Semantics & Pragmatics Volume 12, Article 23, 2019
https://doi.org/10.3765/sp.12.23

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Goldstein, Simon. 2019. Free choice and homogeneity. *Semantics and Pragmatics* 12(23). 1–47. https://doi.org/10.3765/sp.12.23.

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Free choice and homogeneity*

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Abstract This paper develops a semantic solution to the puzzle of Free Choice permission. The paper begins with a battery of impossibility results showing that Free Choice is in tension with a variety of classical principles, including Disjunction Introduction and the Law of Excluded Middle. Most interestingly, Free Choice appears incompatible with a principle concerning the behavior of Free Choice under negation, Double Prohibition, which says that Mary can't have soup or salad implies Mary can't have soup and Mary can't have salad. Alonso-Ovalle 2006 and others have appealed to Double Prohibition to motivate pragmatic accounts of Free Choice. Aher 2012, Aloni 2018, and others have developed semantic accounts of Free Choice that also explain Double Prohibition.

This paper offers a new semantic analysis of Free Choice designed to handle the full range of impossibility results involved in Free Choice. The paper develops the hypothesis that Free Choice is a homogeneity effect. The claim possibly A or B is defined only when A and B are homogenous with respect to their modal status, either both possible or both impossible. Paired with a notion of entailment that is sensitive to definedness conditions, this theory validates Free Choice while retaining a wide variety of classical principles except for the transitivity of entailment. The homogeneity hypothesis is implemented in two different ways, homogeneous alternative semantics and homogeneous dynamic semantics, with interestingly different consequences.

Keywords: free choice, homogeneity, alternative semantics, dynamic semantics

1 Introduction

In the last few decades there has been a flurry of research into the apparent validity of the 'Free Choice' inference, where sentences like (1) and (3) appear to imply sentences like (2) and (4).[1]

(1) You may have soup or salad.

* Thanks to David Boylan, Simon Charlow, Melissa Fusco, Jeff King, Matt Mandelkern, Paolo Santorio, and Dan Waxman. Special thanks to Fabrizio Cariani and Jacopo Romoli.

1 See von Wright 1968 and Kamp 1974, Kamp 1978.

(2) You may have soup and you may have salad.

(3) Mary might be in New York or Los Angeles.

(4) Mary might be in New York and Mary might be in Los Angeles.

FREE CHOICE $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

Free Choice is a surprising inference from the perspective of a classical possible worlds semantics for modals, and a Boolean semantics for disjunction. On that theory, (1) simply says that there is some accessible world where either you have soup or you have salad. This does not imply that there is both an accessible world where you have soup and an accessible world where you have salad. For this reason, semantic analyses of Free Choice have all offered some kind of revision to this classical semantics.[2]

Free Choice also challenges a classical logic for possibility modals and disjunction, because it is incompatible with the validity of some other compelling inferences. Kamp 1974 showed there is serious tension between the validity of Free Choice, Disjunction Introduction, and the Upwards Monotonicity of possibility modals:

DISJUNCTION INTRODUCTION $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$

UPWARDS MONOTONICITY If $\mathsf{A} \models \mathsf{B}$, then $\Diamond\mathsf{A} \models \Diamond\mathsf{B}$

For suppose the entailment relation is transitive:

TRANSITIVITY If $\mathsf{A} \models \mathsf{B}$ and $\mathsf{B} \models \mathsf{C}$, then $\mathsf{A} \models \mathsf{C}$

With these assumptions, Free Choice implies the equivalence of any two possibility claims.[3]

EXPLOSION $\Diamond\mathsf{A} \models \Diamond\mathsf{B}$

Fact 1 (Kamp)**.** Transitivity, Disjunction Introduction, Upwards Monotonicity and Free Choice imply Explosion.

After all, $\Diamond\mathsf{A}$ implies $\Diamond(\mathsf{A} \vee \mathsf{B})$ by Disjunction Introduction and Upwards Monotonicity, which by Free Choice implies $\Diamond\mathsf{B}$. So by Transitivity $\Diamond\mathsf{A}$ implies $\Diamond\mathsf{B}$. For

2 For semantic accounts of free choice, see among others Asher & Bonevac 2005, Aher 2012, Aloni 2007, Aloni 2018, Barker 2010, Ciardelli et al. 2009, Charlow 2015, Fusco 2015, Geurts 2005, Roelofsen 2013, Simons 2005, Starr 2016, Willer 2017a, and Zimmermann 2000.

3 Throughout I assume classical introduction and elimination rules for conjunction.

this reason, all existing accounts of Free Choice have rejected the inference from $\Diamond A$ to $\Diamond(A \vee B)$, by giving up either Disjunction Introduction or Upwards Monotonicity.

A semantic account of Free Choice requires major revision to a classical logic and semantics for disjunction and modals. Any such account also faces serious problems stemming from negation. Alonso-Ovalle 2006 presented what may be the largest challenge for semantic accounts of Free Choice. Like scalar implicatures, Free Choice seems to disappear under negation. While (1) implies (2), the negation of (2) does not appear to imply the negation of (1). Quite the opposite: (5) actually appears to imply (6):

(5) You can't have soup or salad.

(6) You can't have soup and you can't have salad.

DOUBLE PROHIBITION $\neg\Diamond(A \vee B) \models \neg\Diamond A \wedge \neg\Diamond B$

While the classical semantics above does not validate Free Choice, it does validate Double Prohibition. If there is no accessible world where you have soup or salad, there cannot be an accessible world where you have soup, or one where you have salad, since you would have soup or salad in either such world.

There are several reasons to think Free Choice and Double Prohibition are incompatible. First, suppose we accept Transitivity and Contraposition.

CONTRAPOSITION If $A \models B$, then $\neg B \models \neg A$.

Then Free Choice and Double Prohibition again imply that any two possibility claims are equivalent.

Fact 2. Transitivity, Contraposition, Free Choice, and Double Prohibition imply Explosion.

Proof. By Free Choice, $\Diamond(A \vee B) \models \Diamond B$. So by Contraposition $\neg\Diamond B \models \neg\Diamond(A \vee B)$. By Double Prohibition, $\neg\Diamond(A \vee B) \models \neg\Diamond A$. So by Transitivity, $\neg\Diamond B \models \neg\Diamond A$ and hence by Contraposition again $\Diamond A \models \Diamond B$. □

For this reason, the few defenders of both Free Choice and Double Prohibition (Starr 2016, Willer 2017a) have given up the rule of Contraposition.

But even giving up Contraposition isn't enough to avoid further trouble. The joint acceptance of Free Choice and Double Prohibition also requires further non-classicality in the logic of disjunction. Consider the Law of Excluded Middle and Constructive Dilemma:

LEM $\models A \vee \neg A$

CONSTRUCTIVE DILEMMA If $A \models C$ and $B \models D$, then $A \vee B \models C \vee D$

These two principles can be accepted independently of Disjunction Introduction. But they also appear incompatible with the validity of Free Choice and Double Prohibition.

For suppose we again accept that entailment is transitive. Then Free Choice, Double Prohibition and the above assumptions lead to an absurd result, which we can call the 'Interconnectedness of All Things':

IAT $\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$

IAT is bizarre, because it seems inconsistent with the intuitive idea that there can be two claims that differ with respect to their modal status. Unfortunately, it follows from the assumptions above:

Fact 3. Transitivity, Free Choice, Double Prohibition, Constructive Dilemma and LEM imply IAT.

Proof. By LEM, $\models \Diamond(A \vee B) \vee \neg\Diamond(A \vee B)$. By Free Choice, Double Prohibition, and Constructive Dilemma, $\Diamond(A \vee B) \vee \neg\Diamond(A \vee B) \models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$. So by Transitivity, $\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$. □

To see the problem informally, suppose IAT is false, so that there is a situation in which A is possible and B is not. It then seems that neither $\Diamond(A \vee B)$ nor $\neg\Diamond(A \vee B)$ can hold, since the former requires that both A and B are possible, while the latter requires that both are impossible.

Summing up, Free Choice looks like an intuitively plausible principle. But validating Free Choice semantically comes with a variety of costs. Previous attempts to validate Free Choice have given up a variety of classical principles, including Disjunction Introduction, Upwards Monotonicity, Contraposition, Double Prohibition, LEM, and Constructive Dilemma.

In the face of these concerns, there are three natural strategies to pursue. Some, like Alonso-Ovalle 2006, have accepted Double Prohibition but rejected the semantic validity of Free Choice, instead offering a pragmatic account of its validity.[4] Others, like Barker 2010, have offered the reverse diagnosis, validating Free Choice while offering a pragmatic account of Double Prohibition. Any such attempt, though, still gives some of the classical assumptions in Fact 1. Finally, a few recent papers (Aher 2012; Starr 2016; Willer 2017a; Aloni 2018) have offered new semantics for negation, possibility, and disjunction that validate both Free Choice and Double

4 For pragmatic accounts of Free Choice, see among others: Alonso-Ovalle 2006; Fox 2007; Franke 2011; Klinedinst 2007; Kratzer & Shimoyama 2002; Romoli & Santorio 2017; and Schulz 2005.

Prohibition simultaneously. Any such semantics requires some response to each of the three incompatibility results above.

In this paper, I develop a new semantics for Free Choice that validates Double Prohibition while avoiding all of the results above in a unified way. Every one of the classical principles above remains valid, except for one: the transitivity of entailment. To achieve this solution, the main theoretical insight is to introduce homogeneity effects (von Fintel 1997; Križ 2015a) into the semantics of Free Choice. The key idea is that disjunctive possibility claims are defined only when either both or neither of A and B are possible.

(7) Mary may have soup or salad.

TRUE iff Mary may have soup and Mary may have salad.
FALSE iff Mary can't have soup and Mary can't have salad.
UNDEFINED otherwise.

By relying on homogeneity in the right way, we can straightforwardly validate both Free Choice and Double Prohibition. Free Choice is valid because whenever $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true, either both or neither of A and B must be possible. If neither were possible, then $\Diamond(\mathsf{A} \vee \mathsf{B})$ would be false; so both must be true. Double Prohibition is valid for the same reason. In order for $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ to be true, either both or neither of A and B must be possible. If both were possible, $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ would be false; so both must be impossible.

Once we incorporate homogeneity effects into our semantics, we can also introduce a definition of entailment that is sensitive to definedness: Strawson entailment. Here, an argument is valid only if it is truth preserving in contexts where the conclusion is defined. Crucially, this notion of entailment allows us to preserve all of the classical principles above, except the transitivity of entailment. To foreshadow, $\Diamond\mathsf{A}$ implies $\Diamond(\mathsf{A} \vee \mathsf{B})$, because to evaluate this inference we hold fixed the homogeneity of the possibility of A and B. For the same reason, $\Diamond(\mathsf{A} \vee \mathsf{B})$ implies $\Diamond\mathsf{B}$. But these inferences together don't require that $\Diamond\mathsf{A}$ implies $\Diamond\mathsf{B}$, because this last inference has no homogeneity effect. The failure of Transitivity responds in a similar way to the incompatibility results regarding Double Prohibition.

On first glance, one might balk at giving up Transitivity in the face of these incompatibility results. One thing worth observing here is that the intransitivity of Strawson validity is not an artifact of the semantics pursued here. For example, if Strawson validity is adopted to characterize entailment in a language with presuppositions, transitivity can fail in garden variety inferences involving definite descriptions, Strawson's original application. Consider:

(8) The King of France is either bald or not bald

This is Strawson valid, and Strawson implies (9):

(9) There is a unique x that is King of France and x is either bald or not bald.

However, (9) is not itself Strawson valid.[5]

The last main feature of the proposal below is its generality. Homogeneity can be incorporated into very different semantic frameworks, with quite similar results. In particular, this paper develops two new semantics: homogeneous alternative semantics, and homogeneous dynamic semantics. Each theory tells a different story about the compositional origins of homogeneity. In homogeneous alternative semantics, the effect is contributed by the semantics of possibility modals. In homogeneous dynamic semantics, the effect is contributed by the meaning of disjunction. The upshot is that homogeneity effects offer an attractive tool for a wide variety of defenders of the semantic validity of Free Choice: many such theories can go on to validate Double Prohibition and preserve much of classical logic by using a single idea.

In §10, we critically compare the accounts developed here with extant pragmatic and semantic accounts of Free Choice. The pragmatic accounts in Chemla 2008 and Fox 2007 differ from the accounts here in at least the treatment of Free Choice when disjunction takes wide scope to possibility modals and the treatment of duals to $\Diamond$ and $\vee$. In addition, we'll review recent experimental work in Romoli & Santorio 2019 and Tieu et al. 2019 that seems to favor the present account. The semantic accounts in Aher 2012 and Aloni 2018 differ from our own by departing further from classical logic, giving up instances of the Law of Excluded Middle, Law of Non-Contradiction, and Explosion. (Along the way, we'll also consider the accounts in Starr 2016 and Willer 2017a.)

2 Alternative semantics

In the first half of this paper, we validate Free Choice and Double Prohibition with a new theory, *homogeneous alternative semantics*, that incorporates homogeneity effects into alternative semantics. Let's start by reviewing how alternative semantics can be used to explain Free Choice, as in Simons 2005 and Aloni 2007.[6] We then enrich alternative semantics with homogeneity effects.

Throughout our discussion of alternative semantics, we'll interpret a simple modal propositional language.

Definition 1. $\mathsf{L} = \mathsf{p} \mid \neg\mathsf{L} \mid \mathsf{L} \wedge \mathsf{L} \mid \mathsf{L} \vee \mathsf{L} \mid \Diamond\mathsf{L}$

5 There is much prior work on failures of Transitivity, although generally not in the context of Strawson entailment. See Smiley (1958), Tennant (1992, 1994), Ripley (2013, 2015).

6 For a helpful overview of alternative semantics, see Rooth 2016.

The one interesting feature of this language is that it contains a special possibility modal $\Diamonddot$, whose interpretation is parametric to a choice of possibility operator $\Diamond$ in the metalanguage. This contributes generality to the discussion below.

The crucial idea we'll rely on from alternative semantics is that a disjunction $\mathsf{A} \vee \mathsf{B}$ presents both of A and B as alternatives. To implement this idea, we let the meaning of the disjunction $\mathsf{A} \vee \mathsf{B}$ be a set of propositions rather than a set of worlds. In particular, the meaning of $\mathsf{A} \vee \mathsf{B}$ is the set containing the proposition that A is true, and the proposition that B is true.

