Estimation of Radionuclide Release Activity Using an Unscented Kalman Filter

Vineet Vajpayee, Victor Becerra, Nils Bausch, Xiange Tian, T. V. Santhosh, Gopika Vinod

***Abstract*—Estimation of radionuclide release is an important problem due to its impact on population and environment. Especially, radioactivity release, plume height, and wind velocity need to be estimated reliably to plan emergency response in case of any unforeseen situation. In this paper, a non-linear estimation technique based on Unscented Kalman Filter has been proposed to estimate radioactivity release, wind velocity, and height of release using environmental data collected from radiation monitors placed in the proximity of release point. The Gaussian plume model has been considered to model atmospheric dispersion phenomenon of radionuclide release and for the calculation of dose rates. The performance of the proposed estimation technique has been evaluated in terms of root mean squared error. The estimation algorithm is found to be performing satisfactorily.**

***Index Terms*—Atmospheric Dispersion, Dose Rate, Gaussian Plume Model, Radionuclide Release, Unscented Kalman Filter**

I. Introduction

Estimation of radioactivity due to release of a radionuclide into the environment from a nuclear reactor under normal operation or accidental conditions is a regulatory requirement. Particularly, at the initial release stage, a good estimate of the radionuclide spread in a range of tens of kms is of paramount importance due to pressing requirements for citizen protection and immediate evacuation arrangement. Release of radionuclide from a reactor creates radioactive plume near the nuclear power plant (NPP). These radioactive plumes got dispersed in the atmosphere and transported by the wind in various directions. This could cause potentially serious radioactive doses in the area covered by the plume and its surrounding. Environmental radiation monitoring detectors placed around an NPP usually track the amount of radioactive dose in the atmosphere. However, sometimes it is uncertain to know the exact amount of radioactive release. Timely prediction of dispersion of radioactivity release is very much essential to evaluate the adequacy of emergency response measures and to know its environmental impact. Therefore, it is necessary to estimate the release activity for atmospheric dispersion of radioactive materials.

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The environmental monitors placed around an NPP employ radionuclide release information to measure dose rates. Calculation of dose rate is dependent on the plume shape and the distribution of radionuclide concentration. In various reported works, an assumption of uniform concentration distribution of radionuclide has been utilized for simplicity as it leads to a simple expression for dose calculation. Nevertheless, a more appropriate choice is to assume a Gaussian concentration distribution. The Gaussian plume model has been widely used in the literature with different simplifications [1]. In this work, the Gaussian plume dispersion model has been considered to model radionuclide release in atmosphere and for calculation of dose rates due its suitability for modelling radionuclide release from point sources.

From the measured readings of environmental radiation monitors, it is possible to estimate release rate, wind speed, and effective height of release for a given scenario. Many works have been reported in the literature where an estimation technique is employed to evaluate different important parameters related to radionuclide release activity. For instance, linear Kalman filter based methods utilizing off-line recorded radiation monitoring data have been applied for on-line source term estimation for short-range atmospheric dispersion of radionuclide release [2], [3]. Kalman filter based data assimilation techniques combining model predictions and measurements have been suggested in the design of nuclear emergency response system [4], [5]. Data assimilation approaches established on ensemble Kalman filter are applied to Monte Carlo atmospheric dispersion model [6], [7]. An atmospheric dispersion module for radioactivity release evaluations using a Kalman filter technique has been developed as a part of diagnostic system for accident management in [8]. Different modified ensemble Kalman filters for atmospheric dispersion modelling have been recently proposed by Zhang et al. [9], [10]. Particle filter based methods have been integrated with atmospheric transport model to improve model predictions [11], [12].

In this work, an Unscented Kalman Filter (UKF) technique is suggested for the estimation of radioactivity release, plume height, and wind velocity using the dose rates measured with the help of underlying modelling relationship. The methodology utilizes Gaussian plume model for atmospheric dispersion and for dose rate calculation. The algorithm uses environmental radiation monitoring data collected from six radiation detectors close to the release point. Performance of the UKF algorithm during estimation has been validated using root mean squared error measure.

The paper is organized as follows: Section II describes mathematical models of atmospheric dispersion and dose rate calculation. Section III briefly presents the principles of UKF technique. Section IV presents the application of non-linear estimation technique to the radioactivity release estimation. Finally, conclusions are drawn in Section V.

II. Modelling of Radioactivity Release Activity

The process of transport and dispersion of radioactive plume with time needs to be modelled correctly to have a good estimate of release rate and release height. The relationship between distribution of radionuclide in air with time and the dose rate is established thereafter.

