Journal of Physics Special Topics
An undergraduate physics journal

S4_3 Dome Sweet Dome

M. J. Duggan, S. M. B. Croxford, J. T. Best

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

October 31, 2017

Abstract

In the instant classic 'The Simpsons Movie' the town of Springfield becomes trapped by a glass dome. This paper investigates whether the residents will survive in the dome, and finds that they have at the most 28 hours to escape the before they suffocate.

Introduction

In the much loved 2007 film 'The Simpsons Movie', the town of Springfield is encapsulated in a giant glass dome, on the orders of the President of the United States of America to prevent Springfield from polluting the neighbouring environments. In this paper we will discuss whether or not the Springfield residents will suffocate in the dome, and if so how long they have to live.

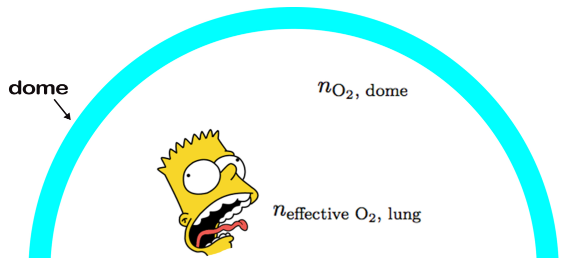

Figure 1: Illustrates dome, captive and the decisive criterion for this paper.

Theory

In order to determine the fate of Springfield's residents we first need to quantify the number of air particles in the dome. The movie portrays the dome to have a circular opening at its apex, but since the diameter of the opening is not much wider than a briefcase, it can be negated from the calculations. Springfield in the Simpsons Universe is modelled after Springfield, Oregon USA, which has an area of 40.79 km^2 [1]. The dome has the geometry of a hemisphere (see Figure 1), therefore has a basin of area $A = \pi r^2$; rearranging for r gives $r = (A/\pi)^{1/2}$. The radius of the dome is 3.6×10^3 m. The volume of the hemisphere is $V = 2\pi r^3/3$ therefore $V = 9.8 \times 10^{10}$ m^3. We can adjust for the altitude pressure using

$$p = p_0(1 - \frac{Lh}{T_0})^{\frac{gM}{RL}}, \qquad (1)$$

where p is the resultant pressure, $p_0 = 101.325$ kPa is the standard pressure at sea level, temperature lapse rate $L = 0.0065$ Km^{-1}, the altitude of the fictional Springfield $h = 482$ m and the sea level standard temperature, $T_0, = 288.15$ K. The terms in the power represent gravitational acceleration g, molar mass of dry air $M = 0.0289644$ kg mol^{-1}, and ideal gas constant, R [2]. The resulting pressure at altitude h is $p = 95.7$ kPa.

The number density of air molecules in the dome can be found by manipulating

$$pV = NkT \qquad (2)$$

to arrive at

$$\frac{p}{kT} = \frac{N}{V}, \tag{3}$$

where $\rho_{\mathrm{N}} = N/V$ is the number density, k is the Boltzmann constant, T is the temperature, N is the number of particles in the median, and V is the volume of the median. The average high temperature for Oregon in July is 300 K [3], therefore $\rho_{\mathrm{N}} = 2.3 \times 10^{25}$ particles m^{-3}. The number of particles in the dome is then $N_{\mathrm{dome}} = \rho_{\mathrm{N}} V = 2.27 \times 10^{36}$ particles.

The average tidal (normal breathing) lung volume for an adult is 0.5 L, with the lower average limit of 12 breaths per minute for respiratory rate [4]. Springfield has a population of $50,720$, with the average individual taking 17280 breaths per day; resulting in population tidal lung volume per day of $V_{\mathrm{lung}} = 4.38 \times 10^5$ m^3 day^{-1}.

Next we need to calculate the number of particles per day entering the population's lungs, n_{lung}. This can be determined by $N_{\mathrm{lung}} = \rho_{\mathrm{N}} V_{\mathrm{lung}}$, resulting in $N_{\mathrm{lung}} = 1 \times 10^{31}$ particles day^{-1}.

Oxygen (O_2) accounts for 20.9% of air [5], therefore the number of O_2 particles in the dome is $N_{\mathrm{O_2,\,dome}} = 0.209 N_{\mathrm{dome}} = 4.73 \times 10^{35}$ particles: this is the initial O_2 particle number immediately after the dome is placed over Springfield. The average human lungs are 27% efficient [5], reducing the effective O_2 consumption to $N_{\mathrm{effective\ O_2,\,lung}} = 0.27 N_{\mathrm{lung}} = 2.7 \times 10^{30}$ particles day^{-1}. This provides the reduction per day of breathable particles in the dome.

Results

Equation 4 allows us to find the number of days remaining immediately after the dome surrounds the residents. Equation 4 is set equal to 0 because within 6 minutes of no oxygen reaching the brain, a person is classed as brain dead. [6]

$$N_{\mathrm{O_2,\,dome}} - {N_{\mathrm{effective\ O_2,\,lung}}}^{d} = 0 \tag{4}$$

where d is days

Rearranging equation 4 for d gives,

$$d = \frac{ln(N_{\mathrm{O_2,\,dome}})}{ln(N_{\mathrm{effective\ O_2,\,lung}})} \tag{5}$$

$$d = 1.17 days. \tag{6}$$

Including the additional 6 minutes for brain death, d can be summarised into hours to be 28 hours and 6 minutes.

Discussion

Our calculations did not take into account the small aperture at the dome's peak, as the effect would be negligible. In addition, we used the lower limit for average adult respiration, which would undoubtedly be higher due to the stress of the situation. It also does not account for carbon dioxide (CO_2) emission during respiration or carbon monoxide, CO, from car emissions. Important assumptions we made are that vegetation does not measurably effect O_2 levels, and that O_2 intake is constant.

Conclusion

The residents have 28 hours and 6 minutes to figure out their escape plan, and so the best they can do is hope that they find Maggie's life-saving sandpit sinkhole.

References

[1] `https://goo.gl/2qrBRB` [Accessed 28 October 2017]

[2] `https://goo.gl/BpmAiw` [Accessed 28 October 2017]

[3] `https://goo.gl/9p54hf` [Accessed 28 October 2017]

[4] `https://goo.gl/CezdCU` [Accessed 30 October 2017]

[5] `https://goo.gl/bzyZJ5` [Accessed 31 October 2017]

[6] `https://goo.gl/RkdsTE` [Accessed 31 October 2017]