Direction of Arrival Estimation Using Hybrid Spatial Cross-Cumulants and Root-MUSIC

Murdifi Muhammad[1, 2, a)], Minghui Li[1], Qammer H. Abbasi[1], Cindy Goh[1], Muhammad Imran[1]

[1]James Watt School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom

[2] Research and Development Department, RFNet Technologies Pte Ltd, Singapore 319319

[a)] Corresponding author: 2167422B@student.gla.ac.uk

Abstract—**This paper presents a novel Direction of Arrival (DOA) estimation technique called Cross Cumulant-MUSIC (CC-MUSIC) which jointly employs higher order cumulant statistics and the root-MUSIC algorithm to perform high-resolution DOA estimation in low Signal-to-Noise Ratio (SNR) scenarios. From the simulation results based out of a 4 element uniform linear array and a far-field narrowband signal source, CC-MUSIC outperforms second-order DOA estimation techniques such as root-MUSIC and ESPRIT with a minimum average of 10.99% to 46.33% depending on the snapshot values at SNR of <15dB for a single signal source scenario and 39.1% to 83.8% for a multi-signal source scenario respectively when contaminated with an Additive White Gaussian Noise (AWGN). The work presented here has implications of future studies for optimization and real-world application where SNR environment is noisy while requiring accurate DOA estimation.**

Index Terms— Antenna array, DOA, Direction of Arrival, Higher-order statistics

I. Introduction

Direction of Arrival (DOA) estimation is considered as one of the most crucial problems of array signal processing with considerable research efforts for developing efficient and effective algorithms. DOA estimation is important in many applications such as radar, sonar and electronic surveillance [1]-[5]. Recent applications include array processing for wireless communication such as adaptive beamforming at both the base station and clients which allows an increase in capacity and efficiency of the systems [5]. In such wireless systems, the computational complexity, robustness and the capability to resolve multiple closely spaced sources plays an important role in determining the performance of the systems.

Recently, there has been a lot of applicable research in using Higher-order Statistics (HOS) [6]-[8]. Audio and wireless signals for example are generated by systems typically with nonlinear dynamics. Therefore, if the prediction and coding quality of such signals is to be improved, then more information of the signal must be exploited.

The fourth order covariance matrix contains more information such as the phase difference at the reference element that the second order covariance matrix [15]. This is calculated via the Kronecker product of the received data and its conjugate value. In other words, by introducing a phase difference to the reference antenna array element, we essentially introduce a virtual array element – therefore extending the antenna array element by $(M+1)$[15]. In theory, this would allow efficient noise suppression and higher resolution accuracy especially in a multi-signal source scenario. It has also been demonstrated that by using higher order statistics applied to the classical MUSIC algorithm as in [16], the effectiveness of the cumulant-based MUSIC can restrain efficiently in the effect of colored noise when compared to the equivalent second order covariance-based version with higher resolution probability and in low SNR scenario for DOA estimation.

While the classical MUSIC has been deemed as a high-resolution DOA estimator, as demonstrated in [15]-[16], there is still the computational load of executing a spectral scan of all possible angles in order to determine the DOAs. One such technique to solve for the high computational load was the introduction of the root-MUSIC [9] technique which encompasses polynomial solving technique to solve for the DOAs. In terms of computational time, root-MUSIC reduces the computational time by half when compared to the classical MUSIC technique [5]. In addition, as the fourth-order cross cumulant has the same matrix size as the second-order covariance matrix, implementation and realization for real world application is relatively simple. To that end, we want to develop an algorithm that has the benefits of using fourth order cumulant-based covariance matrix for its good performance in low SNR scenario as well as root-MUSIC's computational efficiency.

In the present work, our focus is to improve the resolution and accuracy of a DOA estimator – particularly in a low SNR scenario. We propose a hybrid fourth-order cross cumulant based technique to determine a higher order sample covariance matrix coupled with the root-MUSIC algorithm and solving the polynomial roots to determine the respective DOAs. We chose the root-MUSIC estimator as the base algorithm as it retains the high-resolution accuracy and it is relatively easy to implement in real-world scenario as demonstrated in [23]-[24].