For this proposal to work, though, we also need a story about the rest of our language. Here we'll follow traditional formulations of alternative semantics and generalize to the worst case, letting all sentences denote sets of propositions rather than sets of worlds. We assume that atomic sentences denote the singleton of the set of worlds where the sentence is true. $[\![\mathsf{A} \wedge \mathsf{B}]\!]$ performs intersection pointwise across the alternatives from $[\![\mathsf{A}]\!]$ and $[\![\mathsf{B}]\!]$.

Both $\Diamonddot$ and $\neg$ are operators that take a set of alternatives and flatten them, producing a singleton set. $[\![\neg\mathsf{A}]\!]$ returns the singleton of the claim that no alternative in $[\![\mathsf{A}]\!]$ is true. To validate Free Choice, we let possibility modals universally quantify over the alternatives in their scope. We can theorize about this quantification in some generality by deriving the meaning of possibility modals ($\Diamonddot$) from an underlying operator $\Diamond$—the 'proto-possibility' operator—which maps a proposition to a new proposition. $\Diamond$ occurs in the meta language rather than the object language, and simply denotes some function or other from propositions to propositions. For example, we might assume that $\Diamond$ is a usual Kripkean modal operator, which existentially quantifies over accessible worlds (Kripke 1963; Kratzer 2012). Then we can let $[\![\Diamonddot\mathsf{A}]\!]$ return the singleton of the claim that $\Diamond$ can truly apply to every alternative in $[\![\mathsf{A}]\!]$. Summarizing, we reach the following first pass theory:

Definition 2.

i. $[\![\mathsf{p}]\!] = \{\{w \mid w(\mathsf{p}) = 1\}\}$

ii. $[\![\neg\mathsf{A}]\!] = \{W - \bigcup[\![\mathsf{A}]\!]\}$

iii. $[\![\mathsf{A} \wedge \mathsf{B}]\!] = \{\mathbf{A} \cap \mathbf{B} \mid \mathbf{A} \in [\![\mathsf{A}]\!], \mathbf{B} \in [\![\mathsf{B}]\!]\}$

iv. $[\![\mathsf{A} \vee \mathsf{B}]\!] = [\![\mathsf{A}]\!] \cup [\![\mathsf{B}]\!]$

v. $[\![\Diamonddot\mathsf{A}]\!] = \{\bigcap\{\Diamond\mathbf{A} \mid \mathbf{A} \in [\![\mathsf{A}]\!]\}\}$

Suppose the set of propositions in $[\![\mathsf{A}]\!]$ is $\{\mathbf{A}_1, ..., \mathbf{A}_n\}$, denoted by the sentences $\mathsf{A}_1, ..., \mathsf{A}_n$. Then $\Diamonddot\mathsf{A}$ is true just in case each of the possibility claims $\Diamond\mathsf{A}_1, \ldots, \Diamond\mathsf{A}_n$ is true. In other words, the alternative sensitive possibility modal is a generalized conjunction of a series of proto-possibility claims, distributed over the prejacent's alternatives. To recycle one of our early examples, the truth-conditions of *Mary may have soup or salad* demands the truth of both: *Mary may have soup* and *Mary may have salad*.

To define entailment, we flatten the meaning of the relevant sentences. Arguments are valid iff the union of the conclusion is true whenever the union of all the premises are true.

Definition 3. $\mathsf{A}_1,\ldots,\mathsf{A}_n \models \mathsf{C}$ iff $\bigcap_{i\in[1,n]}(\bigcup[\![\mathsf{A}_i]\!]) \subseteq \bigcup[\![\mathsf{C}]\!]$

This proposal guarantees that disjunction behaves as classically as possible. Since entailment is only sensitive to the flattened form of a sentence, the alternative sensitive disjunction *or* must satisfy Disjunction Introduction, LEM, and Constructive Dilemma. In addition, the proposal yields the DeMorgan equivalence of $\mathsf{A}\vee\mathsf{B}$ and $\neg(\neg\mathsf{A}\wedge\neg\mathsf{B})$.

A further consequence of the above is that logical equivalence is less fine grained than equivalence of meaning. While $\mathsf{A}\vee\mathsf{B}$ and $\neg(\neg\mathsf{A}\wedge\neg\mathsf{B})$ are co-entailing, they do not have the same meaning. The disjunction, but not the negated conjunction, denotes a set of alternatives. For this reason, our logic for possibility modals is hyperintensional in the sense that substituting logical equivalents in modal prejacents does not guarantee equivalence of the resulting possibility claims. Possibility modals with disjunctive prejacents go in for Free Choice, while modals with negated conjunctions in the prejacent do not.[7]

Now let's turn to the principles in §1. To consider Free Choice, note that we have the following identity, regardless of what $\Diamond$ means:

$$[\![\Diamond(\mathsf{A}\vee\mathsf{B})]\!] = [\![\Diamond\mathsf{A}]\!] \cap [\![\Diamond\mathsf{B}]\!]$$

This guarantees that Free Choice is valid. In addition, we already saw that Disjunction Introduction is valid. Furthermore, since the consequence relation simply involves preservation of truth, it is transitive. So to avoid the problems from Fact 1, this semantics gives up Upwards Monotonicity. In particular, although A implies $\mathsf{A}\vee\mathsf{B}$, $\Diamond\mathsf{A}$ does not imply $\Diamond(\mathsf{A}\vee\mathsf{B})$. The reason for this is that in the inference from A to $\mathsf{A}\vee\mathsf{B}$, only the single alternative which combines A and B worlds is relevant, while in the inference from $\Diamond\mathsf{A}$ to $\Diamond(\mathsf{A}\vee\mathsf{B})$ the operator $\Diamond$ can access each alternative A and B. (Below we develop a family of theories which validate Upwards Monotonicity.)

While Free Choice is valid in this semantics, Double Prohibition is not. For consider a scenario in which $\Diamond\mathsf{A}$ is true and $\Diamond\mathsf{B}$ is false. In this case we have $\neg\Diamond(\mathsf{A}\vee\mathsf{B})$, and so Double Prohibition fails. More generally, the validity of Double

7 Although see Willer 2017a for arguments that Free Choice occurs even in this case, and Ciardelli et al. 2018 for a response.

Prohibition corresponds to the trivializing condition that any possibility claim implies any other. Summarizing:

Observation 1.

i. For any operator $\Diamond$, $\Diamonddot(\mathsf{A} \vee \mathsf{B}) \models \Diamonddot\mathsf{A} \wedge \Diamonddot\mathsf{B}$.

ii. For any operator $\Diamond$, if $\Diamond\mathsf{A}; \neg\Diamond\mathsf{B}$ is consistent, then $\Diamonddot$ does not validate Double Prohibition.

One last remark: the alternatives approach does not require that possibility modals always go in for Free Choice. To avoid this result, we can introduce a closure operation which flattens alternatives.

Definition 4. $[\![!\mathsf{A}]\!] = [\![\neg\neg\mathsf{A}]\!] = \{\bigcup[\![\mathsf{A}]\!]\}$

We can then allow this closure operator occur in the prejacent, generating the form $\Diamonddot!(\mathsf{A} \vee \mathsf{B})$. This would account for some localized failures of Free Choice, such as (10) and (11).[8]

(10) Mary may have soup or salad, but I don't know which.

(11) Mary may have soup or salad, but I won't tell you which.

3 Homogeneity

In §2, we reviewed how alternative semantics handles Free Choice. We saw that in addition to giving up the Upwards Monotonicity of possibility modals, the semantics also invalidated Double Prohibition. Now we introduce our main idea: homogeneity effects.

Homogeneity effects have been used to explain apparent violations of excluded middle for both conditionals and plurals.[9] In von Fintel 1997, homogeneity is treated as a kind of semantic presupposition, involving undefinedness. In Križ 2015a homogeneity is also treated as a kind of undefinedness, but with a different projection pattern. For our results, we need not take a stand on this issue; but we need to rely on Strawson entailment (as in von Fintel 1997).

Here is the problem: observe first that predications involving plural definites, like (12), plausibly license inferences to universal claims like (13).

(12) The cherries in my yard are ripe.

8 See Zimmermann 2000 among others for discussion.

9 For discussion, see among others von Fintel 1997 and Križ 2015a.

(13) All the cherries in my yard are ripe.

If some but not all cherries are ripe, one would not be in a position to assert (12). Furthermore, plural definites plausibly accept the law of excluded middle. That is, the following sounds like a logical truth:

(14) Either the cherries in my yard are ripe or they (=the cherries in my yard) are not ripe.

If someone were to utter (14), they would sound just about as informative as if they had made a tautological statement. The problem is that, starting with (14) and exploiting entailments like the one from (12) to (13) as well as Constructive Dilemma, we can infer (15):

(15) Either all the cherries in my yard are ripe or all the cherries in my yard are not ripe.

That seems puzzling: did we just prove from logical truths and valid inferences that my yard cannot have some ripe cherries and some non-ripe ones? Of course, something must have gone wrong. The homogeneity view of plural definites explains what that is: first, plural definites are defined only when either the F's are either homogeneously G's or homogeneously not G's. If this condition is satisfied, their content is that all F's are G's. The sense in which (14) sounds tautological is that it cannot be false if its definedness condition is satisfied. Similarly, the sense in which (12) entails (13) is that if the definedness conditions of (12) is satisfied and (12) is true, (13) cannot fail to be true. But even if we can exploit these to deduce (15) we do not have license us to claim that (15) is valid: our justification for (14) and for the (12)-(13) entailment did not discharge the homogeneity condition.[10]

There are a variety of reasons to think that Free Choice involves some kind of homogeneity effect. The first observation here (paralleling Lobner 2000, Fodor 1970 on plural definites), is simply that free choice constructions generate truth value gaps.

(16) Mary may have soup or salad.

TRUE iff Mary may have soup and Mary may have salad.
FALSE iff Mary can't have soup and Mary can't have salad.
UNDEFINED otherwise.

Second, Križ 2015a suggests we can observe truth value gaps in plurals by considering the proper response to plural assertions when undefined. In such cases, the hearer cannot respond with *yes* or *no*, but must instead clarify with *well*:

10 See von Fintel 1997 for a similar analysis of conditionals.

(17) Context: *Half of the professors smiled.*
 A: The professors smiled.
 B: Well / #yes / #no, half of them.

We can observe a parallel effect in the case of Free Choice.

(18) Context: *Soup is permitted; salad is not.*
 A: Mary may have soup and she may have salad.
 B: Well / #yes / no, she can have soup (but she can't have salad).

(19) Context: *Soup is permitted; salad is not.*
 A: Mary may have soup or salad.
 B: Well / #yes / #no, she can have soup (but she can't have salad).

(18) is a conjunction of possibility claims. When one of those conjuncts is false, a speaker can felicitously deny the conjunction by saying *no*. By contrast, in the Free Choice claim (19) it is harder to use *no* (unless metalinguistically) to deny Mary's permission to have soup or salad in a scenario where one of these items is permitted and the other is not.

Finally, Tieu et al. 2019 has found further empirical evidence for a homogeneity effect with Free Choice in particular. Tieu et al. 2019 ran an experiment in which they gave participants a scenario in which Sue is only allowed to buy a boat, and asked them to consider:

(20) Sue is allowed to buy the boat or the car.
(20) Sue is not allowed to buy the boat or the car.

In addition, they gave participants a related scenario in which a character bought both a boat and a car, and asked them to consider:

(21) a. Sue bought the boat or the car.
 b. Sue didn't buy the boat or the car.

In each case, participants heard a puppet assert one of the relevant sentences, and then had to reward the puppet with a small, medium, or large strawberry depending on whether the puppet was 'totally wrong', 'in between, not totally right but not totally wrong', or 'totally right'. They found that in the Free Choice vignette, participants overwhelmingly rewarded the medium strawberry. By contrast, in the case of ordinary disjunction, participants gave a mixed pattern of response to (21-a), but overwhelmingly rejected (21-b). This has at least two upshots. First, in the case of Free Choice Tieu et al. 2019 found exactly the pattern of responses predicted by a homogeneity account. Second, Free Choice patterned quite differently than the kind of exclusivity implicature on display in (21-a). In this way, Tieu et al. 2019 offers both evidence of a homogeneity effect in the case of Free Choice, and a way of distinguishing the homogeneity and implicature based accounts of Free Choice (of which more in §10).

Križ 2015a argues that homogeneity in plural predication involves a distinctive pattern of projection, involving a special kind of supervaluation over undefined individuals which treats them alike. A natural next step would be to explore whether Free Choice exhibits a similar projection profile. However, this paper brackets that question. The problem is that it is unclear whether even homogeneity effects in conditionals pattern like plurals with respect to projection (see Santorio 2018 for discussion in the case of nonmonotonic quantifiers). So Free Choice might behave more like conditionals and less like plurals in this respect, in a way that doesn't affect the present investigation. At any rate, this paper uses some kind of undefinedness to solve the puzzle of Free Choice permission. This project could succeed even if the pattern below isn't quite the one for plural definites.

We now incorporate homogeneity effects into the semantics of Free Choice.

4 Homogeneous alternative semantics

We now develop a new semantics for Free Choice by integrating homogeneity effects into alternative semantics. The key idea is that $\Diamond(\mathsf{A} \vee \mathsf{B})$ is defined only when either all of the alternatives associated with $\mathsf{A} \vee \mathsf{B}$ are possible, or all of them are impossible. The alternatives must be homogeneous with respect to modal status.

In homogeneous alternative semantics, we depart from alternative semantics by introducing an undefined truth value #. Now instead of considering sets of worlds, we consider total functions from worlds to $\{0,\#,1\}$. The meaning of a sentence is now a set of such total functions. Throughout, we'll assume that the undefined value # obeys a weak Kleene logic in the metalanguage. So in our semantic clauses we lift the usual Boolean operators over sets of worlds into Boolean operators over total functions from worlds to $\{0,\#,1\}$. We suppose that whenever a function maps a world to #, any Boolean combination of that function with another function also maps the world to #.[11]

Definition 5. Where p and q are total functions from worlds to $\{0,\#,1\}$:

i. $p - q = \lambda w : p(w) \neq \# \neq q(w)\ .\ p(w) = 1$ and $q(w) \neq 1$

ii. $p \cap q = \lambda w : p(w) \neq \# \neq q(w)\ .\ p(w) = 1$ and $q(w) = 1$

11 '$\lambda x : f(x).\ g(x)$' denotes the smallest function mapping any x which satisfies f to 1 or 0, depending on whether x is g, and which maps any other x to #.

iii. $p \cup q = \lambda w : p(w) \neq \# \neq q(w) \,.\, p(w) = 1$ or $q(w) = 1$

Now we can retain basically our semantics from before for the fragment not containing $\Diamond$, except in letting our meanings be defined relative to these more complex Boolean operators.[12] We can implement our theory of possibility modals by injecting them with definedness conditions. $\Diamond\mathsf{A}$ is defined only if either all alternatives in $[\![\mathsf{A}]\!]$ are possible, or all are impossible.