A. Atmospheric Dispersion Model

Atmospheric dispersion models have a significant role in the accident management of an NPP. The spatio-temporal diffusion of a radionuclide is modelled using atmospheric dispersion model, which formulates a dynamical model of radionuclide contaminant transport in the atmosphere. In this work, the adopted modeling technique for atmospheric dispersion phenomenon is built on the Gaussian plume model [13]. The main reasons for its selection over other atmospheric dispersion model is that the Gaussian plume model is suitable for modelling radionuclide release from point sources. The model is applicable for plain and homogeneous terrain conditions for distances in the range of tens of kms. In addition, it is suitable for short-term as well as long-term impacts. The model requires few parameters and its straightforward representation allows for quick calculation of the radionuclide release activity. The Gaussian plume model assumes that the concentrations of the dispersed radionuclide are normally distributed along the plume centreline. It further assumes homogeneous and stationary meteorological condition with constant source term data [14]. Inputs to the Gaussian plume model are meteorological parameters like wind direction, mean wind speed, atmospheric stability class, and release parameters such as release rate, release height, mean energy of the release etc. The radionuclide activity concentration can be written as

$$\zeta(x,y,z) = \frac{\dot{q}}{2\pi\sigma_y\sigma_z u}\exp\left(-\frac{y^2}{2\sigma_y(x)^2}\right). \left\{\exp\left(-\frac{(z-h)^2}{2\sigma_z(x)^2}\right)+\exp\left(-\frac{(z+h)^2}{2\sigma_z(x)^2}\right)\right\} \quad (1)$$

where x, y, and z denote downwind distance from the release point, crosswind distance from centreline of plume, and height above the ground respectively. $\dot{q}$ denotes the release rate of radionuclide, u is the average wind speed, h is the height of release. $\sigma_y(x)$ and $\sigma_z(x)$ denote plume dispersion parameters in y and z direction respectively. These parameters are associated with travel distance and atmospheric turbulence. The plume dispersion parameters can be calculated from wind velocity measurements fluctuations and can also be determined from atmospheric stability classification based on Pasquill-Gifford.

B. Dose Rate Calculation Model

The calculation of dose rates from release of radioactivity depends on the knowledge of the release such as amount and composition and on the information of meteorological dispersion parameters. Generally, dose rate monitors are placed around an NPP to supply real time data that is utilized to compute important atmospheric dispersion parameters. The absorbed dose rate in air at a receptor position by a Gaussian plume is given by:

$$D(r) = \frac{E_\gamma \mu_a}{4\pi\rho}\int_{0}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\frac{B(\mu r)\exp(-\mu r)\,\zeta(x,y,z)}{r^2}dxdydz \quad (2)$$

where $\zeta(x,y,z)$ denotes activity concentration at point (x,y,z) in the plume with plume origin at $(0,0,0)$. E_γ denotes photon energy. μ and μ_a denotes linear mass attenuation factor and energy absorption factor for photons in air respectively. For simplicity it is assumed that the photon yield to be equal to one. r is the distance between plume point and receptor point and is given by

$$r^2 = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2. \quad (3)$$

The linear dose build-up factor for Compton scattered radiation is given by

$$B(\mu r) = 1 + k\mu r, \quad k = \frac{\mu - \mu_a}{\mu_a} \quad (4)$$

III. Non-linear Estimation Technique

A general non-linear discrete-time system can be written as

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}) + w_{k-1} \\ y_k &= h(x_k, u_k) + v_k \end{aligned} \quad (5)$$

where f and h denote state prediction and observation function respectively. $u \in \mathbb{R}^{n_u}$, $x \in \mathbb{R}^{n_x}$, and $y \in \mathbb{R}^{n_y}$ are input, state, and output vector respectively. $w \in \mathbb{R}^{n_w}$ and $v \in \mathbb{R}^{n_v}$ are process noise and measurement noise vectors respectively. The process and measurement noise covariances are assumed to be white, uncorrelated, and Gaussian with zero mean and covariance $Q_k = E\left[w_k w_k^T\right]$ and $R_k = E\left[v_k v_k^T\right]$ respectively,

$$\begin{aligned} w_k &\sim N(0, Q_k) \\ v_k &\sim N(0, R_k) \\ E\left[w_k v_k^T\right] &= 0 \end{aligned} \quad (6)$$

A Kalman filter based state estimation algorithm involves two recursive steps, state prediction and state correction. Initially, the current estimate of state vector is used along with the dynamical model of the system for estimation of state vector at next instant. Predicted estimates of the state vector are then fused with the measurement data in the second step such that the covariance of the error becomes minimum.