One potential application that can make use of this technique is in direction-finding of wireless signals such as in the transportation market like the high-speed rail (HSR) communication systems, intelligent transportation systems and maritime tracking application where SNR scenarios tend to be much lower in reality [18]-[20].

The rest of the paper is presented as follows. Section II presents the general data model, derivation of the cross cumulants and a brief summary of the root-MUSIC DOA estimator. Section III introduces the proposed CC-MUSIC technique while section IV presents the simulation results for both single-source and multi-source SNR performance in terms of Root Mean-Squared Error (RMSE) analysis for a range of snapshot values. Finally, section V presents the conclusion and future works.

II. DATA MODEL

A. General Data Model

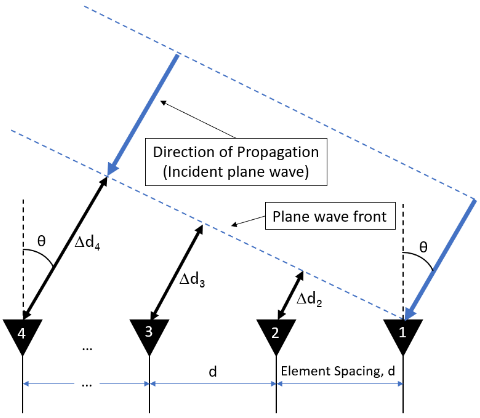

Figure 1: ULA Illustration

Fig. 1 shows a general data model illustration of a simple linear array [5]. Assume that there are N far-field narrowband signals impinging on a Uniform Linear Array (ULA) of $M(> N)$ sensors with an inter-element spacing, d typically at $\frac{\lambda}{2}$ wavelength of its operating frequency. In this case, it is assumed that the signals are uncorrelated with noise. Under this assumption, the received signal at the array output is expressed as [17]:

$$\boldsymbol{x}(k) = \boldsymbol{A}s(k) + \boldsymbol{N}(k) \quad k = 1,2, \dots, K \tag{1}$$

Where K is the number of snapshots, $\boldsymbol{x}(k)$ is a M-by-1 vector of the received signal data consisting of signals and additive noise, $\boldsymbol{N}(k)$ while $\boldsymbol{A} \triangleq [a(\theta_1) \quad a(\theta_2) \quad \cdots \quad a(\theta_N)]$ is an M-by-N matrix containing the signal arrival vector information which consists of the relative phase shifts at the array elements. In addition, $a(\theta)$ in $\boldsymbol{A}$ is the steering vector which is defined as

$$\boldsymbol{a}(\theta) = \begin{bmatrix} 1 & e^{j2\pi\left(\frac{d}{\lambda}\right)\sin\theta} & \dots & e^{j2\pi(M-1)\left(\frac{d}{\lambda}\right)\sin\theta} \end{bmatrix}^T \tag{2}$$

$\boldsymbol{s}(k) \triangleq [s_1(k) \quad s_2(k) \quad \cdots \quad s_N(k)]^T$ is a N-by-1 vector of N incident signal source values and n(k) is an M-by-1 vector of sensor noise values [17]. Finally, $\theta = \{\theta_1, \quad \cdots \quad \theta_N\}$ are the parameters of interest which contains the DOA information which are required to be estimated.

B. Spatial Cross Cumulants

It has also been demonstrated that cross cumulants-based covariance matrix has higher resolution probability especially in low SNR scenario for DOA estimation [6][16]. The second order complex valued cross cumulant for two generic variables X and Y is the cross-power spectrum which is expressed as [15]:

$$C_2(X, Y^*) = E[XY^*] - E[X]E[Y^*] \tag{3}$$

A fourth-order complex valued cross cumulant for two generic zero-mean processes X and Y can be extended from (3) and is expressed as [15]:

$$C_4(X, X, Y^*, Y^*) = E[X^2Y^{*2}] - E[X^2]E^*[Y^2] - 2E[XY^*]^2 \tag{4}$$

To that end, it should be noted there are many variants of cross cumulants techniques for complex-valued random processes. Generally, for a complex-valued cross cumulant of order N, there are 2^N ways to configure the complex conjugate on the parameters to C_N which results in different definitions.

