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A2_7 Breeding Like Rabbits

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Abstract

We investigate an augmented logistics model for determining the lifetime of an emerging human population in a world where all resources are finite. We find that populations with large growth rates will allow the populations sizes to grow to proportionately larger volumes but will ultimately cause the population to die faster because of it. By introducing a variable consumption rate of resources dependent on the population size, the lifetime of such populations decrease drastically in some cases by over 1000 years.

Introduction

We investigate the lifetime of an emerging human population on a world where all resources are finite. We use a basic logistic model as the basis for the behaviour of the population and augment this model [1] to account for environmental factors. We finally amend the augmented model ourselves to account for the limited lifetime of resources therefore treating all resources as finite and consumable.

Theory

The logistics model is determined by the following differential equation

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}) \qquad (1)$$

where N is the total population, r is the growth rate per year and K is the carrying capacity of the environment. The carrying capacity of the environment is defined as an environments maximum load i.e. the maximum amount of humans the Earth can sustain at any one time. In the case of the basic logistics method this is assumed to be a constant. In reality this is not the case as resources need to be worked, and so the carrying capacity is dependent on the population size. In our world we must also consider how resources decay over time. This can be done by considering the carrying capacity. As resources are consumed the maximum carrying capacity will decrease.

Thus we can introduce an augmented logistics model that considers a variable carrying capacity and then alter this model to also include the degradation of resources. When we do this we acquire Eq. (2)

$$\frac{dK}{dt} = \frac{L}{N(t)}\frac{dN}{dt} - \alpha K(1 + \frac{N}{K}) \qquad (2)$$

where L is a constant defined by the environment called the Mill parameter which contributes mainly in determining the maximum possible carrying capacity. $N(t)$ denotes the population size at a given time, t. The α is a consumption rate with units per year. The first part of Eq.(2) considers the variable carrying capacity due to available population to work. The second

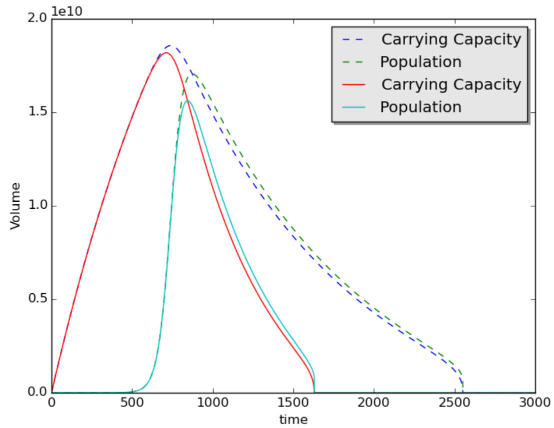

Figure 1: The dashed lines represent the carrying capacity and populations where a constant consumption rate is used and the solid lines denote what happens when a variable consumption rate is considered

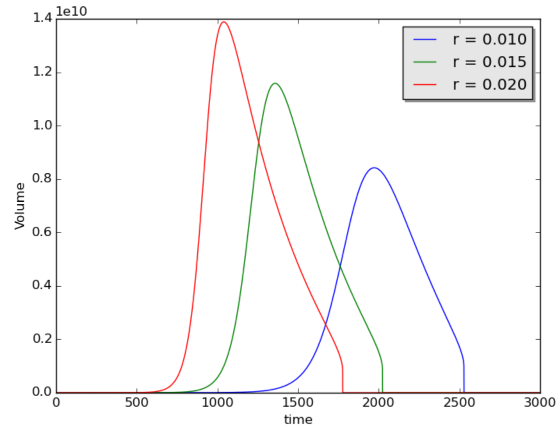

Figure 2: The figure shows how the human population behaves at different growth rates. Mill parameter value is 15×10^8.

part considers the degradation of resources. The $1 + N/K$ term causes the rate of consumption to vary with the population size (As the population increases so does the rate of consumption). In our model the minimum possible rate of consumption is α which can be considered analogous to the rate at which resources naturally decay.

Discussion

Figures (1) and (2) are produced by solving the differential equations. Figure (1) shows how the population and carrying capacity sizes vary for a variable consumption rate and a constant consumption rate. We set the Mill parameter to 15×10^8 though this is arbitrary as it does not affect how the population and carrying capacity behave but instead influences the size to which they can grow. By including a variable consumption rate we can see that the maximum population size reached is noticeably decreased and that the lifetime of the population is much shorter. We can see that once the population becomes greater than the maximum carrying capacity the human populations rapidly decrease however in the variable consumption rate case the rate at which the human population decreases is dramatically increased. Figure (2) shows how the population changes for different growth rates that are close to what has been viewed in human history [2]. We can see that populations with larger growth rates manage to achieve a proportionally larger populations but ultimately die off faster than human populations with smaller growth rates. In the case of a growth rate of 0.020 the population has almost died off before a population of 0.010 has grown to any significant size.

Conclusion

In this paper we have discovered that by including a population dependent consumption rate of resources the lifetime of our human population drastically decreases. Once a population exceeds that of the carrying capacity, both of their respective sizes decrease very rapidly. We find that large growth rates will allow populations sizes to grow to proportionately larger maximum volumes but will inevitably die faster because of it.

References

[1] Joel. E. Cohen *Population Growth and Earth's Human Carrying Capacity*, 1995

[2] https://ourworldindata.org/world-population-growth/ accessed on 07/11/16