A. Unscented Transformation

The unscented Kalman filter is based on unscented transformation which is a statistical alternative to the analytical linearisation approach. The unscented transformation determines a set of sigma points from the assumed prior Gaussian distribution using scaling parameters and weight vectors. The scaling parameter can be computed as,

$$\lambda = \alpha^2(L+\kappa) - L \quad (7)$$

and weight factors can be obtained as,

$$\begin{aligned}\eta_0^m &= \lambda/(L+\lambda)\\ \eta_0^c &= \lambda/(L+\lambda) + \left(1-\alpha^2+\beta\right)\\ \eta_i^m &= \eta_i^c = 1/[2\,(L+\lambda)],\ i = 1,2,\ldots,2L\end{aligned} \tag{8}$$

where α, β, and κ are primary, secondary, and tertiary scaling parameters respectively. L defines the length of state vector.

B. Unscented Kalman Filter

The UKF algorithm is implemented after computing the unscented transformation. The main steps involved in the UKF algorithm are summarized as follows:

1) **Calculate sigma points** Sigma points are computed using the prior state estimate ($\hat{x}_k$) and covariance (P_k) at each discrete time step, as follows

$$\chi_{k-1} = \left[\hat{x}_{k-1}\Gamma \quad \hat{x}_{k-1}\Gamma + \sqrt{P_{k-1}\,(L+\lambda)} \quad \hat{x}_{k-1}\Gamma - \sqrt{P_{k-1}\,(L+\lambda)}\right] \tag{9}$$

where Γ is a $1 \times n_x$ vector of ones.

2) **Prediction Transformation** Each generated sigma points are fed to the non-linear state prediction function and then the a priori state and covariance are estimated using weighted averages of the transformed sigma-points.

$$\chi_{k|k-1}^{(i)} = f\left(\chi_{k-1}^{(i)}, u_{k-1}\right),\quad i = 0,1,\ldots,2L \tag{10}$$

Calculate the mean of predicted state

$$\hat{x}_{\,k|k-1} = \sum_{i=0}^{2L} \eta_i^m \chi_{k|k-1}^{(i)} \tag{11}$$

Calculate the covariance of predicted state

$$P_{k|k-1} = Q_{k-1} + \sum_{i=0}^{2L} \eta_i^c \left(\chi_{k|k-1}^{(i)} - \hat{x}_{\,k|k-1}\right) \left(\chi_{k|k-1}^{(i)} - \hat{x}_{\,k|k-1}\right)^T \tag{12}$$

3) **Observation Transformation** Transformed sigma-points are then passed to the observation function and are then utilized to compute predicted output, output covariance matrix, and cross-covariance between state and output.

$$\psi_{k|k-1}^{(i)} = h\left(\chi_{k|k-1}^{(i)}, u_k\right),\ i = 0,1,\ldots,2L \tag{13}$$

Calculate the mean of predicted output

$$\hat{y}_{\,k|k-1} = \sum_{i=0}^{2L} \eta_i^m \psi_{k|k-1}^{(i)} \tag{14}$$

Calculate the covariance of predicted output

$$P_k^{yy} = R_k + \sum_{i=0}^{2L} \eta_i^c \left(\psi_{k|k-1}^{(i)} - \hat{y}_{\,k|k-1}\right) \left(\psi_{k|k-1}^{(i)} - \hat{y}_{\,k|k-1}\right)^T \tag{15}$$

Calculate the cross covariance between state and output

$$P_k^{xy} = \sum_{i=0}^{2L} \eta_i^c \left(\chi_{k|k-1}^{(i)} - \hat{x}_{\,k|k-1}\right) \left(\psi_{k|k-1}^{(i)} - \hat{y}_{\,k|k-1}\right)^T \tag{16}$$

4) **Measurement Update** This step involves calculation of Kalman gain and updating state estimate and error covariance matrix.

Calculate the Kalman gain

$$K_k = P_k^{xy}(P_k^{yy})^{-1} \tag{17}$$

Update state estimate

$$\hat{x}_k = \hat{x}_{\,k|k-1} + K_k\left(y_k - \hat{y}_{\,k|k-1}\right) \tag{18}$$

Update error covariance

$$P_k = P_{k|k-1} - K_k P_k^{yy} K_k^T \tag{19}$$

These estimates are then employed as the previous $(k-1)$ estimates for the next time step of the UKF. The procedure is repeated at each time step of the UKF for the desired number of time steps.