From (1), the second-order sample covariance matrix can be expressed as [17]:

$$\boldsymbol{R_{2X}} = \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{X}(k)\boldsymbol{X}^H(k) \tag{5}$$

The fourth-order covariance matrix $\mathbf{R_{4X}}$ based on the received signal data matrix can be extended from (5) with reference from (4) and is expressed as follows [15]:

$$\begin{aligned} \boldsymbol{R_{4X}} = &\, E\{(\boldsymbol{X} \otimes \boldsymbol{X}^*)(\boldsymbol{X} \otimes \boldsymbol{X}^*)^H\} \\ &- E\{\boldsymbol{X} \otimes \boldsymbol{X}^*\}E\{\boldsymbol{X} \otimes \boldsymbol{X}^*\}^H \\ &- E\{\boldsymbol{X}\boldsymbol{X}^H\} \otimes E\{(\boldsymbol{X}\boldsymbol{X}^H)^*\} \end{aligned} \tag{6}$$

Where $(\cdot)^*$ denotes the complex conjugate, $E\{\cdot\}$ denotes the expectation operator and $\otimes$ denotes the Kronecker product.

C. Root-MUSIC

Root-MUSIC DOA estimation technique is a modification to the MUSIC algorithm initially proposed in [9] and is based on polynomial rooting from the eigen-decomposed noise subspace from the second-order sample covariance matrix which provides significantly higher angular resolution [5][10]. It has also been shown in past literatures that root-MUSIC presents lower computational costs as compared to the spectral scan technique like MUSIC as the technique does not require a maxima peak search to determine the DOAs [5][11]. The expression of the root-MUSIC technique as derived in [5] can be expressed as:

$$P_{rootmusic}(\theta) = \frac{1}{|\boldsymbol{a}(\theta)^H \boldsymbol{C} \boldsymbol{a}(\theta)|} \tag{7}$$

Where $\boldsymbol{a}(\theta)$ is the steering vector and $\boldsymbol{C}$ is a Hermitian matrix given as:

$$\boldsymbol{C} = \boldsymbol{U_n}\boldsymbol{U_n^H} \tag{8}$$

Where $\boldsymbol{U_n}$ is the noise subspace derived through the eigen-decomposition of the second-order sample covariance matrix. The poles of the MUSIC pseudospectrum is the corresponding roots that lie closest to the unit circle. The roots may not necessarily be exactly on the unit circle due to noise. The polynomials is of order $2(M-1)$ which has been derived from the sum of the diagonal elements $\boldsymbol{C}$. For example, a M-element linear array's covariance matrix is of dimension *M-by-M* and will have $2(M-1)$ diagonals. Thus, each root can be written as [17]:

$$z_i = |z_i|e^{jarg(z_i)} \quad i = 1,2,\ldots,2(M-1) \tag{9}$$

Where $z = e^{j\frac{2\pi}{\lambda}d\sin\theta_i}$ and $arg(z_i)$ is the phase angle of z_i. By comparing $e^{jarg(z_i)}$ and $e^{j\frac{2\pi}{\lambda}d\sin\theta_i}$, the n^{th} roots closest to the unit circle are mapped and converted into the estimated DOAs of interest by[14][17]:

$$\theta_{i(n)} = sin^{-1}\left(\frac{\lambda}{2\pi d}arg\left(z_{i(n)}\right)\right) \tag{10}$$

Where $\theta_{i(n)}$ are the estimated DOAs of interest. Note that the range of i values are dependent on the number of signal source. For example, if there are 2 signal sources, then the 2 roots closest to the unit circle are the estimated DOAs of interest.

III. PROPOSED METHOD

Instead of using the second-order sample covariance matrix to determine the noise subspace and the polynomial roots to obtain the DOAs, we propose to use a modified fourth order cross cumulant matrix derived from the incoming received signal data. It has been shown in past literature such as in [7][8][16] that using cross cumulants can resolve multiple closely spaced sources in a noisy environment. The proposed method consists of 10 simple stages.

