Journal of Physics Special Topics

An undergraduate physics journal

P2_3 Killing Two Birds with One Stone

H.Biddle, K.Golsby, K.Hinchcliffe, J.Mooney

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

October 23, 2019

Abstract

The famous idiom 'to kill two birds with one stone' is often used throughout everyday life to describe achieving two things with only one action. We are considering the physical scenario in which this phrase could be possible. We found that a stone thrown at a velocity of at least 15.7 m s^{-1} and 77.4° to the horizontal would kill a hummingbird hovering at 12 m, and from here found that the falling stone would kill a second hummingbird positioned at least 0.519 m away at an angle of 15.3° down and right from the horizontal with respect to the first bird.

Introduction

The unfortunate idiom 'to kill two birds with one stone' is defined as to 'accomplish two different purposes by the same act or proceeding' [1]. From this idiom, we have decided to take the literal sense, and have calculated the velocity a stone would need to be thrown with to hit and kill a hovering hummingbird, and the position a second hovering bird would need to be in to also be killed by the motion of this stone. We have done this using the equations of projectile motion, as well as the conservation laws of energy and momentum that influence the collisions of objects. No birds were harmed in the making of this article.

Theory and Results

We have considered the scenario in which a hummingbird of mass $m_{\mathrm{B}} = 1.95 \times 10^{-3}$ kg [2] is hovering above the ground at a height of 12 m (the height at which some hummingbirds nest in trees) [3]. We have assumed, as this is the case in many scenarios, that a collision with a window at maximum flight speed is enough to kill a bird, and given that this speed for a hummingbird is 30 m p h (13.41 m s^{-1}) [4], we have calculated the impact energy E required to do so to be 0.175 J using Equation 1:

$$E = \frac{1}{2}mv^2. \tag{1}$$

This is therefore the energy the stone must have if it is to kill a hovering bird, so from this we find that a small stone of assumed mass m_{s}= 0.03 kg would need to collide with the bird at a minimum speed $v_{s||} = 3.42$ m s^{-1}. If we take the altitude of the collision as 12 m (the maximum height s in the stone's projectile motion), Equation 2,

$$v^2 = u^2 + 2as, \tag{2}$$

can be used to calculate the vertical component of the velocity the stone must be thrown at. Given that its horizontal component will be constant if we assume negligible air resistance, we use trigonometry to find a required initial throw velocity of $v_s = 15.7$ m s^{-1} at an angle 77.4° from the horizontal. The scenario proposed to hit the first bird can be seen in Figure 1. The veloc-

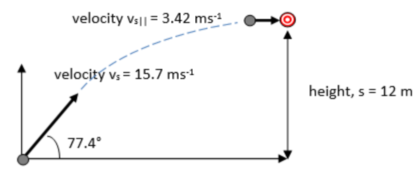

Figure 1: Projectile motion of stone thrown at a hovering bird.

ity of the stone at the point of collision will only have a non-zero horizontal component, so we can make use of the conservation of momentum and energy, described by Equations 3 and 4 respectively, where the inital values are on the left of the equals sign and the final values on the right, assuming this is an approximately elastic collision [5].

$$v_{s||}m_s = v_{sa}m_s + v_Bm_B \tag{3}$$

$$\frac{1}{2}m_s{v_{s||}}^2 = \frac{1}{2}m_s{v_{sa}}^2 + \frac{1}{2}m_B{v_B}^2 \tag{4}$$

Combining and rearranging these gives Equation 5, allowing for calculation of the resultant velocity $v_{sa} = 3.00$ m s^{-1} of the stone after the collision.

$$v_{sa} = \frac{\frac{m_s}{m_B} - 1}{\frac{m_s}{m_B} + 1} v_{s||} \tag{5}$$

The stone at this point has potential energy which will be converted to kinetic as it falls back to the ground, so the following equations of projectile motion can be used,

$$v = u + at \tag{6}$$

$$s = ut + \frac{at^2}{2} \tag{7}$$

where v_{sa} is being substituted in place for u for the horizontal component of velocity, and u is 0 for the vertical component. The vertical and horizontal distances travelled in the 0.167 s (calculated using Equation 6) it takes for gravity to accelerate the stone enough to reach the 3.42 m s^{-1} required to kill another bird can be found using Equation 7. Again using trigonometry, the required position of the second bird relative to the first is found to be 0.519 m at 15.3° down and right from the horizontal for a kill shot. This scenario can be seen in Figure 2.

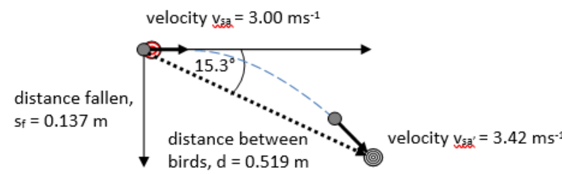

Figure 2: Projectile motion of stone after collision to hit a second bird.

Discussion and Conclusion

We have assumed that air resistance can be ignored, and the collision has been considered elastic. This is feasible to a degree, given the percentage of energy dissipated during the collision likely being very small compared to the initial energy of the stone, and the deceleration due to air resistance is relatively small due to the stone's size and density. However, a more thorough consideration of this may be required. Hence, given these specific conditions, it is possible for a person to kill two birds with one stone. Further study into this topic could evaluate the limit of distance or maximum bird mass that this would be possible for, and the more likely scenario that the target would be flying with its own horizontal velocity, which would call for more complex consideration of the conservation laws.

References

[1] *Definition of to Kill Two Birds with One Stone*, Cambridge University Press, 2019.

[2] A.Glick, *Mellisuga Helenae*, University of Michigan Museum of Zoology, 2017.

[3] `https://www.allaboutbirds.org/guide/Ruby-throated_Hummingbird/lifehistory`, accessed 09/10/19.

[4] `https://onekindplanet.org/animal/bee-hummingbird/`,accessed 22/10/19.

[5] P. A. Tipler, G. Mosca, *Physics For Scientists and Engineers, Sixth Edition*, Page 264, 2007.