

Fiscal Policy with Limited-Time Commitment*

[Short title: Limited-Time Commitment]

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Abstract

We propose a theory of optimal fiscal policy with “Limited-Time Commitment” (LTC). In our framework, successive governments have commitment only over finite, overlapping horizons. We first show that key results in the Full Commitment (FC) literature can often be sustained with a single period of commitment. We then solve a calibrated model in which LTC fails to implement the FC policy, and find that one year (three years) of commitment recovers one third (two thirds) of the welfare losses relative to No Commitment. Finally, we investigate the tradeoff between commitment and flexibility in response to shocks.

Keywords: limited commitment, time inconsistency, optimal fiscal policy

JEL codes: E61, E62, E02, H21

Governments in advanced economies tend to formulate their macroeconomic policies as plans for a finite future horizon. In particular, fiscal policy is typically decided upon and announced before or at the beginning of the fiscal year and remains fixed for the duration of the year.¹ Reforms such as tax rate changes and fiscal consolidation plans are also announced before their implementation and typically contain details of short-to-medium run policy plans.² Moreover, the political process in representative democracies makes it hard to change contemporaneous policies, with the result that policy changes are typically implemented with a delay. Hence, while clearly there are examples of sudden changes following large economic shocks, the way fiscal policy is conducted in normal circumstances features significant lags between decisions and execution.

In contrast, a large part of the literature on optimal fiscal policy assumes either that a single government at the beginning of time has Full Commitment (FC) into the infinite future, or that in each period there is a government with No Commitment (NC)

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¹For example, governments in the Euro area announce their fiscal budget plans – the Annual Draft Budgetary Plans – for the following year. In the UK, the budget is announced on “Budget Day” in March, before the start of the fiscal year, and typically passed shortly afterwards.

²Examples of significant pre-announced VAT reforms include Japan (announced in 1996, implemented in 1997) and Germany (announced in 2005, implemented in 2007). Alesina et al. (2015) document many examples of multi-year fiscal consolidation plans across different countries.

at all, only able to choose contemporaneous policies. In models where optimal policy is time inconsistent, because of the presence of forward-looking private agents (Kydland and Prescott, 1977), these different assumptions on the government’s ability to commit may have large effects on optimal policies and equilibrium outcomes. However, both the assumptions underlying FC and NC policies appear hard to reconcile with the fact that policymakers are in power for a limited amount of time, inherit their predecessors’ plans, and communicate as if they possessed a degree of commitment over a finite future horizon.

Motivated by this apparent distance between observation and theory, in this paper we study fiscal policy when successive benevolent governments inherit the plans of their predecessors and formulate plans for a finite future horizon. In this formulation, which we call Limited-Time Commitment (LTC), governments cannot commit into the infinite future, but instead only possess the ability to commit for a finite horizon. Specifically, we define governments as having $L \geq 1$ periods of commitment if the time- t government cannot change policies dated time t to $t + L - 1$, and chooses policies dated time $t + L$. We then apply our LTC equilibrium concept to make two key contributions.

Our first contribution is to provide a sufficient condition for equivalence between the outcomes that arise under LTC and FC in a general stochastic framework with time inconsistency arising from forward-looking competitive-equilibrium constraints. In this framework, the government under FC has an incentive to make promises about future policies in order to influence private-sector expectations. Ex-post, the government would like to renege on these (past) promises, if a reoptimisation was allowed. We show that if a finite sequence of future policies is sufficient to uniquely pin down all the future variables that enter the current constraint set, then the time-inconsistency problem can be resolved with LTC, because the future variables become, effectively, current choices for the government. The next government will then inherit this policy plan as a state variable, ensuring that equilibrium allocations are not changed relative to the promised plan, and creating a chain of commitment that sustains the FC equilibrium path. This condition can be checked model-by-model. Hence, we provide guidance for future researchers on the cases in which assuming FC, even if seemingly “unrealistic”, leads to the same outcomes as a more empirically plausible amount of commitment.

We apply this equivalence result in a benchmark model of capital and labour taxation following Chari and Kehoe (1999). In the most general formulation of the model, no finite amount of commitment can support the FC solution. However, we present several specialisations of the model, associated with restrictions on the environment or with fiscal rules, where the FC solution can be exactly supported by a finite degree of commitment. These special cases have been studied extensively in the literature. In a model without capital (as in Lucas and Stokey, 1983), FC outcomes can be supported with commitment equal to the maximum government debt maturity. In models with capital, we uncover a key role for constitutional budget restrictions and highlight which tax instruments help to sustain FC outcomes. Importantly, we show that the Chamley (1986)-Judd (1985) result of zero long run capital taxes can often be supported with only a single period of commitment.

Our second contribution is to propose LTC as a positive model of fiscal policy that incorporates realistic commitment assumptions, even in models where exact equivalence with FC does not hold. We explore in depth the properties of our Markov-Perfect

equilibrium concept, which includes pre-existing policy plans as state variables, in the context of a model of public good provision and capital taxation following Klein et al. (2008). In a calibrated version of the model, we find that a single year (three years) of fiscal commitment is sufficient to recover approximately one third (two thirds) of the welfare difference between FC and NC in steady-state. Accordingly, we find that the largest marginal welfare gains come from the ability to commit over short horizons, and that the marginal gains from longer commitment horizons are smaller. We also simulate the effects of a “constitutional reform”, imposing one year of commitment to taxes and spending, starting from the steady-state of an economy with NC. The transitional dynamics following this reform involve a non-monotone path for the size of the government (retrenchment followed by government expansion as the whole economy expands) and lead to significant welfare gains (approximately 1.8% of permanent consumption).

We then introduce shocks to the valuation of government spending, calibrated to match postwar US data. This exercise allows us to assess the importance of state contingency in fiscal commitments in a model that features a trade-off between commitment and state-contingency. We solve two versions of the model, one where the government makes state-contingent plans for the future and one where it can only make non-contingent plans. We find that the high persistence of the calibrated shocks leads to similar outcomes in these two cases. However, we also find that in a counterfactual scenario with i.i.d. shocks, there would be large gains from state-contingency in LTC plans.

Our work connects two main strands of literature on fiscal policy, under FC and NC respectively. Specifically, our equivalence result contributes to the literature on recursive formulations of FC policies (Kydlund and Prescott, 1980, Abreu et al., 1990, Chang, 1998, Marcet and Marimon, 2019) by directly focusing on policy instruments as state variables. At the same time, we build on the literature on time-consistent fiscal policy (Klein and Ríos-Rull, 2003, Krusell et al., 2004, and Klein et al., 2008), by adding pre-committed policy plans as additional elements of the state vector in Markov-Perfect equilibria and showing that in some important cases this approach recovers FC outcomes.³

This paper is also closely related to the Quasi-Commitment and Loose-Commitment approaches (Schaumburg and Tambalotti, 2007, Debortoli and Nunes, 2010, 2013), which build on an early contribution by Roberds (1987). Relative to this literature, which assumes stochastic policy reoptimisations, our results point towards significant differences in outcomes depending on whether commitment is stochastic over an infinite horizon, or deterministic, but limited in time.⁴ Relatedly, we find large effects of small changes in

³See also the related approach to lack of commitment of Chari and Kehoe (1990).

⁴In related work, Brendon and Ellison (2018) propose substituting the assumption of plans formulated from the perspective of time-0 with a Pareto criterion, according to which a plan is selected if in every period there is no plan that can make all current and future governments better off. An alternative approach is given in the reputational equilibria literature, starting with the seminal contribution of Barro and Gordon (1983). Bassetto (2019) distinguishes the communication and implementation of future policies, and shows that there is no separate role for communication if the government has the same information as the private sector. Other papers explore the extent to which FC outcomes can be supported by adding extra policy instruments, e.g. Alvarez et al. (2004) and Laczó and Rossi (2019). Conesa and Domínguez (2012) and Domínguez (2019) focus on constructing suitable one-period contingent bond portfolios to sustain FC outcomes in absence of commitment. In this context, Domínguez (2019) additionally shows that adding implementation lags to fiscal instruments can help sustain FC if insufficient varieties of contingent bond are available.

the timing of commitment in the context of capital income taxes. Klein and Ríos-Rull (2003) assume that the government can set capital taxes one period ahead, but only set current labour taxes. They show that capital taxes are on average high, compared to the FC equilibrium. In a deterministic version of the same model, we show that if instead the time- t government chooses both the time- $t + 1$ capital *and* labour taxes, then LTC sustains the FC outcome.⁵ In Section 5, we expand on the connections between our results and the existing approaches on NC (specifically, the Generalised Euler Equation approach), recursive methods for FC, and Loose Commitment.

Furthermore, the paper is related to the political economy literature on optimal rules in presence of time inconsistency (Amador et al., 2006, Halac and Yared, 2014). While these papers focus on present bias, our source of time inconsistency is the presence of forward-looking agents.⁶ Finally, our work takes a first step to connect the literature on optimal fiscal policy with the literature on the economic effects of anticipated policy changes. Mertens and Ravn (2012) emphasise the importance of anticipation effects in their empirical analysis of tax changes in U.S. post-war data. House and Shapiro (2006), Mertens and Ravn (2011) and Leeper et al. (2012) study fiscal anticipation effects in the context of DSGE models of the economy. We see LTC as a natural framework to study how governments may strategically exploit the link between promised policies in the near future and current equilibrium outcomes.

1 Equilibrium with LTC

In this section, we describe a generic model economy, and define two notions of optimal policy: Full-Commitment Ramsey equilibrium and Limited-Time Commitment equilibrium. LTC additionally nests the No Commitment equilibrium as a special case without any degree of commitment.

1.1 Environment and competitive equilibrium

Time is discrete and indexed by $t = 0, 1, \dots$. The economy is populated by continuums of households and firms, and a government or a sequence of governments. There is a vector of exogenous variables $z_t \in Z$ which evolves stochastically with the Markov property that z_t is sufficient information to calculate the probability distribution over z_{t+1} . We restrict ourselves to settings where the shocks are drawn from either finite or countably infinite distributions.⁷ That is, the set of possible values for z_t can be expressed as a, possibly infinite, list $Z = \{\bar{z}^1, \bar{z}^2, \dots\}$. Denote by P^z the associated Markov transition matrix.

⁵Domeij and Klein (2005) study a tax reform in the presence of implementation lags where they solve the Full Commitment problem under the assumption that policies can only be changed starting from some period T onwards. They assume Full Commitment from time T onwards, whereas we consider limited commitment in all periods. Martin (2015) studies the effects of within-period commitment to fiscal policy.

⁶Relatedly, Athey et al. (2005) study the optimal design of monetary institutions in the context of time inconsistency stemming from the role of expectations on inflation and output determination.

⁷This is to avoid technical discussions which would not add to the substance of the paper. We conjecture that all results go through, suitably modified, in a model with uncountably infinite distributions.