Definition 6 (Homogeneous alternative semantics)**.** Let $[\![\cdot]\!]$ map sentences to sets of total functions from worlds to $\{0, \#, 1\}$:

i. $[\![\mathsf{p}]\!] = \{\lambda w \,.\, w(\mathsf{p}) = 1\}$

ii. $[\![\neg\mathsf{A}]\!] = \{W - \bigcup[\![\mathsf{A}]\!]\}$

iii. $[\![\mathsf{A} \wedge \mathsf{B}]\!] = \{\mathbf{A} \cap \mathbf{B} \mid \mathbf{A} \in [\![\mathsf{A}]\!], \mathbf{B} \in [\![\mathsf{B}]\!]\}$

iv. $[\![\mathsf{A} \vee \mathsf{B}]\!] = [\![\mathsf{A}]\!] \cup [\![\mathsf{B}]\!]$

v. $[\![\Diamond\mathsf{A}]\!] = \{\lambda w : \exists v \in \{0,1\}\ \forall \mathbf{A} \in [\![\mathsf{A}]\!]\ \Diamond\mathbf{A}(w) = v \,.\, \forall \mathbf{A} \in [\![\mathsf{A}]\!]\ \Diamond\mathbf{A}(w) = 1\}$

Finally, to get our desired predictions, we need a definition of consequence. One leading candidate for languages involving definedness conditions is Strawson-validity (Strawson 1952, von Fintel 1997, 1999, 2001). According to this notion, an argument is valid just in case the conclusion is true whenever the conclusion is defined and the premises are true. As in §2, we also assume that entailment is sensitive to the closed forms of sentences.

Definition 7. $\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{C}$ iff $\bigcup[\![\mathsf{C}]\!](w) = 1$ if:

i. $\forall i \in [1, n]\ \bigcup[\![\mathsf{A}_i]\!](w) = 1$

ii. $\bigcup[\![\mathsf{C}]\!](w) \in \{0, 1\}$

We now have all of the tools necessary in order to solve the problems from §1. In the next section we turn to this task.

5 Results

With all of our assumptions in place, let's turn to the principles from §1. First, the semantics validates Free Choice.

12 The precise projection patterns above aren't necessary for our results, which would go through just the same with Strong Kleene projection, or with the connectives in Heim 1992.

Observation 2. $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

Whenever $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true, it is defined. If the sentence is defined, either $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$ are both true, or they are both false. But the truth conditions for $\Diamond(\mathsf{A} \vee \mathsf{B})$ require that at least one of $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$ is true; so together this all requires that both are true.

Observation 2 follows from a more general property. Any inference that was valid in the alternative semantics from §2 remains valid when enriched with homogeneity effects. After all, homogeneous alternative semantics makes possibility sentences true in the same worlds as in §2, provided that the sentence is defined. Furthermore, it takes less for an inference to be valid in homogeneous alternative semantics, since we can restrict our attention to the worlds where the conclusion is defined.

Since homogeneous alternative semantics validates Free Choice, it must respond to Fact 1, which concerned the classical properties of disjunction. As in ordinary alternative semantics, Disjunction Introduction remains valid, and for the same reason as before.

Observation 3. $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$

Again, when we consider whether A implies $\mathsf{A} \vee \mathsf{B}$, we restrict attention to the closed form of $\mathsf{A} \vee \mathsf{B}$ (achieved by applying !), the worlds where one disjunct is true.

At this point, we have our first departure from traditional alternative semantics. Unlike the theory in §2, homogeneous alternative semantics validates Upwards Monotonicity. In particular, since Disjunction Introduction is valid, this theory validates the inference from $\Diamond\mathsf{A}$ to $\Diamond(\mathsf{A} \vee \mathsf{B})$, which some have thought is directly incompatible with Free Choice.[13]

Observation 4. $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$

The definedness of the conclusion requires that either both or neither of A and B are possible. Combined with the truth of the premise $\Diamond\mathsf{A}$, this means that $\Diamond\mathsf{B}$ is true, and so $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true.

One might worry that this last result is too strong, because it fails to distinguish the validity of $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ from the validity of Free Choice, which one might worry is more intuitively compelling. In fact, our semantics can make this distinction. For Free Choice is not merely Strawson valid. It is also flat out truth preserving, so that whenever $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true, $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$ is also true. So although both Free Choice and $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ are both Strawson valid, the marginal appeal of Free Choice could still be explained.

Homogeneous alternative semantics validates Free Choice, Disjunction Introduction, and Upwards Monotonicity. To escape Fact 1, this semantics gives up the

13 See Asher & Bonevac 2005 for discussion.

transitivity of entailment. In particular, while $\Diamonddot \mathsf{A}$ entails $\Diamonddot(\mathsf{A} \vee \mathsf{B})$, and this last entails $\Diamonddot \mathsf{B}$, the premise $\Diamonddot \mathsf{A}$ does not by itself entail $\Diamonddot \mathsf{B}$.

Observation 5. $\Diamonddot \mathsf{A} \not\models \Diamonddot \mathsf{B}$

Crucially, the first inference is only valid because we restrict attention to worlds where the conclusion $\Diamonddot(\mathsf{A} \vee \mathsf{B})$ is defined. But this sentence appears nowhere in the argument from $\Diamonddot \mathsf{A}$ to $\Diamonddot \mathsf{B}$, and so this latter argument remains invalid.

We've now diagnosed how homogeneous alternative semantics escapes the first incompatibility result from §1. Now let's turn to the other incompatibility results, involving Double Prohibition. Here, the first point to notice is that for any proto-possibility operator $\Diamond$, $\Diamonddot$ satisfies Double Prohibition.

Observation 6. $\neg\Diamonddot(\mathsf{A} \vee \mathsf{B}) \models \neg\Diamonddot \mathsf{A} \wedge \neg\Diamonddot \mathsf{B}$

When $\neg\Diamonddot(\mathsf{A} \vee \mathsf{B})$ is true, $\Diamond!(\mathsf{A} \vee \mathsf{B})$ is false. This implies that A and B are both impossible.

In §1, we saw that the joint validity of Free Choice and Double Prohibition raises the danger of Explosion: that any two possibility claims are equivalent. In particular, given Transitivity and Contraposition, Free Choice and Double Prohibition immediately imply Explosion. For by Free Choice, $\Diamonddot(\mathsf{A} \vee \mathsf{B})$ implies $\Diamonddot \mathsf{B}$; so by Contraposition $\neg\Diamonddot \mathsf{B}$ implies $\neg\Diamonddot(\mathsf{A} \vee \mathsf{B})$, which by Double Prohibition implies $\neg\Diamonddot \mathsf{A}$. The semantics above accepts each of the relevant instances of Contraposition.

Observation 7. $\neg\Diamonddot \mathsf{B} \models \neg\Diamonddot(\mathsf{A} \vee \mathsf{B})$.

Here again, the key is that the definedness of the conclusion requires A and B to be treated symmetrically.

To avoid Fact 2, homogeneous alternative semantics gives up Transitivity. While $\neg\Diamonddot \mathsf{B}$ implies $\neg\Diamonddot(\mathsf{A} \vee \mathsf{B})$ and $\neg\Diamonddot(\mathsf{A} \vee \mathsf{B})$ implies $\neg\Diamonddot \mathsf{A}$, we do not have the first premise $\neg\Diamonddot \mathsf{B}$ imply the last conclusion $\neg\Diamonddot \mathsf{A}$. This is exactly parallel to our treatment of Fact 1 above. When we remove the intermediate step $\neg\Diamonddot(\mathsf{A} \vee \mathsf{B})$, we are no longer entitled to restrict our attention to worlds where A and B are treated symmetrically.

Now let's turn to Fact 3. This showed that Free Choice and Double Prohibition seem to require further revisions to the logic of disjunction, giving up either the Law of Excluded Middle or Constructive Dilemma. In particular, the concern was that without giving up one of these principles, we would be forced to accept the absurd 'Interconnectedness' principle IAT, that $(\Diamonddot \mathsf{A} \wedge \Diamonddot \mathsf{B}) \vee (\neg\Diamonddot \mathsf{A} \wedge \neg\Diamonddot \mathsf{B})$ is a logical truth.

Our semantics offers the same solution to this problem as with Facts 1 and 2. First, LEM remains valid. In particular, we validate the potentially problematic instance involving Free Choice:

Observation 8. $\models \Diamond(\mathsf{A} \vee \mathsf{B}) \vee \neg\Diamond(\mathsf{A} \vee \mathsf{B})$

Here, one thing to consider is that Strawson entailment considerably lowers the standards on being a logical truth. A logical truth doesn't need to always be defined; it simply needs to be true whenever defined. The instance above is only defined when A and B are homogeneous, either both possible or both impossible. But when defined, the sentence is guaranteed to be true.[14]

Likewise, the semantics above validates Constructive Dilemma, including the instance used in the derivation of Fact 3. Here, the point is that applying Free Choice and Double Prohibition to the two disjuncts above, we reach IAT.

Observation 9. $\Diamond(\mathsf{A} \vee \mathsf{B}) \vee \neg\Diamond(\mathsf{A} \vee \mathsf{B}) \models (\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}) \vee (\neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B})$

Crucially, however, while the premise of the above is a logical truth, the conclusion is not. That is, we have another failure of Transitivity:

Observation 10. $\not\models (\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}) \vee (\neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B})$

The IAT principle does not contain a disjunction under the scope of a possibility modal. So the IAT principle does not trigger a homogeneity effect. So in order for it to be a validity, it must be true at every world. But it fails at exactly the worlds where the LEM instance above is undefined: where A and B differ in modal status.

In this section, we have seen that homogeneous alternative semantics provides an elegant treatment of the incompatibility results from §1. With each result, the semantics manages to validate all classical principles, with the exception of Transitivity. Most importantly, the semantics is able to deliver the joint validity of Free Choice and Double Prohibition.

Before continuing, one note about the status of homogeneity is in order. Above, we stayed agnostic about whether homogeneity is a presupposition, or instead a

14 Here we differ from Križ 2015a, who gives up the Law of Excluded Middle (see p. 47). Križ 2015a motivates the denial of LEM on empirical grounds, suggesting that the following is invalid:

(i) Adam either read the books or he didn't read them.

(ii) Well, what if he read half of the books?

Interestingly, the analogous argument in our case is unsuccessful:

(iii) Either you can have soup or salad, or you can't.

(iv) ?Well, what if I can have soup but I can't have salad?

So perhaps Križ 2015a's approach is right about plurals, but not Free Choice. On the other hand, nothing prevents Križ 2015a from accepting Strawson validity, where LEM would be valid.

different kind of undefinedness condition as in Križ 2015a. But in the case of Free Choice, there is at least some pressure to side with Križ 2015a against treating the phenomenon as presuppositional. In particular, the homogeneity effect does not seem to project from the antecedent of conditionals.[15] In particular, (22-a) does not seem to imply (22-b):

(22) a. If Mary can have cake or ice cream, we will have a blast.
b. Either Mary can have cake and Mary can have ice cream, or she can have neither.

To predict this, we could rely on a conditional that differs from our Boolean operations above by treating cases where the antecedent is undefined in the same way as cases where the antecedent is false. Under any such flattening operation, homogeneity effects are expected to disappear.[16] [17]

We've now developed a plausible strategy for validating Free Choice with homogeneity effects. But the strategy developed so far has one significant deficit. Free Choice occurs not only when possibility modals scope over disjunction, but also when disjunction at least appears to take wide scope to possibility modals.[18] For example, (23) seems to imply (24):

(23) Mary might be in New York or she might be in Los Angeles.

(24) Mary might be in New York and she might be in Los Angeles.

WIDE FREE CHOICE $\Diamond \mathsf{A} \vee \Diamond \mathsf{B} \models \Diamond \mathsf{A} \wedge \Diamond \mathsf{B}$

Homogeneous alternative semantics invalidates this inference, because Wide Free Choice never allows an alternative-denoting sentence to scope below a possibility

15 Thanks to several independent referees for help here.

16 The relevant flattening operation is defined as follows:

Definition 8. $[\![!\mathsf{A}]\!] = \lambda w.\ [\![\mathsf{A}]\!](w) = 1$

$[\![!\mathsf{A}]\!]$ and $[\![\mathsf{A}]\!]$ map the same worlds to true; but any world mapped to # by $[\![\mathsf{A}]\!]$ is mapped to 0 by $[\![!\mathsf{A}]\!]$. Then we can invalidate the inference from (22-a) to (22-b) by giving a semantics for the conditional which applies ! to the antecedent.

17 Another aspect of homogeneity is non-maximality: the truth of *the team members look happy in this picture* is consistent with there being an ordinarily grumpy team member who is not happy in the picture. For recent work, see Malamud 2012, Križ 2015b, Križ & Spector 2017. Interestingly, Free Choice effects do not exhibit non-maximality. Križ 2015b observes that non-maximality also disappears for small pluralities and explicit conjunctions.

18 See Zimmermann 2000 for a semantic account of Wide Free Choice, Simons 2005 for an argument that apparent instances of Wide Free Choice actually involve narrow scope disjunction (and Hoeks et al. 2018, Meyer & Sauerland 2016 for a response), and Willer 2017b for a pragmatic account of Wide Free Choice.

modal. For this reason, no homogeneity effects are triggered by Wide Free Choice in this system. So $\Diamond A \vee \Diamond B$ is always defined, and true at a world just in case there is some accessible A world or some accessible B world.

We now show homogeneity effects are a tool that a variety of theories of Free Choice can employ. In addition to alternative semantics, homogeneity effects can also be incorporated into dynamic semantics. In this case, we can let disjunction itself rather than possibility modals contribute the homogeneity effect, requiring that either both or neither disjuncts are possible. This is not merely a proof of concept. It validates Wide Free Choice.

