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Relative Positionalism and Variable Arity Relations

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ABSTRACT: Maureen Donnelly's (2016) relative positionalism correctly handles any fixed arity relation with any symmetry such a relation can have, yielding the intuitively correct way(s) in which that relation can apply. And it supplies an explanation of what is going on in the world that makes this the case. But it has at least one potential shortcoming — one that its opponents are likely to seize upon: it can only handle relations with *fixed* arities. It is unable to handle relations with *variable* arities. I argue that, all else being equal, relative positionalism ought nonetheless to be preferred to its closest competitors — at least to the extent that the explanation it supplies of relational application is plausible — even though those competitors *can* handle variable arity relations in addition to fixed arity relations.

Keywords: relation, arity, adicity, symmetry, completion, positionalism, antipositionalism

1. Introduction

Some relations can apply to their relata in different ways while some cannot. *Loving*, for example, can apply to two things in two ways, while *being next to* can apply to two things in only one way. Maureen Donnelly's (2016) relative positionalism correctly handles any fixed arity relation with any symmetry such a relation can have, yielding the intuitively correct way(s) in which that relation can apply. And it supplies an explanation of what is going on in the world that makes this the case. But it has at least one potential shortcoming — one that its opponents are likely to seize upon: it can only handle relations with *fixed* arities. It is unable to handle relations with *variable* arities. In what follows, I argue that, all else being equal, relative positionalism ought nonetheless to be preferred to its closest competitors, viz., Kit Fine's (2000) antipositionalism and Fraser MacBride's (2014) ostrich realism — at least to the extent that the explanation it supplies of relational application is plausible — even though those competitors *can* handle variable arity relations in addition to fixed arity relations.

I begin, in section 2, by introducing relative positionalism, providing all the information the reader will need to understand the discussion that follows. In section 3, I explain why relative positionalism is unable to handle variable arity relations. This involves considering and rejecting a way Donnelly proposes to amend relative positionalism to handle them. I begin section 4 by explaining why relative positionalism's inability to handle variable arity relations means that the relative positionalist cannot comfortably accommodate their existence (e.g., by providing a non-uniform account of fixed and variable arity relations) by introducing relative positionalism's closest competitors and showing that they can provide uniform accounts of both sorts of relation. I then turn to my defense of relative positionalism from the problem of variable arity relations. I explicate a way the relative positionalist can explain the satisfaction of any predicate which putatively expresses a variable arity relation in terms of the application of fixed arity relations, opening the door for her to deny the existence of variable arity relations altogether. I note that whether the relative positionalist may successfully deploy this strategy depends both on the strength of the reasons we have to believe in variable arity relations and on whether relative positionalism has sufficient theoretical advantages over its closest competitors to offset those reasons. But I argue that the reasons we have to believe in variable arity relations are at best defeasible in the remainder of section 4. And in section 5, I argue that relative positionalism has an important virtue that its closest competitors lack, which is sufficient to offset its deficiency in connection with variable arity relations. In particular, I argue that it supplies an explanation of relational application of the sort that a theory of relations ought to supply, while none of its closest competitors does so. I conclude, in section 6, by explaining why this means we ought, all else being equal, to prefer relative positionalism to its closest competitors, at least to the extent that the explanation it supplies of relational application is plausible.

2. Relative Positionalism

Relative positionalism is the view that, when some things stand in a relation, they do so by occupying certain positions of the relation relative to one another. Positions are understood, not as holes or slots in a relation that relata occupy, as they are on absolute positionalist views (such as those defended in Gilmore 2013 and 2014, Orilia 2011 and 2014, and Dixon 2018), but as unary properties that relata instantiate relative to one another. Donnelly calls these properties 'relative properties' (or, when the context is clear, simply 'relatives' or 'properties'). She takes them to be of a sort of entity with which we are already familiar — the same sort of entity as, for example, *being north* (Donnelly 2016: 91). *Being north* is a property, but it is a property that a thing may instantiate only relative to something (e.g., a location). In general, a relative property is a property which can be instantiated by something only relative to something or some things. They have been invoked in other areas of philosophy for various theoretical purposes, and Donnelly enlists them to play a central role in her account.[1]

Relative positionalism is able to correctly handle any relation with fixed arity regardless of its symmetry, which provides her view with an important advantage over absolute positionalist theories, which cannot handle relations with certain symmetries, such as cyclic symmetries. Donnelly achieves this by using group theory to represent the symmetries (or symmetry structures, in Donnelly's parlance) of relations. She begins by imposing the following basic constraints on the behavior of the relative properties she takes to be involved in the application of relations.

> **Donnelly's Base Thesis.**
> Every n-ary relation R has between 1 and $n!$ relative properties, each of which is instantiated by something (relative to anything else) only if that thing stands in R. When some things $x_1, \ldots, x_n$ instantiate R,
>
> (i) each of R's relative properties is instantiated by one of $x_1, \ldots, x_n$, relative to another, ..., relative to the remaining one,
>
> and
>
> (ii) every ordering of $x_1, \ldots, x_n$ is such that at least one of R's relative properties is instantiated by the first relative to the second, ..., relative to the nth.
>
> (Adapted from Donnelly 2016: 91.)

So, for example, if Goethe and Charlotte Buff stand in the binary relation *loving*, then (i) each of *loving*'s relative properties is instantiated by one of Goethe and Buff relative to the other, and (ii) at least one of its relative properties is instantiated by Goethe relative to Buff, and at least one is instantiated by Buff relative to Goethe.

Donnelly represents the symmetry of each relation by its *symmetry group*, which is a group of permutations, i.e., a set of permutations that is closed under an associative operation (function composition, $\circ$) with an identity element (the identity permutation) and an inverse for each element (an inverse permutation). A relation's symmetry group is determined by the particular identities (if any) amongst its relative properties. We can identify it by observing how the relation behaves, which we can do by observing the behavior of certain predicates which express that relation. These predicates must be such that the implications of a relational claim involving the predicate "concerning the order of relational application are completely determined in some fixed way by the order of the terms denoting the relata" relative to the predicate (*Ibid.*: 84, fn. 13). I'll call such predicates 'order-determined'. In particular, permutations P and Q of terms denoting objects in the domain of the relation with respect to an order-determined predicate which expresses it will yield equivalent claims exactly when they are in the symmetry group of the predicate, which is, on Donnelly's account, also the symmetry group of the relation.

> **Definition of Symmetry Groups.**
>
> For any [order-determined] n-place predicate '$\boldsymbol{R}$' standing for any n-ary relation R, ... $\mathrm{SYM}_{\boldsymbol{R}}$ (the symmetry group for '$\boldsymbol{R}$' [and for R]) [is] the set of permutations such that for any terms 'x_1', ..., 'x_n' referring to objects in the domain of R,
>
> (*) $Rx_1 \ldots x_\mathrm{n}$
>
> is equivalent to
>
> ($*_\mathrm{P}$) $Rx_{\mathrm{P(1)}} \ldots x_{\mathrm{P(n)}}$.
>
> (*Ibid.*: 83)

As Donnelly notes (*Ibid.*), this set is always a subgroup of the group of all of the possible permutations of $\{1, \ldots, n\}$ — the *symmetric group of degree* n, or S_n. That is, the set is a subset of that group and itself forms a group under the group operation of function composition. These sets also correspond to the possible identities amongst the relative properties of an arbitrary relation. The condition under which the relatives associated with two given orderings of terms/relata are identical is given by

> ($\bigtriangledown$) $\tau_\mathrm{Q} = \tau_{\mathrm{Q}^*}$ iff there is some $\mathrm{S} \in \mathrm{SYM}_{\boldsymbol{R}}$ such that $\mathrm{Q}^* = \mathrm{SQ}$, where for $\mathrm{Q} \in \mathrm{S}_n$, τ_Q is the property which (*) entails that $x_{\mathrm{Q(1)}}$ has relative to $x_{\mathrm{Q(2)}}$, ..., relative to $x_{\mathrm{Q}(n)}$. (*Ibid.*: 94)

That is, such relatives are identical just in case "they can be transformed into one another by a permutation in the symmetry group" of the predicate/relation (*Ibid.*). It is through the use of this principle and the definition of symmetry groups that we can come to know exactly how many relative properties a given relation has. It is important to emphasize, however, that it is not *because* of its symmetry group (in a metaphysical sense of 'because') that a relation has the number of relative properties it has. A relation's symmetry group is just a mathematical representation of the way(s) it can apply.