We call $b_t \in B$ the endogenous state variables, $c_t \in C$ the remaining variables constituting allocations (for instance consumption and hours worked), $p_t \in P \subset \mathbb{R}^{N_p}$ the prices and $\tau_t \in T$ the policy instruments chosen by the government(s). Households' preferences are represented by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t r(c_t, b_t, z_t, \tau_t), \quad (1)$$

where $r : C \times B \times Z \times T \mapsto \mathbb{R}$ is the instantaneous return function, and $\beta \geq 0$ is the discount factor. \mathbb{E}_t is the expectation operator conditional on information up to time t . Let $z^t = (z_0, \dots, z_t)$ denote the history of shocks up to time t . Any sequence of government policies and equilibrium prices can be written as functions of the histories of shocks: $\{\tau_t(z^t)\}_{t=0}^{\infty}$ and $\{p_t(z^t)\}_{t=0}^{\infty}$. Households and firms take these functions as given.

Governments choose policies subject to competitive-equilibrium conditions. Following the formulation in Marcet and Marimon (2019), we summarise these equilibrium conditions with three sequences of constraints: a transition equation for the endogenous states, a set of constraints involving only contemporaneous allocations, and a set of constraints involving future variables. For any time $t = 0, 1, \dots$ these constraints are

$$b_{t+1} = l(b_t, z_t, c_t, p_t, \tau_t) \quad (2)$$

$$k(b_t, z_t, c_t, p_t, \tau_t) \leq 0 \quad (3)$$

$$\mathbb{E}_t \sum_{n=0}^N h_n(b_{t+n}, z_{t+n}, c_{t+n}, p_{t+n}, \tau_{t+n}) = 0. \quad (4)$$

$N \geq 1$ is a parameter governing the horizon over which future variables affect time- t constraints via (4).⁸ As is well known in the literature, the presence of these future variables in the constraint set defining competitive equilibria is the reason for the time inconsistency of FC policies. We wrote (4) such that it potentially contains all future allocations, prices, policies and so on up to N periods ahead. However, in practice, only some subset of these variables will actually appear in (4) in a given model. Thus, in Definition 1 we explicitly label the future variables which enter into constraint (4) as “problematic”:

Definition 1. Let $i = 1, \dots, N_c$ index the individual elements of c_t , such that $c_t \equiv (c_t^1, \dots, c_t^{N_c})$. We call element i **problematic** if, for any $s > 0$, c_{t+s}^i appears in constraint (4). The same definition applies to elements of p_{t+s} and τ_{t+s} , and to b_{t+s} (with $s > 1$).

Intuitively, any variable which has a future value appearing in a forward-looking constraint is labeled as problematic. For the endogenous state b_t , it is allowed for an element of b_{t+1} to appear in the constraints without it being labelled problematic. The definition of a variable as problematic does not depend on time: it is “consumption” which is problematic, and not “consumption at time t ”.

⁸We write (4) as a sum of separate h_n functions, but all results go through for h functions which are not time separable. It is often possible to rewrite the competitive equilibrium conditions of a model in different, equivalent ways, featuring different versions of (4) with different values of N . If one wants to identify the minimum amount of commitment required for LTC to support FC in a given model, one must first identify the representation of the model with the smallest N .

We define competitive equilibrium in the standard way and relegate the definition to Appendix A. We summarise the variables of the economy as $y_t(z^t) \equiv (c_t(z^t), p_t(z^t), b_t(z^t), \tau_t(z^t))$, and use the notation $\mathbf{y} \equiv \{y_t(z^t)\}_{t=0}^\infty$ to denote plans. We suppress the functional notation ($c_t(z^t)$ and similar) unless it will lead to confusion. (b_0, z_0) are initial conditions, and we restrict our analysis to the subset of initial conditions for which a competitive equilibrium exists for at least one policy sequence, and denote this set $B^* \subset B \times Z$. We maintain a boundedness assumption on r , and that the discount factor satisfies $\beta < 1$ (**Assumption 1**, discussed further in Appendix A).

1.2 Full-Commitment Ramsey equilibrium

In a Full Commitment setup, a single benevolent infinitely-lived government endowed with the ability to credibly commit into the infinite future announces a contingent plan at $t = 0$ and then implements it. Denote a policy plan by $\boldsymbol{\tau} \equiv \{\tau_t(z^t)\}_{t=0}^\infty$. We maintain as an assumption (**Assumption 2**, discussed in Appendix A) that any policy plan pins down a unique competitive equilibrium. This allows us to state the FC government's problem, for any $(b_0, z_0) \in B^*$, as

$$V^{FC}(b_0, z_0) = \max_{\{\tau_t, c_t, b_{t+1}, p_t\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t r(c_t, b_t, z_t, \tau_t), \quad (5)$$

subject to (2), (3) and (4), and initial conditions (b_0, z_0) . This maximization problem is solved by a policy sequence $\boldsymbol{\tau}^{FC}(b_0, z_0) \equiv \{\tau_t^{FC}(z^t)\}_{t=0}^\infty$. A **Full-Commitment (FC) Ramsey equilibrium** is the competitive equilibrium, $\mathbf{y}^{FC}(b_0, z_0)$, associated with the optimal policy.⁹

1.3 Limited-Time Commitment equilibrium

The Limited-Time Commitment setup can be described as a game, where successive one-period-lived governments indexed by t choose only a finite sequence of policy instruments, taking as given the strategies of the following governments. Each government is benevolent and maximises (1) subject to the competitive equilibrium conditions (2), (3) and (4). As is standard in the literature, governments thus act as Stackelberg leaders with respect to the private sector, internalising the effects of policy choices on competitive equilibrium outcomes. We follow the literature (for instance Klein et al., 2008) in restricting attention to symmetric Markov-Perfect equilibria, where each government chooses a common best-response function mapping a small set of “natural” state variables into the chosen sequence of policy instruments.

Let $L = 0, 1, \dots$ index the duration of commitment. The case $L = 0$ coincides with No Commitment. In presence of a commitment horizon $L > 0$, the government at time t is not able to change policies from time t to time $t + L - 1$, and chooses policies to be implemented at time $t + L$. This choice is made given a state variable $s_t \in S$, which will feature past policies as states, depending on the exact form of the game.

⁹For expositional simplicity, we maintain the assumption that the optimal policy is unique for any initial conditions, but all results go through if there are multiple optimal policies inducing the same value.

In a stochastic environment, we must take a stand on whether the government is able to make its choice of τ_{t+L} contingent on future shocks or not. We thus consider two polar assumptions. We denote by “non-contingent LTC” the game where the government has to choose a non-contingent value for τ_{t+L} conditional only on the information available to it at time t . We denote by “contingent LTC” the game where the government is able to commit to a whole menu of values for τ_{t+L} , to be executed conditional on the shocks that occur between t and $t+L$. Contingent LTC is consistent with the view that governments are able to commit to near-term fiscal plans, but harder to reconcile with the notion of implementation lags, which would make state-contingency almost impossible. On the other hand, non-contingent LTC is consistent both with our motivation based on fiscal announcements and with the presence of implementation lags in fiscal policy. Thus, we explore both alternatives, both in our theoretical analysis and by comparing them numerically in Section 4.

1.3.1 Special case: LTC in a deterministic economy

We begin by describing the LTC game in deterministic economies where there is no uncertainty, and $\{z_t\}_{t=0}^{\infty}$ follows a known path. This simplifies the exposition because the government’s policy does not need to be described in terms of a contingent value for the policy instrument depending on the future realisations of the shocks, allowing us to focus on the simpler non-contingent LTC formulation.

In the non-contingent LTC game, the government chooses a value for the future policy τ_{t+L} given the current state variables, giving policy function $\tau_{t+L} = \tau(s_t)$. The government takes as state variables the endogenous and exogenous states b_t and z_t . However, it must also take the previously pre-committed policies $(\tau_t, \dots, \tau_{t+L-1})$ as given, since these were chosen by the governments from time $t-L$ to $t-1$ and cannot be changed. These are summarised in the variable τ_t^L , which is defined as

$$\tau_t^L \equiv \begin{cases} (\tau_t, \dots, \tau_{t+L-1}) & L > 0 \\ \emptyset & L = 0. \end{cases} \quad (6)$$

For $L > 0$, this summarises the vector of pre-committed policies. For $L = 0$, which corresponds to the case of No Commitment, there are no pre-committed policies to take as states and we let τ_t^L equal the empty set, \emptyset . Let $s_t \equiv (b_t, z_t, \tau_t^L)$ denote the state variables at time t . In the simplest case of non-contingent LTC, with $L = 1$, each government takes the current policy, τ_t , as given as part of its state $s_t = (b_t, z_t, \tau_t)$, and makes a non-contingent choice for next period’s policy, τ_{t+1} . Next period, the government takes $s_{t+1} = (b_{t+1}, z_{t+1}, \tau_{t+1})$ as its state variable, and then chooses a new policy τ_{t+2} , and so on.

We next define the Markov-Perfect equilibrium of the non-contingent LTC game for a deterministic economy.¹⁰ Consider the government in power in period t and suppose that all governments from time $t+1$ onwards are expected to play a common policy function $\tau_{t+j+L} = \tau(s_{t+j})$ for $j = 1, 2, \dots$. Given a policy function τ played by all future

¹⁰Throughout the paper, we refrain from using recursive notation and continue to index variables by time in recursive formulations. This is to improve clarity, given the importance of timing assumptions in the LTC game.

governments, a state $s_t = (b_t, z_t, \tau_t^L)$, and a current policy choice τ_{t+L} , we maintain as an assumption that the competitive equilibrium system given by (2), (3) and (4) for $s = t, t+1, \dots$ defines a unique time-invariant solution for the vector (b_{t+1}, c_t, p_t) given by the function $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau_{t+L}; \tau)$.¹¹ We restrict governments to choose policies for which competitive equilibria exist (see Appendix A.3 for details on how to construct the set of admissible policies).

Following Klein et al. (2008), any symmetric Markov-Perfect equilibrium of the deterministic non-contingent LTC game can be written as functions v and τ such that, for all $s_t \in S$, the policy function $\tau_{t+L} = \tau(s_t)$ solves

$$\max_{\tau_{t+L}} r(c_t, b_t, z_t, \tau_t) + \beta v(s_{t+1}) \quad (7)$$

subject to $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau_{t+L}; \tau)$ and the value function $v(s_t)$ satisfies

$$v(s_t) = r(c_t, b_t, z_t, \tau_t) + \beta v(s_{t+1}), \quad (8)$$

where $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau(s_t); \tau)$. The future state, $s_{t+1} = (b_{t+1}, z_{t+1}, \tau_{t+1}^L)$, can be computed given the transition rules for b_{t+1} and z_{t+1} , and using (6) taken one period forward to construct τ_{t+1}^L .

Given an equilibrium policy function τ of the LTC game and an initial condition s_0 , we can iterate the policy function forwards to solve for an implied policy plan $\tau^*(s_0) = \{\tau_t\}_{t=0}^\infty$. Importantly, when $L > 0$, we must specify as part of our initial conditions the pre-committed policies from the point of time 0: τ_0^L . A **symmetric Markov-Perfect Limited-Time-Commitment (LTC) equilibrium** is the competitive equilibrium, $\mathbf{y}^*(s_0)$, associated with an equilibrium policy plan.

1.3.2 General case: Contingent LTC and stochastic economies

The statement of the contingent LTC game follows closely the non-contingent formulation from the previous section, and so we describe it only briefly here. The reader not interested in the technical details may skip directly to Sections 2 and 3.