6 Homogeneous dynamic semantics

In this section, we validate Free Choice and Double Prohibition within dynamic semantics. To do so, we'll rely on one simple idea: disjunctions require each disjunct to be possible.[19] That is, we need a semantics on which the following 'ignorance inference' is valid.[20]

MODAL DISJUNCTION $A \vee B \models \Diamond A \wedge \Diamond B$

There is an intimate connection between Modal Disjunction and the validity of Free Choice. To see why, suppose that we accept the T axiom, which says that anything true is possible:

T $A \models \Diamond A$

Given the duality of *might* and *must*, the T axiom says that *must* is strong, so that $\Box A$ implies A.[21]

Modal Disjunction follows from Free Choice, holding fixed the T axiom and the transitivity of entailment. After all, the T axiom implies that $A \vee B \models \Diamond(A \vee B)$. By Free Choice, this immediately implies $\Diamond A$ and $\Diamond B$.

We just saw that Free Choice implies Modal Disjunction, given a few assumptions. Similarly, given a few more assumptions Modal Disjunction implies Free Choice. For suppose we accept the 4 axiom, so that anything possibly possible is itself possible:

4 $\Diamond\Diamond A \models \Diamond A$

19 See Zimmermann 2000 and Geurts 2005 for other implementations of this idea.
20 See Cariani 2017.
21 For discussion with epistemic modals see Karttunen 1972; Kratzer 1991; von Fintel & Gillies 2010; and Lassiter 2016.

If we combine the 4 axiom with the Upwards Monotonicity of $\Diamond$, Modal Disjunction implies Free Choice.[22] By Modal Disjunction, $\mathsf{A} \vee \mathsf{B}$ implies $\Diamond\mathsf{B}$. So by Upwards Monotonicity, $\Diamond(\mathsf{A} \vee \mathsf{B})$ implies $\Diamond\Diamond\mathsf{B}$, which by the 4 axiom implies $\Diamond\mathsf{B}$. So, summing up, if our possibility operators satisfy the T and 4 axioms, then Free Choice and Modal Disjunction are equivalent.

Fact 4. Suppose Transitivity, Upwards Monotonicity, T and 4. Then Free Choice is valid iff Modal Disjunction is valid.

We've now given some justification for our motivating idea, that disjunctions require both disjuncts to be possible. Now we implement this idea within dynamic semantics (in particular, update semantics).[23] According to dynamic semantics, the meaning of a sentence is not its truth conditions. Rather, the meaning of a sentence is its ability to change the context in which it is said—its *context change potential*.

To give an update semantics, we need two things: a definition of information states (or contexts), and an interpretation function which assigns a context change potential to each sentence in our language. Veltman 1996 models an information state as a set of possible worlds. Then an interpretation function assigns every sentence a context change potential—a function from sets of worlds to sets of worlds. We also follow Heim 1992 by relying on definedness conditions on updating. However, for continuity with the above we model definedness with a special undefined state #, and let our context change potentials remain total functions on the set of states. Finally, we define an analogue of truth in a dynamic setting. Say that a state s supports a claim A just in case A has no effect on s, so that $s[\mathsf{A}] = s$. Support figures in our semantics for disjunction and in our definition of entailment.

Definition 9.

i. W is the set of worlds w assigning truth values to every atomic sentence p.

ii. The set of states S contains every subset of W and the undefined state #.

iii. $[\cdot]$ assigns every sentence A a total function from S to S.

iv. s supports A ($s \models \mathsf{A}$) iff $s[\mathsf{A}] = s$.

Analogous with the previous section, we can extend the usual Boolean operations over states to include sensitivity to the undefined state #. As before, we rely on a weak Kleene notion of undefinedness in the metalanguage.

22 See Zimmermann 2000 for a similar observation.

23 See Stalnaker 1973; Karttunen 1974; Heim 1982; Heim 1983; Veltman 1985; Groenendijk & Stokhof 1990; Groenendijk & Stokhof 1991; and many others.

Definition 10. Let $-$, $\cap$, and $\cup$ be weak Kleene extensions of the usual Boolean operations:

i. a. $\forall s, s' \neq \# : s - s' = \{w \mid w \in s, w \notin s'\}$
   b. $\forall s : \# - s = s - \# = \#$

ii. a. $\forall s, s' \neq \# : s \cap s' = \{w \mid w \in s, w \in s'\}$
   b. $\forall s : \# \cap s = s \cap \# = \#$

iii. a. $\forall s, s' \neq \# : s \cup s' = \{w \mid w \in s \text{ or } w \in s'\}$
   b. $\forall s : \# \cup s = s \cup \# = \#$

We now extend a usual dynamic semantics for the connectives so that the undefined state behaves properly. First, we assume that atomic sentences simply narrow down an information state to the worlds where they are true. Next, we can hold fixed the usual dynamic semantics for negation. On this proposal, updating an information state with $\neg\mathsf{A}$ returns exactly the worlds that would not survive updating with A. As in §2, our Boolean definition of set complement above leads to a smooth account of the definedness conditions for negation. $s[\neg\mathsf{A}]$ is just $s - s[\mathsf{A}]$, which by the definition of complement above returns # whenever s or $s[\mathsf{A}]$ are #. For simplicity, we work with a relatively static conception of conjunction, which intersects the result of updating with either A or B, so that $s[\mathsf{A} \wedge \mathsf{B}] = s[\mathsf{A}] \cap s[\mathsf{B}]$. Again, given the definition of intersection above, this is undefined whenever $s[\mathsf{A}]$ or $s[\mathsf{B}]$ are.

The operations we've considered so far are in the business of narrowing down the worlds in an information state. By contrast, modal sentences don't give us new information about what world we are in. Instead, they are tests, exploring properties of the current state. $\Diamond\mathsf{A}$ explores whether s can be consistently updated with A. If so, the initial state is unchanged by updating. Otherwise, the absurd state $\emptyset$ results. Again, however, we allow some updates in this system to be undefined. To get the right predictions about Free Choice, we assume that updating with $\Diamond\mathsf{A}$ is undefined when updating with A is undefined. To model this using our Boolean operations from above, we simply union our usual update with $s[\mathsf{A}]$, which in this setting merely imposes the extra definedness condition that $s[\mathsf{A}] \neq \#$. This gives us the meaning $s[\Diamond\mathsf{A}] = s \cap \{w \mid s[\mathsf{A}] \neq \emptyset\} \cup s[\mathsf{A}]$.[24]

All that is left is to give a semantics for disjunction, which implements the idea that disjunctions require each disjunct to be possible, so that Modal Disjunction and Free Choice are valid. Here is where we can again appeal to homogeneity. We can say that the disjunction $\mathsf{A} \vee \mathsf{B}$ is defined only when A and B share the same

24 Proof: $s[\mathsf{A}]$ is either #, $\emptyset$, or something else. In the first case, $s[\Diamond\mathsf{A}]$ is #. In the second case, $s[\Diamond\mathsf{A}] = \emptyset$. In the final case, $s[\Diamond\mathsf{A}] = s$.

modal status. Given our dynamic framework, this means that either the information state can be consistently updated with each of A and B, or it can be consistently updated with neither of them. Finally, when defined, $[\mathsf{A} \vee \mathsf{B}]$ then narrows down the state to worlds where one of A or B is true. This theory of disjunction incorporates homogeneity effects into the normal dynamic theory of disjunction, according to which $\mathsf{A} \vee \mathsf{B}$ unions together the result of updating with each of A and B. When A and B are non-modal, this results in classical truth conditions for disjunction, so that updating an information state with $\mathsf{A} \vee \mathsf{B}$ narrows down the state to the worlds where one of A and B are true. Now, however, we've enriched this idea with a homogeneity effect, so that $\mathsf{A} \vee \mathsf{B}$ is only defined at information states that contain both A worlds and B worlds, or contain neither A nor B worlds. Summarizing all we have said, we reach the following definition:

Definition 11 (Homogeneous dynamic semantics)**.**

i. $s[\mathsf{p}] = s \cap \{w \mid w(\mathsf{p}) = 1\}$

ii. $s[\neg \mathsf{A}] = s - s[\mathsf{A}]$

iii. $s[\mathsf{A} \wedge \mathsf{B}] = s[\mathsf{A}] \cap s[\mathsf{B}]$

iv. $s[\Diamond \mathsf{A}] = s \cap \{w \mid s[\mathsf{A}] \neq \emptyset\} \cup s[\mathsf{A}]$

v. $s[\mathsf{A} \vee \mathsf{B}] = \begin{cases} s[\mathsf{A}] \cup s[\mathsf{B}] & \text{if } s \models \Diamond \mathsf{A} \wedge \Diamond \mathsf{B} \text{ or } s \models \neg\Diamond \mathsf{A} \wedge \neg\Diamond \mathsf{B} \\ \# & \text{otherwise} \end{cases}$

With our semantics in place, we need a definition of entailment. Here, we rely on the notion of support ($\models$), and introduce a dynamic version of Strawson entailment, which says that an argument is valid just in case any state where the premises are supported and the conclusion is defined is a state which supports the conclusion.[25]

Definition 12. $\mathsf{A}_1, \ldots \mathsf{A}_n$ entail C ($\mathsf{A}_1, \ldots \mathsf{A}_n \models \mathsf{C}$) just in case $s \models \mathsf{C}$ whenever:

i. $s \models \mathsf{A}_1; \ldots; s \models \mathsf{A}_n$.

ii. $s[\mathsf{C}] \neq \#$.

With our theory in place, we can now turn to the puzzles from §1.

25 See Veltman 1996 and van Benthem 1996 139-41 for other options.

7 Results

Free Choice is valid in homogeneous dynamic semantics.

Observation 11. $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

Whenever s supports $\Diamond(\mathsf{A} \vee \mathsf{B})$, $s[\mathsf{A} \vee \mathsf{B}]$ is defined. This in turn requires that either s supports both $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$, or neither. But since s supports $\Diamond(\mathsf{A} \vee \mathsf{B})$, we also know that $s[\mathsf{A}] \cup s[\mathsf{B}]$ is non-empty. So s supports both $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$.

Since homogeneous dynamic semantics validates Free Choice, it must also respond to Fact 1, concerning the classical properties of disjunction. As in homogeneous alternative semantics, Disjunction Introduction is valid, although for a different reason.

Observation 12. $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$

Here, the proof relies on Strawson validity, which allows us to restrict our attention to information states in which $\mathsf{A} \vee \mathsf{B}$ is defined. When any such state supports A, it also supports $\mathsf{A} \vee \mathsf{B}$, since we can ignore the possibility that s contains no B worlds.

This system also shares another property with homogeneous alternative semantics: Upwards Monotonicity is valid. Of particular interest for Fact 1: this theory validates the instance of Upwards Monotonicity that uses Disjunction Introduction.

Observation 13. $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$

As in homogeneous alternative semantics, the definedness of the conclusion requires that either both or neither of A and B are possible. Combined with the support of the premise $\Diamond\mathsf{A}$, this means that $\Diamond\mathsf{B}$ is supported, and so $\Diamond(\mathsf{A} \vee \mathsf{B})$ is supported.

Again, as in homogeneous alternative semantics this theory also allows us to distinguish the validity of $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ from the validity of Free Choice, which is more compelling. Free Choice is not merely Strawson valid; it is also flat out support preserving.

Homogeneous dynamic semantics validates Free Choice, Disjunction Introduction, and Upwards Monotonicity. To escape Fact 1, this semantics also gives up the transitivity of entailment. While $\Diamond\mathsf{A}$ implies $\Diamond(\mathsf{A} \vee \mathsf{B})$, and this last implies $\Diamond\mathsf{B}$, the premise $\Diamond\mathsf{A}$ does not by itself imply $\Diamond\mathsf{B}$.

Observation 14. $\Diamond\mathsf{A} \not\models \Diamond\mathsf{B}$

Again, the first inference is only valid because we restrict attention to information states where the conclusion $\Diamond(\mathsf{A} \vee \mathsf{B})$ is defined. But this sentence appears nowhere in the argument from $\Diamond\mathsf{A}$ to $\Diamond\mathsf{B}$, and so this latter argument remains invalid.

Homogeneous dynamic semantics has so far had the same results as homogeneous alternative semantics. The two frameworks differ, however, in the features of bare disjunctions. Homogeneous dynamic semantics validates not only Free Choice but also Modal Disjunction:

Observation 15. $\mathsf{A} \vee \mathsf{B} \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$.

We saw with Fact 4 that Modal Disjunction is equivalent to Free Choice given Upwards Monotonicity, the T axiom, the 4 axiom, and Transitivity. All of the above hold in this system except Transitivity. But while Transitivity fails in general, the particular instances relevant to Fact 4 do not fail.

We've now diagnosed how homogeneous dynamic semantics escapes the first incompatibility result from §1. Now let's turn to the other incompatibility results, involving Double Prohibition. First, one of the signature properties of this system is that Double Prohibition is valid:

Observation 16. $\neg\Diamond(\mathsf{A} \vee \mathsf{B}) \models \neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}$

Whenever $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ is supported, it is also defined. When it is defined, either A and B are both possible or both impossible. So if $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ is supported, both A and B are impossible.

In §1, we saw that the joint validity of Free Choice and Double Prohibition increases the danger of Explosion, where any two possibility claims are equivalent. In particular, given Transitivity and Contraposition, Free Choice and Double Prohibition immediately imply Explosion. For by Free Choice, $\Diamond(\mathsf{A} \vee \mathsf{B})$ implies $\Diamond\mathsf{B}$; so by Contraposition $\neg\Diamond\mathsf{B}$ implies $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$, which by Double Prohibition implies $\neg\Diamond\mathsf{A}$.

A first natural reaction to this result is to note that homogeneous dynamic semantics gives up Contraposition in general. As in ordinary versions of update semantics, A implies $\Box\mathsf{A}$ (letting $\Box$ be the dual of $\Diamond$), but $\Diamond\mathsf{A}$ does not imply A. Nonetheless, this failure of Contraposition is not relevant to our second impossibility result. The semantics above accepts each of the relevant instances of Contraposition in that result.

Observation 17. $\neg\Diamond\mathsf{B} \models \neg\Diamond(\mathsf{A} \vee \mathsf{B})$

Here again, the key is that the definedness of the conclusion requires A and B to be treated symmetrically.

As in homogeneous alternative semantics, this theory avoids Fact 2 by giving up Transitivity. While $\neg\Diamond\mathsf{B}$ implies $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ and $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ implies $\neg\Diamond\mathsf{A}$, we do not have the first premise $\neg\Diamond\mathsf{B}$ imply the last conclusion $\neg\Diamond\mathsf{A}$. This is exactly parallel to our treatment of Fact 1 above. When we remove the intermediate step $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$, we are no longer entitled to restrict our attention to states where A and B are treated symmetrically.