Stage	Procedure
1	Obtain received signal data matrix, $\boldsymbol{X_{raw}}$
2	Detrend X by removing the mean values, $\boldsymbol{X}$
3	Evaluate fourth-order moments, $\boldsymbol{Z}$
4	Evaluate fourth-order cross moments, $\boldsymbol{C_{4M}}$
5	Evaluate correlation matrix, $\boldsymbol{R_{corr}}$
6	Evaluate moment matrix, $\boldsymbol{R_{mom}}$
7	Evaluate cross-cumulant matrix, $\boldsymbol{R_{4X}}$
8	Perform eigen-decomposition to determine noise subspace.
9	Solve for polynomial roots
10	Convert polynomials roots into respective DOAs.

Table 1: Proposed DOA Algorithm

Stage 1 is to obtain the received signal data matrix of size K-by-N. stage 2 is to detrend $\boldsymbol{X_{raw}}$ by removing the mean values of the data. By removing the mean values of the data, we can perform an analysis based on the fluctuation in the data. Stage 3 is to determine the fourth-order moment which is expressed as [6][15]:

$$\boldsymbol{Z} = \boldsymbol{X} \odot \boldsymbol{X}(\boldsymbol{X}^*) \tag{11}$$

Where $\odot$ denotes the element-wise multiplication or Hadamard product. Stage 4 evaluates the fourth-order cross moments which is expressed as [15]:

$$\boldsymbol{C_{4M}} = \frac{1}{K}\sum_{k=1}^{K}(\boldsymbol{Z}^H\boldsymbol{X}(k))^* \tag{12}$$

Then, stage 5 & 6 evaluates the correlation and moment matrix respectively which is presented as:

$$\boldsymbol{R_{corr}} = \frac{1}{K}\sum_{k=1}^{K}(\boldsymbol{X}^H\boldsymbol{X}(k))^* \tag{13}$$

$$\boldsymbol{R_{mom}} = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{X}^H(k)\boldsymbol{X}^*(k) \tag{14}$$

Next, stage 7 evaluates the cross-cumulant matrix which is expressed as [15]:

$$\begin{aligned}\boldsymbol{R_{4X}} = \boldsymbol{C_{4M}} - 2diag\big(diag(\boldsymbol{R_{corr}})\big)\boldsymbol{R_{corr}} \\ -diag\big((diag(\boldsymbol{R_{mom}}^*)\big)\boldsymbol{R_{mom}}\end{aligned} \tag{15}$$

Where diag(·) represents the diagonal elements of the matrix. Finally, stages 8-10 evaluates the polynomial roots to determine the respective DOAs as in (10).

IV. SIMULATION RESULT

The proposed technique for DOA estimation which is denoted as Cross-Cumulant MUSIC (CC-MUSIC) is evaluated and the performance compared to the root-MUSIC and ESPRIT algorithm. Mainly, this is to present the benefits of using fourth-order cross cumulants for the sample covariance matrix as compared to the one utilizing the second-order sample covariance matrix technique such as those typically employed for root-MUSIC and ESPRIT. We chose root-MUSIC and ESPRIT as the comparison because it all employs the same polynomial technique to determine the DOAs [14].

To evaluate the performance of the DOA algorithms, we modelled a simple single-user Wi-Fi antenna configuration that is employed typically between vehicular clients and access points for wireless communication [18]-[22]:

- Carrier Frequency: 5500MHz

- Number of elements: 4
- Element spacing: λ/2

There are many algorithms and statistical techniques that is able to predict the number of signal sources such as the Akaike Information Criterion (AIC) and Minimum Description Length (MDL) such as in [12]-[13]. This however is beyond the scope of this paper and we assumed that the number of narrowband signal sources is known a priori for simplicity.