In contingent LTC, the government at time t makes a plan for τ_{t+L} contingent on the history of shocks between $t+1$ and $t+L$. Denoting this partial history of shocks by $z_{t+1}^{t+L} \equiv (z_{t+1}, \dots, z_{t+L})$, the government chooses a policy function $\tau : S \times Z^L \mapsto T$ such that $\tau_{t+L} = \tau(s_t, z_{t+1}^{t+L})$. Define \mathcal{T}_{t+L} as a vector listing all the values of τ_{t+L} across the possible shock realisations between $t+1$ and $t+L$. This is the vector of contingent values for τ_{t+L} which the time t government must choose. Similarly, define $\mathcal{T}(s_t)$ as the vector-valued policy function summarising $\tau(s_t, z_{t+1}^{t+L})$. Thus, we equivalently can write the policy function as $\tau_{t+L} = \tau(s_t, z_{t+1}^{t+L})$ or $\mathcal{T}_{t+L} = \mathcal{T}(s_t)$.

Given this structure for choices, the policy state variable, τ_t^L , needs to be augmented to recognise the state-contingent nature of the pre-committed policies. Specifically, τ_t^L

¹¹Even though **Assumption 2** pins down a unique competitive equilibrium for an *infinite sequence* of government policies, the choice of a *single* policy τ_{t+L} combined with a future policy function τ might not. This could make it impossible for a government to know which equilibrium its policy choice pins down. This issue arises in both the LTC and No Commitment games, and to state the LTC game we must maintain as an assumption that the environment and permissible policy functions uniquely pin down equilibrium in this sequence game. This assumption is implied by the stronger **Assumption 3** used in our equivalence proof.

is now given by

$$\tau_t^L \equiv \begin{cases} (\tau_t, \tau_{t+1}(z_{t+1}), \tau_{t+2}(z_{t+1}^{t+2}), \dots, \tau_{t+L-1}(z_{t+1}^{t+L-1})) & L > 0 \\ \emptyset & L = 0 \end{cases} \quad (9)$$

for some value τ_t and functions $(\tau_{t+1}, \dots, \tau_{t+L-1})$ which are (with some abuse of notation) truncations of previous governments' policy functions, and which the time t government takes as given.¹² In the simplest case of contingent LTC, with $L = 1$, each government takes the current policy, τ_t , as given and makes a state-contingent plan for next period's policy, $\tau_{t+1} = \tau(s_t, z_{t+1})$. Next period, the shock z_{t+1} is revealed and the relevant policy is implemented, leaving $s_{t+1} = (b_{t+1}, z_{t+1}, \tau_{t+1})$ as the state variable for the $t + 1$ government, and so on.

Symmetric Markov-Perfect equilibria of the contingent LTC game in stochastic economies can be written as functions v and τ such that, for all $s_t \in S$, the policy function $\tau_{t+L} = \tau(s_t)$ solves

$$\max_{\tau_{t+L}} r(c_t, b_t, z_t, \tau_t) + \beta \mathbb{E}_t v(s_{t+1}) \quad (10)$$

subject to¹³ $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau_{t+L}; \tau)$ and the value function $v(s_t)$ satisfies

$$v(s_t) = r(c_t, b_t, z_t, \tau_t) + \beta \mathbb{E}_t v(s_{t+1}), \quad (11)$$

where $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau(s_t); \tau)$. The future state, $s_{t+1} = (b_{t+1}, z_{t+1}, \tau_{t+1}^L)$, can be computed given the transition rule for b_{t+1} , and using (9) taken one period forward to construct τ_{t+1}^L given the policy choice τ_{t+L} and the realised value of z_{t+1} .

Finally, it is also possible to define the non-contingent LTC game in stochastic economies. In this game, the government is restricted to only making non-contingent choices for the policy τ_{t+L} despite the presence of uncertainty. The statement of this game is identical to the statement of the contingent LTC game in stochastic economies, but with the policy functions restricted to be non-contingent ($\tau_{t+L} = \tau(s_t)$) in the maximisation (10).

2 Equivalence result: When LTC sustains FC

In this section we provide a sufficient condition under which the unique equilibrium of the LTC game coincides with the Full-Commitment outcome. We prove this result for both stochastic economies (contingent LTC) and deterministic economies. For ease of exposition we discuss here only the case of LTC in deterministic economies, and provide proofs for both cases in Appendix A.4.

¹²Specifically, τ_t is the value implied by the policy function $\tau_t = \tau(s_{t-L}, z_{t+1-L}^t)$ and the realised values of (s_{t-L}, z_{t+1-L}^t) . $\tau_{t+1}(z_{t+1})$ is the function implied by the policy function $\tau_{t+1} = \tau(s_{t+1-L}, z_{t+2-L}^{t+1})$ and the realised values of $(s_{t+1-L}, z_{t+2-L}^{t+1})$, and similarly for the remaining functions.

¹³The previously defined function ϕ is appropriately redefined in the case of stochastic economies and contingent LTC to accept the contingent policy functions and states as arguments.

2.1 Special case: LTC in a deterministic economy

The key requirement for equivalence of LTC and FC is that a finite sequence of government policies uniquely pins down certain allocations through competitive equilibrium restrictions. We first formally define what it means to pin down a variable:

Definition 2. *In a deterministic economy, for any t and $t' \geq t$, we say that the natural states (b_t, z_t) and a partial policy sequence $\{\tau_s\}_{s=t}^{t'}$ uniquely determine a variable $x_{t'}$ from time t if all possible competitive equilibria from time t onwards feature the same value of $x_{t'}$ regardless of the future policy choices $\{\tau_s\}_{s=t'+1}^{\infty}$.*

Recalling that we have relegated two regularity assumptions to the appendix, this allows us to state our third and main assumption in the deterministic case:

Assumption 3. *Consider a deterministic economy. There exists an L , with $N \leq L < \infty$, such that the following holds for all $t = 0, 1, \dots$*

1. *From time t , the state variables $s_t = (b_t, z_t, \tau_t^L)$ uniquely determine all problematic variables dated time t to $t + N - 1$.*
2. *From time t , the state variables $s_t = (b_t, z_t, \tau_t^L)$ and the time- t government's choice τ_{t+L} uniquely determine c_t, p_t and b_{t+1} .*

Intuitively, the first part of this assumption states that there is a finite commitment horizon, L , which removes the ability of governments to alter the values of the variables they disagree upon with past governments. The second part allows us to compute the whole contemporaneous allocation, and hence the value of the state tomorrow, given the government's choice.¹⁴ It is worth stressing that this assumption does not imply that each government in the LTC game has only one feasible choice. In all of the examples we present below, the time- t government has many feasible choices of τ_{t+L} , and will *optimally* choose the value consistent with the FC plan. Our main proposition shows that this implies that, under the right initial conditions, the unique equilibrium of the contingent LTC game supports the FC solution:

Proposition 1. *Consider a deterministic economy where there exists an L such that Assumption 3 holds, and fix initial conditions $(b_0, z_0) \in B^*$. If, in the non-contingent LTC game, either*

1. *$\tau_0^L = (\tau_0^{FC}, \tau_1^{FC}, \tau_2^{FC}, \dots, \tau_{L-1}^{FC})$, such that the initial L periods of policies are restricted to be the optimal values from the FC solution, or*
2. *the time-0 government, in addition to choosing τ_L , is also allowed to choose τ_0^L*

then the unique equilibrium of the LTC game induces the value $V^{FC}(b_0, z_0)$ and generates the Full Commitment policy sequence $\boldsymbol{\tau}^{FC}(b_0, z_0)$, and the Full Commitment Ramsey equilibrium path, $\mathbf{y}^{FC}(b_0, z_0)$.

¹⁴Splitting the assumption this way highlights that we are not assuming that the government at time t has no control over the time- t allocation. It is only required to assume that the government is not able to change problematic variables, but its choices are allowed to influence contemporaneous variables which are not problematic.

The formal proof of this proposition is relegated to Appendix A.4, but we provide an intuitive sketch of the proof here to outline the main steps. Recall the function $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau_{t+L}; \tau)$, which gives the competitive equilibrium values of (b_{t+1}, c_t, p_t) in the LTC game. In general, these values could depend on the policy function τ played by future governments. However, under **Assumption 3**, we have assumed that we have enough periods of commitment such that, in this environment, the dependence of current outcomes on τ disappears.

In this case we are left with a map $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau_{t+L})$. This constraint is consistent with standard dynamic-programming results, and allows us to rewrite the equilibrium of LTC game as a Bellman equation:

$$v(s_t) = \max_{\tau_{t+L}} r(c_t, b_t, z_t, \tau_t) + \beta v(s_{t+1}) \quad (12)$$

subject to $(b_{t+1}, c_t, p_t) = \phi(s_t, \tau_{t+L})$. Applying standard proofs of recursivity, we prove that the unique solution to this equation coincides with the solution to a sequence problem where the government takes $s_0 = (b_0, z_0, \tau_0^L)$ as given and chooses a path for τ . This problem, which we refer to as the “Modified Problem”, is similar, but not equivalent, to the FC problem. In the FC problem, the government only takes as initial states the “natural” states (b_0, z_0) , and chooses the optimal path τ , *including* the initial values of the policy instruments in τ_0^L . Hence, the final step of the proof is to prove that the Modified Problem and FC problem have identical solutions if the initial policies τ_0^L are appropriately chosen. The proposition considers two cases. In point 1, equivalence holds if the initial policies are arbitrarily chosen to be the FC policies. In point 2, we show that equivalence also holds if the time-0 government in the LTC game is also allowed to choose the initial policies in addition to its choice of τ_L .

This completes the discussion of **Proposition 1**, which shows that if **Assumption 3** holds for some L , the solution to the FC problem in a deterministic economy can be supported as the unique Markov-Perfect equilibrium of the non-contingent LTC game with L periods of commitment.

2.2 General case: Contingent LTC and stochastic economies

In stochastic economies, we prove that contingent LTC can support the FC solution with suitably modified versions of **Definition 2** and **Assumption 3**. The result is given as **Proposition 1*** in Appendix A.4.

In a stochastic environment, it is not feasible for non-contingent LTC to support the FC solution. This is because the optimal FC policies may depend on contemporaneous shocks, while policies at time $t + L$ under non-contingent LTC can only be chosen based on shocks observed up to period t . Hence, we study numerically the implications of non-contingent LTC in stochastic environments in Section 4.

2.3 Additional results

In the interest of space, we relegate several additional results to Appendix A. Firstly, we provide a discussion of the necessity of **Assumption 3** in allowing LTC to exactly support FC allocations. Secondly, we investigate extensions of the LTC setup to multi-period governments and stochastic changes of government. As long as any government

at time t cannot change policies dated t to $t + L - 1$, then the rest of the power structure is irrelevant and the FC solution will arise in equilibrium. Thirdly, our results imply that whenever exact equivalence between LTC and FC holds, the FC path can be computed by solving for the LTC equilibrium. We provide a numerical algorithm for these cases. Finally, we discuss the robustness of our results to arbitrary initial conditions in Appendix B.4.

3 LTC in a benchmark model of fiscal policy

In this section we provide an investigation of the conditions under which a finite amount of commitment can sustain the FC policy in a standard class of dynamic models of optimal fiscal policy. We consider a benchmark model of optimal fiscal policy that features labour income taxes, capital income taxes, and government debt. We characterise restrictions on the environment and fiscal rules that break the link between policies in the far future and current agents' decisions, thus allowing LTC to sustain FC outcomes with a finite (often short) commitment horizon.

