 60 W lightbulb non-stop for almost a month.

For future work we would recommend a look at how much energy could be produced by installing turbines into turnstiles in public places, such as the London Underground entrances.

References

[1] https://en.climate-data.org/europe/united-kingdom/england/leicester-17/[Accessed 11 October 2019]

[2] https://www.leafguard.com/gutter-resources/rain-gutters [Accessed 1 October 2019]

[3] https://power-calculation.com/hydroelectricity-energy-calculator.php [Accessed 1 October 2019]

[4] https://www.renewablesfirst.co.uk/hydropower/hydropower-learning-centre/how-much-power-could-i-generate-from-a-hydro-turbine/ [Accessed 1 October 2019]

[5] https://www.google.com/maps/place/University+of+Leicester/@52.6222482,-1.1235543,18z/data=!4m5!3m4!1s0x4877612fa55d947f:0xf8da17fed74e0044!8m2!3d52.6211393!4d-1.1246325 [Accessed 11 October 2019]