The relative positionalist identifies the way(s) a relation R's relative properties can be assigned to n objects (given the constraints imposed by Donnelly's Base Thesis, $\mathrm{SYM}_{\boldsymbol{R}}$, and ($\triangledown$)) with the way(s) R can apply to n objects. And, in general, if R has k relative properties, then they can be so assigned in k ways.[2] So, for example, any relation with a single relative property will be able to apply in only one way to n objects. Such a relation has *complete symmetry*. Its symmetry group contains all $n!$ of those orderings, since every permutation of terms flanking any order-determined predicate which expresses it results in an equivalent claim. *Being next to*^{2} is an example of such a relation. (Henceforth, I indicate the arity of a fixed arity relation denoted by a term with a superscripted positive integer in the way I have just done.) This can be seen by considering the order-determined predicate '... is next to ...', which expresses it. This predicate has the symmetry group

$$\mathrm{SYM}_{\ldots\text{ is next to }\ldots} = \{[1\,2], [2\,1]\},$$

where $\ulcorner[x_1\, x_2 \ldots x_n]\urcorner$ denotes the permutation of $\{1, 2, \ldots, n\}$ that maps 1 to x_1, 2 to x_2, ..., and n to x_n. This can be seen by noting that corresponding instances of the following schemas are equivalent to one another.

$$\begin{array}{rl} (*) & x_1 \text{ is next to } x_2 \\ (*_{[2\,1]}) & x_2 \text{ is next to } x_1 \end{array}$$

By the Base Thesis, when *being next to*^{2} applies to two things, one of its relative properties (for all we currently know, not necessarily the same one) is instantiated by each of them relative to the other in each of the two possible orderings of them. And because

$$[2\,1] \in \mathrm{SYM}_{\ldots\text{ is next to }\ldots} \text{ and } [2\,1] \circ [1\,2] = [2\,1],$$

we know by ($\triangledown$) that the relative property that x_1 instantiates relative to x_2 is the same as that which x_2 instantiates relative to x_1.[3] So we know that *being next to*^{2} has a single relative property, τ_1, and hence that there is, according to relative positionalism, only one way it can apply to two objects, such as Goethe and Buff (figure 1).

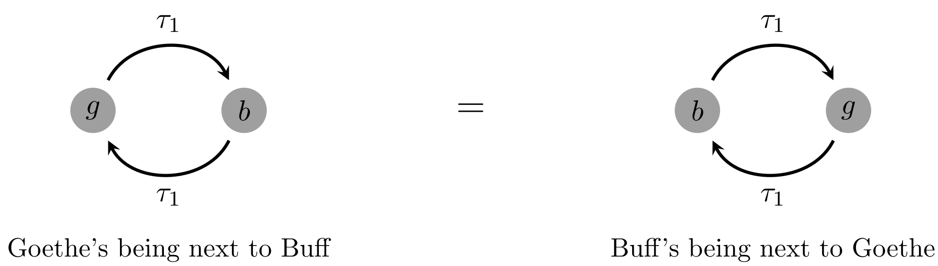


Figure 1. The fixed arity relation *being next to*^{2}

In this diagram and the ones to follow, a relative property instantiated by one thing relative to another is represented by an arrow going from the first thing to the second.

This is as it should be. There is, intuitively, only one way in which *being next to*^{2} can apply to two objects; there is only one completion of a given sort (fact, state of affairs, or proposition) that can result from it applying to two objects.[4]

An n-ary relation with $n!$ relative properties, on the other hand, will be able to apply in $n!$ ways to n objects. Such a relation has *complete non-symmetry.* Its symmetry group contains only the identity permutation, since the only permutation of terms flanking any order-determined predicate which expresses it that results in an equivalent claim is the one which leaves the terms where they are. *Loving*^{2} is an example of such a relation. This can be seen by considering the order-determined predicate '... loves ...', which expresses it. This predicate has the symmetry group

$$\mathrm{SYM}_{\ldots\text{ is next to }\ldots} = \{[1\,2]\}.$$

This can be seen by noting that corresponding instances of the following schemas are not equivalent to one another.

$$\begin{array}{rl} (*) & x_1 \text{ loves } x_2 \\ (*_{[2\,1]}) & x_2 \text{ loves } x_1 \end{array}$$

By the Base Thesis, when *being next to*^{2} applies to two things, one of its relative properties (for all we currently know, not necessarily the same one) is instantiated by each of them relative to the other in each of the two possible orderings of them. And because

$$\text{there is no } \mathrm{P} \in \mathrm{SYM}_{\ldots\text{ loves }\ldots} \text{ such that } \mathrm{P} \circ [1\,2] = [2\,1],$$

we know by ($\bigtriangledown$) that the relative property that x_1 instantiates relative to x_2 is distinct from that which x_2 instantiates relative to x_1. So we know that *loving*^{2} has two relative properties, τ_2 and

τ_3, and hence that there are, according to relative positionalism, two ways *loving*^{2} can apply to two objects (figure 2).

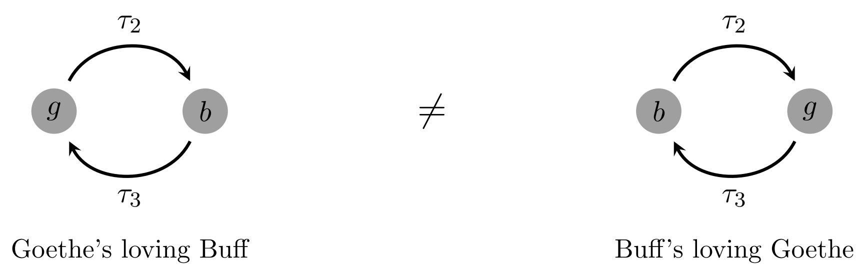


Figure 2. The fixed arity relation *loving*^{2}

Again, this is as it should be. There are, intuitively, two ways in which *loving*^{2} can apply to two objects; there are two completions of a given sort that can result from it applying to two objects.

Donnelly (*Ibid.*: 91) strives to provide interpretations of the relative properties invoked in the cases she discusses. While they are not required for formal purposes, they are helpful in making sense of why each relative is assigned in the way it is. Without such interpretations, one might worry that the assignment of relative properties among the relevant relata is arbitrary. Why should τ_2 be the relative property that Goethe instantiates relative to Buff and τ_3 be the one that Buff instantiates relative to Goethe when Goethe loves Buff and not vice versa? In this case, Donnelly interprets τ_2 and τ_3 as *being a lover* (*relative to*) and *being beloved* (*relative to*), respectively. Now the reason is clear. It is because Goethe is a lover relative to Buff while Buff is beloved relative to Goethe. And while Donnelly does not explicitly provide an interpretation of the single relative of *being next to*^{2}, she makes a remark (*Ibid.*: 86) that suggests that it could be interpreted as something like *being adjacent* (*relative to*).

Many of relative positionalism's competitors, including some absolute positionalist theories, are able to correctly handle relations with complete symmetry and complete non-symmetry just as well as relative positionalism. Relative positionalism's primary asset over such views is its ability to handle any relation with a *partial symmetry.* The symmetry group of such a relation is a proper non-trivial subgroup of its symmetric group, since some, but not all, non-identity permutations of terms flanking any order-determined predicate which expresses it results in an equivalent claim. Only relations with arities greater than 2 may have partial symmetries. *Being between*^{3}, for example, permits of transpositions between two of its arguments but not of any permutations involving the other argument, assuming those arguments are pairwise distinct. And while some absolute positionalist theories can handle relations with this sort of symmetry, they cannot handle relations with other sorts of partial symmetries, most notably, those with cyclic symmetries, like *being arranged clockwise in that order*^{3}. I will not discuss why they cannot, since it is not relevant to my purpose here. But see Fine 2000: 17–18 (esp. fn. 10) and Donnelly 2016: §5.3 for discussions. I will nonetheless discuss how relative positionalism handles this last relation, since it will help the reader understand the problem of variable arity relations, which I present in the next section.

In general, an n-ary relation with partial symmetry has, according to relative positionalism, k relatives, where $1 < k < n!$ and k is a factor of $n!$. An order-determined predicate that expresses *being arranged clockwise in that order*^{3}, such as '…, …, and … are arranged clockwise', has the symmetry group

$$\mathrm{SYM}_{\text{..., ..., and ... are arranged clockwise in that order}} = \{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}.$$

This can be seen by noting that corresponding instances of the following schemas are equivalent to one another.