IV. Result and Discussion

In this study, a non-linear estimation technique based on Unscented Kalman Filter is proposed to evaluate the release rate, wind speed, and release height in case of radionuclide release. Six radiation detectors are placed at 60 degrees apart at a distance of 1.5 km from the source. These detectors are placed at the ground level and their placement is shown in Figure 1. The coordinate axes are fixed, and the mean wind direction is assumed to be constant at 30 degrees to X-axis. The measurement data is generated using the model given in Section II. The varying release rate and the wind speed data were first generated. Using the wind speed and release rate data, a set of dose rates for the 6 detectors (D1 to D6) were generated with height of release taken as 100 m.

The state and measurement vector for the system are represented by a random walk model and they are given as follows,

$$x_k = \begin{bmatrix}\dot{q}_k\\ u_k\\ h_k\end{bmatrix} = \begin{bmatrix}\dot{q}_{k-1}\\ u_{k-1}\\ h_{k-1}\end{bmatrix} + \begin{bmatrix}w_{\dot{q},k-1}\\ w_{u,k-1}\\ w_{h,k-1}\end{bmatrix} \tag{20}$$

$$y_k = \begin{bmatrix}D_{1,k}\\ D_{2,k}\\ D_{3,k}\\ D_{4,k}\\ D_{5,k}\\ D_{6,k}\end{bmatrix} + \begin{bmatrix}v_{1,k}\\ v_{2,k}\\ v_{3,k}\\ v_{4,k}\\ v_{5,k}\\ v_{6,k}\end{bmatrix} \tag{21}$$

The algorithm is applied to estimate different states using environmental radiation monitoring data collected close to the release point. To validate the efficacy of the proposed approach in tracking step variations, the wind speed has been changed at 250 s from an initial value of 4 m/s to 6 m/s. Measured and estimated dose rates at six different detector locations are shown in Figures 2–7. The corresponding estimated and true

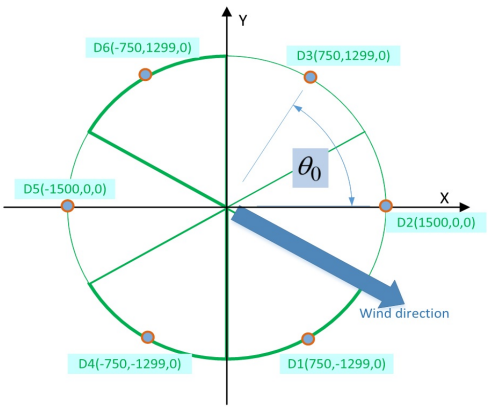

Fig. 1. Location of different dose rate detectors.

states, i.e. release rate, wind speed, and height of release are shown in Figures 8, 9, and 10 respectively. The algorithm is able to track the step change in wind speed. The filter shows very small perturbation in the estimation of height around the time of application of perturbation in wind speed. From the simulation results, it is evident that the estimated states from UKF are in good agreement with the true values. The algorithm is able to track the step change in wind speed in addition to other states and measurements.

With regards to the application of the UKF algorithm, the process noise and measurement noise covariance matrices are selected after analysing the filter performance at different values of Q and R. A tuning measure based on computing the variance of innovations have been adopted here. The values of Q matrices have been varied for a fixed value of R and the variance of innovation have been calculated. The range of Q and R matrices are varied from $Q = 1 \times 10^{-6} I_3$ to $Q = 1 \times 10^{-1} I_3$ and $R = 1 \times 10^{-5} I_6$ to $R = 1 \times 10^{1} I_6$ respectively. For all the six detectors these values are plotted in Figure 11. The values of Q and R are selected as $Q = 1 \times 10^{-3} I_3$ and $R = 1 \times 10^{-4} I_6$, where I_3 and I_6 denotes identity matrices of order 3 and 6 respectively.

To quantitatively assess the performance of UKF algorithm, a measure based on Root Mean Squared Errors (RMSE) has been adopted. It is given by

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2} \tag{22}$$

where N denotes the total number of samples. x_i and $\hat{x}_i$ are true and estimated quantities respectively. RMSE between true values and estimated values of states are computed. They are given in Table I. The error values indicate that the estimated values are close to the measured values.