3.1 A benchmark model

We focus on a deterministic version of the neoclassical growth model with distortionary taxes based on Chari and Kehoe (1999), and we refer to it as the “benchmark model”. All the results in this section generalise to stochastic environments as long as the government is able to make state-contingent LTC plans. We restrict the set of exogenous states to just productivity for simplicity. We assume that governments can commit to always repaying their debt, even if they cannot commit to levels of taxes and government spending.¹⁵

To avoid notational clashes with the general framework presented in Section 2, here and wherever we refer to a specific model we use upright text to denote variables. Preferences of a representative household over consumption, c_t , labour, l_t , and government spending, g_t , are represented by the following separable utility function

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t) + w(g_t)], \quad (13)$$

with standard assumptions $u'(c) > 0$, $u''(c) < 0$, $w'(g) > 0$, $w''(g) < 0$, $v'(l) > 0$, and $v''(l) < 0$. We restrict ourselves to separable preferences only for expositional simplicity. Our results go through for non-separable preferences, under conditions that we discuss in more detail in the text. The household's budget constraint is

$$c_t + k_{t+1} + q_t b_{t+1} = w_t l_t (1 - \tau_t^l) + (1 + r_t (1 - \tau_t^k)) k_t + b_t. \quad (14)$$

b_{t+1} is household saving in a government bond which trades at price q_t and pays one unit of consumption at $t + 1$. w_t is the wage, r_t is the return on capital net of depreciation,

¹⁵In a stochastic environment with costly default, our results also hold if we consider government default choices symmetrically with the other policy instruments and allow L periods of commitment to default decisions. See, for example, Adam and Grill (2017) for an analysis of sovereign default under Full Commitment, where only one period of commitment would be needed to recover the FC solution. Our results also hold if we add endogenous capital utilisation of the, e.g., Greenwood et al. (2000) form.

k_{t+1} is capital chosen at t and productive at $t + 1$. Finally, τ_t^l is a proportional labour tax and τ_t^k is a proportional capital tax paid on income net of depreciation.

The production function is Cobb-Douglas and constant returns to scale in capital and labour, $y_t = z_t k_t^\alpha l_t^{1-\alpha}$, where $0 \leq \alpha \leq 1$ controls the capital share in production.¹⁶ z_t is a neutral productivity process, which follows an exogenous sequence $\{z_t\}_{t=0}^\infty$ which has the Markov property that z_t is sufficient to predict z_{t+1} . Firms are competitive, hence factor prices are equal to their marginal products: $w_t = (1 - \alpha)z_t k_t^\alpha l_t^{-\alpha}$ and $r_t = \alpha z_t k_t^{\alpha-1} l_t^{1-\alpha} - \delta$. The household's optimality conditions for labour supply, investment, and bond purchases are, respectively:

$$\frac{v'(l_t)}{u'(c_t)} = (1 - \tau_t^l) (1 - \alpha) z_t k_t^\alpha l_t^{-\alpha} \quad (15)$$

$$u'(c_t) = \beta u'(c_{t+1}) [1 + (\alpha z_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} - \delta) (1 - \tau_{t+1}^k)] \quad (16)$$

$$q_t u'(c_t) = \beta u'(c_{t+1}). \quad (17)$$

The resource constraint of the economy reads:

$$c_t + k_{t+1} - (1 - \delta) k_t + g_t = z_t k_t^\alpha l_t^{1-\alpha}. \quad (18)$$

The government's budget constraint is implied by the household's budget constraint and the resource constraint, and reads: $\tau_t^l w_t l_t + \tau_t^k r_t k_t + q_t b_{t+1} = g_t + b_t$. Replacing the factor prices using the firm's first order conditions and the bond price using (17) gives

$$[\alpha \tau_t^k + (1 - \alpha) \tau_t^l] z_t k_t^\alpha l_t^{1-\alpha} - \tau_t^k \delta k_t + \beta \frac{u'(c_{t+1})}{u'(c_t)} b_{t+1} = g_t + b_t, \quad (19)$$

where $b_{t+1} > 0$ represents government borrowing.

Chari and Kehoe (1999) formulate the FC problem as a “primal” problem, solving out for both prices and taxes, and directly choosing competitive equilibrium allocations, subject to a single “implementability constraint” and the sequence of resource constraints. Since we are explicitly interested in the role of partial commitment on policy instruments, we refrain for solving out for taxes and switching to the primal problem. Thus, after solving out for factor prices, the bond price, and output, we can summarise the model with equations (15), (16), (18), and (19). In the general notation of Section 1, we have $b_t = (k_t, b_t)$, $c_t = (c_t, l_t)$, $\tau_t = (\tau_t^k, \tau_t^l, g_t)$, and $z_t = z_t$. We have solved out for all prices, thus leaving p_t empty. Government debt is treated as a residual, determined as the level required to balance the government's budget constraint conditional on its choices of taxes and spending. The role of constraints on government debt is discussed extensively in Section 3.3.3.

3.1.1 FC and sources of time inconsistency

We impose an upper bound on initial capital taxation so that the government cannot finance all spending with an initial tax which is non-distortionary. For symmetry with the LTC game, we impose this upper bound in all periods, so that $\tau_t^k \leq \bar{\tau}^k$ for all t .

¹⁶The results extend to generic neoclassical constant-returns-to-scale production functions.

The problem of a government with Full Commitment is then to choose paths for taxes and government spending, $\{\tau_t^k, \tau_t^l, g_t\}_{t=0}^\infty$ to maximise (13) subject to the competitive equilibrium constraints above and $\tau_t^k \leq \bar{\tau}^k$.

The FC policy typically features a capital tax that converges to zero, which is a version of the classic results of Chamley (1986) and Judd (1985).¹⁷ The model has problematic variables c_t , l_t , and τ_t^k , since these variables enter one period ahead in the forward-looking constraints (16) and (17) at date t . Hence, we have $N = 1$, as variables one period ahead appear in the time- t competitive equilibrium constraints.

There are two main sources of time inconsistency of the FC policy in this model, both of which have been studied in the literature. First, the presence of future consumption in the bond Euler equation (17) implies that the government has an ex-post incentive to implement a policy that gives a lower interest rate (assuming a positive level of government debt) by increasing current consumption relative to past promises. This incentive is present in the seminal work of Lucas and Stokey (1983). Second, because of the presence of capital, the government has an ex-post incentive to tax capital income at a higher rate than promised, because this represents a form of lump-sum taxation from an ex-post perspective (i.e., after investment has taken place). Because of these two forces, future allocations appear in the implementability constraint of the FC government.

3.1.2 Assumption 3 in the benchmark model

The two sources of time inconsistency described above imply that expectations on allocations and policies far in the future pose binding constraints on current government choices. Consistent with this intuition, we now argue that no finite amount of commitment can support the Full Commitment solution in the benchmark model. That is, **Assumption 3** fails for any L in the absence of further restrictions. Intuitively, this means that policies to be implemented in the infinite future matter for current agents' decisions. Consider the LTC game in the model when governments have $L \geq 1$ periods of commitment. The time- t government inherits pre-committed policies $\tau_t^L = \{\tau_{t+s}^k, \tau_{t+s}^l, g_{t+s}\}_{s=0}^{L-1}$ and chooses $\tau_{t+L} = (\tau_{t+L}^k, \tau_{t+L}^l, g_{t+L})$. The state vector is $s_t = (k_t, b_t, z_t, \tau_t^L)$, and governments respect all equilibrium conditions (15), (16), (18), and (19), and $\tau_t^k \leq \bar{\tau}^k$ from time t onwards.

For **Assumption 3** to hold with $L = 1$, the predetermined policies along with the state variables must uniquely determine the problematic variables c_t , l_t , τ_t^k , and g_t . The two policy variables are determined by construction, but what about the endogenous variables c_t and l_t ? It turns out that these variables are not uniquely determined by any finite sequence of policies. One way to see this is simply to count equations and unknowns. After fixing $(\tau_t^k, \tau_t^l, g_t)$, the competitive equilibrium conditions at time t , equations (15), (16), (18), and (19), provide only four equations in six unknowns: $c_t, l_t, k_{t+1}, b_{t+1}, c_{t+1}$, and l_{t+1} . Thus, the values of c_t and l_t cannot be determined by these equations alone. To determine the values of these variables it is required to add more competitive equilibrium restrictions from future periods, and to consider a longer commitment horizon.

¹⁷Straub and Werning (2018) provide conditions under which the capital tax does not converge to zero. The characterization of the FC solution does not affect our results, which concern the ability of Limited-Time Commitment to sustain Full Commitment outcomes.

However, there is no way to do this in a way which “closes” the system, that is, without adding further unknowns. Considering $L = 2$ and adding the labour optimality condition, (15), at time $t + 1$ gives us one more equation in k_{t+1} , c_{t+1} , and l_{t+1} . This brings us to five equations in six unknowns, meaning that only one more appropriate equation would be required to potentially pin down these variables, and hence uniquely determine the problematic variables as a function of a finite sequence of policies.¹⁸

Adding the resource constraint, (18), from time $t + 1$ adds one more equation, but also the extra unknown k_{t+2} . Adding the government budget constraint, (19), from time $t + 1$ adds one more equation, but also the extra unknowns b_{t+2} and c_{t+2} . Adding the Euler equation, (16), from time $t + 1$ adds one more equation, but also the extra unknowns k_{t+2} , c_{t+2} , and l_{t+2} . The same logic holds for adding any competitive equilibrium restrictions from time $t + 2$, $t + 3$, and so on.

Accordingly, without enough equations to explicitly pin them down, the problematic variables must be determined by the whole future sequence of competitive equilibrium constraints and government policies. Intuitively, in this model households’ consumption, investment, and labour supply decisions depend on announced policies even arbitrarily far in the future through both classic income and substitution effects. Policymakers will always be tempted to exploit these effects to distort the economy in the presence of time inconsistency, no matter how long the delay imposed on them by partial commitment.

Thus, we now turn to the following question: what restrictions on the economic environment or on government choices would allow the model to sustain FC outcomes with a finite commitment horizon? We find two key conditions, each of which is sufficient for **Assumption 3**. The intuition in both cases is that these restrictions insulate the competitive equilibrium today from policy announcements made sufficiently far in the future. The first condition specialises the benchmark model to a model without capital. In so doing, this condition prevents agents from shifting resources across time, delinking the economy from future policy announcements. The second condition places limits on government debt, which insulate the economy from future policy announcements by endogenously restricting the set of feasible policies.¹⁹

3.2 Case 1: Equivalence without capital

The first specialisation of the benchmark model for which we can prove that LTC supports FC is the canonical Lucas and Stokey (1993) labour taxation model. This is a special case of the benchmark model without capital, where output is produced only with labour. As there is no possibility for the aggregate economy to shift resources across periods, current consumption and hours become independent of fiscal policies to be implemented in the distant future. Hence, this is a leading case of a restriction on the economic environment that makes short-term fiscal commitment quite powerful.

¹⁸Of course, since our equations are non-linear, having as many equations as unknowns is not sufficient to pin down unique values of the unknowns which solve the equations. There could be no, or multiple, solutions. However, this is not a problem for this exercise, which simply says that we do not even have the minimal requirement of an equal number of unknowns and equations.

