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P3_11 Smallest Violin

A. Phong , M. McNally, R. Pierce, T. Searle

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract

This paper investigates the length of violin string needed for "the world's smallest violin playing the world's saddest song." It is found that the length required for an E string is 0.0108m and a violin of this size capable of producing the required frequency is unlikely to be possible.

The world's saddest song

There exists a phrase, "the world's smallest violin playing the world's saddest song" or other similar variations. A smaller instrument will produce a higher frequency, therefore, it will be assumed that the parameter for "the world's saddest song" is the highest frequency audible by the human ear and this has a value of around 20kHz [1].

The world's smallest violin

The violin string that has the highest frequency is the E string, so this string is taken as the sample and its properties are used to determine the length string that is required to play the target frequency.

It is assumed that the violin will play an "open" note, which means that no finger will be pressed on the string. When a string is played on a violin, it vibrates at its fundamental frequency and at the harmonics of that frequency. The pitch that is heard is made up of the many harmonics of the fundamental and consists of a range of frequencies, with the fundamental frequency being the loudest. To simplify the calculation, it will be assumed that the pitch that will be heard is 20kHz and that this will be produced purely by the fundamental mode of the string vibrating at a frequency of 20kHz. The fundamental mode of the string will have a length (L) equal to half the wavelength (λ) as shown in Fig. 1.

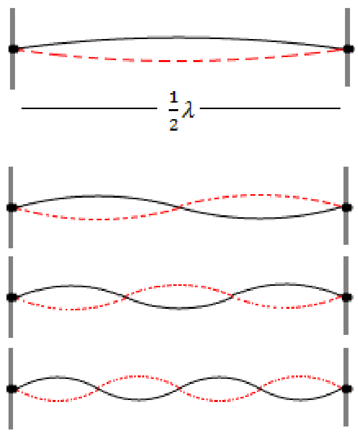

Figure 1. Illustration of the fundamental (top) and the next three harmonic standing waves on a vibrating string.

The speed of a wave (v) is given by its wavelength multiplied by its frequency f,

$$v = \lambda f. \qquad (1)$$

Therefore,

$$v = 2Lf. \qquad (2)$$

The wave speed for a uniform string can be determined from the physical properties of the string and is given by [2],

$$v = \sqrt{\frac{T}{\mu}}, \qquad (3)$$

where T is the tension of the string and μ is the mass per unit length. Combining equations (2) and (3) and rearranging for length gives,

$$L = \frac{1}{2f}\sqrt{\frac{T}{\mu}}. \qquad (4)$$

A typical E string has values of $T = 78\text{N}$ [3] and $\mu = 0.000417\text{kg m}^{-1}$ [3], this gives a string length equal to 0.0108m.

Discussion

It would be extremely difficult to make a traditional violin this small and that can play 20kHz on an E string, as the material may not support the tension required. Even if such a violin could be made, the vibrations produced may be too quiet to be audible. In this case, the other harmonics are neglected, but they may be capable of producing an audible frequency of 20kHz with a longer string that would allow the violin to be larger and stronger. Another possible method would be to make a custom designed string that can play at 20kHz with a lower tension.

Conclusion

The length of violin string required to produce the highest frequency that the human ear can hear is found to be 1.08cm. However, this result only considers the fundamental mode of a particular string. The world's smallest functional violin is about 4.1cm long [4]. If the previous values and assumptions are used, and it is approximated that the length of string equals the length of the violin, then the frequency played by this violin is 5.27 kHz. Perhaps this value should be considered as "the world's saddest song."

References

[1] Sensitivity of the human ear, http://hyperphysics.phy-astr.gsu.edu/hbase/sound/earsens.html (08/03/11)

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[3] E. Jansson, *Acoustics for violin and guitar makers* (Royal Institute of Technology, 2002), chapter 4, page 4.13. http://www.speech.kth.se/music/acviguit4/part4.pdf (08/03/11)

[4] World's Smallest Violin, http://www.therainiervalley.com/violin.html (08/03/11)