V. Conclusion

A non-linear estimation technique utilizing the unscented Kalman filter has been proposed in the paper to estimate

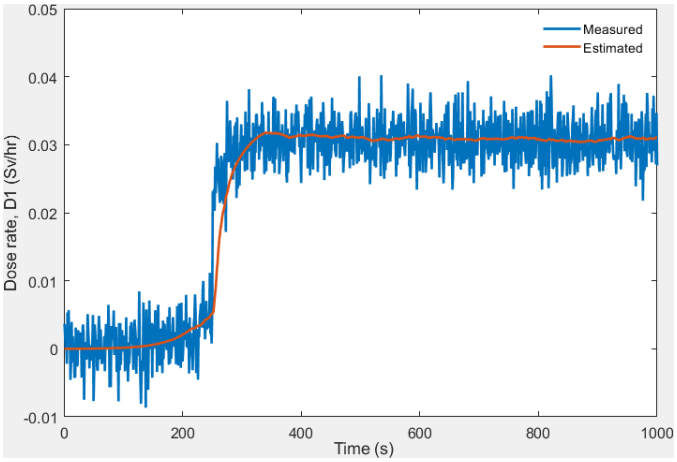

Fig. 2. Measured and estimated dose rates at detector 1.

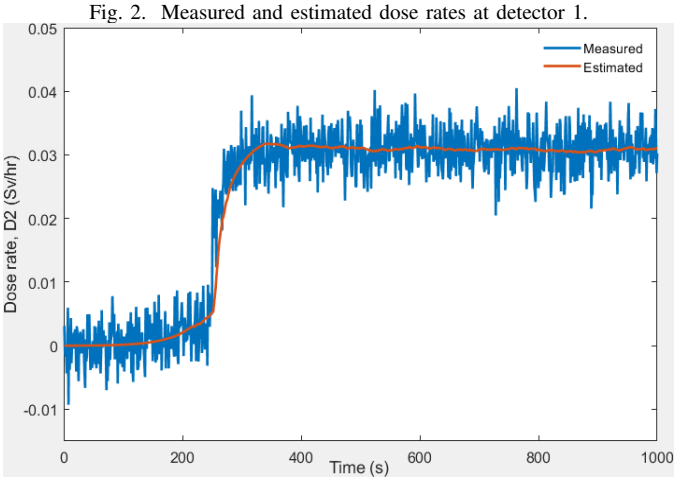

Fig. 3. Measured and estimated dose rates at detector 2.

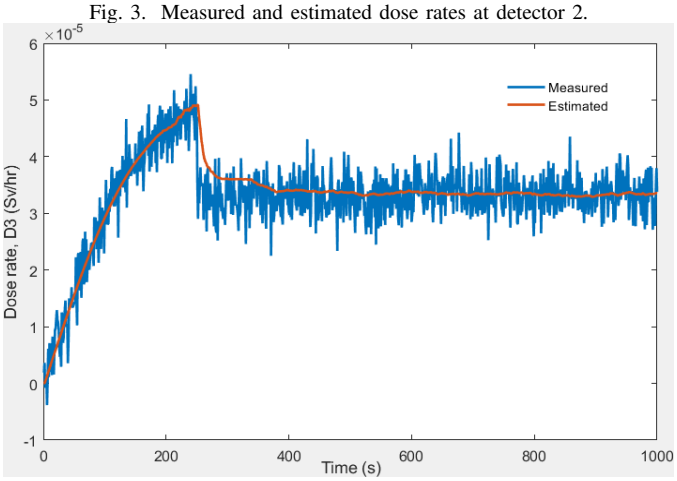

Fig. 4. Measured and estimated dose rates at detector 3.

radionuclide release estimation. The proposed method employs the Gaussian plume model of atmospheric dispersion for calculating the radionuclide concentration and dose rates. Measurement data obtained from six different environmental radiation monitoring detectors close to the release point is utilized for estimation purposes. The performance of filter

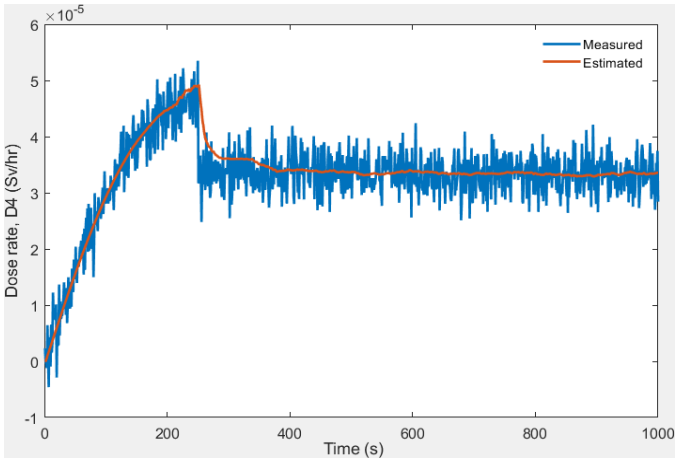

Fig. 5. Measured and estimated dose rates at detector 4.

