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A5_1 Super Sonic (the Hedgehog)

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Abstract

In the popular game franchise *Sonic The Hedgehog*, Sonic has several powers. He can move at the speed of sound, and is able to transform into Super Sonic- a gold version of himself who can move close to the speed of light [1]. In this paper we use the equations of relativity to theorise what would happen if Super Sonic was running towards, and then past a stationary observer. We find he would have to travel between 0.1400 c and 0.2633 c to be seen as golden, and would appear to be between 2.7107 m and 2.703 m shorter due to length contraction. Due to time dilation he would also experience between 1.010 s and 1.037 s of time for every second an observer experiences.

Introduction

When an object is moving close to the speed of light, the wavelength (λ) of light it emits changes. If said object is moving away from a stationary observer its wavelength gets longer- a phenomenon known as the Doppler Effect. Conversely, if the object is moving towards a stationary observer at these speeds its wavelength will be measured as being shorter. We use these facts to analyse a situation in which Sonic becomes 'Super Sonic', a transformation in which he turns from his usual blue colour to a gold/yellow.

We calculate the range of velocities required for Sonic to achieve this Doppler shift and the wavelengths of light he would emit to a stationary observer as he moved towards, past, and then away from them. We further this analysis by calculating the time dilation (a slowing of time in accordance to the theory of relativity) and length contraction (an apparent shortening of a relativistic object from the perspective of an observer) of Sonic as he moves at these velocities. We present a description of what would occur if 'Super Sonic' existed in real life, and how he would look to a stationary observer.

Theory

When speeds close to the speed of light are achieved it is no longer accurate to use non-relativistic physics. At speeds where $v << c$ the equation $\frac{\lambda_0}{\lambda} = \frac{v}{c}$ is used [2], however in this case we must use:

$$\lambda_{observed} = \lambda_{source} \sqrt{\frac{1-\beta}{1+\beta}} \tag{1}$$

where $\beta = \frac{v}{c}$; the relativistic Doppler shift equation [3]. This can be rearranged to find the velocity required for a specific wavelength change in the form:

$$v = c\frac{1-x}{x+1} \tag{2}$$

where $x = [\frac{\lambda_{observed}}{\lambda_{source}}]^2$.

We also require the Lorentz factor, γ, which is defined by $\frac{1}{\gamma} = \sqrt{1-(\beta)^2}$. This is used to calculate time dilation and length contraction:

$$\text{t}=\gamma\, t' \text{ and } l= \frac{l'}{\gamma} \tag{3}$$

where *t'* and *l'* represent the 'proper time' and 'proper length' of an object's reference frame- in this case Sonic's. By calculating the Lorentz factors for Sonic's range of velocities we are able to calculate t and l, which is the time and length experienced in a stationary observer's reference frame [4].

Results

Due to the nature of the wavelengths of visible light, a range of results was obtained. The wavelength of blue light is 450.0 nm - 495.0 nm, and the wavelength of yellow light is 570.0 nm - 590.0 nm [5]. For Sonic to be moving fast enough to Doppler shift between these wavelengths, the largest and smallest differences were taken and substituted into Equation (2) to get a velocity range between -42.05 $\times 10^6$ ms^{-1} and -79.33 $\times 10^6$ ms^{-1}, or 0.1400 c and 0.2633 c. The velocities were negative due to Sonic moving away from the observer.

The modulus of these values were taken and substituted into Equation (1) to obtain the observed wavelengths as Sonic moves towards the observer. It was found that his observed wavelength would range between 343.2 nm and 429.9 nm, meaning he would emit in both the violet and ultraviolet range.

The Lorentz factors for both directions (towards and away) were found to be 1.010 and 1.037, which were then used to calculate time dilation and length contraction. (These quantities are not dependant on direction due to the v^2 term in the equation for Lorentz factor.)

For every second of time passing in Sonic's reference frame a range of 1.010 s to 1.037 s would pass for an observer, and assuming his width is 0.300 m he would appear to have a length between 0.2893 m and 0.2970 m.

Discussion

From these results we can theorise what would be seen if a stationary observer watched 'Super Sonic' run towards and past them at the calculated velocities. Whilst running towards them, Sonic would appear violet, while also emitting ultraviolet light. Time would be moving slower for him and he would appear thinner. Once moving away from the observer Sonic would appear yellow as his 'Super' form states, and his time and length contraction would stay the same.

Conclusion

We have shown the circumstances required for Sonic to enter his 'Super Sonic' form due to Doppler shift. We can conclude that Sonic would only turn yellow when moving close to the speed of light away from a stationary observer. Therefore when he turns 'Super' it must be due to something other than relativistic effects, otherwise the game would be rendered unplayable.

Further Work

Objects moving fast enough past a stationary observer with a single side facing them will be seen as 3D instead of 2D. This is due to the time difference between light moving to an observer from the object's closest and furthest sides [6]. Further work could be done to quantify this 3D effect related to Sonic, and to see whether 2D Sonic would actually appear 3D as he ran past at Super speeds.

References

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