Non-Gaussian Colored Noise Generation for Wireless Channel Simulation with Particle Swarm Optimizer

Shaowei Dai*, Minghui Li, Qammer H. Abbasi and Muhammad Imran
School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom
*Contact email: s.dai.2@research.gla.ac.uk

***Abstract*—Random Variable (RV) with different Probability Density Function (PDF) and Power Spectral Density (PSD) is a critical component for simulation of different wireless channel fading profile. To get a specific PSD for simulation of different multi-path scenario, the usual method is to pass a white noise through a filter with the required shape. But the filtering process will cause the change of random variable's PDF unless the input noise follows Gaussian Distribution. In this paper, a Particle Swarm Optimization (PSO) based method to generate Non-Gaussian noise by a pre-distortion filter and Inverse Transform Sampling (ITS) that meets both the requirement of PSD and PDF is described. As the solution is based on filtering, after the filter weight is found using PSO, the simulation could be carried out in a real-time manner compared to block-based methods. The numerical simulation confirms that it can generate the required PDF and more than 90% similar to the required PSD.**

***Index Terms*—Wireless, Channel, Non-Gaussian, Noise, Simulation**

I. Introduction

Wireless channel simulation is critical for verification of transceivers with different algorithms in various channel situations. Multipath is a unique character of wireless channel that causes fading. To simulate the multipath effect, mainly there are two approaches namely Ray Tracing[1] and Stochastic Modeling method[2].

Ray tracing method is a site-specific algorithm where multiple communication paths are traced directly for a particular site model. Although it is computing-intensive, the channel's characteristics like coherence bandwidth, coherence time etc is reflected in the ray modeling. On the other hand, Stochastic Model method is generic and simpler for simulating different fading scenarios by modeling the channel gain as a Random Variable (RV) with different Power Spectral Density (PSD) and Probability Density Function (PDF) which could be considered as an aggregated effect from multipath. There are many random distributions like Rayleigh, Rician, Nakigami, and Weibull etc which have been proposed, tested and verified in the field with various levels of accuracy. This paper focuses on the Stochastic Modeling method.

The PSD of the channel gain random variable represents the doppler spectrum of wireless channel model. The wider the Doppler Spread, the faster the change of the channel gain, hence the shorter the coherence time of the channel. So to properly model a wireless channel characteristic, it is necessary to implement specific PSD for the channel gain random variable. Different Doppler spectrum like U-Shaped [3, p.148], Bell shaped[4, p.45] etc. have been proposed in the literature for various fading model.

II. Motivation

The challenge for stochastic modeling method is that changing of PSD and PDF of a random variable is highly related to each other except for the Gaussian distribution case. In Fig. 1, two random noise with uniform and Gaussian distribution is illustrated for the impact of filtering. It is clear from the comparison that although the output of filtering a Gaussian random noise still follows Gaussian distribution, filtering a uniform distribution random variable to get a specific PSD would transform the random variable to a different distribution. According to Central Limit Theorem, the filter actually makes the distribution more Gaussian-like.

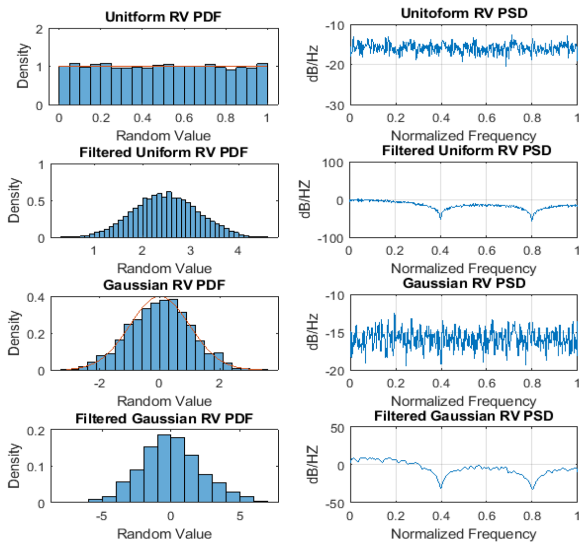

Fig. 1. Changing PSD of a Random Variable by Filtering affects Its PDF

But in most of the wireless channel simulation, the channel gain doesn't follow Gaussian distribution. Many system design and simulation need to consider Non-Gaussian noise [5], [6],