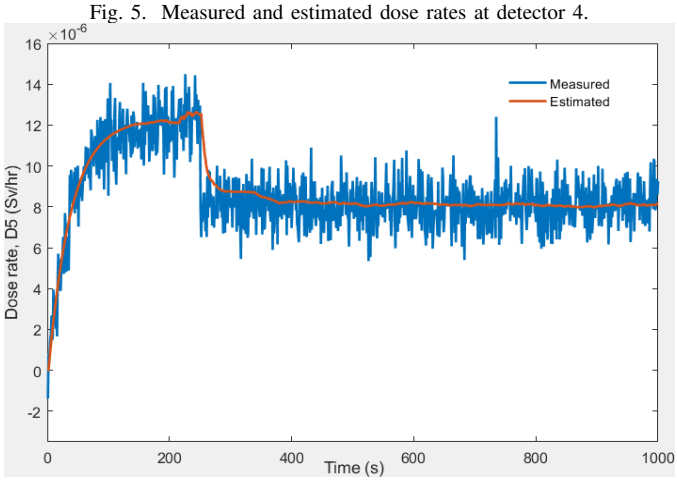

Fig. 6. Measured and estimated dose rates at detector 5.

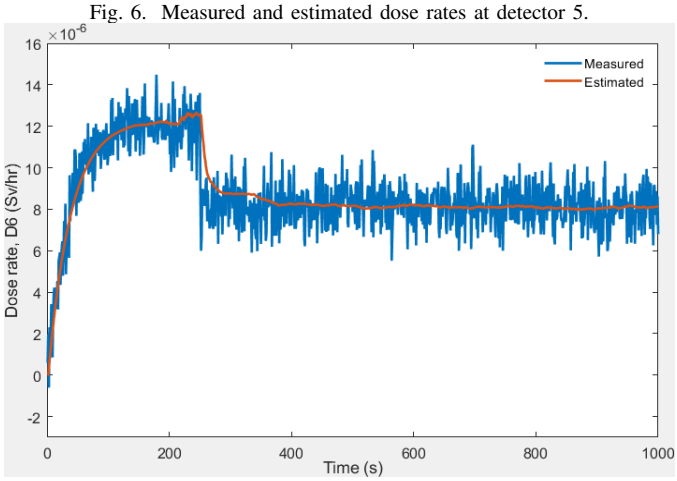

Fig. 7. Measured and estimated dose rates at detector 6.

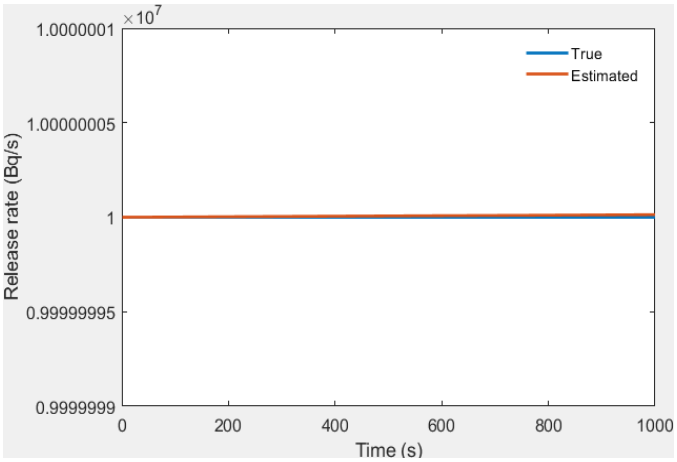

Fig. 8. True and estimated release rate.

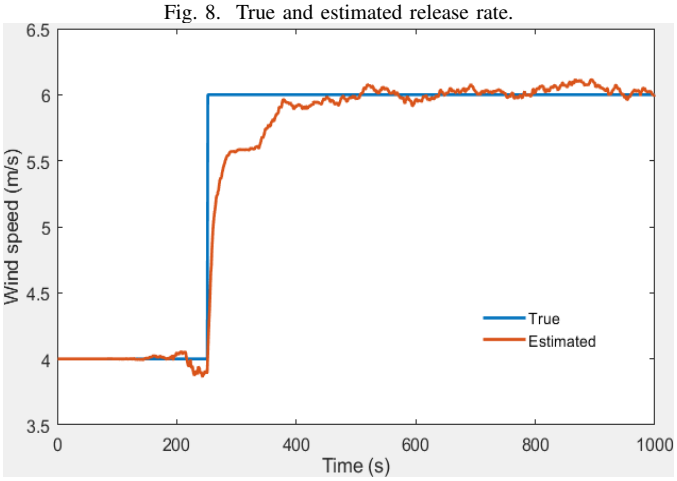

Fig. 9. True and estimated wind speed.

