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P2_7 Leaning Tower of London

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Abstract

In this paper, we calculate the maximum angle that Elizabeth Tower could lean before toppling over, which was found to be 7.13°. We also find the length of time it would take for this to happen as approximately 4380 years, and the minimum force that would be needed if one were to push the tower until it fell. The force needed to push the tower until it toppled was found to be 3.50×10^7 N if applied perpendicular to the tower, or a minimum force of 3.48×10^7 N when applied most effectively.

Introduction

Much like the infamous Leaning Tower of Pisa, Elizabeth Tower (formerly known as Big Ben), also leans. Considering the Centre of Mass (COM) of the tower, we can find the maximum angle with which the tower can lean until toppling to the ground. We can also deduce the length of time the tower would take to reach this maximum angle by calculating the current lean rate. Finally, investigating the torque of the towers' tipping point can allow us to find the force needed to push the tower until it topples.

Theory and Results

To find the maximum angle of lean that Elizabeth Tower can have, we first need to understand the position of the COM of the tower. If we approximate the tower to be of uniform mass and dimensions, we can assume the tower to be a cuboid with dimensions 12 m x 12 m x 96 m[1]. Due to our assumptions, the COM of the tower would appear in the centre of the tower, at point 6 m x 6 m x 48 m. Using the principle of the COM, an object topples once the vertical line from the COM falls outside of the objects base. We assume the tower is constantly leaning perpendicular to the ground on one face as it falls, and can therefore approximate the scenario as a 2D rectangular model. This scenario can be seen in Figure 1, where θ is the maximum angle (to the vertical) that the tower can lean. We can find θ through simple trigonometry, where $\theta = tan^{-1}(\frac{6}{48})$, giving us a maximum angle of 7.13°. As Elizabeth Tower is currently leaning at an angle of 0.26° [2] and was built 160 years ago [1], we can crudely approximate a constant rate of lean as $1.63 \times 10^{-3°}$ yr^{-1}. Using this rate, we can find that to reach the maximum angle of 7.13°, it would take roughly 4380 years. This is consistent with the approximate time scale calculated by experts during a Transport for London report, where the figure was stated as 4000 - $10,000$ years [2].

We can then find the force required to push the tower in its leaning direction in order to topple. We can calculate two values of force for this to occur; one scenario in which the force is applied at an angle (horizontally) to the leaning tower

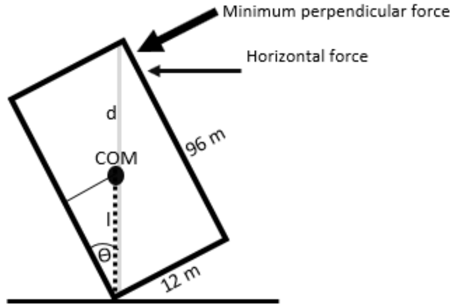

Figure 1: The scenario in which Elizabeth Tower is considered a 2D rectangle with a lean angle θ and COM in the centre of the assumed uniform mass distribution.

in the direction of the tower's lean, and another in which the force is applied at perpendicular to the tower in order to maximise the effect of the force and create the biggest 'lever' of which to use. These two scenarios are also illustrated in Figure 1.

To consider the first scenario in which the force is applied horizontally, we must consider the torque created by gravity at the towers' tipping point. This torque T is equal to mgl, where m is the mass of the tower, g is the acceleration due to gravity and l is the distance from the tipping point to the COM. The force required to topple Elizabeth Tower would have to counteract this torque, and is given by Fh, where F is the horizontal force required to topple the tower, and h is the height of the tower. By equating the torque and the horizontal force and rearranging for F, we find Equation 1,

$$F = \frac{mgl}{h} \tag{1}$$

. We can substitute our previous value for the height of the tower into the equation, along with a value for l of which has been calculated using Pythagoras' Theorem to be $l = 48.4$ m. We then need to find the mass of the tower to substitute into Equation 1. As Elizabeth Tower was constructed using 850 cubic metres of stone and 2600 cubic metres of brick [1], we can find the mass of these building materials using $m = \rho V$, where ρ is the density of the material (on average, 2.5×10^3 kg m^{-3} and 1.9×10^3 kg m^{-3} for stone and brick respectively[3]). We therefore find our total mass of the tower, including the mass of the bell 'Big Ben' (13 tonnes), to be 7.08×10^6 kg, and hence find our value of the horizontal force required to topple Elizabeth Tower to be 3.50×10^7 N.

In order to consider the scenario in which the minimum force is applied perpendicular to the tower, we need to maximise the length of the 'lever' with which we can apply the force over. We therefore utilise Equation 1 however in place of h we can substitute d, the diagonal length of the towers face, where $d = 2l$. Hence, we find $d = 96.7$ m. Thus, we find the minimum force to topple the tower as 3.48×10^7 N.

Discussion

In this paper, assumptions have been made regarding the COM of Elizabeth Tower. We have assumed the tower to be a 2D rectangle. We have also assumed a constant rate of lean, however this would more likely be an exponential, with the tower falling at a faster rate as the lean angle increases. Further consideration could take this exponential rate into account in order to approximate the time taken to fall. However, we believe our value found to be within reason, due to the consistency with expert opinion.

Conclusion

In order for Elizabeth Tower to topple to the ground, a minimum force of 3.48×10^7 N would need to be applied perpendicularly to the tower in the direction of lean, or you would have to wait approximately 4380 years for the towers lean to reach the maximum permissible angle of 7.13°.

References

[1] *Facts and Figures*, Big Ben, UK Parliament

[2] https://www.bbc.co.uk/news/uk-england-15243118, [Accessed 30/10/19]

[3] https://www.engineeringtoolbox.com/density-solids-d_1265.html, [Accessed 30/10/19]