[7]. Thus many literatures have investigated how to generate colored Non-Gaussian noise. Non-Gaussian distributed random variable could be generated from well know distribution like Gaussian or uniform variables using Inverse Transform Sampling (ITS) [10, p.104] method. Since generally the inverse of the required Cumulative Distribution Function (CDF) is non-linear, complicated methods like Hermite Expansion is needed to compensate for the effect of the non-linear transformation. In [8], the concept of rearranging the random sequence to follow a reference sequence is explored to reshape the PSD without modifying the PDF. It is referred to as Sequence Rearrangement Block Method in this paper since it can only generate random variable in a block manner. since the rearranged sequence only asymptotically approaches the required PSD, the block size needs to be set to relatively large. This makes it ineffective for wireless communication simulation. In [12], the summation of real sinusoid is explored where all the amplitude, phase and frequency of the sinusoids could be used to derive the final random variable. So that the PDF could be controlled by amplitude and phase. And the PSD could be controlled by the selection of frequency component. But it can only be used to generate symmetric PSD.

To eliminate the limitations of existing methods, this paper uses Gaussian random variable filter method so that once the filter coefficients are determined, the channel gain random variable can be generated in real time. But in contrast to using complex Hermite Expansion to find the non-linear effect of Inverse Transform Sampling, our proposed method considers the ITS non-linear effect for PSD as distortion for the filter response. By taking into consideration of the non-linear effect of the ITS directly, Particle Swarm Optimization (PSO) is then used to find the pre-distorted filter solution. PSO is widely used to optimize various non-linear problems since it is first proposed in [13] due to its simpleness and effectiveness for implementation. As a population based stochastic algorithm, the concept of PSO is simple and inspired by birds flocking and fish schooling. It uses a preset number of random particles which represents a point in the solution space to search for a globally optimized answer to minimize a cost function.

This paper uses PSO to search for a pre-distorted filter that can correct the distortion caused by the non-linear effect of ITS so that the combined output generates the required PSD while the ITS will guarantee the generated variable meets the required PDF. Compared to existing methods, it has no assumption for PSD and PDF and it can generate the channel gain random variable in real time after the filter coefficients are found by PSO. The rest of the paper is organized as follows. Firstly a wireless channel model is illustrated where the channel gain could be modeled as a random variable following the Non-Gaussian distribution with a specific Doppler spectrum. Then the proposed solution for the generation of the Non-Gaussian variable is discussed in detail. Finally the numerical simulation for random variable with specific distribution and required PSD is presented to validate the proposed solution.

III. Problem Formulation

A Single Input Single Output (SISO) wireless channel model is shown in Fig. 2.

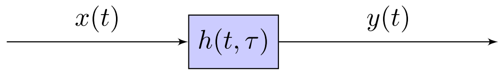

Fig. 2. SISO Wireless Channel Model

The channel convolution effect could be described as [9, p.44]

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(t,\tau)d\tau \tag{1}$$

where $y(t), x(t), h(t,\tau)$ is the received signal, transmitted signal and channel impulse response at time t respectively. For a SISO channel with N multiple paths, the $h(t,\tau)$ is modeled as (2) [9].

$$h(t,\tau) = \sum_{i=1}^{N} a_i(t)\delta(\tau - \tau_i(t)) \tag{2}$$

where $a_i(t), \tau_i(t)$ is the gain and delay at time t for i^{th} path respectively. The received signal could then be described in (3)

$$y(t) = \sum_{i=1}^{N} a_i(t)x(t - \tau_i(t)) \tag{3}$$

To simplify the discussion, we assume the channel is one tap wideband channel which is a valid assumption nowadays due to the wide use of Orthogonal Frequency Division Multiplexing (OFDM) in wireless communication [14]. Then (3) could be further simplified as

$$y(t) = g(t)x(t-\tau) \tag{4}$$

where $g(t) = \sum_{i=1}^{N} a_i(t)$ is the aggregated channel gain and τ is the one tap delay. The channel gain $g(t)$ is assumed to be a Wide Sense Stationary Process (WSS) which is normally valid for wireless channel simulation for symbol recovery.