(*) x_1, x_2, and x_3 are arranged clockwise in that order
($*_{[2\,3\,1]}$) x_2, x_3, and x_1 are arranged clockwise in that order
($*_{[3\,1\,2]}$) x_3, x_1, and x_2 are arranged clockwise in that order

But none is equivalent to any instance which results from any other permutation of x_1, x_2, and x_3 with respect to the predicate. By the Base Thesis, when *being arranged clockwise in that order*^{3} applies to three things, some relative property of that relation (for all we currently know, not necessarily the same one) is instantiated by each of them relative to the others in each of the six possible orderings of them. And because

(i) $[2\,3\,1] \in \mathrm{SYM}_{\text{..., ..., and ... are arranged clockwise in that order}}$ and $[2\,3\,1] \circ [1\,2\,3] = [2\,3\,1]$
and
(ii) $[3\,1\,2] \in \mathrm{SYM}_{\text{..., ..., and ... are arranged clockwise in that order}}$ and $[3\,1\,2] \circ [1\,2\,3] = [3\,1\,2]$,

we know by ($\bigtriangledown$) that the relative property that x_1 instantiates relative to x_2, relative to x_3 is the same as (i) that which x_2 instantiates relative to x_3, relative to x_1, and (ii) that which x_3 instantiates relative to x_1, relative to x_2. Furthermore, since

(i) $[2\,3\,1] \in \mathrm{SYM}_{\ldots,\ \ldots,\ \text{and}\ \ldots\ \text{are arranged clockwise in that order}}$ and $[2\,3\,1] \circ [1\,3\,2] = [2\,1\,3]$ and

(ii) $[3\,1\,2] \in \mathrm{SYM}_{\ldots,\ \ldots,\ \text{and}\ \ldots\ \text{are arranged clockwise in that order}}$ and $[3\,1\,2] \circ [1\,3\,2] = [3\,2\,1]$,

we know by ($\triangledown$) that the relative property x_1 instantiates relative to x_3, relative to x_2 is the same as (i) that which x_2 instantiates relative to x_1, relative to x_3, and (ii) that which x_3 instantiates relative to x_2, relative to x_1. Moreover, because

> there is no permutation $\mathrm{P} \in \mathrm{SYM}_{\ldots,\ \ldots,\ \text{and}\ \ldots\ \text{are arranged clockwise in that order}}$ such that, e.g., $\mathrm{P} \circ [1\,3\,2] = [1\,2\,3]$,

we know by ($\triangledown$) that the relative property that x_1 instantiates relative to x_2, relative to x_3 must be distinct from that which x_1 instantiates relative to x_3, relative to x_2. So we know that the relation has two relative properties, τ_4 and τ_5, which Donnelly interprets, respectively, as *being clockwise in front of* (*relative to*) and *being clockwise behind* (*relative to*).

This means that the relative positionalist will say that there are two ways in which *being arranged clockwise*[2] can apply to three objects, such as Larry, Curly, and Moe (figure 3).

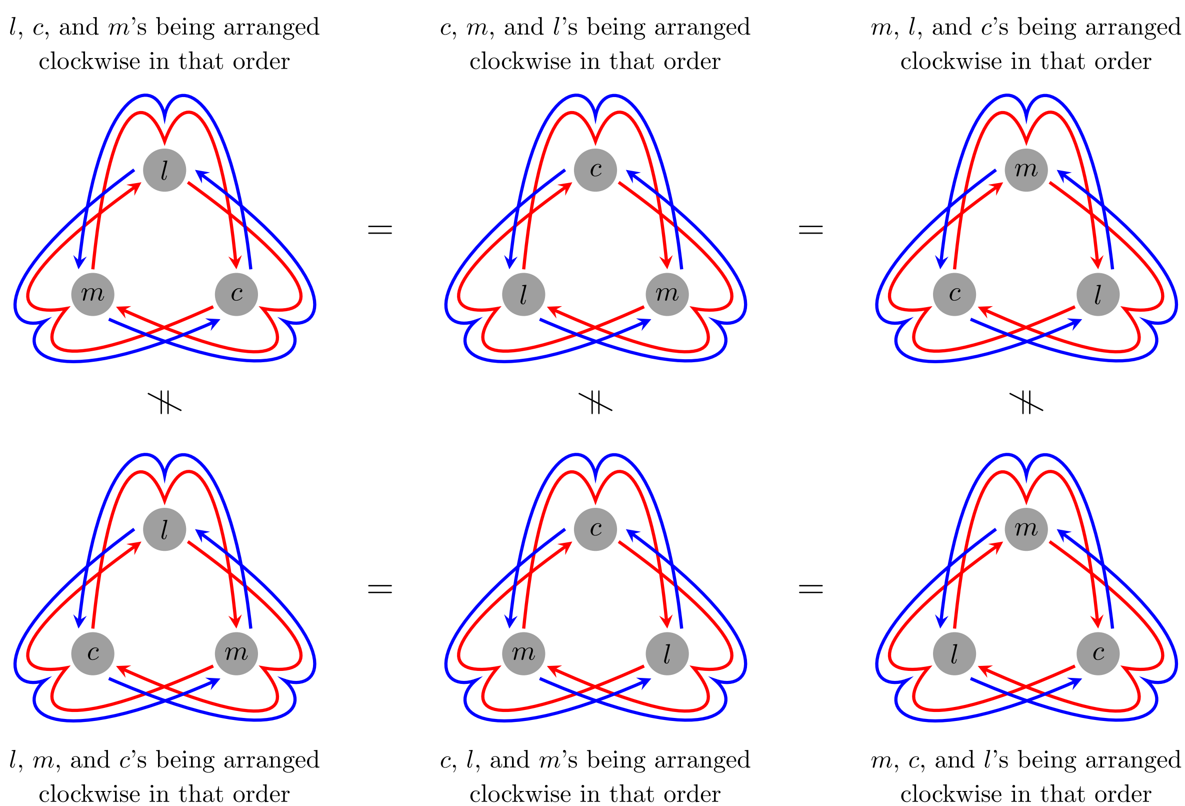


Figure 3. The fixed arity relation *being arranged clockwise in that order*[3]

In this diagram and the ones to follow, a relative property instantiated by one thing relative to another, relative to another is represented by an arrow going from the first thing to the second, then to the third thing. In this diagram, red arrows depict assignments of τ_4, while blue arrows depict assignments of τ_5.

On Donnelly's view, the completions depicted in the top row of the diagram are identical to one another, as are those depicted in the bottom row. But any in the top row is distinct from any in the bottom row. This is because those in the top row are the result of the same relative property assignments amongst Larry, Curly, and Moe (left column below), while those in the bottom row are the result of a different set (right column).

- l has τ_4 relative to c, relative to m
- c has τ_4 relative to m, relative to l
- m has τ_4 relative to l, relative to c
- l has τ_5 relative to m, relative to c
- c has τ_5 relative to l, relative to m
- m has τ_5 relative to c, relative to l

- l has τ_4 relative to m, relative to c
- c has τ_4 relative to l, relative to m
- m has τ_4 relative to c, relative to l
- l has τ_5 relative to c, relative to m
- c has τ_5 relative to m, relative to l
- m has τ_5 relative to l, relative to c.[5]

Again, relative positionalism gets things right. There are, intuitively, two ways for Larry, Curly, and Moe, to complete this relation. First, they may do so in the order just specified. Or Larry, Moe, and Curly can do it in *that* order instead. These are the ways depicted on the top and bottom rows of figure 3, respectively. In general, there are, intuitively, two ways in which *being arranged clockwise in that order*[3] can apply to three objects; there are two completions of a given sort that can result form it applying to two objects.

The remarkable thing about relative positionalism, as opposed to absolute positionalist theories in particular, is that, for *any* n-ary relation, it provably yields, without exception, the correct number of ways in which it can apply to n objects. For a proof of this claim, see Donnelly 2016: 94–96.[6] While I will not go through Donnelly's proof, I hope the examples I have discussed have gone some way toward convincing the reader that this is so. They are also sufficient to provide the background necessary to see why the view cannot handle variable arity relations. I turn now to that topic.

3. Variable Arity Relations

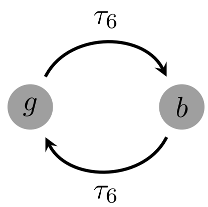
While a fixed arity relation can take only a single number of arguments, a variable arity (or multigrade) relation can take more than one number.[7] A plausible example of a variable arity relation is *meeting for lunch*. (I indicate that a relation has a variable arity by assigning it no superscript.) This is something two people can do. But three can also do it. As can four. And so on. It seems that, when Goethe and Buff meet for lunch, they stand in the very same relation to one another as do Larry, Curly, and Moe when *they* meet for lunch. There is a very simple reason that relative positionalism cannot accommodate variable arity relations: it is formulated in such a way that it applies only to fixed arity ones. This comes out particularly clearly by looking back to the Definition of Symmetry Groups, which applies only to fixed arity predicates expressing fixed arity relations.

