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P2_7 Not So "Incy Wincy"

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Abstract
This paper takes the abilities of spider silk to the extreme by examining how thick a single strand would have to be in order to stop a commercial airliner mid-flight. We found that a 50m long strand with a diameter of 12.57cm would be able to bring the aircraft to rest without exceeding the limit of elasticity.

Introduction

Spider's silk is known as one of the world's strongest materials, its strength per weight exceeds that of man-made materials such as steel [1]. Scientists interested in "Biomimicry" have long been interested in spider silk due to its balance of strength and toughness, and attempts have been made to commercialise the production of it [2]. It is often claimed that a single strand of spider silk could stop a commercial plane in mid-flight, in the same way as a spider web can catch a fly.

Below we will investigate how thick a strand of spider silk would need to be to stop a Boeing 737 travelling at cruising speed without exceeding the limit of elasticity.

Theory

We start by making some assumptions. Firstly, we assume that the strand remains within its elastic limit. Secondly, we neglect the thrust of the plane, by assuming the engines have stopped before impact. We also assume that the plane is at cruising altitude and speed (35000ft, 0.785mach) [3] and that it does not deform or break apart on impact.

The equation relating the amount of strain experienced by a material for a given stress below the limit of elasticity is the equation for Young's modulus [4],

$$Y = \frac{(F/A)}{(\Delta L/L)}, \qquad (1)$$

where Y is the Young's modulus, F force applied to the strand, ΔL is the change in length of the strand, L is the original length and A its cross-sectional area. The quantity (F/A) is the stress, and $(\Delta L/L)$ is the strain. The Young's modulus is a measured quantity for a given material. For this investigation, we have used the silk of the *Caerostris Darwini* spider with a Young's modulus of 11.5GPa [5].

The quantity $\Delta L/L$ is also measurable and is known as the strain. Up to the elastic limit, the maximum strain for the silk is 0.52 [5]. As we are looking for the thickness of strand required, we rearrange equation (1) for the area, A.

$$A = \frac{F}{Y(\Delta L/L)} \qquad (2)$$

The next step is to find the force required to stop the Boeing 737, F. This is done using Newton's 2nd law $F=ma$. The acceleration a is found by,

$$a = \frac{-u^2}{2s} \qquad (3)$$

where u is the initial speed and s is the distance over which the acceleration takes place.

Results and Discussion

The cruising speed of a 737 is 0.785mach at 35000ft, giving 231.9ms^{-1} [6]. The distance is found by first assuming that the plane impacts the centre-point of the strand. The length of the strand must also be known, for this investigation a length of 50m was chosen. Using the value of strain for spider silk, and the length of the strand, the maximum change in length ΔL is 26m. This gives the length of the fully stretched strand to be 76m. Using trigonometry, we can find the distance the strand is deflected by, s, is 28.6m.

Substituting these values of u and s into equation (3) gives an acceleration a of -940.17ms^{-2}. Using the mass of a Boeing 737 of 79000kg [3] and applying Newton’s 2nd law gives a force F of 7.43×10^{7}N.

Substituting the calculated values into equation (1) gives a cross-sectional area of spider silk of 0.0124m^2 (We have assumed that the cross-section of the strand is circular). This gives a thickness of strand of 12.57cm.

The cross-sectional area required does depend on the length of the strand used, as a longer strand allows the plane to decelerate over a longer distance. Performing the calculation using a strand length of 1km results in a strand thickness of 2.81cm.

Conclusion

The results shown above show that it is theoretically possible to arrest a commercial aircraft mid-flight using a strand of spider silk. The reality is however, that the aircraft would most likely break apart due to the massive forces involved. As such, we would not recommend replacing the runways at Heathrow airport with giant spider webs any time soon!

References

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