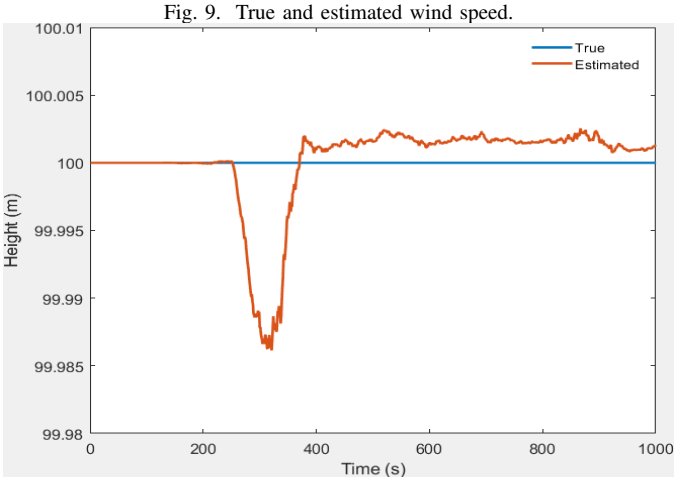

Fig. 10. True and estimated effective height of release.

is enhanced by tuning covariance matrices associated with process and measurement noise. The estimation algorithm is found to reliably estimate radioactivity release, wind velocity, and height of release. It can be clearly verified from the simulation results that the estimation algorithm is performing satisfactorily.

REFERENCES

[1] S. S. Raza, R. Avila, and J. Cervantes, "A 3-d Lagrangian (Monte Carlo) method for direct plume gamma dose rate calculations," *Journal of Nuclear Science and Technology*, vol. 38, no. 4, pp. 254–260, 2001.

[2] M. Drews, B. Lauritzen, H. Madsen, and J. Q. Smith, "Kalman filtration of radiation monitoring data from atmospheric dispersion of radioactive