A. Single Source SNR-RMSE Performance

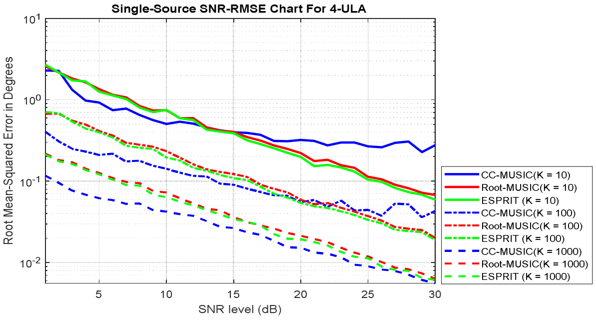

Figure 2: Single-Source SNR-RMSE Performance

Fig. 2 presents the Signal-to-Noise Ratio Root Mean-Squared Error (SNR-RMSE) performance comparison between CC-MUSIC, Root-MUSIC and ESPRIT of snapshot values of 10, 100 and 1000 with a single narrowband signal source impinging onto the array at 40°. In addition, the simulation is carried out with an SNR value ranging from 1dB to 30dB. The simulation loops 300 samples to determine the mean average of its performance at each time step. Noise is assumed to be AWGN for the varying SNR values. Lastly, the RMSE is calculated as:

$$y_{rms} = \sqrt{\frac{1}{Q}\sum_{q=1}^{Q} |\theta_{in} - \theta_{est}|^2} \qquad (16)$$

Where Q is the number of simulation samples, θ_{est} is the estimated DOA and θ_{in} is the simulated true DOA.

With reference to Fig. 2, it can be seen that at low SNR (<15dB), CC-MUSIC outperforms root-MUSIC and ESPRIT by a minimum average of 10.99%, 39.64% and 46.33% in terms of RMSE for K = 10, 100 and 1000 respectively. We also observe that at higher SNR (>15dB) for low K snapshot values, root-MUSIC and ESPRIT outperforms CC-MUSIC. At high K values, CC-MUSIC performs similarly when compared to root-MUSIC and ESPRIT with an average RMSE difference of 10%. One of the reasons as to why high SNR effects the performance of CC-MUSIC is due to the fluctuations and over-saturation of phase information in the moment matrix, $\boldsymbol{R_{mom}}$. However, as it is unlikely that in real-world application that SNR will be as high as >15dB, it can be considered as a compromise – especially if there is a need to perform statistically well in low SNR scenarios.

B. Multi-Source SNR-RMSE Performance

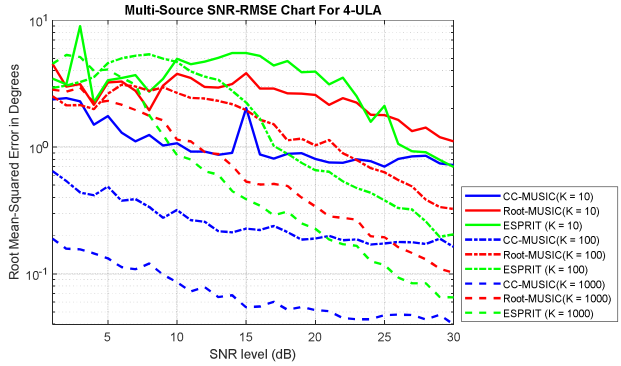

Figure 3: Multi-Source SNR-RMSE Performance

Fig. 3 presents the SNR-RMSE performance for a multi-signal source scenario. In this scenario, there are 2 narrowband signals impinging onto the array set 5° apart at 40° and 45°. All other parameters are the same as in the previous simulation as defined for the single-source scenario. For a multi-signal source scenario, the RMSE is averaged across the and calculated as:

$$y_{multi-rms} = \frac{\left(y_{rms_1} + y_{rms_2} + \cdots + y_{rms_N}\right)}{N} \qquad (17)$$

Where y_{rms_N} is the nth signal source based on (15). For the simulation scenario of having 2 signal sources, from (16) and (17), the RMSE is calculated as:

$$y_{rms_2} = \frac{\left(\sqrt{\frac{1}{Q}\sum_{q=1}^{Q}|\theta_{in1}-\theta_{est1}|^2}\right)+\left(\sqrt{\frac{1}{Q}\sum_{q=1}^{Q}|\theta_{in2}-\theta_{est2}|^2}\right)}{2} \qquad (18)$$

Where θ_{in1} and θ_{in2} are the simulated true DOAs which in this case are 40° and 45°, whereas θ_{est1} and θ_{est2} are the estimated DOAs.