¹⁹These conditions provide restrictions on the competitive equilibrium system and break the links between policies in the far future and current allocations. In the framework of Chari and Kehoe (1999), the first condition makes the resource constraint “static”, whereas the second condition invalidates the use of a single infinite-horizon implementability constraint in the FC problem.

3.2.1 Specialised model equations

Formally, we recover this special case by setting $\alpha = 0$ and $\delta = 1$.²⁰ With this specialisation, the production function is now simply $y_t = z_t l_t$. Since there is no capital in the model, the capital Euler equation, (16), is dropped from the list of competitive equilibrium constraints. The labour first order condition, (15), simplifies to

$$\frac{v'(l_t)}{u'(c_t)} = (1 - \tau_t^l) z_t, \quad (20)$$

and the resource constraint, (18), simplifies to

$$c_t + g_t = z_t l_t. \quad (21)$$

Finally, the government budget constraint, (19), becomes

$$\tau_t^l z_t l_t + \beta \frac{u'(c_{t+1})}{u'(c_t)} b_{t+1} = g_t + b_t. \quad (22)$$

This incorporates the household's bond Euler equation, (17), which is used to price government debt. The competitive equilibrium is now summarised by equations (20), (21), and (22). Accordingly, the only problematic variable is c_t , since c_{t+1} appears in the government budget constraint, (22), through its affect on the bond price, q_t . This is the only source of time inconsistency in this model, since there is no capital and hence there are no issues with capital taxation. Hence, we have $N = 1$.²¹ At $t = 0$, the FC government has an incentive to use the initial level of consumption to decrease the market value of outstanding initial debt b_0 , and hence reduce the distortions required to finance expenditure.

3.2.2 Checking Assumption 3

We now show that the FC equilibrium can be supported in the LTC game with $L = 1$ periods of commitment. In this game, the government inherits the pre-committed labour tax and government spending, $\tau_t^L = (\tau_t^l, g_t)$ as states, and then chooses (τ_{t+1}^l, g_{t+1}) . The overall state vector is $s_t = (b_t, z_t, \tau_t^l, g_t)$. To prove equivalence, we need to show that **Assumption 3**, holds in this model. In other words, we need to show that (i) if we fix s_t then we pin down the problematic variable c_t , and (ii) s_t and (τ_{t+1}^l, g_{t+1}) additionally pin down l_t and b_{t+1} . To prove part (i), combine the labour supply condition, (20), and resource constraint, (21) to form the single equation

$$v' \left(\frac{c_t + g_t}{z_t} \right) = u'(c_t) (1 - \tau_t^l) z_t. \quad (23)$$

²⁰Consistent with the general framework, we allow government spending to be a choice variable of the government, but it is straightforward to show that the results also hold in the formulation of the model of Lucas and Stokey (1983), with exogenous government spending.

²¹This is an example in which the constraints can be equivalently formulated in different ways, with different values of N . For instance, the implementability condition can be written as either $u'_t b_t = u'_t c_t - v'_t l_t + \beta u'_{t+1} b_{t+1}$ for $t = 0, 1, \dots$ or (by iterating forward on this equation) as $u'_t b_t = \sum_{s=t}^{\infty} \beta^{s-t} [u'_s c_s - v'_s l_s]$, also for $t = 0, 1, \dots$. The first gives $N = 1$ and the second gives $N = \infty$. We focus on the representation with $N = 1$.

Given a predetermined tax rate τ_t^l and level of spending g_t , this is one equation pinning down one unknown, namely c_t . The solution is unique under the assumed regularity conditions, since v' and u' are strictly increasing and decreasing functions respectively. Hence the only problematic variable is pinned down by the state and predetermined policies. For part (ii), c_t is already pinned down, and l_t can be recovered from the resource constraint (21). The policy choice (τ_{t+1}^l, g_{t+1}) pins down c_{t+1} by the same logic as in part (i), and hence the bond issuance b_{t+1} from the budget constraint (22).

Having shown that our key assumption holds, we have proven that the FC solution can be supported by the LTC game. The intuition is very simple. Consumption, the only problematic variable, is entirely pinned down by the intratemporal labour supply condition and resource constraint in this model, which is now static. Therefore consumption only depends on the contemporaneous policy choices, and with one period of commitment, the time- t government effectively chooses c_{t+1} . This converts the problematic variable into a standard choice variable, unalterable by future governments who disagree over its optimal value. In the benchmark model, we did not have enough equations to pin down the problematic variables using a finite sequence of policy instruments. By removing capital from the model, the resource constraint becomes static, and we can construct one equation, (23), which pins down the problematic variable, c_t . In summary, we have shown that the commitment requirements in the Lucas and Stokey (1983) model are far less strict than they initially appear. With one-period debt, the Full Commitment solution can be supported with a single period of commitment. The result holds for standard non-separable utility functions and for other forms of taxation.²²

3.2.3 Long-maturity bonds

In order to highlight the key driver of the required length of commitment L in this model, we now consider an extension with long-maturity bonds. The model is identical to the previous section, except that governments at t now issue a bond, b_{t+N} at price q_t , with maturity $N \geq 1$, which is repaid in period N .²³ The Euler equation for an N -period bond is $q_t u'(c_t) = \beta^N u'(c_{t+N})$, and the government's budget constraint is now

$$\tau_t^l z_t l_t + \beta^N \frac{u'(c_{t+N})}{u'(c_t)} b_{t+N} = g_t + b_t. \quad (24)$$

The remaining equations are the same as the model with one-period bonds, so the competitive equilibrium is summarised by (20), (21), and (24). The choice of the label N for maturity is not accidental: variables N periods ahead, specifically c_{t+N} , now appear

²²Consider non-separable utility functions of the form $u(c_t, l_t, g_t)$. (23) now becomes $-u_l \left(c_t, \frac{c_t + g_t}{z_t}, g_t \right) = u_c \left(c_t, \frac{c_t + g_t}{z_t}, g_t \right) (1 - \tau_t^l) z_t$. A sufficient condition is that the marginal rate of substitution (with l_t replaced with $(c_t + g_t)/z_t$) is either always strictly increasing or always strictly decreasing in c_t . This is true for standard non-separable preferences, such as CES, and GHH (Greenwood, Hercowitz and Huffman, 1988). Consider now the case of additional tax instruments. The key requirement for equivalence is simply that we can pin down c_t using (20) and (21). This continues to be true if the government uses other taxes, such as consumption taxation.

²³The analysis can easily be extended to multiple maturities, but we restrict ourselves to one bond for expositional simplicity. Moreover, in the case $N > 1$ we consider a long-bond with “no buy-back” as Faraglia et al. (2018): governments cannot repurchase bonds before maturity. This is for expositional purposes, and our results also apply in the case of buy-back.

in the equilibrium conditions. It is easy to prove, using the same logic as above, that N periods of commitment are now required to support the FC solution in this model. Thus, the degree of commitment necessary to achieve FC outcomes is tied to an economically meaningful feature of the model: the longer the maturity of debt, the higher the number of periods of commitment required. This result arises because in the presence of long-maturity bonds, allocations far in the future affect current bond prices.

A policy implication of this analysis is that real-world limits to the degree of commitment to fiscal policy may make short-term debt more desirable than long-term debt, other things being equal. Moreover, our analysis suggests that future work on lack of commitment in fiscal policy should devote more attention to long bonds, as in Debortoli et al. (2017), because lack of commitment appears unlikely to be a large concern in economies with one-period debt only, as long as there is a minimal degree of commitment.²⁴

3.3 Case 2: Equivalence with capital and balanced budgets

We next consider specialisations of the model which allow for investment in productive capital.²⁵ We assume that the government cannot issue debt, and must follow a balanced budget. That is, we require that $b_t = 0$ for all $t = 0, 1, \dots$ ²⁶ As we will show, this restriction effectively limits the incentives for governments to deviate from the FC path of fiscal policy. Hence, this is a leading case of a restriction on the fiscal constitution that allows FC outcomes to be achieved with a finite commitment horizon.

3.3.1 Specialised model equations

The model equations are all the same as in the benchmark model of Section 3.1, except for the government budget constraint. Without access to government debt, this constraint reduces to

$$[\alpha\tau_t^k + (1 - \alpha)\tau_t^l] z_t k_t^\alpha l_t^{1-\alpha} - \tau_t^k \delta k_t = g_t, \quad (25)$$

which says that current spending must be fully financed from current tax revenue. The competitive equilibrium is now described by equations (15), (16), (18), and (25). The problematic variables are still c_t , l_t , and τ_t^k , since these variables enter dated $t + 1$ in the capital Euler equation, (16). These capture the sources of time inconsistency in the model. Most directly, the government has an incentive to promise a low capital tax, τ_{t+1}^k , to encourage investment, and then increase the tax ex-post.

²⁴Lucas and Stokey (1983) argue that the FC outcome can be sustained even in the absence of commitment, through a suitable maturity choice, provided that the maturity structure is sufficiently rich. Debortoli et al. (2018) discuss limitations of this result.

²⁵Following the standard assumption in the literature, we assume a single period of time to build for capital. In Appendix B.3.2, we discuss the effects of a longer time to build on our results.

²⁶The importance of balanced-budget rules was first studied by Stockman (2001) under the assumption of FC. While Stockman (2001) allows for an arbitrary constant level of debt, we set this level to zero and explore the role of balanced-budget rules for the possibility of sustaining FC outcomes under LTC. Almost all existing papers on limited commitment in optimal policy study either debt or capital, but not both in the same model. The recent work by Azzimonti et al. (2016) constitutes a noticeable exception.

3.3.2 Checking Assumption 3

We now show that LTC sustains the FC equilibrium with $L = 1$ periods of commitment. In this game, the government inherits the pre-committed policies, $\tau_t^L = (\tau_t^k, \tau_t^l, g_t)$ as states, and then chooses $\tau_{t+1} = (\tau_{t+1}^k, \tau_{t+1}^l, g_{t+1})$. The overall state vector is $s_t = (b_t, z_t, \tau_t^L)$. To prove equivalence, we need to show that **Assumption 3** holds in this model. In other words, we need to show that (i) if we fix s_t then we pin down the problematic variables (c_t, l_t, τ_t^k) , and (ii) s_t and $(\tau_{t+1}^k, \tau_{t+1}^l, g_{t+1})$ additionally pin down the remaining variable k_{t+1} . To prove part (i), notice that, given the government's state, $(k_t, z_t, \tau_t^k, \tau_t^l, g_t)$, equations (15) and (25) form a system of two (non-linear) equations in two unknowns, (c_t, l_t) . If this system admits a unique solution, then we satisfy the first requirement. This is simple to prove for the separable preferences assumed in the baseline model: (25) pins down a unique value of l_t which balances the government budget, and then, for a given l_t , (15) pins down a unique c_t consistent with labour-market optimality since u' is strictly decreasing. Finally, the problematic policy variable τ_t^k is trivially pinned down as part of the policy state τ_t^L . For part (ii), given the values of (c_t, l_t) , k_{t+1} can be backed out from the resource constraint, (18).