Donnelly considers a modification of relative positionalism in an attempt to extend it to cover variable arity relations. But while this modification can handle *some* variable arity relations (like *meeting for lunch*), it cannot handle *all* such relations. Donnelly proposes that the relative positionalist "suppose that some of the relative properties ... may hold relative to sequences of varying lengths" (2016: 81, fn. 5). In implementing this proposal, the relative positionalist would proceed by regarding any predicate which putatively expresses a single variable arity relation as a member of a class of fixed arity predicates, each expressing an appropriate fixed arity relation. Those expressing *meeting for lunch* are indicated by the following elliptical list.

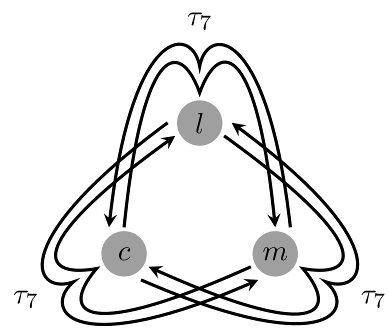
- ... and ... are meeting for lunch
- ..., ..., and ... are meeting for lunch
- ..., ..., ..., and ... are meeting for lunch

  ⋮

The relative positionalist would then proceed as she normally would with any other set of fixed arity predicates. Applying the Definition of Symmetry Groups to each would yield the result that the nth predicate in this list, and so the relation it presumably expresses, has a symmetry group identical to the symmetric group of degree $n+1$. That is, its symmetry group includes every possible permutation of terms with respect to the predicate (since, when some things meet for lunch, they do so in every possible ordering of them). ($\bigtriangledown$) would then tell us that each of the relations expressed by these predicates has one relative property, which applies across every possible ordering of objects to which each relation applies. The binary predicate, for example, would be akin to '... is next to ...', insofar as relative positionalism would say that it expresses a binary relation with a single relative property, τ_6, which, when the relation applies to two objects, is instantiated by each relative to the other (left side of figure 4). The ternary predicate would be akin to '. . . , . . . , and . . . are triplets' (if this is a relational predicate), insofar as it would express a ternary relation with a single relative property, τ_7, which, when the relation applies to three objects, each instantiates relative to each of the others, relative to the remaining one (right side of figure 4).



Goethe and Buff's meeting for lunch



Larry, Curly, and Moe's meeting for lunch

Figure 4. The variable arity relation *meeting for lunch*

Donnelly's amendment to relative positionalism would then become operative. Effectively, her proposal is to allow relative properties to be able to be instantiated by something relative to more than one number of things. This would allow the relative positionalist to identify the relative properties associated with these predicates with one another (so that $\tau_6 = \tau_7$). This would enable the relative positionalist to identify the relations expressed by them. (Absent Donnelly's amendment, one might take the fact that some relatives are instantiated relative to different numbers of things to be enough to distinguish them.) The relative positionalist would then be able to countenance the variable arity relation *meeting for lunch*, and say that it is expressed by each of the predicates in the list above.

One might attempt to undermine Donnelly's proposal by arguing in various ways that the relations expressed by the predicates in the above list must be pairwise distinct. In the interest of space, however, I will ignore such arguments, since there is a more pressing problem with Donnelly's proposed modification. While nothing may stand in the way of the relative positionalist identifying the relatives associated with the various predicates in the list above, and thus identifying the relations expressed by them, this is an artifact of the example; each of these predicates and relations is completely symmetric. Completely symmetric predicates and relations stand apart from their counterparts with incomplete symmetries (i.e., symmetries that are anything other than complete) in that each has a single relative property, but, more importantly, in that the number of relatives each has is the same no matter its arity. Relations with incomplete symmetries which differ in arity may differ in the number of relative properties they have. Thus a variable arity relation which has incomplete symmetry in some of its manifestations of arity may cause problems for relative positionalism.

A plausible candidate for such a relation, and one that causes problems for relative positionalism, is the variable arity counterpart of the fixed arity relation which Donnelly uses to motivate her view over absolute positionalist theories, viz., *being arranged clockwise in that order*. Three things can be arranged clockwise in a given total order. But so can four. And five. And so on. Arguably, two can as well, and in the interest of keeping the discussion that follows as simple as possible, I will suppose that this is so. Moreover, as in the case of *meeting for lunch*, it seems that it is the same relation that things stand in, no matter their number. When Goethe and Buff are arranged clockwise in that order, they seem to stand in the same relation as do Larry, Curly, and Moe when *they* are arranged clockwise in that order. And they seem to stand in the same relation as do Leonardo, Donatello, Raphael, and Michelangelo when *they* are arranged clockwise in that order. But the relative positionalist can't identify the clockwise arrangement relations which hold between these different numbers of entities, since relative positionalism demands that they have different numbers of relative properties. And it is not clear how a single relation could have different numbers of relative properties. Let me explain why these things are the case.

In her attempt to accommodate the allegedly variable arity relation *being arranged clockwise in that order*, the relative positionalist would, as in the case of *meeting for lunch*, proceed by regarding any predicate which putatively expresses this variable arity relation as a member of a class of fixed arity predicates, each expressing an appropriate fixed arity relation.

- ... and ... are arranged clockwise in that order
- ..., ..., and ... are arranged clockwise in that order
- ..., ..., ..., and ... are arranged clockwise in that order

  ⋮

The relative positionalist would then apply the Definition of Symmetry Groups to each predicate in the list. Applying it to the first would yield the result that the relation it expresses has the same symmetry group as *being next to*^{2}. Its symmetry group contains all of the possible orderings of its relata, since every permutation of terms flanking the predicate results in an equivalent claim. Thus that relation, like *being next to*[2], will have one relative property, τ_8 (left side of figure 5). Applying the Definition of Symmetry Groups to the second predicate in the list would yield the result that the relation it expresses (unsurprisingly) has the same symmetry group as the ternary fixed arity relation *being arranged clockwise in that order*[3] (right side of figure 5). Thus that relation will have two relative properties. (Since this variable arity relation would presumably replace the ternary fixed arity relation in the relative positionalist's ontology, I'll reassign its relative properties, τ_4 and τ_5.)

Goethe and Buff's being arranged clockwise in that order

Larry, Curly, and Moe's being arranged clockwise in that order

Figure 5. The variable arity relation *being arranged clockwise in that order*

As before, red arrows depict assignments of τ_4, while blue arrows depict assignments of τ_5.

But Donnelly's suggestion that relative properties can apply relative to more than one number of things is of no help in this case, since it is not just the number of things relative to which a relative property of the putative variable arity relation can be instantiated that differs when the relation applies to different numbers of relata. The *number of relative properties itself* differs when the relation applies to different numbers of relata as well. The problem this poses for relative positionalism is that it is not clear how a single relation — the variable arity *being arranged clockwise in that order* — could have both one and two relative properties.[8] The problem is akin to that raised by Fine (2000: 22) concerning absolute positionalism's prospects of handling variable arity relations. Though he puts the point a bit differently, it would require that such a relation has both n and m positions for some $n \neq m$. Thus the relative positionalist has a problem analogous to that which certain absolute positionalist views have handling variable arity relations.

The relative positionalist might reply by insisting that there is a single variable arity relation expressed by each of these predicates, which has all of the relative properties for each of the arities the relation can manifest. In general, a variable arity relation which can apply to n objects for any $n \in \{2, 3, \ldots\}$ would have $\alpha_2 + \alpha_3, + \ldots$ relative properties, where α_i is the number that are, according to relative positionalism, instantiated whenever the relation applies to i things for every $i \in \{2, 3, \ldots\}$. *Meeting for lunch* would have $1 + 1 + \ldots$ relative properties, while *being arranged clockwise in that order* would have $1 + 2 + \ldots$ Relative properties could be likened, on this account, to differently sized yokes attached to a single plow (the relation) that can be used to attach the plow to different numbers of oxen (the relata). But putting the somewhat baroque character of the resulting view aside, Donnelly makes a modification of relative positionalism in order to address another concern, which prevents the relative positionalist from availing herself of this reply. She notes (2016: 98–99) that relative positionalism, as she originally formulates it and as I have formulated it in section 2, is committed to two forms of instantiation: (i) non-relative instantiation — the relation between some relata and the relation that relates them, and (ii) relative instantiation — the relation between some relata and the relative properties of the relation that relates those relata. Donnelly concedes that this is a cost of her view, as originally formulated, particularly since other theories of relations, particularly MacBride's ostrich realism, which I will discuss more in the next section, are committed only to a single one of these relations (usually non-relative instantiation).