Now let's turn to Fact 3, which placed Free Choice and Double Prohibition in tension with the Law of Excluded Middle and Constructive Dilemma. Again, the problem was that all together these principles implied IAT, that $(\Diamond \mathsf{A} \wedge \Diamond \mathsf{B}) \vee (\neg \Diamond \mathsf{A} \wedge \neg \Diamond \mathsf{B})$ is a logical truth.

Here, our semantics uses the same solution strategy as with Facts 1 and 2. First, LEM remains valid, including the following instance:

Observation 18. $\models \Diamond(\mathsf{A} \vee \mathsf{B}) \vee \neg \Diamond(\mathsf{A} \vee \mathsf{B})$

As in homogeneous alternative semantics, the key observation is that Strawson entailment requires much less of logical truths. Rather than support at all states, we only require the sentence to be supported at the states in which it is defined. The above state is only defined at states which either contain both A worlds and B worlds, or which contain neither kind of worlds. At any such state, the test imposed by one of the disjuncts is passed. Likewise, the semantics above validates Constructive Dilemma, including the instance used in the derivation of Fact 3. Here, the point is that applying Free Choice and Double Prohibition to the two disjuncts above, we reach IAT.

Observation 19. $\Diamond(\mathsf{A} \vee \mathsf{B}) \vee \neg \Diamond(\mathsf{A} \vee \mathsf{B}) \models (\Diamond \mathsf{A} \wedge \Diamond \mathsf{B}) \vee (\neg \Diamond \mathsf{A} \wedge \neg \Diamond \mathsf{B})$

Crucially, while the premise of the above is a logical truth, the conclusion is not. That is, we again have another failure of Transitivity:

Observation 20. $\not\models (\Diamond \mathsf{A} \wedge \Diamond \mathsf{B}) \vee (\neg \Diamond \mathsf{A} \wedge \neg \Diamond \mathsf{B})$

IAT fails spectacularly in this system: it is not only invalid, but inconsistent. That is: whenever it is defined, its negation is supported.

For the same reason, Observation 19 only holds vacuously, because there is no state where the conclusion is defined and the premise is true.

One might worry that homogeneous dynamic semantics rejects IAT in too strong a fashion. Although IAT should not be a validity, it should be consistent. In the next section, we generalize the system to account for other flavors of modals, like deontic modals. This generalization predicts that IAT is consistent.

Finally, the semantics above generalizes smoothly to nested disjunctions. The claim $\Diamond(\mathsf{A} \vee (\mathsf{B} \vee \mathsf{C}))$ is defined only if either all of A-C are possible or all are impossible, and when defined asserts that the former condition obtains.

In this section, we have seen that homogeneous dynamic semantics provides an elegant treatment of the logical incompatibility results from §1. With each result, the semantics manages to validate all classical principles, with the exception of

Transitivity. Most importantly, the semantics is able to deliver the joint validity of Free Choice and Double Prohibition. So far, however, the reader may have a concern. Homogeneous dynamic semantics seems tailor-made for epistemic versions of Free Choice. But how can it extend to Free Choice for other flavors of modality? In the next section we achieve this result.

8 Extension to other modals

So far, we've focused our discussion on epistemic modals. Here, we achieve similar results for other flavors of modals. The upshot is that we can treat modals in a truth conditional way and still reap the benefits of the above, provided that disjunctions themselves use a dynamic kind of possibility operator.

Above, we appealed to possibility operators in two separate ways. First, Free Choice involved embedding disjunction under some sort of possibility modal. Second, our semantics had disjunctions themselves require that each disjunct be possible. We'll now see that these two appeals to possibility modals can be treated separately. We can integrate a static semantics for deontic modals with our dynamic semantics for disjunction in a way that validates Free Choice for deontic modals.

We introduce a possible worlds modal $\blacklozenge$ into this system that is parameterized to an underlying accessibility relation R, where R^w denotes the information state made up of the accessible possibilities from w. Then we let $\blacklozenge\mathsf{A}$ narrow down an information state to the worlds w where R^w can be consistently updated with A. We impose an extra condition so that this update is defined only if $R^w[\mathsf{A}]$ is also defined. To implement this last condition, we union the overall update with $R^w[\mathsf{A}] \cap \emptyset$, which has no effect when $R^w[\mathsf{A}] \neq \#$ and otherwise produces #.

Definition 13. $s[\blacklozenge\mathsf{A}] = s \cap \{w \mid R^w[\mathsf{A}] \neq \emptyset\} \cup (R^w[\mathsf{A}] \cap \emptyset)$

This semantics for modals allows us to model arbitrary flavors of modality. For example, we can model Free Choice for deontic modals by appeal to a deontic accessibility relation R_d, which relates any world w to the worlds that are consistent with the normative rules at w.

Now we can retain our earlier semantics for disjunction, defined in terms of the test operator $\Diamond$. But we can allow this dynamic disjunction operator to embed under $\blacklozenge$. The resulting theory validates Free Choice:

Observation 21. $\blacklozenge(\mathsf{A} \vee \mathsf{B}) \models \blacklozenge\mathsf{A} \wedge \blacklozenge\mathsf{B}$

For s supports $\blacklozenge(\mathsf{A} \vee \mathsf{B})$ only if for every world w in s, $R^w[\mathsf{A} \vee \mathsf{B}] \neq \emptyset$. But given the semantics for $\vee$, this in turn requires that $R^w \models \Diamond\mathsf{A}$ and $R^w \models \Diamond\mathsf{B}$, so that $R^w[\mathsf{A}] \neq \emptyset$ and $R^w[\mathsf{B}] \neq \emptyset$. Since w is arbitrary in s, this implies that $s \models \blacklozenge\mathsf{A}$ and $s \models \blacklozenge\mathsf{B}$.

To summarize, the crucial property needed here is that the following principle is valid:

OUTER POSITIVE INTROSPECTION $\blacklozenge\lozenge\mathsf{A} \models \blacklozenge\mathsf{A}$.

The reason that we are able to validate this principle regardless of the structure of R is that $\lozenge$ is a dynamic operator. $\lozenge\mathsf{A}$ predicates possibility of a body of information. But because $\lozenge\mathsf{A}$ is dynamic, it can in principle predicate possibility of different bodies of information, depending on the environment in which it occurs. So if $\lozenge\mathsf{A}$ applies to the whole information state s, $\lozenge\mathsf{A}$ predicates epistemic possibility of A—consistency with what is common knowledge, say. By contrast, when $\lozenge\mathsf{A}$ is embedded under an information shifting operator, like $\blacklozenge$, it has a different effect. Under $\blacklozenge$, $\lozenge\mathsf{A}$ says A is consistent with R^w. That is, $\lozenge$ says the same thing under the scope of $\blacklozenge$ as $\blacklozenge$ says when unembedded. This sensitivity to local context is a hallmark of dynamic semantics.

We've now seen that a truth conditional semantics for possibility modals can be integrated into our dynamic semantics for disjunction in a way that also validates Free Choice. Similarly, this revised semantics validates Double Prohibition. More generally, this semantics has the same results as that in the previous section, with one exception. In the previous section, we predicted that IAT ($[\lozenge\mathsf{A} \wedge \lozenge\mathsf{B}] \vee [\neg\lozenge\mathsf{A} \wedge \neg\lozenge\mathsf{B}]$) was not only invalid, but inconsistent. This seemed too strong, since for example (25) seems like it could be true:

(25) Mary may have soup and Mary may have salad; or Mary may not have soup and Mary may not have salad.

Here, our new semantics makes just the right prediction. IAT is still invalid, but is also consistent:

Observation 22. $(\blacklozenge\mathsf{A} \wedge \blacklozenge\mathsf{B}) \vee (\neg\blacklozenge\mathsf{A} \wedge \neg\blacklozenge\mathsf{B}) \not\models \bot$

For example, consider a state made up of two worlds. At the first world, A and B are both deontically permissible. At the second world, neither A nor B are deontically permissible. This state supports $(\blacklozenge\mathsf{A} \wedge \blacklozenge\mathsf{B}) \vee (\neg\blacklozenge\mathsf{A} \wedge \neg\blacklozenge\mathsf{B})$, because each disjunct holds at one of the two worlds.[26]

26 To reach similar results for epistemic modals, we could use the dynamic semantics in Goldstein 2018, where epistemic modals are sensitive not only to the input information state but also to a world of evaluation.

9 Comparing the two accounts

We've now developed two different semantics that use homogeneity to validate Free Choice and Double Prohibition. This section explores two factors that might decide between these two systems.

9.1 Wide free choice

As we saw above, Free Choice also occurs when disjunction takes wide scope to possibility modals.

WIDE FREE CHOICE $\Diamond\mathsf{A} \vee \Diamond\mathsf{B} \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

While this inference is invalid in homogeneous alternative semantics, the inference is valid in homogeneous dynamic semantics. In homogeneous dynamic semantics, homogeneity effects are contributed by disjunction itself. So $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ is itself defined in s only if s supports $\Diamond\Diamond\mathsf{A}$ and s supports $\Diamond\Diamond\mathsf{B}$. Since $\Diamond$ satisfies the 4 axiom, Wide Free Choice is valid for the epistemic test operator $\Diamond$.

Things are a bit more complicated for versions of Wide Free Choice with other modals. In homogeneous dynamic semantics, this involves the inference from $\blacklozenge\mathsf{A} \vee \blacklozenge\mathsf{B}$ to $\blacklozenge\mathsf{A} \wedge \blacklozenge\mathsf{B}$. Unlike the narrow scope version, this inference's validity turns on properties of the accessibility relation associated with $\blacklozenge$. In particular, the inference is valid only if R is universal, so that whenever v is accessible from any w, v is accessible from every w. For suppose v is accessible from w but not u. Now let $s = \{w, u\}$, and let A be a claim true at v uniquely. s supports $\Diamond\blacklozenge\mathsf{A}$, since $s[\blacklozenge\mathsf{A}] = \{w\} \neq \emptyset$. But s does not support $\blacklozenge\mathsf{A}$, since s contains u, which cannot see v.

For this reason, homogeneous dynamic semantics allows that Wide Free Choice is cancellable for deontic, but not epistemic modals.[27]

(26) You may take an apple or you may take a pear, but I don't know which.

(27) You make take an apple or you may take a pear, but I won't tell you which.[28]

In (26) and (27), Free Choice is cancelled: these sentences seem to have deontic modals under the scope of disjunction. In homogeneous dynamic semantics, any case with this form involves a modal being interpreted relative to a non-universal accessibility relation. Here our account makes the kinds of predictions that were tested in Hoeks et al. 2018, which found experimentally that narrow scope Free Choice is accepted for deontic modals independently of speaker knowledge of deontic facts, while wide scope Free Choice only seems to hold when the speaker is

27 A similar prediction is achieved in Zimmermann 2000.
28 See Kamp 1978 and Willer 2017b for discussion.

treated as an expert about the rules. Finally, Simons 2005 suggests there may be a few cases in which Free Choice can be cancelled even for epistemic modals. We can explain these cases by proposing that sometimes the relevant epistemic modal is not the test operator $\Diamond$, but rather $\blacklozenge$ under an epistemic accessibility relation.

Finally, in homogeneous dynamic semantics there is no analogous way to cancel narrow scope Free Choice inferences. These are valid regardless of the choice of accessibility relation. But the narrow scope analogues of (26) and (27) (the sentences (10) and (11) from §2) are equally compelling. We can then explain these cases by suggesting that at the level of logical form they have disjunction taking wide scope to $\blacklozenge$, with a non-universal accessibility relation.

It is no coincidence that homogeneous dynamic semantics validates Wide Free Choice. We know that homogeneous dynamic semantics is a modal theory of disjunction. But it turns out that Wide Free Choice is intimately connected to Modal Disjunction. Given a variety of principles we have already explored, these two principles are equivalent.

Fact 5. Suppose Transitivity, T, the 4 axiom and Constructive Dilemma. Then Wide Free Choice is valid iff Modal Disjunction is valid.

Proof. For the left to right direction: by T, A implies $\Diamond\mathsf{A}$ and B implies $\Diamond\mathsf{B}$. So by Constructive Dilemma $\mathsf{A} \vee \mathsf{B}$ implies $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$. So by Transitivity and Wide Free Choice, $\mathsf{A} \vee \mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$. For the right to left direction: by Modal Disjunction, $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ implies $\Diamond\Diamond\mathsf{A} \wedge \Diamond\Diamond\mathsf{B}$. By the 4 axiom, $\Diamond\Diamond\mathsf{A} \wedge \Diamond\Diamond\mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$. So by Transitivity, $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$. □

Together, Facts 4 and 5 create a bridge from Free Choice to Wide Free Choice. If we hold fixed all of the assumptions from Facts 4 and 5, Free Choice and Wide Free Choice are equivalent.

Just like Free Choice, Wide Free Choice also disappears under negation. To test for this effect, we can embed instances of Wide Free Choice under negative attitude verbs like *doubts*, which are intuitively equivalent to *believes not*. For example, (28) appears to entail (29):

(28) Alex doubts that Billie might be in New York or Billie might be in Los Angeles.

(29) Alex believes that Billie can't be in New York and Billie can't be in Los Angeles.

WIDE DOUBLE PROHIBITION $\neg(\Diamond\mathsf{A} \vee \Diamond\mathsf{B}) \models \neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}$

Homogeneous dynamic semantics handles this data with ease. A state supports $\neg(\Diamond\mathsf{A} \vee \Diamond\mathsf{B})$ only if either both A and B are possible, or both are impossible. The ordinary update of $\neg(\Diamond A \vee \Diamond B)$ rules out the former condition; so both must be impossible.

The following table summarizes the pros and cons of the theories above.

		AS	HAS	HDS
Free Choice	$\Diamond(A \vee B) \models \Diamond A \wedge \Diamond B$	+	+	+
Double Prohibition	$\neg\Diamond(A \vee B) \models \neg\Diamond A \wedge \neg\Diamond B$	-	+	+
Wide Free Choice	$\Diamond A \vee \Diamond B \models \Diamond A \wedge \Diamond B$	-	-	+
Disjunction Introduction	$A \models A \vee B$	+	+	+
LEM	$\models A \vee \neg A$	+	+	+
IAT	$\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$	-	-	-
Modal Disjunction	$A \vee B \models \Diamond A \wedge \Diamond B$	-	-	+
UM	$A \models B \Longrightarrow \Diamond A \models \Diamond B$	-	+	+
Transitivity	$A \models B \;\&\; B \models C \Longrightarrow A \models C$	+	-	-
Contraposition	$A \models B \Longrightarrow \neg B \models \neg A$	+	+	+

So far, we've considered one advantage of homogeneous dynamic semantics over homogeneous alternative semantics: that the former but not the latter can validate Wide Free Choice. Now we turn to one potential disadvantage: the interaction between disjunction and quantifiers.