Intuitively, one period of commitment directly removes the ability of the government to renege on promises about next period's capital taxes. However, in order to exactly sustain FC, the time- t government must not be able to affect the values of c_t and l_t either. Since the resource constraint is no longer static, as it was in the Lucas and Stokey (1983) model, the time- t government could, in principle, affect c_t by using $(\tau_{t+1}^k, \tau_{t+1}^l, g_{t+1})$ to affect investment. This suggests that no finite amount of commitment could ever sustain FC in this model, following the general argument from Section 3.1.2. However, the balanced budget assumption, (25), plays a crucial role in stopping the time- t government from being able to influence c_t and l_t . Studying (25), we see that if taxes and spending are fixed, then there is only a single value of l_t which balances the budget. Labour supply affects output and hence tax revenues, and so a decline in labour supply would reduce tax revenues, leading to a shortfall in the government's budget. Once this value of l_t is known, the government must also ensure that c_t takes the correct value to "induce" this level of labour supply, as given in the labour optimality condition, (15). Hence the government is forced to choose its future policies so as not to unbalance its current budget, which incidentally stops it from altering any problematic variables, allowing LTC to support the FC allocation.

Once again, in the general model we did not have enough equations to pin down the problematic variables using a finite sequence of policy instruments. By removing debt from the model, the government budget constraint becomes static, and we are left with two equations, (15) and (25), which pin down the two problematic variables, c_t and l_t . This allows us to show that the Chamley-Judd result can be sustained with a single period of commitment in models with balanced budget rules.

3.3.3 Policy implications: Building fiscal rules that promote commitment

The results above highlight how adding a single fiscal rule to the model, in this case a balanced budget rule, allowed the FC solution to be supported with a limited amount of commitment. Hence, introducing a balanced budget rule may improve welfare (if governments possess a limited amount of commitment) by reducing welfare losses associated

with a lack of commitment. We now address the more general question of how to design fiscal constitutions that allow limited commitment to sustain FC outcomes.

Firstly, what restrictions on government debt help sustain FC outcomes? As discussed above, the key feature allowing LTC to support FC was the balanced budget assumption. However, we can also identify looser fiscal restrictions that give the same result. First, assume instead that the government is allowed to raise resources via one period bonds, b_{t+1} , with equilibrium price q_t . Define $d_t = q_t b_{t+1}$ as the revenue raised through government debt at time t . Consider a rule, in the spirit of the US debt ceiling, which states that issued debt must equal $\bar{d}_t = d_t$ at time t . As long as the government cannot instantaneously change the debt limit, for example if it is set one period in advance by the previous government symmetrically with the other policy instruments, then again one period of commitment can support FC.²⁷ Second, we show in Appendix B.3.1 that if the government only has to balance its budget every M periods, then LTC can support FC if the government has M periods of commitment. Thus, the economically meaningful feature of the model which ties down the required length of commitment to support FC is in fact the length of time over which the budget must be balanced. This result may be relevant for the design of medium-run budget targets.

Secondly, what tax instruments help sustain FC outcomes? The second feature which allowed LTC to support FC was that labour l_t , affected tax revenue, and hence entered the government budget constraint. This required the government to set policy to achieve a given value of l_t in order to balance its budget. Can we draw more general insights on what type of tax instruments are useful to restrict governments' incentives to reoptimise? To investigate this, we extend the model to include a proportional consumption tax, τ_t^c , and a proportional wealth tax, τ_t^a (a tax on the *stock*, k_t), two instruments that are commonly used in the real world. These instruments lead to the following government budget constraint and leisure-consumption condition:

$$[\alpha\tau_t^k + (1 - \alpha)\tau_t^l] z_t k_t^\alpha l_t^{1-\alpha} + (\tau_t^a - \tau_t^k \delta) k_t + \tau_t^c c_t = g_t, \quad (26)$$

$$\frac{v'(l_t)}{u'(c_t)} = \frac{(1 - \tau_t^l)}{(1 + \tau_t^c)} (1 - \alpha) k_t^\alpha l_t^{-\alpha}. \quad (27)$$

Suppose there is one period of commitment to all tax rates and to government spending. As long as some form of income (τ_t^k or τ_t^l) or consumption (τ_t^c) is taxed, (26) and (27) continue to give two equations pinning down unique values of c_t and l_t . However, notice that if the only instrument available to the government was the wealth tax, τ_t^a , then the argument would break, because the budget constraint (26) no longer involves c_t or l_t , and the system no longer pins down unique values of the problematic variables consumption and labour. It is possible to show that with wealth taxes only, LTC could still sustain FC, but with a longer required commitment horizon, $L = 2$.²⁸ Thus, we find that income or consumption taxes help sustain the FC outcome with a smaller degree

²⁷Note that this rule is stricter than a debt limit, which would only specify an upper limit on debt, $d_t \leq \bar{d}_t$. To prove the result, modify the balanced budget rule to $[\alpha\tau_t^k + (1 - \alpha)\tau_t^l] (v_t k_t)^\alpha l_t^{1-\alpha} + d_t = g_t + b_t$ and repeat the steps in the original proof.

²⁸The argument uses a similar logic to the proof in Appendix B.2.2. With two periods of commitment and wealth taxes only, the predetermined values of τ_{t+1}^a and g_{t+1} force the government to induce a specific level of investment, k_{t+1} , to ensure balance in the $t + 1$ budget, which (by time- t market clearing) fixes the problematic variable c_t .

of commitment than wealth taxes. Intuitively, this is because the tax base of a wealth tax is predetermined in this model, and hence does not tie the government's hands with respect to the current values of the problematic variables.²⁹

In summary, our results show how a combination of (i) fiscal rules placing restrictions on government borrowing and (ii) tax revenue tied to income and consumption help promote commitment. These results may be useful in helping design fiscal rules, and provide a motivation for placing discipline on government budgets, even in the absence of other political-economy constraints.

3.3.4 The role of preferences

While we emphasised the role of physical environment and fiscal constitutions in sustaining FC outcomes, a natural question is whether our results depend on restrictions on preferences. We find that our equivalence result survives most types of non-separable preferences.³⁰ Preferences of the GHH (Greenwood, Hercowitz and Huffman, 1988) form do not satisfy the conditions of the original proof, but in Appendix B.2.1 we show that LTC sustains FC outcomes with $L = 2$ periods of commitment in this case.

Finally, in our two key specialisations (Cases 1 and 2), we showed that LTC could support FC if either the resource constraint, (18), or government budget constraint, (19), is converted into a static equation. In Appendix B.2.3, we present a special case showing that it is possible to support FC in the presence of time inconsistency if neither condition holds. This example features production with capital, no balanced-budget, and linear utility in consumption.

4 Numerical results

In this section we use numerical methods to study a specification of the benchmark model calibrated to the US economy. In this specialisation of the model, LTC does not support FC outcomes for any length of commitment, and we use this model as a laboratory to study LTC as a positive theory of fiscal policy. Our key finding is that, even in the absence of equivalence between LTC and FC, a short commitment horizon leads to substantial welfare gains relative to the absence of any fiscal commitment.

4.1 Model setup

We focus on the source of time inconsistency related to capital taxation, and abstract from government debt. In particular, we consider a specialisation of the model of Section 3.3, based on Klein et al. (2008). We assume that labour supply is inelastic and governments choose the level of government spending to be financed using only capital income

²⁹Relatedly, automatic stabilisers on the spending side, such as unemployment benefits linked to the level of employment, may also deliver similar restrictions on the allocation in the presence of balanced-budget constraints.

³⁰Consider non-separable utility functions of the form $u(c_t, l_t, g_t)$. The condition pinning down consumption now becomes $-u_l(c_t, l_t, g_t) = u_c(c_t, l_t, g_t) (1 - \tau_t^l) (1 - \alpha) z_t k_t^\alpha l_t^{-\alpha}$. A sufficient condition is that the marginal rate of substitution for given (l_t, g_t) is either always strictly increasing or always strictly decreasing in c_t . This is true, for example, with CES preferences in consumption and leisure.

taxes and subject to a balanced budget rule.³¹ The equations of the model correspond to (16), (18), and (25), with the restrictions that $z_t = 1$, $l_t = 1$, $b_t = 0$, and $\tau_t^l = 0$ for all t . In order to ensure that LTC cannot sustain the FC outcome, we restrict the government to only have commitment to future spending, and assume that taxes are set equal to the level required to balanced the government's budget.³²

The source of time inconsistency in this model is that government spending, which is valued by households, can only be financed with capital income taxation, which distorts the incentives to invest. Governments have the incentive to promise low public good provision ex-ante to encourage investment, and then raise capital taxes ex-post to fund higher government spending.³³

Details of the recursive formulation of the game are relegated to Appendix C, which also includes a description of a simple global algorithm to solve for smooth LTC equilibria based on projection with third-order Chebyshev polynomials. We also provide the first order conditions of the FC Ramsey plan as well as a characterisation of the smooth NC equilibrium as analysed by Klein et al. (2008). In order to obtain quantitative results on the effects of LTC in this model, we follow the calibration choices of Klein et al. (2008), with the difference that we set hours worked equal to an exogenous constant. We parameterise utility as $u(c_t) = \log(c_t)$ and $w(g_t) = \xi \log(g)$, and set $\beta = 0.96$, $\xi = 0.5$, $\alpha = 0.36$, and $\delta = 0.08$. Table E.1 in the appendix reports the parameter values. One period in the model corresponds to one year.

4.2 Steady-state comparison: FC, NC, and LTC

In Table 1 we compare steady-state allocations, policies and welfare when the government has FC, NC, and LTC with one, two, and three years of commitment.³⁴ Because the whole infinite sequence of future tax rates matters for the allocation at t in this model, the LTC equilibrium does not coincide with FC, and is characterised by higher taxes. Hence, as shown in the table, LTC taxes and public good provision in steady-state are

³¹Since labour taxes are now effectively lump sum, we set them to zero to keep the government's problem meaningful. Considering a single tax instrument and abstracting from debt also simplifies the computation of the model with LTC significantly. We also solved the model under the alternative assumption that the tax base is overall output, instead of only capital. In this case, consistent with the findings of Klein et al. (2008), we find that the FC policy is close to being time consistent. In a similar model with endogenous labour supply, Klein and Rios-Rull (2003) also assume one year of commitment to capital taxes, but no commitment to labour taxes, and compute linear approximations of optimal policy in presence of shocks. We consider a deterministic economy with endogenous government spending, abstract from labour taxes and focus on a global solution that allows us to characterise transitional dynamics after a constitutional reform.

³²We could equivalently assume that the government commits to a tax rate, and balanced budget determines the level of spending. If instead the government could commit to the future levels of both taxes and spending, it would pin down a single level of future capital consistent with balanced budget at $t + 1$, and LTC would support FC with $L = 2$ periods of commitment. For more details, see Appendix B.2.2. In Appendix C.3, we also endogenize labour supply, and use this extension to study the differential effects of committing to taxes or to spending when the tax base is not predetermined.

³³No upper bound on capital taxation needs to be imposed in this model, since balanced budgets mean that a capital levy in period 0 cannot be saved to reduce distortionary taxation in future periods.

³⁴The assumption of exogenous labour drives the difference between the statistics reported in our NC column and the corresponding column in Table 1 of Klein et al. (2008). We verified that our method delivers the same steady-state results as theirs in the case of NC and endogenous labour supply.

set at an intermediate level between FC and NC.³⁵ The table shows two key results.