To address this concern, Donnelly proposes that the relative positionalist abandons relations altogether, and supposes instead that relational predicates are associated directly with a certain number of relative properties. This "relationless" relative positionalism is committed only to relative instantiation. There are no relations, and hence there is nothing that stands in the non-relative instantiation relation. So there is no reason for the relative positionalist to posit this relation at all. Indeed, doing so would be needlessly extravagant. Answering the "two forms of instantiation" concern in this way, however, prevents the relative positionalist from making use of the "one plow, many yokes" reply to the variable arity problem, according to which the relative positionalist says that each variable arity relation has all of the relative properties needed for each of the arities the relation can exhibit. First of all, according to relationless relative positionalism, there are no relations, and so there is no *single* relation that can possess the numerous different relative properties associated with the different arities a variable arity predicate can manifest. But more importantly, the absence of relations in general means that there are no *variable arity* relations, and so the view obviously cannot handle variable arity relations any better than Donnelly's original formulation can.

4. Do Variable Arity Relations Exist?

The simple fact that relative positionalism cannot handle variable arity relations doesn't mean that the relative positionalist must reject their existence. She might treat them in an alternate way. But this would yield a non-uniform account of relations, which would be awkward at best, and at worst will put relative positionalism at a decided theoretical disadvantage relative to its closest competitors. The only two views out there that can properly handle relations with all of the symmetries that relative positionalism can handle (including relations with cyclic symmetries), viz., Fine's (2000: §§4–6) antipositionalism and MacBride's (2014) ostrich realism, can also handle every variable arity relation we might take to exist. Moreover, they offer a *uniform* treatment of both sorts of relation.

Antipositionalism (*Ibid.*: §§4–6), as its name suggests, does not posit positions in relations. The ways (or *manners*, in Fine's parlance) in which a given relation may apply to some things are determined not by facts about the internal structure of the completions that result from its application. Instead, they are fixed by identity and distinctness relationships which exist between completions of it by different sets of objects. For example, the manner in which Goethe and Buff complete *loving*[2] in Goethe's loving Buff is the same, on Fine's view, as exactly one of the two manners in which W. B. Yeats and Maud Gonne complete that relation in Yeats's loving Gonne and Gonne's loving Yeats, and distinct from the other. Which identity and distinctness relationships hold of these two possible but mutually exclusive sets is, according to the antipositionalist, a matter of brute fact.[9] This approach affords antipositionalism a lot of flexibility when it comes to handling relations with any possible symmetry, including variable arity relations. It can explain why any relation with any symmetry can apply in the way(s) it can to some objects as long as there are some other objects to which it applies in each of those ways.

For various reasons, MacBride rejects antipositionalism and endorses a view that he calls 'ostrich realism'. According to his view, there is no explanation whatsoever for why any relation can apply in the way(s) that it can. Each such fact is taken as primitive. Ostrich realism has even more flexibility than Fine's antipositionalism when it comes to correctly handling relations with any possible symmetry, including variable arity relations. There need be no objects to which a given relation applies (such objects are required on Fine's account) for MacBride to explain why it can apply in the way(s) that it can, since he supplies no explanation of this fact at all.[10] What is common to antipositionalism and ostrich realism is that they have no special problem correctly handling any relation with any symmetry one might throw at them — even if that relation has a variable arity. Neither view posits machinery internal to completions, akin to the positions of absolute positionalism or the relative properties of relative positionalism, that might give incorrect results about the number of ways a given relation can apply, or prevent the view from being able to accommodate the relation altogether.

Relative positionalism's deficiency concerning variable arity relations is a particularly tempting basis on which the antipositionalist or ostrich realist might build an argument against their rival. Because antipositionalism and ostrich realism can provide uniform accounts of fixed and variable arity relations, they enjoy a theoretical advantage over relative positionalism. If the relative positionalist is going to have a fighting chance against these two competitors, she would be better off simply denying the existence of variable arity relations. But she must find an alternative explanation for the truth of relational claims that allegedly involve commitment to them. There must be *something* going on in the world that makes it true that Goethe and Buff are arranged clockwise in that order, and that Larry, Curly, and Moe are arranged clockwise in that order, even if it is not the application of a variable arity relation that does so. Fortunately, there is a way for the relative positionalist to provide such an explanation. She can say that any predicate which allegedly involves a commitment to a variable arity relation actually expresses a relation with an appropriate fixed arity. Instead of there being, for example, one variable arity relation *meeting for lunch*, there are many fixed arity meeting for lunch relations — one for each of the numbers of things that can possibly meet for lunch. And instead of there being one variable arity relation *being arranged clockwise in that order*, there are as many fixed arity clockwise arrangement relations as there are numbers of things that can possibly be arranged clockwise in a given total order.[11]

It is good news for the relative positionalist that it seems that this strategy can always be implemented, no matter the predicates allegedly expressing a variable arity relation with which one is confronted. For any such predicate one happens upon, one can always posit a class of fixed arity relations in terms of which one can analyze its satisfaction, and reject the existence of the alleged variable arity relation. But if the relative positionalist is going to be justified in making use of the strategy, the following two conditions must be met.

(C1) Any reasons we have for believing in the existence of variable arity relations must be defeasible.

(C2) Relative positionalism must have advantages over its closest competitors, viz., antipositionalism and ostrich realism, that are sufficient to defeat the reasons we have for believing in variable arity relations, all else being equal (i.e., assuming that relative positionalism does not have any disadvantages relative to its competitors that would tip the balance back in their favor).

If (C1) were not met, then the strategy just outlined to do away with variable arity relations would be decidedly unacceptable, since it implies that variable arity relations do not exist. And if (C2) were not met, then the strategy would be unmotivated and thus *ad hoc*, given that antipositionalism and ostrich realism can handle variable arity relational claims in a non-revisionary way.[12] Fortunately for the relative positionalist, both of these conditions appear to be met. I argue that (C1) is met in the remainder of this section. I argue that (C2) is met in the section that follows.

What reasons do we have for believing in the existence of variable arity relations? The strongest (and only) one that I know of is discussed by MacBride (2005: 568–93). He notes first that "universals are given to us (in one guise) as the entities to which we are ontologically committed by our use of predicates" (*Ibid.*: 571). Second, he notes that "there is a wide class of predicate — predicates that are employed not only in ordinary usage but also in a wide variety of theoretical contexts — that appear to be ontologically committed to the existence of multigrade universals" (*Ibid.*). The predicates that MacBride seems to have in mind are *collective* variable arity predicates, i.e., variable arity predicates which are not distributive, where, roughly, a distributive predicate is a predicate such that if it is satisfied by some things (jointly), then it is satisfied by each of them individually. The satisfaction of a distributive variable arity predicate, like '____ are spherical' by some things (for example, the earth and the sun) can be explained by the instantiation of a property by each of those things. (I adopt the convention that an occurrence of a baseline segment represents an argument *place* of a predicate, where each place represents a single grouping of a variable number of serially adjacent argument positions, which form a list when filled with arguments.[13]) This is not so for collective variable arity predicates. The members of the two sets of fixed arity predicates

I discussed in the previous section, which allegedly express the variable arity relations *meeting for lunch* and *being arranged clockwise in that order*, could be understood as instances of collective variable arity predicates, viz., '____ are meeting for lunch' and '____ are arranged clockwise in that order'.

Thus, MacBride thinks that our use of relational predicates, in general, commits us to the existence of corresponding relations. So, MacBride concludes, our use of collective variable arity relational predicates, in general, commits us to the existence of corresponding variable arity relations. I grant that MacBride's argument is strong enough to establish that, absent the presence of countervailing considerations, one should recognize the existence of variable arity relations (if one recognizes the existence of relations at all). But I do not grant that it *conclusively* establishes their existence. My reason for thinking this is that MacBride's operative premise (that our use of relational predicates, in general, commits us to the existence of corresponding relations), isn't even intended to entail that every predicate expresses a relation. Theories abound which claim that certain relational predicates do not express relations. An eliminativist about the mental, for example, might deny that relational mental predicates like '... believes ...' express relations. Nor is MacBride's premise even intended to entail that every predicate expresses the relation we are pretheoretically inclined to think it does. Other theories abound which supply a revisionary account of what certain relational predicates express. The mind-brain identity theorist, for example, might hold that '... believes ...' expresses not a *sui generis* mental relation, but instead a (perhaps complex) physical relation.

So the fact that the relative positionalist supplies a revisionary account of an entire class of predicate, viz., collective variable arity predicates, is not in principle objectionable in light of MacBride's argument. If it were, then widely accepted views like eliminativism and the identity theory would have to be rejected. But of course, the relative positionalist must have good reasons to deny the existence of variable arity relations, just as the eliminativist must provide good reasons to deny that relational mental predicates express relations, and just as the identity theorist must provide good reasons to hold that they express physical relations rather than mental ones. Fortunately for relative positionalism, it has an important theoretical virtue which both antipositionalism and ostrich realism lack, which is substantial enough to override MacBride's argument for the existence of variable arity relations. I turn now to a discussion of this virtue.[14]

5. Why Do We Theorize about Relations?

I now set out to establish that (C2) is met — that relative positionalism has an advantage over antipositionalism and ostrich realism that is, other things being equal, sufficient to defeat the reason we have to believe in variable arity relations. That advantage consists of the fact that relative positionalism supplies the sort of explanation of relational application that a theory of relations ought to supply, while neither antipositionalism nor ostrich realism does so. This advantage is quite substantial, since supplying such an explanation is plausibly one of the primary goals of a theory of relations and their application. I begin by spelling out this explanatory target.