9.2 Quantifiers

Homogeneous dynamic semantics, and modal theories of disjunction in general, face a *prima facie* difficulty when disjunction is embedded under quantifiers. For example, consider (33):

(30) Every philosopher or linguist went to the party.

One might worry that homogeneous dynamic semantics makes a bad prediction about (33): that it only quantifies over people who might be a philosopher and also might be a linguist. After all, on the account above the open formula *x is a philosopher or x is a linguist* should be defined only if either both claims are possible, or both are impossible. Intuitively, however, (33) is false if someone who must be a linguist and not a philosopher did not go to the party.

Of course, to understand this prediction we have to integrate the dynamic semantics above with a semantics for quantification. I'll now argue that when we do so, the problem above disappears. If we extend our semantics in the right way, we can predict that the disjunctive restrictor in (33) quantifies over all philosophers and all linguists.[29]

29 This relates to observations in Büring 1998.

Here we'll consider a semantics for quantifiers that is a simplification of the dynamic semantics in Groenendijk et al. 1996. For brevity, we'll omit many details of the semantics; but a full implementation can be found in Carter & Goldstein 2017. One of the goals of this semantics is to make the right predictions about donkey anaphora, so that (31) and (32) are equivalent:

(31) If a donkey is beaten, it is sad.

(32) Every donkey that is beaten is sad.

We'll model universal quantification using a generalized quantifier $\forall\mathsf{x}$ that takes two sentences as input, and we'll model indefinites like *a donkey* with a special operator $\exists\mathsf{x}$, which occurs within the antecedent of the conditional in (31). This gives us:

REDUCTION $\exists\mathsf{x}\mathsf{F}\mathsf{x} \to \mathsf{G}\mathsf{x} \dashv\models \forall\mathsf{x}(\mathsf{F}\mathsf{x})(\mathsf{G}\mathsf{x})$

Our semantics lets contexts be sets of assignment-world pairs rather than just sets of worlds. Then $[\mathsf{Fx}]$ narrows down the context to the assignment-world pairs where the referent of x satisfies F. Crucially, $[\exists\mathsf{x}]$ performs a non-eliminative update, expanding the context so that any $\langle g, w\rangle$ pair in the current context is joined by any pair $\langle g', w\rangle$ where g and g' differ only in the value of x. The operators $\to$ and $\Diamond$ retain their usual test semantics, so that $\mathsf{A} \to \mathsf{B}$ tests that the context is unchanged by B after being updated with A. Finally, we can simply define the sentence $\forall\mathsf{x}(\mathsf{A})(\mathsf{B})$ directly in terms of $\exists\mathsf{x}$ and $\to$, to achieve Reduction by brute force. Where $g[x]g'$ iff g' differs from g in at most the value of variable x:

Definition 14.

i. $c[\mathsf{Fx}] = \{\langle g, w\rangle \in c \mid g(\mathsf{x}) \in V(\mathsf{F})(w)\}$

ii. $c[\exists\mathsf{x}] = \{\langle g', w\rangle \mid g[x]g' \;\&\; \langle g, w\rangle \in c\}$

iii. $c[\exists\mathsf{x}\mathsf{A}] = c[\exists\mathsf{x}][\mathsf{A}]$

iv. $c[\mathsf{A} \vee \mathsf{B}] = \begin{cases} c[\mathsf{A}] \cup c[\mathsf{B}] & \text{if } c \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B} \text{ or } c \models \neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B} \\ \# & \text{otherwise} \end{cases}$

v. $c[\Diamond\mathsf{A}] = \begin{cases} c & \text{if } c[\mathsf{A}] \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

vi. $c[\mathsf{A} \to \mathsf{B}] = \begin{cases} c & \text{if } c[\mathsf{A}] \models \mathsf{B} \\ \emptyset & \text{otherwise} \end{cases}$

vii. $c[\forall\mathsf{x}(\mathsf{A})(\mathsf{C})] = c[\exists\mathsf{x}\mathsf{A} \to \mathsf{C}]$

With a simple dynamic theory of quantifiers and modals in place, let's return to the meaning of disjunctive quantified claims like $\forall \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx})(\mathsf{Hx})$. This sentence has the same meaning as $\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx}) \rightarrow \mathsf{Hx}$. The key property here is that $c[\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx}) \rightarrow \mathsf{Hx}]$ explores whether $c[\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx})]$ is unchanged by $[\mathsf{Hx}]$. Now $c[\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx})]$ is in turn decomposed into $c[\exists \mathsf{x}][(\mathsf{Fx} \vee \mathsf{Gx})]$. $c[\exists x]$ expands c to allow x to refer to any individual. $c[\exists \mathsf{x}][(\mathsf{Fx} \vee \mathsf{Gx})]$ is then defined only if $c[\exists \mathsf{x}][\mathsf{Fx}]$ and $c[\exists \mathsf{x}][\mathsf{Gx})]$ are each non-empty. This requires that $c[\exists \mathsf{x}]$ include some $\langle g, w \rangle$ where $g(\mathsf{x})$ is F at w, and include some $\langle g', w' \rangle$ where $g'(\mathsf{x})$ is G at w'. Since $c[\exists \mathsf{x}]$ includes any possible value of x, this reduces to the requirement that c include some world where something is F, and some world where something is G. In other words, the disjunctive restrictor only applies a possibility test to the global context; it does not place this requirement on each individual one at a time. Finally, $c[\exists \mathsf{x}][(\mathsf{Fx} \vee \mathsf{Gx})]$ narrows down $c[\exists \mathsf{x}]$ to the assignment-world pairs where x is either F or G. Summarizing, the whole sentence is then supported by a context just in case (i) the context contains a world where something is F, and a world where something is G; and (ii) every individual who is F or G at any world in the context is also H. This is a perfectly reasonable prediction, and avoids the concerns above. When someone who must be a linguist and not a philosopher fails to go to the party, condition (ii) fails and (33) is false.

This theory makes the right predictions about the interaction between quantifiers and disjunction. But without saying more, the theory makes the wrong prediction about the interaction between quantifiers and epistemic modality. Beaver 1994 and Groenendijk et al. 1996 note that if the overt epistemic possibility modal *might* is simply understood as the test operator $\Diamond$ above, then we make some bad predictions. For example, (33) and (34) quantify over everyone in the domain; not simply those who might be linguists.

(33) Someone who might be a linguist went to the party.

(34) Everyone who might be linguist went to the party.

In a context where both linguists and philosophers are under discussion, bad predictions result. To make the right predictions here, we can keep the same semantics for disjunction, but offer a different semantics for overt epistemic possibility claims.

The crucial idea, building on a proposal in Büring 1998, is too let overt epistemic possibility function as a kind of 'quasi-distributive' operator, $\Diamond^{\#}$, which updates a state by first partitioning it into cells which agree on assignment functions, and then applying the test operator $\Diamond$ to each cell. The result is that update proceeds 'cell-wise' rather than point-wise: cells which pass the $\Diamond$ test remain in the state, while cells which fail the test are eliminated.

Definition 15. $c[\Diamond^{\#}\mathsf{A}] = \bigcup\{\{\langle g', w \rangle \in c \mid g' = g\}[\Diamond \mathsf{A}] \mid \exists w' : \langle g, w' \rangle \in c\}$

Now (33) and (34) can restrictedly quantify over the domain of quantification, because the restrictor $\Diamond^{\#}\mathsf{L}x$ updates c to the union of cells where the assignment function maps x to an individual that is a linguist at some world. (For more on this meaning for possibility modals, and how it might be decomposed into more basic operations, see Carter & Goldstein 2017.)

The crucial observation for our purposes is that while overt possibility claims are quasi-distributive, the kind of possibility at work in disjunction is not. When a disjunction $\mathsf{F}x \vee \mathsf{G}x$ occurs in the restrictor of a quantifier, the resulting quantification is not restricted to individuals who might be F and also might be G. So even if the test operator is not a good model of overt epistemic possibility, it still has an application in the case of disjunction.

All that remains is to verify that Free Choice is still valid once we enrich the meaning of overt possibility modals. The question is whether $\Diamond^{\#}(\mathsf{A} \vee \mathsf{B}) \models \Diamond^{\#}\mathsf{A} \wedge \Diamond^{\#}\mathsf{B}$. To see that this is so, note that $\Diamond^{\#}\Diamond\mathsf{A} \models \Diamond^{\#}\mathsf{A}$. After all, the premise $\Diamond^{\#}\Diamond\mathsf{A}$ partitions c into cells that agree on the value of all variables, and then checks whether that cell supports $\Diamond\Diamond\mathsf{A}$. It does so just in case the cell also supports $\Diamond\mathsf{A}$, exactly what is required by the conclusion $\Diamond^{\#}\mathsf{A}$. On the other hand, this account by itself doesn't guarantee Wide Free Choice. Here, the crucial question is whether $\Diamond\Diamond^{\#}\mathsf{A} \models \Diamond^{\#}\mathsf{A}$. This condition fails exactly when $\Diamond$ and $\Diamond^{\#}$ come apart. If the context contains to candidates for x, d and d', where the former might be F and the latter cannot be, $\Diamond\Diamond^{\#}\mathsf{F}x$ leaves c unchanged, while $\Diamond^{\#}\mathsf{F}x$ removes d' as an option. Thus $\Diamond^{\#}\mathsf{F}x \vee \Diamond^{\#}\mathsf{G}x$ implies $\Diamond\mathsf{F}x \wedge \Diamond\mathsf{G}x$, but only implies $\Diamond^{\#}\mathsf{F}x \wedge \Diamond^{\#}\mathsf{G}x$ in contexts where either every candidate for the value of x either can be F and can be G, or no candidate can. One question, left for further research, is whether this requirement could itself be achieved by imposing another kind of homogeneous definedness requirement on the domain of quantification. Of course, when Wide Free Choice is tested on the fragment of the language without variables, this subtlety can be ignored.

If we have a sophisticated treatment of quantification in dynamic semantics, we can allow disjunction to be modal without making bad predictions about the way in which disjunction embeds under quantifiers.

10 Comparison with existing accounts

10.1 Semantic accounts

This section briefly compares this paper's theories with some existing semantic accounts of Free Choice, including Aher 2012, Aloni 2018, Starr 2016, and Willer 2017a. While Aher 2012 and Willer 2017a resemble homogeneous alternative semantics, Aloni 2018 and Starr 2016 instead resemble homogeneous dynamic semantics. Each account differs in some interesting ways from my own.

Aher 2012 gives an inquisitive semantics for deontic modals, in which sentences are recursively assigned supporting and rejecting states by $\models$ and $=\!|$. Deontic modals are interpreted in terms of a special *violation* atom v, true at a world just in case the rules are violated there (and supported[/rejected] by a state just in case true[/false] at every world in it.)

Definition 16.

i. a. $s \models \mathsf{p}$ iff $\forall w \in s : w(\mathsf{p}) = 1$
   b. $s =\!| \ \mathsf{p}$ iff $\forall w \in s : w(\mathsf{p}) = 0$

ii. a. $s \models \neg\mathsf{A}$ iff $s =\!| \ \mathsf{A}$
   b. $s =\!| \ \neg\mathsf{A}$ iff $s \models \mathsf{A}$

iii. a. $s \models \mathsf{A} \vee \mathsf{B}$ iff $s \models \mathsf{A}$ or $s \models \mathsf{B}$
   b. $s =\!| \ \mathsf{A} \vee \mathsf{B}$ iff $s =\!| \ \mathsf{A}$ and $s =\!| \ \mathsf{B}$

iv. a. $s \models \mathsf{A} \wedge \mathsf{B}$ iff $s \models \mathsf{A}$ and $s \models \mathsf{B}$
   b. $s =\!| \ \mathsf{A} \wedge \mathsf{B}$ iff $s =\!| \ \mathsf{A}$ or $s =\!| \ \mathsf{B}$

v. a. $s \models \Diamond\mathsf{A}$ iff $\forall s' \subseteq s$: if $s' \models \mathsf{A}$ then $s' =\!| \ \mathsf{v}$
   b. $s =\!| \ \Diamond\mathsf{A}$ iff $\forall s' \subseteq s$: if $s' \models \mathsf{A}$ then $s' \models \mathsf{v}$

Within this framework, the relevant notion of entailment for validating Free Choice is preservation of support.

Definition 17. $\mathsf{A}_1; \ldots; \mathsf{A}_n \models \mathsf{B}$ iff $\forall s$: if $s \models \mathsf{A}_1$, ..., and $s \models \mathsf{A}_n$, then $s \models \mathsf{B}$.

Here, Aher 2012 is similar to homogeneous alternative semantics: Free Choice and Double Prohibition are valid. To see why $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A}$, suppose that $s \models \Diamond(\mathsf{A} \vee \mathsf{B})$. Then every $s' \subseteq s$ which supports $\mathsf{A} \vee \mathsf{B}$ rejects v. Crucially, any state which supports A supports $\mathsf{A} \vee \mathsf{B}$. So every $s' \subseteq s$ which supports A also rejects v. So s supports $\Diamond\mathsf{A}$. To see why Double Prohibition is valid, suppose that s rejects $\Diamond(\mathsf{A} \vee \mathsf{B})$. Then every $s' \subseteq s$ which supports $\mathsf{A} \vee \mathsf{B}$ supports v. So s rejects $\Diamond\mathsf{A}$.

Since preservation of support is transitive, we know from our earlier results that it doesn't contrapose. In particular, although $\neg\Diamond(\mathsf{A} \vee \mathsf{B}) \models \neg\Diamond\mathsf{A}$, we do not have that $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$. For this very reason, possibility modals are not upward monotonic. The inference $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ fails despite the validity of Disjunction Introduction.[30]

30 That said, we could enrich Aher 2012 with Strawson entailment, where an inference is valid when any premise that supports the premises does not reject the conclusion.

Aher 2012 differs from homogeneous alternative semantics in several respects. One difference concerns the Law of Non-Contradiction.