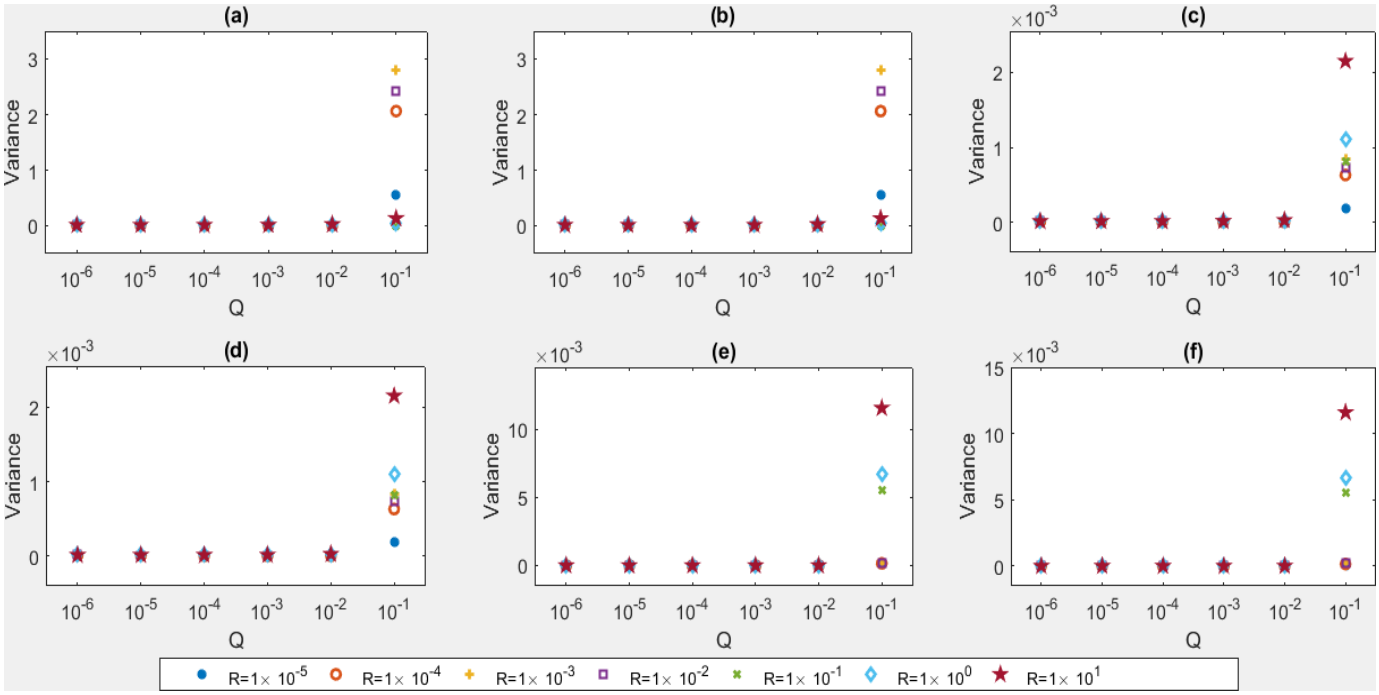

Fig. 11. Variance of innovations of detectors (a) D1, (b) D2, (c) D3, (d) D4, (e) D5, (f) D6, for different values of Q and R matrices.

TABLE I
ROOT MEAN SQUARED ERROR

Parameters	RMSE
Release Rate	7.7624×10^{-3}
Wind Velocity	2.1397×10^{-1}
Height of Release	3.3285×10^{-3}

materials," *Radiation Protection Dosimetry*, vol. 111, no. 3, pp. 257–269, 2004.

[3] M. Drews, B. Lauritzen, and H. Madsen, "Analysis of a Kalman filter based method for on-line estimation of atmospheric dispersion parameters using radiation monitoring data," *Radiation Protection Dosimetry*, vol. 113, no. 1, pp. 75–89, 2005.

[4] P. Astrup, C. Turcanu, R. Puch, C. Rojas-Palma, and T. Mikkelsen, "Data assimilation in the early phase: Kalman filtering RIMPUFF," *Ris National Laboratory, Roskilde, Denmark*, pp. 78–89, 2004.

[5] C. Rojas-Palma, H. Madsen, F. Gering, R. Puch, C. Turcanu, P. Astrup, H. Mller, K. Richter, M. Zheleznyak, D. Treebushny, M. Kolomeev, D. Kamaev, and H. Wynn, "Data assimilation in the decision support system RODOS," *Radiation Protection Dosimetry*, vol. 104, pp. 31–40, 2003.

[6] D. Q. Zheng, J. K. C. Leung, B. Y. Lee, and H. Y. Lam, "Data assimilation in the atmospheric dispersion model for nuclear accident assessments," *Atmospheric Environment*, vol. 41, no. 11, pp. 2438–2446, 2007.

[7] D. Q. Zheng, J. K. C. Leung, and B. Y. Lee, "Online update of model state and parameters of a Monte Carlo atmospheric dispersion model by using ensemble Kalman filter," *Atmospheric Environment*, vol. 43, no. 12, pp. 2005–2011, 2009.

[8] T. V. Santhosh, M. H. Prasad, V. Gopika, R. K. Singh, and K. K. Vaze, "Development of atmospheric dispersion module and its integration with diagnostic system for radioactivity release evaluations," in *Proceedings of the International Workshop on New Horizons in Nuclear Reactor Thermal Hydraulics and Safety*, Jan 2014.

[9] X. L. Zhang, G. F. Su, H. Y. Yuan, J. G. Chen, and Q. Y. Huang, "Modified ensemble Kalman filter for nuclear accident atmospheric dispersion: Prediction improved and source estimated," *Journal of Hazardous Materials*, vol. 280, pp. 143–155, 2014.

[10] X. L. Zhang, G. F. Su, J. G. Chen, W. Raskob, H. Y. Yuan, and Q. Y. Huang, "Iterative ensemble Kalman filter for atmospheric dispersion in nuclear accidents: An application to kincaid tracer experiment," *Journal of Hazardous Materials*, vol. 297, pp. 329–339, 2015.

[11] P. H. Hiemstra, D. Karssenberg, and A. van Dijk, "Assimilation of observations of radiation level into an atmospheric transport model: a case study with the particle filter and the ETEX tracer dataset," *Atmospheric Environment*, vol. 45, no. 34, pp. 6149–6157, 2011.

[12] P. H. Hiemstra, D. Karssenberg, A. van Dijk, and S. M. de Jong, "Using the particle filter for nuclear decision support," *Environmental Modelling & Software*, vol. 37, pp. 78–89, 2012.

[13] R. Macdonald, *Theory and objectives of air dispersion modelling*, Dec 2003.

[14] *Modeling pollutant dispersion in the environment*. Environmental Sciences and Engineering, BARC Highlights.