The problem could be summarized as to generate a random variable G which represents the channel gain $g(t)$ that satisfies the two requirements at the same time: a specific Cumulative Distribution Function (CDF) F_G and a specific autocorrelation function $R_G(\tau)$. This is equivalent to meet the requirement of PSD and PDF at the same time.

IV. Proposed Solutions

A. Proposed Architecture for Noise Generation

Fig. 3 shows the model for the generation of the required non-Gaussian variable from a Gaussian distributed variable W.

The Gaussian random variable sequence $w(t)$ is filtered before feed into the ITS block. Since the filtered version of Gaussian variable is also Gaussian distributed, the ITS could be designed with the assumption that its input is Gaussian random variable. So that the ITS could be designed as:

$$G = F_G^{-1}(F_W(w)) \tag{5}$$

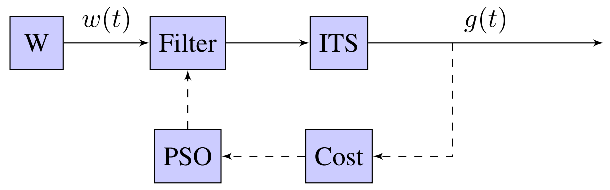

Fig. 3. Architecture of PSO based Non-Gaussian Noise Generation

where G is the generated random variable, F_G^{-1} is the inverse of the desired CDF of G, F_W is the CDF of Gaussian variable and w is the sample from a Gaussian random Variable W.

The generated random number $g(t)$ follows distribution function F_G no matter what filter coefficients are used since the input to ITS is always Gaussian. So we can focus on getting the right filter that can generate the right PSD for the output of ITS.

The proposed solution works in two stage. In the first stage, PSO is used to find the optimum filter coefficients. In the Second stage, the Filter process the input of Gaussian random numbers with the optimum coefficients.

B. Searching for Optimum Filter Coefficient by PSO

The number of filter coefficients defines the search space for PSO. For filters with n coefficients, the i^{th} particle p_i could be represented by the coefficients vector as:

$$p_i = [a_{i1}\ a_{i2}\ \cdots\ a_{in}]^T \tag{6}$$

where a_{i1}, a_{i1}, a_{in} is the filter coefficients for the i^{th} particle and $[\cdot]^T$ denotes the transpose operator. Each particle then tracks its local and global best location based on a performance cost measurement function.

The cost function in PSO is to measure the error caused by the deviation of each particle position to the optimum position. In this paper, the error e is designed to be the Mean Squared Error of generated PSD compared with the required PSD as follows:

$$e = \frac{1}{K}\sum_{k=1}^{K}(P_g(k) - P_r(k))^2 \tag{7}$$

Where K is the number of frequency point to be measured, $P_g(k)$ is the k^{th} frequency point of the generated random variable PSD and $P_r(k)$ is the k^{th} frequency point of the required PSD.

The location of each particle is then updated iteratively. The $(m+1)^{th}$ iteration of position p_i^{m+1} is then updated by its m^{th} position p_i^m and $(m+1)^{th}$ velocity v_i^{m+1} [15]:

$$p_i^{m+1} = p_i^m + v_i^{m+1} \tag{8}$$

The velocity v_i^{m+1} is also derived iteratively from the m^{th} iteration of velocity v_i^m, the detected local best location p_{i_lbest}, the global best location p_{gbest}^m and their weight and random number as follows[15]:

$$v_i^{m+1} = w_i v_i^m + w_l r_l (p_{i\_lbest}^m - p_i^m) + w_g r_g (p_{gbest}^m - p_i^m) \tag{9}$$

where w_i is the inertial weight, w_l is the self adjustment weight, w_g is the social adjustment weight. The uniform distributed random numbers r_l and r_g are used to explore the solution space by adjust the steps and direction randomly.