Ideally, a theory of relations will supply not only an account of their nature, but also an account of their application to things. The latter involves accomplishing at least two things. First, such a theory will ideally correctly handle the application of any relation we take to exist. That is, it must say, for any given relation in our ontology, that it can apply to some things in the ways that we think it can. Thus the theory should say, for example, that the binary relation *being next to*^{2} can apply to two objects in only one way, and that *loving*^{2} can apply to two objects in two ways. As we have seen, antipositionalism and ostrich realism fare just as well as (and, if we include variable arity relations, better than) relative positionalism with respect to this issue. The second thing a theory of relations must do if it is to provide an (adequate) account of relational application is to supply answers to the following two questions.

- (Q1) Why can each relation apply in the way(s) it can?
- (Q2) Why are the ways in which some relations can apply to their relata the same as one another, and those in which others can apply to their relata different from one another?

As far as (Q1) goes, a theory of relations will ideally explain why, for example, *being next to*^{2} can apply to two things in only one way. It will also explain why *loving*^{2} can apply to two things in two ways. I expect little skepticism about the importance of the role (Q1) has played in the literature on relations. It is, after all, one of the central questions which preoccupies Fine (2000), MacBride (2014), Donnelly (2016), and others in their developments of their respective views about relations. (While MacBride ultimately adopts a theory of relations which, as I will explain below, does not supply an answer to (Q1), it is evident that he takes the question seriously, and adopts such a theory only as a last resort, after finding other theories deficient in various ways.)

(Q2) deserves more discussion, and not just in its defense as an essential part of developing a theory of relations. It is important to get clear first on exactly what the question is. Examples will help. According to (Q2), a theory of relations will ideally explain why, for example, the two ways in which *loving*^{2} can apply to two things are the same as the two ways in which *hating*^{2} can do so, and why they are different from the way in which *being next to*^{2} can apply to two things.[15] But this example fails to illustrate that there is more to (Q2) than just the question of why the ways in which some n-ary relations can apply to their relata are the same in number, while those in which others can do so are different in number. Distinct n-ary relations which can apply to n objects in the same number of ways can nonetheless apply in different ways. A different example will illustrate this.

The quaternary relations *being arranged clockwise in that order*^{4} and *being closer together than*$^{2\text{-}2}$ can apply in the same number of ways as one another to four objects. (Here the superscript '$^{2\text{-}2}$' indicates that the relation relates the distances between two pairs of objects, as in 'Alice and Bob are closer together than Carol and Diane'.) But, I will argue, they are not the same ways, as are, for example, the ways in which *being arranged clockwise in that order*^{4} and *being arranged counterclockwise in that order*^{4} can apply to four objects. The reader can use the same method from section 2 to confirm that the symmetry groups of these relations are

$$\begin{aligned}
&\text{SYM}_{\ldots,\ \ldots,\ \ldots,\ \text{and}\ \ldots\ \text{are arranged clockwise in that order}} = \{[1\,2\,3\,4], [2\,3\,4\,1], [3\,4\,1\,2], [4\,1\,2\,3]\}\\
&\text{SYM}_{\ldots\ \text{and}\ \ldots\ \text{are closer together than}\ \ldots\ \text{and}\ \ldots} = \{[1\,2\,3\,4], [1\,2\,4\,3], [2\,1\,3\,4], [2\,1\,4\,3]\},
\end{aligned}$$

and that each has six relative properties, and thus that each can apply to four objects in six ways. When *being closer together than*$^{2\text{-}2}$ applies to four objects, there are pairs of these objects which can be transposed without resulting in a new completion. But there are no such pairs in the case of any application of *being arranged clockwise in that order*^{4} to four objects. Any such transposition will result in a new completion. For example,

- Alice and Bob's being closer together than Carol and Diane

is the same state of affairs as

- Bob and Alice's being closer together than Carol and Diane

But

- Alice, Bob, Carol, and Diane's being arranged clockwise in that order

is a distinct state of affairs from any of the following (which represent all of the possible ways to transpose exactly two of Alice, Bob, Carol, and Diane, as so ordered, with one another).

- Bob, Alice, Carol, and Diane's being arranged clockwise in that order
- Carol, Bob, Alice, and Diane's being arranged clockwise in that order
- Diane, Bob, Carol, and Alice's being arranged clockwise in that order
- Alice, Carol, Bob, and Diane's being arranged clockwise in that order
- Alice, Diane, Carol, and Bob's being arranged clockwise in that order
- Alice, Bob, Diane, and Carol's being arranged clockwise in that order

This suggests that the ways these two relations can apply to four objects are different, despite the fact that they are the same in number. Another way to put the point is that there is a substantive difference in the symmetry structures of these two relations that has nothing to do with the number of ways in which they can apply to four objects.[16] (Q2), therefore, should be understood as asking more than why the ways in which some relations can apply to their relata are the same *in number*, while those in which others can apply to their relata are different *in number*. It also asks about the identity and distinctness of the ways *themselves* in which two relations can apply to their relata.

Now that I have explicated (Q2), I turn to the question of why a theory of relations should supply an answer to it. (Q2) is equivalent to the question of why some relations have the same symmetry and others have different symmetries. Of course, relations with different (fixed) arities can't have the same symmetry (nor can they apply in the same ways). But differences in symmetry (and manners of applicability) in these cases can be explained by appealing to the fact that the relations differ in arity, whether one is a relative positionalist or not. As the preceding discussion shows, however, even among relations of the same arity, there is more to the symmetry structure of any given one than just the number of ways in which it can apply. That is, there is more to the character of the way(s) such a relation can apply than just the *number* of ways it can apply. Why can some relations apply in the same ways, and some in different ways, *the latter of which include some which can apply in the same* number *of ways*? This question seems interesting in its own right, and seems to be among the right questions one should seek to answer when thinking about relational application in the context of a metaphysics of relations. Moreover, there are precedents in the literature, which are especially obvious when (Q2) is understood in terms of symmetry structures rather than in terms of manners of applicability. One of Donnelly's main goals, for example is to give an account of the various symmetry structures relations can have, which yields answers to all questions about their individuation. But even Bertrand Russell (1903: §§94–95) appears to be addressing it to some degree when he distinguishes symmetric and non-symmetric (binary) relations from one another by noting that the former are identical to their converses while the latter are distinct from theirs. While he does not explicitly use explanatory language, he seems to be appealing to differences between these two types of relation at least to help illustrate their characters.

Having explicated and motivated (Q1) and (Q2), what remains is to show that relative positionalism provides answers to both of these questions while neither antipositionalism nor ostrich realism does so. I begin by showing the former. As far as (Q1) goes, the relative positionalist can explain why each relation can apply in the way(s) it can in terms of the specific number of relative properties it has, and the way(s) those relative properties can be instantiated by its relata. *Being next to*^{2}, for example, can apply to two things in only one way, since it has only a single relative property, which must be instantiated by each of its relata relative to the other whenever it applies. *Loving*^{2}, on the other hand, can apply to two things in two ways, since it has two relative properties, one of which must be instantiated by one of its relata relative to the other while the other must be instantiated by the other relatum relative to the one whenever it applies.

Concerning (Q2), the relative positionalist can explain identities and differences between the way(s) in which distinct relations can apply by appealing to identifies and differences in the number of relative properties each relation has and identities and differences in the ways those relative properties can be instantiated by the relata of each relation. Differences of the former sort suffice to explain why certain n-ary relations which can apply in different numbers of ways can apply in different ways. Differences of the latter sort are required to explain why certain such relations which can apply in the same numbers of ways can nonetheless apply in different ways. So, for example, the relative positionalist can explain the difference between the two ways in which *loving*^{2} can apply to two objects on the one hand and the one way in which *being next to*^{2} can do so on the other by appealing to the fact that they have different numbers of relative properties. But to explain the difference between the six ways in which each of *being arranged clockwise in that* *order*^{4} and *being closer together than*$^{2\text{-}2}$ can apply, she must appeal to the fact that the six ways in which the six relative properties of the former relation can be instantiated by four objects relative to one another are different from the six ways in which the six relative properties of the latter relation can be instantiated by four objects relative to one another. This can be established using the same reasoning I employed above to show that the ways in which these two relations can apply to four objects are different, despite their being the same in number. (Remember that the relative positionalist identifies the ways in which the relative properties of an n-ary relation can be instantiated by n objects relative to one another with the ways in which that relation can apply to them.)