Firstly, increasing the number of periods of commitment with LTC quickly brings the equilibrium closer to FC. Accordingly, taxes fall, and capital, consumption and output increase with the number of periods of commitment.³⁶ We compute the welfare losses relative to FC as the fraction of steady-state consumption that would make the representative household indifferent between living in these different economies and living in the FC economy. Going from FC to NC is equivalent to a drop in permanent consumption of 9.3%. Importantly, just three years of commitment brings the model remarkably close to FC, recovering two thirds of the welfare losses from NC.

Table 1: *Deterministic model: steady-state comparison*

Variable	FC	NC	LTC(1)	LTC(2)	LTC(3)
y	1	0.866	0.904	0.926	0.944
k/y	1.733	1.342	1.450	1.513	1.565
c/y	0.712	0.693	0.698	0.703	0.705
g/c	0.209	0.288	0.267	0.250	0.241
τ	0.674	0.790	0.764	0.737	0.724
welfare loss	–	0.093	0.058	0.044	0.031

Steady-state results for the baseline calibration of the model in Section 4. We consider five versions of the economy (FC, NC and LTC with one, two and three years of commitment) and report steady-state output, capital-output ratio, private consumption-output ratio, public consumption-private consumption ratio, tax rate and welfare loss, measured as the fraction of permanent consumption that would make the representative household indifferent between the economy considered and the FC economy. $LTC(x)$ refers to LTC with x years of commitment.

Secondly, we find that the largest welfare gains from limited commitment are from introducing the first period of commitment. Over a third of the welfare loss can be recovered by imposing a single year of commitment to fiscal policy. As extra periods of commitment are added, welfare continues to increase, although at a decreasing marginal rate: the marginal welfare gain from adding the third year of commitment is 37% of the gain from adding the first year of commitment. Thus, we find that the largest marginal welfare gains come from the ability to commit over short horizons, and that the marginal gains from longer commitment horizons are smaller. The reason is intuitive: adding one period of commitment directly addresses the time inconsistency stemming from the appearance of τ_{t+1}^k in the Euler equation, (16). The remaining time inconsistency arises because c_{t+1} also appears in the Euler equation through the intertemporal marginal rate of substitution, $\beta \frac{u'(c_{t+1})}{u'(c_t)}$, and hence affects investment. However, future policies, τ_{t+2}^k , τ_{t+3}^k , ..., have increasingly smaller effects on this equilibrium object, making commitment to them less valuable.

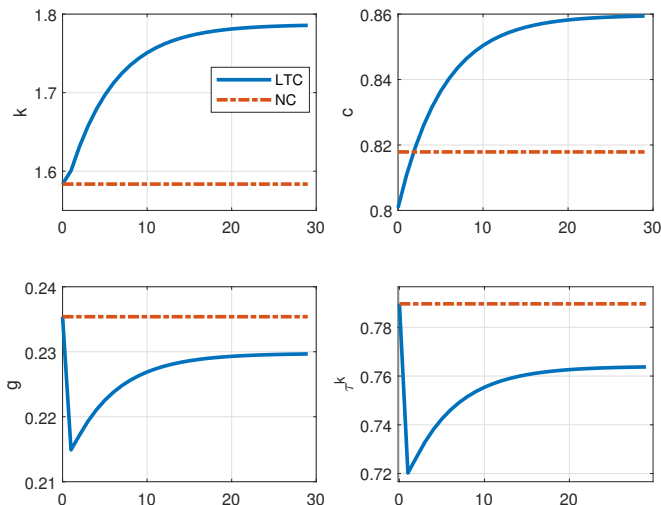
³⁵In all economies, taxes are substantially higher than in the US economy. This is because the only tax base in the model is capital income.

³⁶We find that most of the difference in the allocation between LTC and NC is driven by the fact that a larger fraction of output is devoted to investment under LTC. This extra investment is financed almost entirely by reducing the fraction of public spending, while the fraction of output devoted to private consumption is remarkably similar across all three economies.

4.3 Transitional dynamics: a constitutional reform

In this section, we investigate the effects of a constitutional reform that imposes fiscal commitment. Suppose that the government initially has no ability to commit, and the economy is in the NC steady-state. We consider an unexpected “constitutional reform” which imposes that governments must announce policies one year in advance, and always respect these plans. In order to evaluate the effects of this reform, we compute the whole transition path to the the LTC steady-state.

Figure 1: A “constitutional reform”: transition from NC to LTC



Deterministic transitional dynamics from NC steady-state (dashed-dotted red) to LTC steady-state with one year of commitment (solid blue). Top left panel: capital stock, k_t ; top right: private consumption, c_t ; bottom left: government spending, g_t ; bottom right: tax rate, τ_t^k .

Figure 1 shows the paths of capital, private consumption, public consumption and the tax rate. The solid red line shows the transition of interest, and the dashed blue line illustrates the counterfactual NC steady-state. Capital gradually increases in response to the lower taxes under LTC. Interestingly, the LTC government decides to overshoot the decrease in taxes (and hence spending) at the beginning of the transition in order to foster faster capital accumulation. The overall welfare benefit of this reform, accounting for the transition, is equal to 1.8% of permanent consumption.

4.4 LTC in the presence of shocks

How important is state-contingency in fiscal commitments? To study this question, we extend the baseline model to study LTC in a stochastic environment, with shocks to the valuation of public spending. We find that the value of state-contingency depends importantly on the degree of persistence of these shocks. In the presence of highly persistent shocks, calibrated to US data, contingent and non-contingent LTC lead to similar outcomes. Moreover, we find that the long-run gains from non-contingent fiscal

commitment outweigh the lost value of flexibility from a lack of commitment.

In order to perform this analysis, we add a shock to the utility households receive from the public good. Specifically, utility is now given by $w(g_t, \xi_t) \equiv \xi_t \log(g_t)$, where ξ_t is a random variable that follows a two-state Markov chain with realisations $\xi^L < \xi^H$ and transition matrix P_ξ . This shock determines the first-best marginal rate of substitution between private and public consumption.

We assume that the realisations of the shock are symmetric around the mean and the transition matrix is symmetric. This leaves two parameter values to choose in order to specify the stochastic process for ξ_t . We calibrate these parameters in order to match the standard deviation and autocorrelation coefficient of the (linearly-detrended) ratio between public spending (specifically, Government Consumption Expenditures) and private consumption (Non-durable Goods and Services), assuming the true model of fiscal policy determination is non-contingent LTC with one year of commitment. We use annual data from the US between 1960 and 2017. In the data, the autocorrelation of this ratio g_t/c_t is 0.915 and the standard deviation is 0.056. This leads to the values $\xi^L = 0.444$, $\xi^H = 0.556$ and a probability of staying in the same state equal to 0.974. We solve and simulate the model under both contingent and non-contingent LTC with $L = 1$ periods of commitment. In Table 2 we present results under FC, NC, non-contingent LTC (given in column $LTC(1)$), and contingent LTC (given in column $LTC_{\xi^L}(1)$).³⁷

The table shows that the long run averages of all variables and the welfare loss under both kinds of LTC lie in between the NC and FC values. The results are remarkably similar for contingent and non-contingent LTC. Thus, while non-contingent LTC is less flexible at responding to shocks than contingent LTC, and could be expected to deliver lower welfare, this does not appear to be a major issue for a shock calibrated to US business-cycle data. This reflects the high persistence of our estimated shocks: even though non-contingent LTC will always react to shocks with a lag compared to contingent LTC, the high persistence of the shocks limits the frequency at which this happens.

The table also displays the variances of key variables (in logs). While LTC induces intermediate outcomes between FC and NC in terms of the volatility of output and government spending, this is not true for the volatility of taxes: tax rates are less volatile under both types of LTC than under NC *or* FC. Overall, this analysis suggests that high volatility of fiscal variables should not be used to infer that a government has a low degree of commitment. In fact, higher commitment allows the government to be more timely in the provision of the public good in response to shocks.

In order to illustrate the equilibrium dynamics, in Figure 2 we plot the responses of the key variables to a shock. For illustrative purposes, we consider a transition from a long sequence of realisations $\xi_t = \xi^L$ to a long sequence of $\xi_t = \xi^H$.³⁸ The dynamics

³⁷Under non-contingent LTC, the government in power at t observes the state variables (k_t, g_t, ξ_t) and commits to a non-contingent level of public spending for the following period. Under contingent LTC, the government can additionally condition its plan on the future realisation of the shock. A comparison of Table 2 with the steady-states of the deterministic model in Table 1 suggests that these economies fluctuate around long-run outcomes that are very close to their deterministic steady-state counterparts.

³⁸Given the high persistence of the shock, the response to this path for the shock is fairly representative of the dynamics that arise in an actual simulation of the model. In Table 2, we show the dynamic properties of the ratio between public and private consumption under the four considered commitment regimes. This ratio is more volatile and more strongly correlated with the shock under FC than in the alternative regimes. Intuitively, the non-contingent LTC government induces a somewhat lower volatility

Table 2: *Stochastic model: key statistics*

Variable	<i>FC</i>	<i>NC</i>	<i>LTC(1)</i>	<i>LTC$_{\xi_t}(1)$</i>
$\mathbb{E}(y)$	1	0.859	0.900	0.899
$\mathbb{E}(k/y)$	1.759	1.343	1.460	1.456
$\mathbb{E}(c/y)$	0.711	0.693	0.699	0.698
$\mathbb{E}(g/c)$	0.209	0.287	0.264	0.265
$\mathbb{E}(\tau)$	0.677	0.788	0.759	0.760
welfare loss	–	0.104	0.069	0.070
$\sigma(y)$	0.010	0.027	0.022	0.023
$\sigma(i)/\sigma(y)$	4.145	4.305	4.137	4.325
$\sigma(c)/\sigma(y)$	1.838	1.257	1.263	1.256
$\sigma(g)/\sigma(y)$	6.104	1.543	1.732	1.888
$\sigma(\tau)$	0.061	0.045	0.040	0.044
$\sigma(g/c)$	0.078	0.062	0.056*	0.060
$\rho(g/c, g_{-1}/c_{-1})$	0.938	0.945	0.915*	0.931
$\rho(g/c, \xi)$	0.998	0.999	0.928	0.997

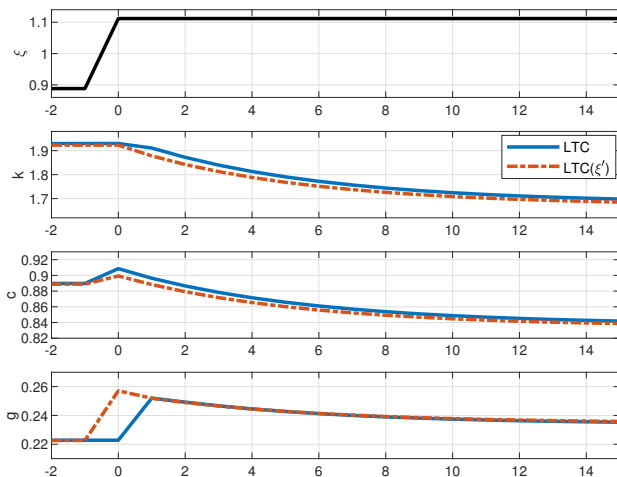
Statistics from the stochastic model. We consider four versions of the stochastic economy: FC, NC, non-contingent LTC (“*LTC(1)*”) and contingent LTC (“*LTC $_{\xi_t}(1)$* ”) (both with one year of commitment). We report means and volatilities of key variables from a simulation of 50,000 periods (volatilities are computed after taking logs). The welfare loss is measured as the fraction of permanent consumption that would make the representative household indifferent between the economy considered and the FC economy. We also report three relevant moments related to the ratio between public and private consumption. Stars denote calibration targets.

of capital, consumption and government spending under non-contingent LTC (solid blue line) and contingent LTC (dashed-dotted red line) are quite similar except for the period in which the shock changes value. In that period, government spending responds and increases instantaneously under contingent LTC, whereas it responds only with a lag under non-contingent LTC. After the change in the value of the shock, the increase in government spending calls for higher taxes, which induces a decline in investment and, eventually, in consumption.

