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A1_2 Duck, duck... Spruce!

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Abstract

The H-4 Hercules was a prototype aircraft built in 1947 which was never properly flight tested. This report explores a theory that the aircraft would never have been able to fly more than a few feet above the ground and in 1947 it only flew due to the ground effect. This will enable us to determine if the plane was suitable for transporting heavy military equipment. By exploring the pressure difference over the wing surfaces, the aircraft is shown to be able to reach a maximum altitude of 7,128 m and to require a minimum takeoff speed of 259.9 km/h. It is therefore shown that the aircraft would have been able to takeoff fully loaded at maximum speed, although this could have put dangerous strain on the engines.

Introduction

In 1947 the American businessman and aviation engineer Howard Hughes completed his prototype seaplane, the H-4 Hercules, popularly named the "Spruce Goose". This aircraft was highly controversial because Hughes was originally tasked with building three by 1944 to help aid the war effort but none were finished in time. Furthermore, at more than 218 ft in length with a wingspan of almost 320 ft this aircraft was the largest ever built and still retains the record for the largest wingspan of any aircraft in history. As a result many critics believed the plane unable to fly and accused Hughes of wasting the government's money on a project which was not only overdue for the war effort but in many opinions unable to fly [1].

The aircraft was only flown once on November 2, 1947 for approximately 1.6 km at an altitude of 21 metres. In 1947 this was suitable evidence to prove that the aircraft could indeed fly [1], although recent advances in science suggest that the aircraft may only have flown due to the *ground effect*. A plane experiences excess lift and a reduction in drag whilst less than one wingspan in altitude above the surface due to this effect [2]. This report determines whether this aircraft could have flown at higher altitudes and if it was designed practically for this purpose.

Discussion

The first question to address is whether the aircraft could have reached its takeoff speed. It did successfully takeoff in 1947, but with only 32 people onboard when the plane was designed to carry 750 soldiers or an M4 Sherman tank weighing 30.3 tonnes [1]. To calculate the takeoff speed we start with the Bernoulli equation,

$$\frac{v_1^2}{2} + \frac{P_1}{\rho} + gz = \text{constant}, \quad (1)$$

where v_1 is the speed of air passing over a wing, P_1 is the pressure on the top of that wing, ρ is the density of air, g is the acceleration due to gravity, and z is the altitude of the aircraft. We assume that the air is incompressible (ρ is constant) and that there is no friction between the air and the plane [3]. Equating Equation 1 (which relates to the air flowing over the top of the wing) with a similar expression for the air flowing underneath the wing, and rearranging for the pressure difference gives,

$$P_2 - P_1 = \frac{\rho}{2}(v_1^2 - v_2^2), \quad (2)$$

where v_1, P_1 relate to the air speed and pressure above the wing and v_2, P_2 relate to the same terms below the wing. The gz terms cancel as the difference in altitude z at the top and bottom of the wing is negligible. We then say that the rate of flow of air over both surfaces of the wing (with path length x), is the same, and hence $v_1/v_2 = x_1/x_2 = m$, a dimensionless constant. The lift force F_L experienced by a wing with pressure difference ΔP and surface area A is given by,

$$F_L = \Delta PA. \quad (3)$$

Substituting the pressure difference $\Delta P = P_2 - P_1$ given by Equation 2 into Equation 3 gives,

$$F_L = \frac{\rho}{2}Av_2^2(m^2 - 1). \quad (4)$$

To determine whether the aircraft could takeoff we equate F_L with the force due to gravity $F_g = Mg$. It is assumed that the wing design of the H-4 Hercules is similar to that of the Blohm & Voss BV 238 which is calculated to have the ratio $m = 1.28$ [4][5]. The area of its wings is $A = 1{,}061\ \mathrm{m}^2$, the density of air at sea level is $\rho = 1\ \mathrm{kg/m}^3$, and the fully loaded mass of the aircraft is $M = 180{,}000\ \mathrm{kg}$ [6]. Putting these values into Equation 4 results in a takeoff speed $v_t \approx v_2 = 259.9\ \mathrm{km/h}$ – higher than the cruise speed of $v_c = 217\ \mathrm{km/h}$ which the plane flew at in 1947 [1]. However, the H-4 Hercules' maximum speed was projected to be 378 km/h [1], so had the plane gone at full speed it could have flown.

The density of air decreases with height above the Earth's surface, and this can be used to determine the maximum altitude the H-4 Hercules could reach if its engines were at full power (we assume a velocity of 378 km/h). If we make the assumption that the atmosphere is isothermal, then the density of the atmosphere will decrease by a factor of $1/e$ (where $e \approx 2.72$) after a scale height H above the Earth's surface [7]. This scale height is defined as,

$$H = \frac{kT}{m_a g}, \quad (5)$$

where $m_a = 4.81 \times 10^{-26}\ \mathrm{kg}$ is the average molecular mass of the air [8], $T = 328\ \mathrm{K}$ is the average temperature of the troposphere and k is the Boltzmann constant. It follows that,

$$\rho = \rho_0 e^{-z/H}, \quad (6)$$

where $\rho_0 = 1\ \mathrm{kg/m}^3$ is the density of air at sea level and $\rho = 0.473\ \mathrm{kg/m}^3$ is the minimum density of air through which the plane can fly (see Equation 4). From Equation 6 we calculate the maximum altitude that the aircraft could achieve is $z = 7{,}182\ \mathrm{m}$.

Conclusion

In conclusion it has been shown that the H-4 Hercules would have been able to successfully reach the takeoff speed of 259.9 km/h when fully loaded. It could have flown at a height of about 7km, however this was at maximum speed which could have put a lot of strain on the engines. In 1947 it flew at 217 km/h: lower than the takeoff speed, and so only the ground effect was responsible for its flight.

References

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