Journal of Physics Special Topics

P3_8 Elastic Launch

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November 26, 2012

Abstract

This is a brief investigation into the energetics of using a stretched elastic based slingshot to launch objects into space. The energy required to reach escape velocity is used to calculate some parameters of a theoretical elastic material that would be used to store the energy. It is found that for an acceleration distance of 828 m a spring constant of 4.01x10^6 Nm^{-1} would be needed to provide the 15385*g* acceleration.

Introduction

Launching objects into space is a very costly process in terms of energy, fuel and also financially. If we are going to be able to continue putting large objects into orbit it is going to become necessary to reduce the fuel use and perhaps more importantly, the cost.

It is proposed that a more efficient method of launch might be to use a fixed structure such as a tower and an stretched elastic material to slingshot things into orbit. A simplistic consideration of the energetics involved is performed to determine if this proposal is a reasonable one or not.

Theory

For a body to leave the gravitational influence of another much larger body (note that this is not the same as being placed into orbit around it) it must reach a velocity greater than or equal to some critical velocity. This is commonly known as the escape velocity, v_e, and is given by

$$v_{\mathrm{e}}=\sqrt{\frac{2\mathrm{GM}}{r}} \quad , \qquad (1)$$

where G is the universal gravitational constant, M is the mass of the larger body and r is the distance from the centre of mass of the larger body to that of the smaller one. This can be used to calculate a kinetic energy, E_k, for a body of mass, m, if it were travelling at this velocity. The critical energy required for escape is simply found to be

$$E_{\mathrm{k}}=\frac{GMm}{r} \quad . \qquad (2)$$

Since the launch is from the surface of the Earth we can replace r with the radius of the Earth, R_E, which has a value of 6371 km.

For this simplified consideration, it is assumed that all of the potential energy that would be stored in the elastic is converted directly into kinetic energy of the body. This will produce an underestimate overall since in reality a non-negligible portion of the energy is lost (not converted directly to kinetic energy). The elastic potential energy, E_s, is given by

$$E_{\mathrm{s}}=\frac{1}{2}kx^2 \quad , \qquad (3)$$

where k is the spring constant of the elastic and x is the displacement from it's equilibrium position. The spring constant is not a fundamental property of the material, it depends on a number of other factors such as the size and shape.

Equating equations 2 and 3 and rearranging for k gives

$$k=\frac{2\,GMm}{x^2\,R_{\mathrm{E}}} \quad . \qquad (4)$$

Using equation 4 it can be determined if this would be a reasonable launch method for a proposed maximum displacement by comparison to known values.

The acceleration can be calculated from the force that could be applied by the elastic using

$$F = ma = kx \quad , \tag{5}$$

where the variables have their previous meaning. When rearranged for *a* this gives

$$a = \frac{kx}{m} \quad . \tag{6}$$

If this method of launch is to be used for human space-flight then the maximum acceleration must be less than that which can be applied to a human body without causing serious injuries.

The maximum acceleration will occur just after the craft is released from the launch position, when the displacement is equal to the height of the tower (or more generally the length of the path it will travel before losing contact with the elastic).

Discussion

First the spring constant is considered with a maximum displacement, to calculate a value for this, a model launch must be defined. The tallest building in the world is the Burj Khalifa skyscraper in Dubai [1], for the purpose of this investigation the height of this building, 828 m [1], is used as the value for *x*. This essentially makes the assumption that we would be able to construct a launch tower of this height. For such large distances the mass of the elastic material may become significant, it is assumed here that this mass remains negligible despite the distance. It is also assumed that the body is accelerated directly upwards and that the effects of air resistance are negligible.

In reality this is likely to be far greater than would be mechanically plausible, due to the huge forces that would need to be supported to store that amount of energy. The mass of the object being launched is taken to be 22000 kg, roughly the typical payload capacity of the Space Shuttle [2]. Using equation 4 the spring constant of the elastic is found to be $4.01\text{x}10^{6}$ Nm^{-1}. By comparison with typical values for the spring constant, usually ~50 Nm^{-1} depending on the application, it can be concluded that since the value required is significantly higher, it would not be possible to produce such a material.

If the acceleration exceeds 100*g* [3] at any point then any passengers on board the spacecraft will be likely to sustain serious injuries. The maximum acceleration can be calculated using equation 6, with the same displacement as above and the calculated spring constant. This is found to be 150922 ms^{-2} (15385*g*), meaning any people, and most equipment, would be seriously damaged.

Conclusion

The energetics of launching a mass into space using a system similar to a slingshot have been considered. A maximum value for a reasonable displacement was proposed, using this it was found that the spring constant would be far greater than practical. In addition to this the acceleration required to achieve the escape velocity over the same distance was found to be 15385*g*, far greater than could be survived by any human.

Not considered here are the potential issues that may arise from the construction of the system, it is unlikely that it would be possible to build a structure of this height that would be able to withstand such huge forces.

References

[1] online.wsj.com/article/SB100014240527487035809045746381116676588806.html accessed on 13/11/2012.

[2] www.nasa.gov/audience/foreducators/rocketry/home/rockets_stack_up_nf_prt.htm accessed on 12/12/2012.

[3] D. Shanahan, *Human Tolerance and Crash Survivability* (Pathological Aspects and Associate Biodynamics in Aircraft Accident Investigation, 2004).