Finkel Was Right: Counter-Examples to Several Conjectures on Variants of Vector Addition Systems

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Abstract

Studying one-dimensional grammar vector addition systems has long been advocated by Alain Finkel. In this presentation, we shall see how research on those systems has led to the recent breakthrough tower lower bound for the reachability problem on vector addition systems, obtained by Czerwiński et al. In fact, we shall look at how appropriate modifications of an underlying technical construction can lead to counter-examples to several conjectures on one-dimensional grammar vector addition systems, fixed-dimensional vector addition systems, and fixed-dimensional flat vector addition systems.

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1 Outline

We shall discuss counter-examples to each of the following conjectures.

In the context of the gap between the ExpSpace membership of the coverability problem for one-dimensional grammar vector addition systems [2] and its PSpace hardness [5], it was attractive to think that there is an exponential bound such that, at least for systems whose ratio is finite and greater than 1, every positive instance has some derivation whose height is at most the bound.

▶ **Conjecture 1.** *One-dimensional grammar vector addition systems whose ratio is finite and greater than* 1 *have coverability witnessing derivations that are at most exponentially high.*

It has been known since the refinement by Rosier and Yen [4] of Rackoff's bounds [3] that fixed-dimensional vector addition systems have coverability witnessing runs that are at most exponentially long, and there seemed to be no reason why the same should not hold for the reachability problem.

▶ **Conjecture 2.** *Fixed-dimensional vector addition systems have reachability witnessing runs that are at most exponentially long.*

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It has also been known that flat vector addition systems in which update vectors are given in unary have reachability witnessing runs that are at most polynomially long, provided the dimension is 1 or 2 [1], and it seemed plausible that this property would be true in every fixed dimension.

▶ **Conjecture 3.** *Fixed-dimensional flat vector addition systems in which update vectors are given in unary have reachability witnessing runs that are at most polynomially long.*

The same work of Englert et al. established NL membership of the reachability problem for two-dimensional flat unary vector addition systems, and it was similarly plausible that the same would be the case in every fixed dimension. However, we shall discuss how to obtain NP hardness already in a specific small dimension.

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