As mentioned, however, neither antipositionalism nor ostrich nominalism supply answers to both (Q1) and (Q2). Neither supplies an answer to (Q2). Neither explains, for example, the fact that the two ways in which *loving*^{2} can apply to two things are the same as the two in which *hating*^{2} can do so. Antipositionalism supplies no way to compare the way(s) in which distinct relations can apply (cf. Donnelly 2016: 99). Completions y and z can be co-mannered, according to Fine's account, only if they are completions of the same relation.[17] And the ostrich realist takes facts about the way(s) any given relation can apply as primitive. Thus any identities or differences amongst the way(s) two relations apply will be inexplicable as well. For the same reason, ostrich realism does not supply an answer to (Q1) either. She can provide no explanation, for example, for the fact that *being next to*^{2} can apply to two things in only one way, or for the fact that *loving*^{2} can apply to two things in two ways. As just stated, she takes these facts as primitive.[18] So, while antipositionalism and ostrich realism are on par with, or superior to, relative positionalism in that they can correctly handle any relation one might throw at them, including variable arity relations, they are decidedly inferior with respect to one of the central purposes of theorizing about relations. Neither supplies a complete explanation of relational application. Each fails to supply an answer to at least one of (Q1) and (Q2).

6. Concluding Remarks

Were it not for the fact that the relative positionalism can supply an explanation of the satisfaction of *any* predicate which putatively expresses a variable arity relation in terms of the application of fixed arity relations, I would admit that antipositionalism and ostrich realism win the day, in spite of the fact that they fail to supply a complete explanation of relational application. But since relative positionalism can supply such an explanation, then, all else being equal, relative positionalism is preferable. Of course, if the explanation of relational application that relative positionalism supplies is inadequate, or if the view has other problems, then the balance may be tipped back in favor of antipositionalism and ostrich realism. But, I might add, these problems would need to be rather significant to offset the rather sizable theoretical advantage constituted by the fact that relative positionalism supplies the sort of explanation of relational application that a theory of relations ought to supply, while antipositionalism and ostrich realism do not. Nonetheless, my conclusion should not be seen as categorical. We have reason to prefer relative positionalism to its closest competitors *to the extent that the explanation of relational application it supplies is plausible*, and *under the assumption that it does not possess other any other problems.* So I agree with MacBride (2014: 15) that ostrich realism may be a pill we would have to swallow if another theory, free of unwholesome consequences, cannot be found. It is just that the fact that the relative positionalist must deny the existence of variable arity relations, while admittedly failing to be *completely* wholesome, is not unwholesome enough to, on its own, force ostrich realism (or antipositionalism) upon us.

Notes

[1]Relative properties can be invoked, for example, by endurantists as a way to avoid the problem of temporary intrinsics (see, e.g., Lewis 1986: 202–04). Rather than being committed to the contradictory result that Lewis is both bent and straight (and so both bent and not bent), the endurantist might take the view that shape is a relation to a time. This would allow the endurantist to say that, while Lewis is both bent and straight, the result is not contradictory, since he is bent in relation to one time and straight in relation to another. As Lewis notes, however, it is awkward at best to regard the shape of something as a relation that it stands in to something. He says, "If we know what shape is, we know that it is a property, not a relation" (*Ibid.*: 204). An endurantist who is moved by this concern might say instead that shape is a relative property. It is a property, not a relation. But it is a property that an object may instantiate only relative to a time, in the same way that *being north* is a property that an object may instantiate only relative to a location. This would allow the endurantist to maintain that shape is in fact a property, while also being able to resolve the contradiction: while Lewis is both bent and straight, the result is not contradictory, since he is bent relative to one time and straight relative to another.

[2]Assume that an n-ary relation R with k relative properties applies to pairwise distinct $x_1, \ldots, x_n$ and consider any relative property τ of R. By Donnelly's Base Thesis, τ is instantiated by $x_{\mathrm{P}(1)}$ relative to $x_{\mathrm{P}(2)}$, ..., relative to

$x_{P(n)}$ for some $P \in S_n$. The Base Thesis, along with $\mathrm{SYM}_{\boldsymbol{R}}$ and ($\bigtriangledown$), ensures that this assignment of τ determines the assignment of each of the relative properties of R (which might include other assignments of τ). This means that this assignment of τ is sufficient to single out one way for R's relatives to be assigned to $x_1, \ldots, x_n$. If R has only one relative property, then this assignment of τ singles out the only such way. If R has any other relative properties, however, we know, by ($\bigtriangledown$), that the instantiation of any one of them by $x_{P(1)}$ relative to $x_{P(2)}$, ..., relative to $x_{P(n)}$ singles out a distinct way for R's relatives to be assigned to $x_1, \ldots, x_n$, since the identity permutation is a member of the symmetry group of any relation. *Mutatis mutandis* for every other relative property of R (if there are any). So, since there are k relatives of R in all, there are k ways in which those relatives can be assigned. The number of relatives a relation has also helps determine the number of ways it may apply to m objects when $m < n$. But this also depends on the fact that certain combinatorial possibilities collapse when objects are permuted with themselves. See Donnelly 2016: 83–84, fn. 11.

[3]Recall that $\circ$ is function composition. The composite $P \circ Q$ of permutations P and Q is the permutation mapping each $i \in \{1, \ldots, n\}$ to $P(Q(i))$, i.e., it is the result of applying Q to i and then applying P to that result. So $[2\,1] \circ [1\,2] = [2\,1]$ because

(i) $([2\,1] \circ [1\,2])(1) = [2\,1]([1\,2](1)) = [2\,1](1) = 2$ and
(ii) $([2\,1] \circ [1\,2])(2) = [2\,1]([1\,2](2)) = [2\,1](2) = 1$.

The permutation identities that follow can be computed using the same general method.

[4]A completion is any object which results from a relation applying to some things in a certain way. See Fine 2000: 4–5. Fine intends completions to include facts, states of affairs, and propositions (at least potentially — if one allows for the existence of each of these sorts of things). There are, of course, important differences between these sorts of completions. For example, the fact that Goethe loves Buff presumably exists only if Goethe loves Buff, while this is often thought not to be the case for the state of affairs of Goethe's loving Buff, or for the proposition that Goethe loves Buff.

[5]The reader can check that the appropriate relative property assignments hold, when interpreted as Donnelly suggests, when Larry, Curly, and Moe are arranged clockwise in the two orders possible. The reader should think of x_1's being clockwise in front of x_2 relative to x_3 as x_1's being in front of x_2 when she (the reader) imagines herself looking around the spatial arrangement of them clockwise from the perspective of x_3, and of x_1's being clockwise behind x_2 relative to x_3 as x_1's being behind x_2 when she so imagines herself.

[6]It can also be proved that it does so for m objects when $m < n$, when the collapse of appropriate combinatorial possibilities is taken into account. See n. 2.

[7]For further discussion of such relations, see Leonard and Goodman 1940: 50 and MacBride 2005: §2.

[8]I have used a binary manifestation of the putative variable arity relation to make my point — one which admittedly is at best a degenerate case of clockwise arrangement. But its n-ary manifestations for $n > 3$ also entail its having different numbers of (in each case, more than two) relative properties.

[9]See Fine 2000: 20–21 and 2007: 61. One might be concerned that it appears to be an arbitrary matter which identity and distinctness relationships hold of these two possible sets. But antipositionalism may have the resources to deal with this worry, by explaining sameness of manner of completion in terms of simultaneous substitution, as Fine

(2000: 25–26) does, and then explaining simultaneous substitution in terms of sequences of applications of singular substitution (see Fine 2000: 26, fn. 15 and MacBride 2007: 45–47). Whether or not this strategy is successful is not relevant to my purpose here. See MacBride 2007: 48–50 and Gaskin and Hill 2012: 177–82 for further discussion.

[10]MacBride (2007: 50 and 2014: 14) regards the fact that antipositionalism requires this as a disadvantage of the view relative to ostrich realism. For other criticisms of Fine's antipositionalism, see MacBride 2007: §8 and 2014: §6 and Gaskin and Hill 2012: §§3–4.

[11]The relationless relative positionalist could employ an analogous strategy, and say that each relational claim, which one might have thought involved commitment to a variable arity relation, actually involves commitment to a number of relative properties appropriate to the arity of the claim's relational predicate. Thanks to an anonymous referee for helping me to appreciate the viability of this alternative.

[12]Moreover, it may make the relative positionalist guilty of a "Russellian bias", assuming that all relations have fixed arities without argument (see MacBride 2005: 568–71).

[13]For more on variable arity predicates, see Oliver and Smiley 2004 and 2013: Ch. 10.