To gain a further understanding of the value of state-contingency in public good provision, we also solve the model under the assumption that the shocks are i.i.d. over time. We maintain the same set of possible realisations for ξ_t . We plot a sample of the equilibrium dynamics obtained in a long simulation in Figure 3. With i.i.d. shocks, non-contingent LTC (solid blue line) does not respond to the realisations of ξ_t . This is because any fiscal-policy response to ξ_t would only be implemented at $t + 1$, but by that time, the distribution of the shock ξ_{t+1} , which would determine the desirability of such response, is independent of the state at t . As a consequence, also the level of capital and consumption are constant over time. In contrast, contingent LTC is highly responsive to the realisations of the shock, inducing higher volatility in the allocations and a closer

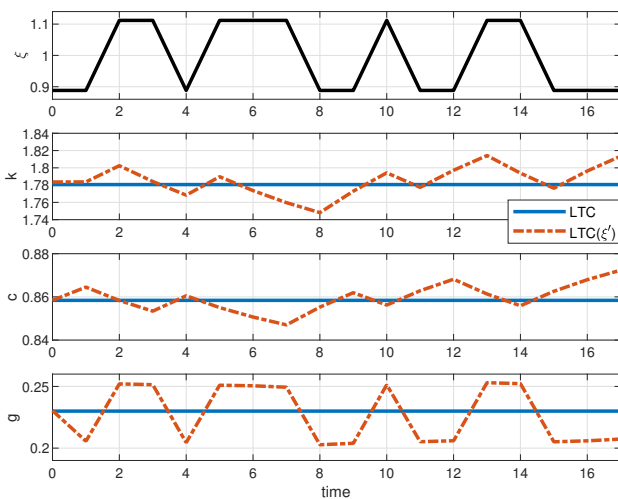
and correlation between g_t/c_t and the shock ξ_t . This result arises because of the one-period lag in the response of government spending to the shock. Consistent with this explanation, contingent LTC, which does not feature this lag, obtains a higher correlation.

Figure 2: *LTC with persistent shocks*



Transition from a long simulation with $\xi_t = \xi^L$ to a long simulation with $\xi_t = \xi^L$ in the stochastic model for non-contingent LTC (solid blue) and contingent LTC (dashed-dotted red). From top to bottom panel: (i) shock, ξ_t ; (ii) capital, k_t ; (iii) private consumption, c_t ; (iv) government spending, g_t .

Figure 3: *LTC with i.i.d. shocks*



Simulation with i.i.d. shocks for non-contingent LTC (solid blue) and contingent LTC (dashed-dotted red). Economy initialized at the ergodic mean of the endogenous state variables. From top to bottom panel: (i) shock, ξ_t ; (ii) capital, k_t ; (iii) private consumption, c_t ; (iv) government spending, g_t .

match between the desirability of the public good and its provision. Consistent with this result, we now find that with i.i.d. shocks contingent LTC leads to a welfare gain of approximately one percent of permanent consumption relative to non-contingent LTC. Hence, we conclude that state-contingency in fiscal plans is highly valuable in “turbulent” economies, while it appears to play a smaller role when the shocks are calibrated to US data on government spending.

5 Relation to existing approaches

In this paper we have proposed a new notion of optimal fiscal policy, with commitment to fiscal instruments over a finite future horizon. Our results bridge several existing strands of research. Thus, we conclude our analysis by highlighting our contributions in connection to three key approaches in the literature: (i) Generalized Euler Equations, (ii) Recursive formulations of FC, and (iii) Loose Commitment.

5.1 Generalised Euler Equations for LTC

The Generalized Euler Equation (GEE) approach of Klein et al. (2008) naturally extends to the LTC game. For concreteness, we refer to the deterministic version of the model used in Section 4. This model admits a tractable GEE, which we also use to cross-validate the accuracy of our numerical results. All derivations are given in Appendix C.

Under LTC with $L \geq 1$ periods of commitment, the optimal government spending choice at time t , $g_{t+L} = g(k_t, g_t, \dots, g_{t+L-1})$, can be characterised by the following GEE for g_{t+L} :

$$w'(g_{t+L}) = u'(c_{t+L}) \left(1 - \frac{\gamma_{t+L-1}}{k_{t+L}} \right) + \gamma_{t+L} u''(c_{t+L}) + \sum_{j=0}^{L-1} \beta^{j-L+1} \gamma_{t+j} u''(c_{t+j+1}) \left(\alpha k_{t+j+1}^{\alpha-1} + 1 - \delta - \frac{g_{t+j+1}}{k_{t+j+1}} \right) \frac{\partial c_{t+j+1}}{\partial g_{t+L}} \quad (28)$$

for $t = 0, 1, \dots$, where γ_{t+s} is the Lagrange multiplier on the Euler equation at time $t + s$, $c_{t+1} = c(k_{t+1}, g_{t+1}, \dots, g_{t+L})$, and $\frac{\partial c_{t+j+1}}{\partial g_{t+L}} \equiv c_{g_{t+L}}(k_{t+j+1}, g_{t+j+1}, \dots, g_{t+j+L})$. The interpretation of this GEE is as follows. The left-hand side is the marginal value of the public good when it is eventually provided at date $t + L$. The right-hand side represents the marginal cost of providing the good, in terms of private consumption at $t + L$, plus the marginal effect this promise has on the discounted *sequence* of Euler equations for investment between t and $t + L$. The GEE features derivatives of future policy functions, here future consumption, which are indicative of the time inconsistency problem.

It is instructive to compare this GEE to the one obtained under NC. In that case, the corresponding condition is simply $w'(g_{t+L}) = u'(c_{t+L}) + \gamma_{t+L}^{NC} u''(c_{t+L})$ for $t = 0, 1, \dots$, where γ_t^{NC} is the multiplier associated with the Euler equation, and we push the time index L periods forward for comparability with the LTC GEE. Since the NC government chooses spending contemporaneously, *after* investment has been chosen, it does not take into account how increased spending affects past investment, and hence the GEE is missing the discounted cost terms from the right hand side of the LTC GEE.

In Appendix C.2, we also derive the first order condition under Full Commitment, which shares a crucial feature of the LTC GEE, namely the internalisation of investment distortions, but importantly differs from the LTC GEE in the valuation of the marginal effect of government spending on future consumption. In the same appendix, we also discuss the optimality condition for capital accumulation under FC, LTC and NC.

5.2 Recursive formulations of Full Commitment

It has long been known that appropriately chosen “auxiliary” state variables, such as marginal utilities or values (Kydlund and Prescott, 1980, Abreu et al., 1990) or Lagrange multipliers (Marcet and Marimon, 2019), can be used to represent the FC solution in a recursive way. These methods do not “solve” the time inconsistency problem, given the set of fiscal instruments, and simply provide a way to recursively compute the time-inconsistent FC plan. In general, it is not clear to what extent the auxiliary state variables, which are not fiscal instruments, can be interpreted as partial government commitments. We consider an alternative approach. Under LTC, we explicitly specify the set of *policy instruments* over which we assume the government has commitment. These are then taken as state variables by future governments, and used to define a Markov-Perfect equilibrium in the style of the NC literature.

Consider the benchmark model of Section 3.1. The FC policy can be recursively calculated following Kydlund and Prescott (1980) by taking $m_t^1 \equiv u'(c_t)$ and $m_t^2 \equiv u'(c_t) [1 + (\alpha z_t k_t^{\alpha-1} 1_t^{1-\alpha} - \delta) (1 - \tau_t^k)]$ as auxiliary state variables. However, m_t^1 and m_t^2 are competitive equilibrium objects, and not fiscal instruments. In the general version of the model, we argued that the FC solution was not implementable with any finite commitment horizon. The two special cases of the benchmark model presented also admit recursive formulations of the FC solution with auxiliary state variables (in Sections 3.2 and 3.3, m_t^1 and m_t^2 are the required state variables, respectively). However, in both cases, our equivalence results imply that the auxiliary state variables can be pinned down with a single period of commitment to fiscal variables. Hence, our results clarify whether the existing recursive methods for FC can be interpreted as positive theories of fiscal policy with limited commitment, depending on whether our equivalence result applies in the models at hand.

5.3 Loose Commitment

Debortoli and Nunes (2013) study a version of the Lucas and Stokey (1983) economy from Section 3.2. They show that under Loose Commitment, an equilibrium exists where government debt optimally converges to a steady state value independent of initial conditions, even if commitment only lasts *on average* for one period. We showed that LTC with one period of commitment is enough to recover the FC solution in this economy. However, the FC solution features a non-stationary long-run level of government debt, which depends on the government’s initial debt position. Hence, perhaps surprisingly, one year of LTC and Loose Commitment which lasts *on average* one year do not lead to similar outcomes. This is because commitment to policies in the nearer future is more valuable than commitment to more distant policies when trying to overcome commitment problems.

Debortoli and Nunes (2010) study the capital and labour taxation problem with balanced budgets from Section 3.3. They find that under Loose Commitment the capital tax rate does not converge to zero, as it does under FC. We showed that LTC supports the FC solution with one period of commitment, and capital taxes *do* converge to zero under LTC. Hence LTC and Loose Commitment again deliver different results in this model. These results highlight that, once we depart from FC or NC, how we do so can have potentially large implications for optimal policy.³⁹

6 Conclusions

In this paper, we developed a theory of optimal fiscal policy, “Limited-Time Commitment”, in which governments are (i) constrained in their current policy stance by their predecessors’ plans and (ii) free to choose plans for a near-future finite horizon. This setup captures the observation that policy-makers act and communicate future plans as if they had a degree of commitment over fiscal policy in the near future. Moreover, changing fiscal policy quickly is typically costly in representative democracies.

Our key insight is that a limited degree of commitment to future fiscal policy often goes a long way in sustaining outcomes associated with high welfare. We first study a benchmark model that nests the seminal papers on optimal taxation with and without capital, and characterise the conditions under which governments achieve Full Commitment outcomes with a finite (often short) commitment horizon. We link the required degree of commitment to fundamentals of the models, such as physical environment and available fiscal instruments. We then consider a calibrated model of public good provision, and find that one third (two-thirds) of the welfare losses from lack of commitment can be recovered with just one year (three years) of commitment to fiscal policy. We believe that our results may provide guidance both for policymakers designing fiscal rules, and for future researchers choosing which assumptions on government commitment are more suitable for a given model and quantitative application.

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³⁹A further connection with the Loose Commitment approach is given by Debortoli and Nunes (2006, 2008). These papers consider models in which a planner is in power for $T > 1$ periods, but does not have power to commit to policies that have to be implemented during the successor’s term in office.

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