Once the error e reaches the required level of accuracy before the number of iterations exceeds threshold, it would be concluded that the PSO has found a proper filter coefficients with the required pre-distortion. Otherwise, manual check or adjustment might be needed on PSO parameters.

V. Numerical Simulation and Result Analysis

A first order AutoRegressive Model is used to verify the effectiveness of the proposed algorithm. The required channel gain is modeled as:

$$g(t) = \rho g(t-1) + w(t) \tag{10}$$

where $\rho = 0.4$ is used for the simulation, w is the Gaussian random variable for reference. The required channel gain needs to follow Rayleigh distribution for simulation. The PSO is designed with the self adjustment weight set to 0.3 and social adjustment weight set to 0.6 for the simulation. 25000 random numbers are generated for the comparison.

Fig. 4 illustrates the PDF of the original Gaussian noise input and the generated Rayleigh noise to simulate the channel gain. It shows that the generated noise follows Rayleigh distribution after ITS.

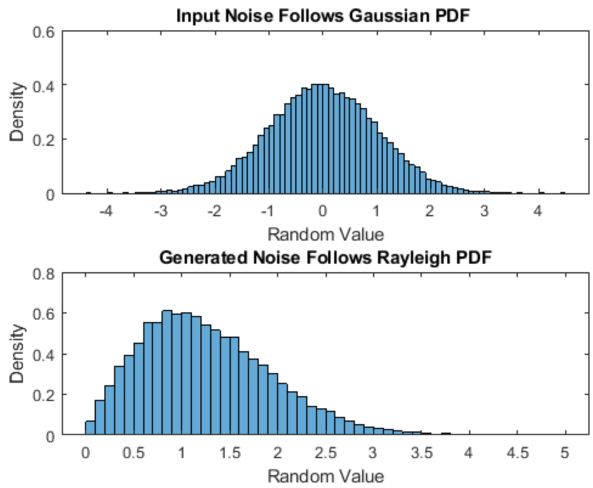

Fig. 4. Generated Noise Follows the Required Rayleigh PDF with Gaussian Noise as Input

Fig. 5 illustrates the PSD of the generated Non-Gaussian noise using the proposed PSO based filter and the Sequence Rearrangement Block Method.

To compare the similarity of the generated noise PSD to the required PSD, the correlation between the generated PSD and the required PSD is calculated in Table I. It shows that the generated random variables by both methods match to the required PSD. PSO based Filter method has slightly lower correlation value. This is also evidenced from Fig. 5 where

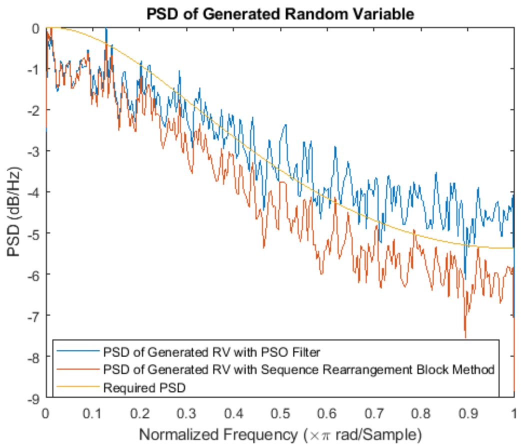

Fig. 5. Generated Noise Follows the Required PSD for Proposed Method

at higher frequency it deviates slightly from the required PSD. Part of the reason could be due to the numerical error generated from the optimization process. But the proposed PSO based filter method could be used to generate continuous noise while the Sequence Rearrangement Block method can only generate noise block by block thus delay is unavoidable.