[14]One might worry that it is unclear how the account I have proposed would handle plurally quantified claims like 'Larry, Curly, and some other things are arranged clockwise in that order'. This concern is due to an anonymous referee. It is worth pointing out that it is not a special problem for the account I have proposed. It is an example of a general sort of limitation of relative positionalism that is a result of the fact that it is formulated in a singular first-order language. As such, it simply does not have the resources to deal with every plurally quantified claim. For this reason, I am tempted to leave this concern aside, writing it off as, for present purposes, being unrelated to a discussion about what the relative positionalist ought to say about variable arity relations. Still, the referee's concern seems to loom larger in the context of such a discussion, since claims like these seem to call more loudly for a treatment in terms of variable arity relations than relational claims involving only singular terms. So I'll say some things in response to it.

There are at least two things one might mean when one asks how an account handles a claim like this. First, one might be wondering how the account explains the truth of claims like these. So construed, I think the relative positionalist has a plausible answer. It is plausible and common in the truthmaker and grounding literatures to say that existential truths are made true by, or grounded in, facts concerning only particulars instantiating fixed arity relations. Armstrong (2004: 54–55), for example, takes any sentence of the form '$\exists x_1 \ldots \exists x_n, Rx_1 \ldots x_n$' to be made true by the state of affairs of $a_1, \ldots, a_n$'s R-ing for each $a_1, \ldots, a_n$ such that $Ra_1 \ldots a_n$. Fine (2012: 59–60) grounds the fact that $\exists x_1 \ldots \exists x_n, Rx_1 \ldots x_n$ in the following facts:

- $Ra_1 \ldots a_n$,
- Ea_1,

  $\vdots$
- Ea_n

for each $a_1, \ldots, a_n$ such that $Ra_1 \ldots a_n$, where 'E' is an existence predicate that is not itself defined in terms of existential quantification. Each of the states of affairs or facts invoked in these accounts can be understood either as a completion of R, which can itself be understood as a fixed n-ary relation, or as a completion of the property E. It is no less plausible that plurally existentially quantified sentences like the one above are made true by or grounded in these same sorts of states of affairs or facts.

Second, however, one might be wondering what the semantic values of the relational predicates are in sentences like 'Larry, Curly, and some other things are arranged clockwise in that order' according to the account — what, for example, the semantic value of '. . . , . . . , and ____ are arranged clockwise in that order' is, and what the internal structure is of the propositions expressed by plurally quantified sentences like this. These questions present more of a problem. The relative positionalist might be tempted to point out that, in the literature on relations, the focus has been on the internal structure of facts and states of affairs rather than propositions, or that, to the extent that the focus has included propositions, it has included only *singular* propositions. But the proponent of a given account of relations should have *some* prospective strategies for dealing with general propositions like existentially quantified propositions, since their account is, in part, intended to give an account of completions of relations, which, *prima facie*, include propositions, singular and general. Fortunately, there appear to be some strategies available. First, the relative positionalist can identify the semantic value of predicates like '. . . , . . . , and ____ are arranged clockwise in that order' not with a relation, but with something else, such as a class or plurality of fixed-arity relations, and say that it is this class or plurality, and not any single relation, which figures into the proposition. Another proposal (due to the same anonymous referee) is that the proposition can be understood as an infinite disjunction: that either

(i) there is an x such that Larry, Curly, and x are arranged clockwise in that order (and $x \neq$ Larry and $x \neq$ Curly), or
(ii) there is an x and a y such that Larry, Curly, x and y are arranged clockwise in that order (and $x \neq$ Larry and . . .), or

$\vdots$

I will endorse none of these strategies today. It may be that this problem is insurmountable for the relative positionalist if these strategies turn out to be unfruitful and no alternative strategies can be found. But, at least for the time being, there appear to be some avenues of reply that the relative positionalist can explore.

[15]One might be concerned that identifying the two ways in which *loving*^{2} can apply to two things with the two ways *hating*^{2} can do so commits one to the claim that each of the ways in which each one of these relations can apply to two things is identical to one of the ways in which the other can do so. This could be seen as problematic, since either possible assignment would seem unmotivated. One might think that there is reason to be found to make one assignment rather than another. Each of *loving*^{2} and *hating*^{2} has an agent role and a patient role, and so one might reasonably identify the way that has x_1 in *loving*^{2}'s patient role and x_2 in its patient role with the way that has x_1 in *hating*^{2}'s agent role and x_2 in its patient role. But one could easily have chosen a non-symmetric binary relation instead of *hating*^{2} that does not obviously possess such roles, such as *being taller than*^{2} or *being to the left of*^{2}. In such cases, there might be no basis for identifying either of the ways in which *loving*^{2} can apply to two things with a particular one of the ways in which the other relation can do so. Thanks to an anonymous referee for raising this concern. I am not convinced, however, that, in identifying ways w and v with w' and v', one needs suppose that each of w and v is identical to a particular one of w' and v'. Consider two non-rotating toroidal space stations that are intrinsic duplicates of one another, aligned in space randomly with respect to one another, and imagine two

astronauts, each inside a different one of them. There seems to be no basis on which one can identify one of the directions in one station with either of the directions in the other. Yet it seems reasonable to say that these astronauts have the same two options concerning the directions they can travel around their respective stations. But in case one is unconvinced by this reply, it is worth noting that (Q2) can be formulated in terms of comparative similarity rather than identity.

(Q2′) Why are the ways in which some relations can apply to their relata more similar to one another than to the ways in which other relations can apply to their relata?

What would otherwise be understood as identical manners of applicability can be understood instead as maximally similar ones, where

> **Maximal Similarity.** The ways in which n-ary relations R and R can apply to n objects are *maximally similar* to one another $=_{df}$ there is no n-ary relation R'', distinct from each of R and R', such that the ways in which it can apply to n objects are more similar to the ways one of R and R' can apply to n objects than they are to the ways in which the other can do so.

It seems that one can safely say that the two ways in which *loving*^{2} and another non-symmetric binary relation can apply are comparatively more similar to one another than they are to the way(s) in which a relation with a different sort of symmetry can apply without having to say anything about which one of the two ways that *loving*^{2} can apply is more similar to which of the two ways the other non-symmetric binary relation can apply. And relative positionalism will be able to provide an answer to (Q2′), while neither antipositionalism nor ostrich realism will be able to, for reasons very similar to those I articulate below which establish that this is true of (Q2).

[16]The same point can be made formally by noting that the symmetry groups of these two relations, while sharing the same index in S_4 (i.e., as having the same number of left cosets in S_4, each left coset representing a way the relation can apply to four objects), are not isomorphic, in the sense that there is no one-to-one correspondence between their elements which respects their respective group operations, which is function composition in both cases (see Donnelly 2016: 83–84, fn. 11). Their non-isormorphism can be established by noting that some elements of the symmetry group of *being arranged clockwise in that order*^{4} has elements of order 4, while none of the elements of the symmetry group of *being closer together than*$^{2\text{-}2}$ has any elements of order 4. It is a theorem that the group-isomorphic image of an element of a group and that element have the same order (see Gallian 2013: 133). So an isomorphic image of any order-four element of any symmetry group of the former relation must be order-four.

[17]This is for the simple fact that Fine defines sameness of manners of completion only for single relations. He does so in terms of simultaneous substitution (see n. 9 above). Specifically, he says,

> to say that s is a completion of a relation R by $a_1, a_2 \ldots, a_{\mathrm{m}}$, in the same manner as t is a completion of R by $b_1, b_2, \ldots, b_{\mathrm{m}}$ is simply to say that s is a completion of R by $a_1, a_2 \ldots, a_{\mathrm{m}}$ that results from simultaneously substituting $a_1, a_2 \ldots, a_{\mathrm{m}}$ for $b_1, b_2, \ldots, b_{\mathrm{m}}$ in t (and vice versa). (Fine 2000: 25–26)

But a modification of this definition to accommodate distinct relations is not forthcoming. It seems arbitrary whether the manner in which, for example, Goethe and Buff complete *loving*^{2} in Goethe's loving Buff is the same as or different from that in which they complete *being to the left of*^{2} in Goethe's being to the left of Buff. (I am assuming here that each of Goethe and Buff is being substituted with itself.) Note that here we are attempting to identifying a single way one relation can apply with a single way a distinct relation can apply, and thus it would be of no help to the antipositionalist to invoke the argument I gave in n. 15 for the view that multiple manners of applicability of distinct relations can be identified without presupposing the identity of each way in which each of the relations can apply with some way in which the other can apply.

[18]The antipositionalist can explain why a given relation can apply in the way(s) it can by appealing to the number of classes of co-mannered completions associated with it.

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The author has no competing interests to declare.

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