LNC $\mathsf{A}; \neg\mathsf{A} \models \bot$

Aher 2012's semantics for possibility modals allows that some states both accept and reject $\Diamond\mathsf{A}$. Consider any state which rejects p and rejects v. In that state $\Diamond\mathsf{A}$ and $\neg\Diamond\mathsf{A}$ are both supported. So LNC fails. Finally, Aher 2012 focuses on deontic possibility, with the analysis focused on a notion of *violation* that has no counterpart in the epistemic domain.[31] Summarizing, Aher 2012 agrees in many respects with homogeneous alternative semantics. On the other hand, the account differs from homogeneous dynamic semantics, since the account invalidates Modal Disjunction and Wide Free Choice.

Aloni 2018 works within the same semantic framework as Aher 2012, but with a different semantics for disjunction and possibility modals. On Aloni 2018's proposal, natural language *or* ($\vee_+$) is decomposed into an underlying disjunction operator $\vee$ enriched with a non-emptiness requirement NE. On the resulting semantics, $\mathsf{A} \vee_+ \mathsf{B}$ is supported by s just in case there are two non-empty subsets of s which jointly exhaust s where one subset supports A and the other supports B. Aloni 2018 then relies on an ordinary Kripke semantics for possibility modals.[32]

Definition 18. Where $R^w = \{v \mid wRv\}$:

i. a. $s \models \mathsf{p}$ iff $\forall w \in s : w(\mathsf{p}) = 1$
   b. $s \rightmodels \mathsf{p}$ iff $\forall w \in s : w(\mathsf{p}) = 0$

ii. a. $s \models \neg\mathsf{A}$ iff $s \rightmodels \mathsf{A}$
   b. $s \rightmodels \neg\mathsf{A}$ iff $s \models \mathsf{A}$

iii. a. $s \models \mathsf{A} \vee \mathsf{B}$ iff $\exists t, t' : t \cup t' = s \;\&\; t \models \mathsf{A} \;\&\; t' \models \mathsf{B}$
   b. $s \rightmodels \mathsf{A} \vee \mathsf{B}$ iff $s \rightmodels \mathsf{A}$ and $s \rightmodels \mathsf{B}$

iv. a. $s \models \mathsf{A} \wedge \mathsf{B}$ iff $s \models \mathsf{A}$ and $s \models \mathsf{B}$
   b. $s \rightmodels \mathsf{A} \wedge \mathsf{B}$ iff $\exists t, t' : t \cup t' = s \;\&\; t \rightmodels \mathsf{A} \;\&\; t' \rightmodels \mathsf{B}$

v. a. $s \models \Diamond\mathsf{A}$ iff $\forall w \in s : \exists t \subseteq R^w : t \neq \emptyset \;\&\; t \models \mathsf{A}$
   b. $s \rightmodels \Diamond\mathsf{A}$ iff $\forall w \in s : R^w \rightmodels \mathsf{A}$

vi. a. $s \models \mathsf{NE}$ iff $s \neq \emptyset$

31 See Aher & Groenendijk 2013 for more on the interaction between deontic and epistemic modals.
32 The framework is enriched with further complexities (for example, a designated information state) to handle epistemic contradictions. For simplicity, I ignore these aspects of the view.

b. $s \dashv \mathsf{NE}$ iff $s = \emptyset$

vii. a. $s \models \mathsf{A} \vee_+ \mathsf{B}$ iff $s \models (\mathsf{A} \wedge \mathsf{NE}) \vee (\mathsf{B} \wedge \mathsf{NE})$

b. $s \dashv \mathsf{A} \vee_+ \mathsf{B}$ iff $s \dashv (\mathsf{A} \wedge \mathsf{NE}) \vee (\mathsf{B} \wedge \mathsf{NE})$

As in Aher 2012, Aloni 2018 defines entailment as preservation of support.

Aloni 2018 shares many of the predictions of homogeneous dynamic semantics. Free Choice, Modal Disjunction, and Double Prohibition are all valid. s supports $\Diamond(\mathsf{A} \vee_+ \mathsf{B})$ just in case there is some $t \subseteq R^w$ which supports $\mathsf{A} \vee_+ \mathsf{B}$, for all $w \in s$. For such a t to support $\mathsf{A} \vee_+ \mathsf{B}$, there must be non-empty subsets u and u' of t which support A and B. Since u and u' are also subsets of R^w, we know that s also supports $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$.

Modal Disjunction is valid because epistemic modals quantify over worlds in the relevant state. So if s supports $\mathsf{A} \vee_+ \mathsf{B}$, there must be non-empty subsets of s which support each of A and B. But this means that every world in s can see an A world, and every world in s can see a B world.

Double Prohibition is valid, for suppose $s \dashv \Diamond(\mathsf{A} \vee_+ \mathsf{B})$. Then $R^w \dashv \mathsf{A} \vee_+ \mathsf{B}$ for all $w \in s$. So $R^w \dashv (\mathsf{A} \wedge \mathsf{NE}) \vee (\mathsf{B} \wedge \mathsf{NE})$. So $R^w \dashv \mathsf{A} \wedge \mathsf{NE}$, and $R^w \dashv \mathsf{B} \wedge \mathsf{NE}$. So $R^w \dashv \mathsf{A}$ and $R^w \dashv \mathsf{B}$, for each $w \in s$. So $s \dashv \Diamond\mathsf{A}$ and $s \dashv \Diamond\mathsf{B}$.

Next, Aloni 2018's semantics has similar properties to our own with respect to Wide Free Choice. Say that R is indisputable at s when any two worlds in s have the same accessible possibilities. Wide Free Choice holds at any states with respect to which R is indisputable. For suppose s supports $\Diamond\mathsf{A} \vee_+ \Diamond\mathsf{B}$ and that R is indisputable with respect to s. From s supporting $\Diamond\mathsf{A} \vee_+ \Diamond\mathsf{B}$, we know there are two worlds w and v in s where R^w is consistent with A and R^v is consistent with B. This immediately implies that $R^{w'}$ is consistent with A and consistent with B for every $w' \in s$.

It is not coincidental that Aloni 2018's semantics has so many similar predictions to those homogeneous dynamic semantics. Both theories embody the idea that disjunctions are homogeneous with respect to modal status. For say that $\mathsf{A} \vee_+ \mathsf{B}$ is defined at s just in case it is either supported or rejected there. In the semantics above, $\mathsf{A} \vee_+ \mathsf{B}$ is only defined in s when either both of A and B or neither of them are consistent with s.

While homogeneous dynamic semantics and Aloni 2018's system embody a similar idea of disjunction, they do so in different ways. The most striking difference concerns the logical status of the NE sentence (itself definable as $\top \vee_+ \top$, with $\top = \neg(\mathsf{p} \wedge \neg\mathsf{p})$). Interestingly, this sentence is not accepted by the absurd state. This gives the logic a curious property: the principle of explosion is invalid.

EXPLOSION $\mathsf{A} \wedge \neg\mathsf{A} \models \mathsf{B}$

In particular, $\mathsf{A} \wedge \neg\mathsf{A}$ does not imply NE, since there is a state ($\emptyset$) which supports $\mathsf{A} \wedge \neg\mathsf{A}$ without supporting NE. Since the disjunction operator $\vee_+$ incorporates NE, a similar failure of Explosion percolates down: for example, $\mathsf{A} \wedge \neg\mathsf{A} \not\models \mathsf{A} \vee_+ \neg\mathsf{A}$. This invalidity is striking, since it means that conjunctions are not always as strong as the corresponding disjunction.

Finally, there's an interesting architectural difference between both of the above accounts and our own. In the accounts above, the properties of Free Choice under negation are derived by appealing to two independent sets of alternatives: positive and negative alternatives. By contrast, in homogenous alternative semantics the meaning of a sentence is modeled in terms of a single set of alternatives: but each alternative is trivalent. This architectural difference has empirical upshots. First, it means that homogeneous alternative semantics makes the fairly strong prediction that Free Choice effects do not occur with negated conjunctions. In a bilateral system, we can simply stipulate that the negative alternatives for conjunctions are the same as the positive alternatives for the relevant disjunction. In homogeneous alternative semantics, this strategy is not available. As discussed in the next section, this may be a desirable prediction, since Free Choice with negated conjunctions is a weaker effect. Second, Aloni 2018 argues against unilateral systems on the basis of recent work by Romoli & Santorio 2019. But our own unilateral system is richer than the ones discussed in Aloni 2018, because our alternatives are themselves trivalent. As discussed in the next section, this extra richness allows us to explain this data, which other unilateral accounts struggle with.

Starr 2016 and Willer 2017a also offer semantic accounts of Free Choice. Starr 2016 offers a theory of deontic Free Choice on which modals dynamically update preferences over possibilities. As in homogeneous dynamic semantics, Starr 2016 seeks to validate both Wide Free Choice and Free Choice, along with Double Prohibition. Starr 2016 lets a context contain a set of preference orderings. Updating a context with a deontic possibility claim $\Diamond\mathsf{A}$ involves two steps. First, the context is tested to make sure that every preference ordering allows some A world. If this test is failed, the absurd state results. Otherwise, the context expands to include not only all the current preference orderings, but also any way of taking a current preference ordering and refining it to include a top ranked A world. This non-eliminative update interacts happily with an ordinary dynamic meaning for disjunction which takes the union of both updates. In a non-eliminative setting, the union of two updates is stronger than each update alone. $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$ because an update with $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ involves non-eliminatively updating with both $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$. Starr 2016 validates ordinary Free Choice in another way, by generating alternatives out of updating, and letting $\Diamond$ universally quantify over the alternatives generated by $\mathsf{A} \vee \mathsf{B}$, which include A and B. Finally, Starr 2016 validates Double Prohibition by appealing to a nonclassical negation $\neg$A which can reverse the preference orderings induced by A.

This theory differs in several respects from our own. First, the analysis only covers Free Choice in deontic modals; it does not extend to epistemic modals. Second, the analysis is less classical than our own with respect to negation. In particular, Starr 2016 seeks to invalidate both double negation elimination and the duality of *must* and *may*. On this proposal, $\neg\neg\Diamond$A and $\neg\Box\neg$A are both weaker than $\Diamond$A. Finally, the theory may face a technical issue when $\Diamond A \vee \Diamond B$ is evaluated at contexts where A is permitted by all preference orderings, while B is not permitted by all preference orderings. In such a context S, $S[\Diamond A]$ works normally while $S[\Diamond B]$ crashes, because the test for $\Diamond$B fails. In that case, $S[\Diamond A \vee \Diamond B] = S[\Diamond A]$, while $S[\Diamond A \wedge \Diamond B] = S[\Diamond A][\Diamond B]$, which is the absurd state. The result is that Wide Free Choice may not actually be valid, since it only holds in contexts where the tests for $\Diamond$A and $\Diamond$B are both satisfied.

On Willer 2017a's proposal, the meaning of a sentence is a pair of a positive and negative update on states, themselves sets of alternatives or classical propositions. When a state is updated with the disjunction $A \vee B$, the result is a new state consisting of the result of updating every input alternative with A, and the result of updating every input alternative with B. Possibility modals test the state to ensure that updating the union of alternatives in the state with the prejacent does not produce absurdity. The result is that Free Choice is valid: $s \uparrow \Diamond(A \vee B)$ is consistent only if the union of alternatives in s is consistent with A, and is also consistent with B. This is just what is required by $s \uparrow \Diamond A \wedge \Diamond B$. Like Aher 2012 and Aloni 2018 ,Willer 2017a validates Double Prohibition by appeal to bilateralism, in this case by offering an explicit definition of positive and negative updating. The negative update with a possibility modal (or the positive update with the negation of a possibility modal) requires that updating the union of alternatives with the prejacent is guaranteed to only produce absurdity. Crucially, $s \uparrow \neg\Diamond(A \vee B)$ therefore requires that the positive update with both of A and B produces absurdity. So $s \uparrow \neg\Diamond(A \vee B)$ coincides with $s \uparrow \neg\Diamond A \wedge \neg\Diamond B$.

Willer 2017a generates a similar empirical profile to homogeneous alternative semantics. While Free Choice and Double Prohibition are valid, Wide Free Choice is invalid. However, one significant difference between Willer 2017a and homogeneous alternative semantics concerns the treatment of the DeMorgan equivalent $\neg(\neg A \wedge \neg B)$. Willer 2017a seeks to validate Free Choice even for this, so that $\Diamond\neg(\neg A \wedge \neg B) \models \Diamond A \wedge \Diamond B$. However, as we discuss below, Ciardelli et al. 2018 argue against exactly this. At any rate, this inference is invalid in homogeneous alternative semantics. Another difference is that Willer 2017a relies on a quite different semantic architecture, folding some kind of alternative semantics into a dynamic system. This architectural choice leads to further differences in the logic. For example, Willer

2017a preserves signature properties of existing dynamic theories, including the inconsistency of the epistemic contradiction $\mathsf{A} \wedge \Diamond\neg\mathsf{A}$.

Some existing semantic accounts come close to homogeneous alternative and dynamic semantics. But extant theories differ in various ways, including with respect to Explosion and the Law of Non-Contradiction. In addition, existing accounts of Free Choice do not rely on Strawson validity.

10.2 Implicature based accounts

This section compares this paper's theories with two accounts of Free Choice, Chemla 2008 and Fox 2007, where the inference resembles a scalar implicature. Chemla 2008 gives a pragmatic account of Free Choice which appeals to sets of alternatives. Utterances are interpreted relative to a variety of sets of alternatives. Interpretation is governed by a pragmatic similarity principle: treat similar alternatives similarly. Two alternatives are similar when they are derived by replacing relevant items in a similar way. An agent treats two alternatives similarly when she believes one iff she believes the other.

Take a disjunction like $\mathsf{A} \vee \mathsf{B}$. This sentence generates two relevant sets of alternatives. Chemla 2008 relies on a linear scale $\langle\top, \vee, \wedge, \bot\rangle$ which orders connectives by strength. The members $\top$ and $\bot$ are superweak and superstrong connectives, which map their inputs to the ordinary tautology and contradiction. This scale generates alternatives for $\mathsf{A} \vee \mathsf{B}$ by replacement of $\vee$ with a scalemate. The resulting alternatives are then partitioned into those stronger and weaker than $\vee$. This gives us two sets of alternatives: $\{\top\}$, and $\{\mathsf{A} \wedge \mathsf{B}, \bot\}$. In addition to a linear scale, Chemla 2008 relies on a second method of alternative generation: connective split. Any sentence with a connective generates the alternatives that result from keeping the left or right side of the connective. This gives us a third set of alternatives: $\{\mathsf{A}, \mathsf{B}\}$.