TABLE I
COMPARISON OF CORRELATION TO THE REQUIRED PSD

Algorithm Name	Correlation to Required PSD
PSO Based Filter Method	0.9126
Sequence Rearrangement Block Method	0.9158

VI. CONCLUSION

A PSO based method for generating colored Non-Gaussian noise that meets both the requirements of PSD and PDF is proposed. The numerical simulation result confirms its effectiveness. The benefit of this new method is that the complexity of deriving the required pre-distortion for the filter is replaced with a simple PSO search method. And once the required filter coefficients are found, the generation of non-Gaussian noise could be done in real time by filtering and ITS. Compared to block based methods or other analytical methods, it is more suitable for real time wireless channel simulation. The limitation for the proposed method is that the inverse of the required PDF needs to be available. Future investigation would be suggested on the verification of the validity for various PDFs and PSDs that might be useful for wireless channel gain simulation.

REFERENCES

[1] S. Arikawa and Y. Karasawa, "A Simplified MIMO Channel Characteristics Evaluation Scheme Based on Ray Tracing and Its Application to Indoor Radio Systems," in *IEEE Antennas and Wireless Propagation Letters*, vol. 13, pp. 1737-1740, 2014.

[2] X. Mao, J. Jin and J. Yang, "Wireless channel modeling methods: Classification, comparison and application," *2010 5th International Conference on Computer Science & Education*, Hefei, 2010, pp. 1669-1673.

[3] F. P. Fontan and P. M. Espineira, *Modelling the Wireless Propagation Channel: A simulation approach with Matlab*, 2008, Wiley

[4] E. Perahia and R. Stacey, 2013, *Next Generation Wireless Lans: 802.11n and 802.11ac (2nd ed.)*, Cambridge University Press, New York, NY, USA.

[5] H. Jiang, Y. Lu and S. Yao, "Random matrix based method for joint DOD and DOA estimation for large scale MIMO radar in non-Gaussian noise," 2016 *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Shanghai, 2016, pp. 3031-3035.

[6] V. G. Chavali and C. R. C. M. da Silva, "Classification of Digital Amplitude-Phase Modulated Signals in Time-Correlated Non-Gaussian Channels," in *IEEE Transactions on Communications*, vol. 61, no. 6, pp. 2408-2419, June 2013.

[7] E. Eweda, N. J. Bershad and J. C. M. Bermudez, "Stochastic Analysis of the LMS and NLMS Algorithms for Cyclostationary White Gaussian and Non-Gaussian Inputs," in *IEEE Transactions on Signal Processing*, vol. 66, no. 18, pp. 4753-4765, 15 Sept.15, 2018.

[8] J. C. S. Santos Filho and M. D. Yacoub, "Coloring Non-Gaussian Sequences," in *IEEE Transactions on Signal Processing*, vol. 56, no. 12, pp. 5817-5822, Dec. 2008.

[9] D. Tse and P. Viswanath, 2005, *Fundamentals of Wireless Communication. Cambridge University Press*, New York, NY, USA.

[10] G. James E, 2004, *Random Number Generation and Monte Carlo Methods. Springer Science & Business Media*,

[11] Kibble, W. (1945). An extension of a theorem of Mehler's on Hermite polynomials. *Mathematical Proceedings of the Cambridge Philosophical Society, 41* (1), 12-15.

[12] S. Kay, "Representation and Generation of Non-Gaussian Wide-Sense Stationary Random Processes With Arbitrary PSDs and a Class of PDFs," in *IEEE Transactions on Signal Processing*, vol. 58, no. 7, pp. 3448-3458, July 2010.

[13] J. Kennedy and R. Eberhart, "Particle swarm optimization," *Proceedings of ICNN'95 - International Conference on Neural Networks, Perth, WA, Australia*, 1995, pp. 1942-1948 vol.4.

[14] A. M. Jaradat, J. M. Hamamreh and H. Arslan, "Modulation Options for OFDM-Based Waveforms: Classification, Comparison, and Future Directions," in *IEEE Access*, vol. 7, pp. 17263-17278, 2019.

[15] D. W. Boeringer and D. H. Werner, "Particle swarm optimization versus genetic algorithms for phased array synthesis," in *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 3, pp. 771-779, March 2004.