Journal of Physics Special Topics

An undergraduate physics journal

P5_6 A Race in Space

Dodds, C. J. C., Benham, T. J., Foxcroft, B., Waters, S. J. W

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 5, 2016

Abstract

By comparing solar and laser radiation sources, each driving a nano-satellite of mass 1 gram, we find that the solar sail is more advantageous up to 9.5 AU, and the laser sail is better beyond this distance. We also find that the laser sail has a constant acceleration throughout, but the solar sail's velocity asymptotes towards $150,000$ ms^{-1}, as it's distance from the sun increases.

Introduction

Nano-satellites are compact satellites capable of carrying only small payloads, often instrumentation for observations, and may be added to the payloads of existing missions with relative ease [1].

In this paper we explore the recent advent of nano-satellites making use of small solar-sails as their primary propulsion system, and further investigate whether purely solar radiation is as effective, or less so, than the use of a laser shone from orbit about the Sun. The case is examined where two craft are part of a hypothetical race, with the two radiation sources incident upon separate, but identical mass, solar sails.

We propose the laser source be in orbit around the Sun, at distance 1 AU, to keep the distances between power source and craft equal for each sail. To also negate the gravitational influence of bodies such as the Earth; and the possibility of eclipsing the power source, we assume the craft travel directly down, perpendicular to the planetary plane. The laser beam will be a continuous, monochromatic and perfectly collimated beam (no spread at large distances). The solar radiation will be assumed to be a continuous stream of homogenous radiation. The solar sail will be 10 m^2, and the laser sail will be large enough to capture only the negligibly small laser beam. The areas of the sails are of no importance, as resistance will be virtually non-existent in a vacuum. The mass of both sails is assumed to be 1 gram.

Theory

Solar sails utilise the momentum exchange brought about when photons are incident upon the surface of the solar sail. We shall neglect particles other than photons and assume that it is the continual stream of photons within the Sun's radiation. The sails are made of a very thin highly reflective material, which we shall assume reflects all photons incident on the sail perfectly, and thereby the momentum change on the sail is twice the momentum change of the incident photons. Radiation pressure by reflection [2] is given by

$$P_{reflect} = \left(\frac{2E_f}{c}\right)\cos^2(\alpha), \tag{1}$$

Where E_f is the energy flux equal to the photon energy per m^2, α is the angle between the surface normal and the incident radiation and c is the speed of light. In this paper we shall assume that all radiation is perpendicular to the solar sail, thereby the cosine term may be ignored. The energy flux [3] from the Sun is given by

$$E_f = \frac{P_\odot}{A_r} = \frac{P_\odot}{4\pi r^2}, \tag{2}$$

Where $P_\odot$ is the power of the Sun and A_r is the surface area at a distance r from the Sun. Therefore, using $P_{reflect} = F/A = 2E_f/c$ and $F = ma$ the acceleration acting on the solar sail craft can be found to be

$$a_{solar} = \frac{2E_f A}{mc} = \frac{2P_\odot A}{4mc\pi r^2}, \tag{3}$$

where A is the surface area of the solar sail (10 m^2), m is the mass of the craft (1 g), c is again the speed of light and r is the distance from the Sun. The acceleration acting on the laser sail craft is given by

$$a_{laser} = \frac{2E}{mc}, \tag{4}$$

where E is the laser power (4 kW [4]). To calculate the distance these two craft have traveled we will use the SUVAT equation for displacement

$$s = ut + \frac{1}{2}at^2 \tag{5}$$

where s is the displacement from the starting point, u is the initial velocity, a is the acceleration on the craft and t is the time to reach that point.

Results

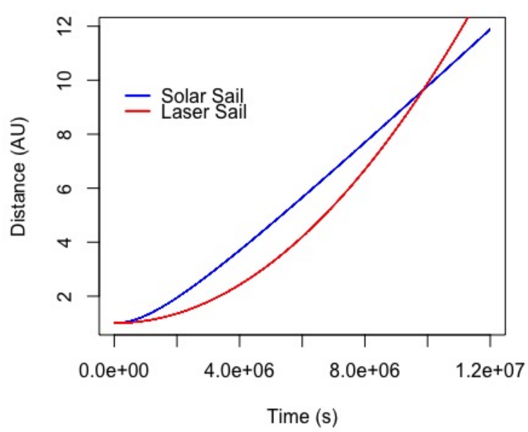

Figure 1: The time, in seconds, it takes for each sail to travel the distance, in AU.

Figure 1 shows a constant velocity for the solar sail (blue line), and the laser sail velocity (red line) starting off slowly and continuously increasing over time. Figure 2 shows the velocity of the craft's sail against the time taken to reach such a velocity. As may be seen by the blue curve, the sail employing solar radiation asymptotically approaches a value of about 150,000 ms^{-1}. The red line shows the acceleration of the craft using laser radiation to be approximately linear, implying a continually increasing velocity.

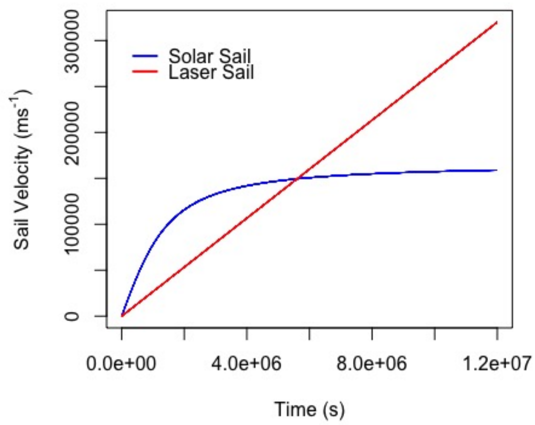

Figure 2: The velocity of the solar sail against travel time.

Conclusion

To a distance of about 9.5 AU, the case of the sail using solar radiation is more advantageous than a 4 kW laser sail, given the higher velocities achieved. For distances greater than approximately 9.5 AU, the sail making use of the laser is more appropriate, as the beam does not diminish to the point the solar radiation does given how intense the laser beam is initially. In practice, the laser beam will have a divergence at such great distances, and subsequently the propulsion will suffer; this effect may be explored in further papers on the subject. It may also be noted that a material of such low mass, 1 gram, with as large an area in the case of the solar sail, 10 m^2, may not be feasible with current technology; significant values may well still be achievable with current technology, and we suggest a combination of the two radiation sources would be the most beneficial method for future missions. As noted by the popular idiom, and for the case of using solar sails with various radiation sources, slow and steady does indeed win the race.

References

[1] `https://goo.gl/9nnY2B` [Accessed 08/11/16]

[2] `https://goo.gl/SlJtVs` [Accessed 08/11/16]

[3] `https://goo.gl/t0bzXi` [Accessed 08/11/16]

[4] `https://goo.gl/xChCfo` [Accessed 08/11/16]