Chemla 2008 applies a similarity rule of interpretation to these sets of alternatives: every member of a set of alternatives must be treated the same by the speaker: either they believe every member, or they believe no member. In the case of a bare disjunction, this first explains why speakers who assert $\mathsf{A} \vee \mathsf{B}$ do not believe $\mathsf{A} \wedge \mathsf{B}$. By the similarity principle, they would believe $\mathsf{A} \wedge \mathsf{B}$ iff they believe $\bot$, which is absurd. In addition, this rule explains why speakers who assert $\mathsf{A} \vee \mathsf{B}$ do not believe A and do not believe B: the similarity principle says they believe A iff they believe B; but if they believed both, they would also believe $\mathsf{A} \wedge \mathsf{B}$, contradicting the first result.

Finally, Chemla 2008 strengthens the similarity principle to derive secondary implicatures. At this secondary step, we require that for any similar alternatives X and Y, the speaker believes: X iff Y. This strengthening is only applied when consistent with the primary implicatures and the truth conditions. For example, applied to $\mathsf{A} \vee \mathsf{B}$, we produce the secondary implicature that the speaker believes $\neg(\mathsf{A} \wedge \mathsf{B})$. For the secondary step this gives us that the speaker believes: $\mathsf{A} \wedge \mathsf{B}$ iff $\bot$. Since they believe $\neg\bot$, this implies belief in $\neg(\mathsf{A} \wedge \mathsf{B})$. Now consider each disjunct individually. The similarity principle implied the speaker believes A iff she believes B. The secondary step would strengthen this inference to believing: A iff B. But this step is blocked. For this step requires either believing $\mathsf{A} \wedge \mathsf{B}$ or believing $\neg\mathsf{A} \wedge \neg\mathsf{B}$. The former state would contradict the primary implicatures of $\mathsf{A} \vee \mathsf{B}$; the second step would contradict its truth conditions.

Now consider Free Choice. $\Diamond(\mathsf{A} \vee \mathsf{B})$ is also associated with three sets of alternatives. First, the linear scale $\langle \top, \vee, \wedge, \bot \rangle$ generates the alternative sets $\{\Diamond\top\}$ and $\{\Diamond(\mathsf{A} \wedge \mathsf{B}), \Diamond\bot\}$ via replacement and ordering by strength. Second, connective split produces the alternative set $\{\Diamond\mathsf{A}, \Diamond\mathsf{B}\}$. The similarity principle first implies the speaker doesn't believe $\Diamond(\mathsf{A} \wedge \mathsf{B})$, since $\Diamond\bot$ is absurd. Likewise, it implies the speaker either believes $\Diamond\mathsf{A}$ and believes $\Diamond\mathsf{B}$, or believes neither. This primary implicature can be strengthened into the secondary implicature that they believe $\Diamond\mathsf{A}$ and believe $\Diamond\mathsf{B}$, since the belief that: $\Diamond\mathsf{A}$ iff $\Diamond\mathsf{B}$ is consistent with the primary implicature that the speaker doesn't believe $\Diamond(\mathsf{A} \wedge \mathsf{B})$.

One significant empirical point of difference between Chemla 2008 and homogeneous dynamic semantics concerns the status of Wide Free Choice. $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ is associated with three sets of alternatives. First, the linear scale $\langle \top, \vee, \wedge, \bot \rangle$ generates the alternative sets $\{\top\}$ and $\{\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}, \bot\}$ via replacement and ordering by strength. Second, connective split produces the alternative set $\{\Diamond\mathsf{A}, \Diamond\mathsf{B}\}$. The similarity principle *prima facie* implies the speaker doesn't believe $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$, since $\bot$ is absurd. This is where the problem arises. Chemla 2008 suggests Wide Free Choice arises exactly when this primary exclusivity implicature is cancelled. Now turn to the connective split alternatives. Similarity implies that the speaker either believes $\Diamond\mathsf{A}$ and believes $\Diamond\mathsf{B}$, or believes neither. When exclusivity is cancelled, strengthening can be applied to reach the result that the speaker believes $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$. So, to summarize, Wide Free Choice is predicted to hold exactly when the wide scope exclusive reading of disjunction is absent. By contrast, in homogeneous dynamic semantics Wide Free Choice is guaranteed. This prediction favors homogeneous dynamic semantics, since *You may have soup or you may have salad* grants permission for both items even when it suggests you can't have both soup and salad.

Another difference between a homogeneous semantics for Free Choice and Chemla 2008 concerns the status of Free Choice when we replace $\Diamond$ and $\vee$ with their duals $\Box$ and $\wedge$:

DUAL FREE CHOICE $\neg\Box(\mathsf{A} \wedge \mathsf{B}) \models \neg\Box\mathsf{A} \wedge \neg\Box\mathsf{B}$

(35) a. You are not required to eat an apple and a banana.

b. So you aren't required to eat an apple.

Chemla 2008 predicts Dual Free Choice has the same status as Free Choice. By contrast, neither homogeneous alternative semantics nor homogeneous dynamic semantics make such a prediction. In each framework, conjunction is treated classically. So any alternative sensitivity contributed by $\Box$ will be confined to disjunction. The status of Dual Free Choice is controversial. For example, Ciardelli et al. 2018 argue against the validity of a closely related principle: $\Diamond\neg(\mathsf{A}\wedge\mathsf{B}) \models \Diamond\neg\mathsf{A}$. Precisely this principle is implicit in Dual Free Choice, since $\Diamond\neg(\mathsf{A}\wedge\mathsf{B})$ and $\neg\Box(\mathsf{A}\wedge\mathsf{B})$ are logically equivalent given the duality of $\Diamond$ and $\Box$. But Ciardelli et al. 2018 suggest the principle is invalid:

(36) a. Mary might not speak both Arabic and Bengali.
b. # So, she might not speak Arabic.

Similarly, Chemla 2009 found that while Dual Free Choice is inferred with some regularity, the effect is not as strong as ordinary Free Choice.

A third difference between Chemla 2008 and homogeneity semantics concerns disjunction under quantifiers. Chemla 2008's theory predicts that existential quantifiers in general generate an analogue of Free Choice:

(37) a. Some students will skip exercise A or exercise B.
b. Some students will skip exercise A and some students will skip exercise B.

In §9.2, we extended homogeneous dynamic semantics with a dynamic semantics for quantifiers. This validates a weakening of the inference above:

(38) a. Some students will skip exercise A or exercise B.
b. Some students might skip exercise A and some students might skip exercise B.

To adjudicate between between Chemla 2008 and homogeneous dynamic semantics, we could determine which of (38-b) or (37-b) follow from (37-a).

Now we'll turn from Chemla 2008's account to the theory in Fox 2007, which explains Free Choice through a syntactic version of pragmatic exhaustification. In particular, Fox 2007's theory of Free Choice revolves around the availability of a covert exhaustification operator exh, which functions to negate a set of alternatives. For example, a simple disjunction like (39) is interpreted with the LF (40):

(39) I ate the cake or the ice-cream.

(40) $\mathsf{exh}(\mathsf{C})(\mathsf{cake}\vee\mathsf{ice\text{-}cream})$

Alternatives are determined recursively from a set of alternative connectives to $\vee$, which includes $\wedge$, L, and R, the latter being constant functions to the left or right input. This produces the alternative set $\mathsf{C} = \{\mathsf{cake} \vee \mathsf{ice\text{-}cream}, \mathsf{cake} \wedge \mathsf{ice\text{-}cream}, \mathsf{cake}, \mathsf{ice\text{-}cream}\}$.

exh says that $\mathsf{cake} \vee \mathsf{ice\text{-}cream}$ is true, but that some alternatives in C are false. It says $\mathsf{cake} \wedge \mathsf{ice\text{-}cream}$ is false, because it is the unique alternative that is innocently excludable. An alternative in C is innocently excludable given $\mathsf{cake} \vee \mathsf{ice\text{-}cream}$ just in case it is in the intersection of the set of maximal subsets of C where negating every member of that subset is consistent with $\mathsf{cake} \vee \mathsf{ice\text{-}cream}$. There are two such maximally strong subsets: $\{\mathsf{cake} \wedge \mathsf{ice\text{-}cream}, \mathsf{cake}\}$ and $\{\mathsf{cake} \wedge \mathsf{ice\text{-}cream}, \mathsf{ice\text{-}cream}\}$: their intersection is $\{\mathsf{cake} \wedge \mathsf{ice\text{-}cream}\}$. The result is that $\mathsf{exh}(\mathsf{C})(\mathsf{cake} \vee \mathsf{ice\text{-}cream})$ is equivalent to $(\mathsf{cake} \vee \mathsf{ice\text{-}cream}) \wedge \neg(\mathsf{cake} \wedge \mathsf{ice\text{-}cream})$.

Validating Free Choice requires double exhaustification. Hold fixed a classical semantics for $\Diamond$ and $\vee$. Where $\mathsf{C}' = \{\mathsf{exh}(\mathsf{C})(\mathsf{p}) \mid \mathsf{p} \in \mathsf{C}\}$, the Free Choice validating logical form is (41):

(41) $\mathsf{exh}(\mathsf{C}')(\mathsf{exh}(\mathsf{C})(\Diamond(\mathsf{A} \vee \mathsf{B}))$

C' includes just three alternatives: $\{\Diamond(\mathsf{A} \vee \mathsf{B}) \wedge \neg\Diamond(\mathsf{A} \wedge \mathsf{B}), \Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}, \neg\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}\}$. Crucially, each of $\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}$ and $\neg\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$ are innocently excludable: their negations guarantee $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$ when combined with the truth of $\Diamond(\mathsf{A} \vee \mathsf{B})$.

This account of Free Choice makes a variety of empirical predictions that depart from homogeneous alternative semantics and homogeneous dynamic semantics. First, this account predicts Dual Free Choice ($\neg\Box(\mathsf{A} \wedge \mathsf{B}) \models \neg\Box\mathsf{A} \wedge \neg\Box\mathsf{B}$) has the same status as ordinary Free Choice. In particular, the logical form $\mathsf{exh}(\mathsf{C}')(\mathsf{exh}(\mathsf{C})(\neg\Box(\neg\mathsf{A} \wedge \neg\mathsf{B}))$ has the same truth conditions as (41), and in particular guarantees $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$. Second, the account differs from homogeneous dynamic semantics in the treatment of Wide Free Choice ($\Diamond\mathsf{A} \vee \Diamond\mathsf{B} \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$). While that inference is valid in homogeneous dynamic semantics, the inference is predicted to fail in Fox 2007. (Here Fox 2007 makes the same prediction as homogeneous alternative semantics).

One question for Fox 2007 concerns the status of Free Choice under negation (see Aloni 2018 for discussion). In principle, negating the logical form $\mathsf{exh}(\mathsf{C}')(\mathsf{exh}(\mathsf{C})(\Diamond(\mathsf{A} \vee \mathsf{B}))$ simply requires that either $\Diamond\mathsf{A}$ or $\Diamond\mathsf{B}$ is false, not both. If this is an available logical form for (42), then Double Prohibition is not explained.

(42) You can't have soup or salad.

To rule out this sort of logical form, Fox 2007 could appeal to an economy principle; for example, perhaps only applying exhaustification when it produces a logically stronger meaning, or only when it avoids extra ignorance implicatures. While this strategy is in principle promising, one challenge is to make sure such an economy principle does not rule out too many logical forms. For example, Simons 2005 observes that Free Choice effects can occur even when both options are compossible. For example, one reading of (43) allows that Jane may sing, Jane may dance, and Jane may do both.

(43) Jane may sing or dance.

To explain this reading, Fox 2007 appeals to a more complex logical form. Where $C'' = \{A, B\}$, the relevant logical form is:

(44) $\mathsf{exh}(C')(\mathsf{exh}(C)(\Diamond(\mathsf{exh}(C'')(A) \vee \mathsf{exh}(C'')(B)))$

A challenge for Fox 2007 is thus to permit both doubly and quadruply exhaustified logical forms without requiring either, and while ruling out exhaustification under the scope of negation.

A final problem for both Chemla 2008 and Fox 2007, solved by either homogeneous alternative semantics or homogeneous dynamic semantics, comes from Romoli & Santorio 2019. Romoli & Santorio 2019 explore the interaction between Free Choice and presupposition projection. Consider (45):

(45) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan.

The first disjunct of (45) is the negation of a Free Choice claim, and the second disjunct presupposes that Mary is permitted to study in Japan. Where $C_{\Diamond A}$ represents a sentence C with the presupposition $\Diamond A$, (45) has the following logical form:

(46) $\neg\Diamond(A \vee B) \vee C_{\Diamond A}$

Romoli & Santorio 2019's observation is that (45) does not presuppose that Maria can study in Japan; this presupposition is filtered by the first disjunct. This suggests Free Choice must be accounted for semantically. For on standard accounts of presupposition projection, a disjunction $A \vee B_C$ presupposes $\neg A \rightarrow C$. Given this assumption, the presupposition of (46) is:

(47) $\neg\neg\Diamond(A \vee B) \rightarrow \Diamond A$

This presupposition is trivial only if $\neg\neg\Diamond(A \vee B) \models \Diamond A$. This is predicted by homogeneous alternative semantics and homogeneous dynamic semantics. By contrast, the implicature based account in Fox 2007 predicts that negated Free Choice sentences do not doubly exhaustify. But if double exhaustification is not present in logical form, a doubly exhaustified logical form cannot provide the local context for interpreting the second disjunct of (45).[33]

11 Conclusion

Building on Santorio 2017, Cariani & Goldstein 2018 develop a version of homogeneous alternative semantics for conditionals in order to validate another combination of principles that appear jointly inconsistent. They develop a battery of incompatibility results, showing that Simplification of Disjunctive Antecedents ($(\mathsf{A} \vee \mathsf{B}) > \mathsf{C} \models (\mathsf{A} > \mathsf{C}) \wedge (\mathsf{B} > \mathsf{C})$) is in *prima facie* conflict with Conditional Excluded Middle ($\models (\mathsf{A} > \mathsf{B}) \vee (\mathsf{A} > \neg\mathsf{B})$). Then using homogeneous alternative semantics, they validate both principles, by giving up the transitivity of entailment. Putting these papers together, the synthesis of homogeneity and alternatives (or dynamics) provides a powerful tool with which to combat various results showing that within classical logic various constellations of plausible principles are jointly incompatible.

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33 Another place to look for concerns about various pragmatic analyses is a growing body of literature suggesting Free Choice differs from scalar implicature with respect to processing time (Chemla & Bott 2014) and acquisition (Tieu et al. 2016).

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