From the simulation results, we can confirm that in a multi-source scenario, the usage of the cross cumulants in CC-MUSIC outperforms the classical second order covariance-based DOA estimators across the SNR range and in varying snapshot values. On average across the whole range of SNR value, CC-MUSIC outperforms root-MUSIC and ESPRIT by 48.1%, 20.37% and 4.5% for K = 10, 100 and 1000 respectively. However, one thing to note that at lower SNR value, CC-MUSIC significantly outperforms to that of root-MUSIC and ESPRIT. If we mark the range from 1dB to 15dB, we determine that the average SNR-RMSE for CC-MUSIC compared against root-MUSIC and ESPRIT is 39.1%, 79.1% and 83.8% for K = 10,100 and 1000 respectively. As demonstrated before in [6][8][15], the introduction of an extended virtual array element allows improved resolution accuracy and noise suppression which has been demonstrated in this simulation scenario.

V. CONCLUSION & FUTURE WORK

The proposed novel DOA estimation technique, CC-MUSIC to use fourth-order cross-cumulant statistics was presented to significantly improve the performance of the estimator particularly in low SNR scenarios without the expense of computational cost based on the RMSE performance. Based on the simulation results, at high SNR scenarios for a single signal source, it is best to determine the DOAs using second order-based sample covariance techniques. However, in a multi-signal source scenario, it is best to use higher-order statistics to resolve multiple signals – particularly if the sources are closely related which has been demonstrated from the simulation results.

One way to overcome this is by implement a detect and switch algorithm that allows automatic switching of the sample covariance matrix from fourth-order to second-order based on either the SNR threshold if the noise power is known or the Received Signal Strength Indicator (RSSI) values if noise power from the emitter is unknown. To achieve this, a correlation between known RSSI and noise values can be determined by mapping the 2 variables together. This would allow a threshold to be set and determine when the switching can occur upon reaching a specific level of RSSI which can be estimated. In terms of computational costs, CC-MUSIC has the same complexity as compared to ESPRIT and root-MUSIC. This switching technique can be implemented as part of future works as well as conducting a performance analysis in colored noise environment.

ACKNOWLEDGMENT

The authors would like to acknowledge and express appreciation to the Singapore Economic Development Board (EDB) and RFNet Technologies Pte Ltd for financing and providing a good environment and facilities to support the project.

REFERENCES

[1] C. Uysal and T. Filik, "Contactless respiration rate estimation using MUSIC algorithm," 2017 10th International Conference on Electrical and Electronics Engineering (ELECO), Bursa, 2017, pp. 606-610.

[2] Â. M. C. R. Borzino, J. A. Apolinário, and M. L. R. de Campos, "Robust DOA estimation of heavily noisy gunshot signals," 2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), South Brisbane, QLD, 2015, pp. 449-453.

[3] R. K, H. F and M. M A, "A comprehensive analysis and performance evaluation of different direction of arrival estimation algorithms," 2012 IEEE Symposium on Computers & Informatics (ISCI), Penang, 2012, pp. 256-259.

[4] D. M. Vijayan and S. K. Menon, "Direction of arrival estimation in smart antenna for marine communication," 2016 International Conference on Communication and Signal Processing (ICCSP), Melmaruvathur, 2016, pp. 1535-1540.

[5] M. Muhammad, M. Li, Q. H. Abbasi, C. Goh and M. Imran, "Performance Evaluation for Direction of Arrival Estimation Using 4-Element Linear Array," 2019 13th European Conference on Antennas and Propagation (EuCAP), Krakow, Poland, 2019, pp. 1-5.

[6] M. Swartling, B. Sallberg and N. Grbic, "Direction of arrival estimation for speech sources using fourth order cross cumulants," 2008 IEEE International Symposium on Circuits and Systems (ISCAS), Seattle, WA, 2008, pp. 1696-1699.

[7] C. Gu, H. Hong, X. Zhu and J. He, "Highly Accurate Two-Dimensional Direction Finding Using Cumulant," in IEEE Access, vol. 6, pp. 70910-70918, 2018.

[8] Y. Zhao, W. Li, B. Yang and Y. Zhang, "Modified direction-of-arrival estimation approach of virtual antenna array based on fourth order statistics," 2017 Progress in Electromagnetics Research Symposium - Fall (PIERS - FALL), Singapore, 2017, pp. 346-349.

[9] R. Schmidt, "Multiple emitter location and signal parameter estimation," in IEEE Transactions on Antennas and Propagation, vol. 34, no. 3, pp. 276-280, Mar 1986.

[10] B. D. Rao and K. V. S. Hari, "Performance Analysis Of Root-MUSIC" Twenty-Second Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, USA, 1988, pp. 578-582.

[11] R. K, H. F and M. M A, "A comprehensive analysis and performance evaluation of different direction of arrival estimation algorithms," 2012 IEEE Symposium on Computers & Informatics (ISCI), Penang, 2012, pp. 256-259.

[12] E. Fishler, M. Grosmann and H. Messer, "Detection of signals by information theoretic criteria: general asymptotic performance analysis," in IEEE Transactions on Signal Processing, vol. 50, no. 5, pp. 1027-1036, May 2002.

[13] A. S. Sekmen and Z. Bingul, "Comparison of algorithms for detection of the number of signal sources," Proceedings IEEE Southeastcon'99. Technology on the Brink of 2000 (Cat. No.99CH36300), Lexington, KY, USA, 1999, pp. 70-73.

[14] C. Ko and J. Lee, "Performance of ESPRIT and Root-MUSIC for Angle-of-Arrival(AOA) Estimation," 2018 IEEE World Symposium on Communication Engineering (WSCE), Singapore, Singapore, 2018, pp. 49-53.

[15] Z. Shan and T. -. P. Yum, "A conjugate augmented approach to direction-of-arrival estimation," in IEEE Transactions on Signal Processing, vol. 53, no. 11, pp. 4104-4109, Nov. 2005.

[16] H. Chu-lin and W. De-shi, "The Fourth Order Cumulant-Based MUSIC Algorithm for Harmonic Retrieval in Colored Noise," 2009 WRI International Conference on Communications and Mobile Computing, Yunnan, 2009, pp. 564-567.

[17] F. Gross, Smart Antennas for Wireless Communications with MATLAB. New York: McGraw-Hill, 2005, pp 193-196.

[18] A. O. Laiyemo, P. Luoto, P. Pirinen and M. Latva-aho, "Higher frequency band beamforming scheme for high speed train," 2017 IEEE Vehicular Networking Conference (VNC), Torino, 2017, pp. 203-208.

[19] M. Zhou, H. Harada and J. S. Pathmasuntharam, "An analysis model of DoA in maritime environment for ship-to-ship/shore wireless communications," 2009 9th International Conference on Intelligent Transport Systems Telecommunications, (ITST), Lille, 2009, pp. 376-380.

[20] V. Y. Vu and A. B. Delai, "Digital Solution for inter-vehicle localisation system by means of Direction-Of-Arrival," 2006 International Symposium on Intelligent Signal Processing and Communications, Tottori, 2006, pp. 875-878.

[21] M. C. Tan, M. Li, Q. H. Abbasi and M. Imran, "A smart and low-cost enhanced antenna system for industrial wireless broadband communication," 12th European Conference on Antennas and Propagation (EuCAP 2018), London, 2018, pp. 1-4.

[22] M. C. Tan, M. Li, Q. H. Abbasi and M. Imran, "A Wideband Beam forming Antenna Array for 802.11ac and 4.9 GHz," 2019 13th European Conference on Antennas and Propagation (EuCAP), Krakow, Poland, 2019, pp. 1-5.

[23] F. Belloni, A. Richter and V. Koivunen, "Performance of root-MUSIC algorithm using real-world arrays," 2006 14th European Signal Processing Conference, Florence, 2006, pp. 1-5.

[24] H. Paaso and M. Hirvonen, "Angular Resolution Improvement by Using Multi-Radar and FBSS MUSIC DoA Estimation Algorithm," 2019 IEEE Intelligent Vehicles Symposium (IV), Paris, France, 2019, pp. 730-735.