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Measuring the coefficient of restitution for all six degrees of freedom

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Abstract The coefficient of restitution is a cornerstone empirical parameter of any model where energy is dissipated by particle collisions. However, completely determining this parameter experimentally is challenging, as upon collision, a particle's material properties (such as roughness, sphericity and shape) or minor imperfections, can cause energy to be shifted to other translational or rotational components. When all degrees of freedom are not resolved, these shifts in energy can easily be mistaken for dissipated energy, affecting the derivation of the coefficient of restitution. In the past, these challenges have been highlighted by a large scatter in values of experimental data for the restitution coefficient. In the present study, a novel experimental procedure is presented, determining all six degrees of freedom of a single, spherical, nylon particle, dropped on a glass plate. This study highlights that only by using all six degrees of freedom, can a single reliable and consistent coefficient of restitution be obtained for all cases and between subsequent collisions.

Keywords Particle Characterisation · Coefficient of restitution · Particle Tracking Velocimetry · Rotational Moment

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1 Introduction

The coefficient of restitution (CoR) is widely used for simulations in many applications, for example: in nature, from molecular to planetary scale (Bizon et al., 1999; Li et al., 2016; Walsh et al., 2008); in modelling chemical reactants and catalyst collisions (Kuwabara and Kono, 1987; Kuo et al., 2002; Li and Guenther, 2012); and for determining trajectories of balls in sports-science (Briggs, 1945; Cross, 2002; Owens et al., 2003; Cross, 2015). The CoR was first derived from kinematics by Newton (1687), defined as the ratio of the relative velocities of the colliding partners after $v_{(b)}$ and prior $v_{(a)}$ impact. Experimentally these are determined from impact on a flat surface e.g. (Seifried et al., 2005; Briggs, 1945; Owens et al., 2003) or between two or multiple impacting spheres e.g. (Kuwabara and Kono, 1987; Foerster et al., 1994) and expressed as either the tangential or normal CoR. This definition can further be extended to be defined as the amount of kinetic energy E_d dissipated upon collision:

$$\epsilon = \left| \frac{v_{(b)}}{v_{(a)}} \right| = \sqrt{\frac{\mathrm{E}_{k(b)}}{\mathrm{E}_{k(a)}}} = \sqrt{1 - \frac{\mathrm{E}_d}{\mathrm{E}_{k(a)}}} \tag{1}$$

where $\mathrm{E}_{k(b)}$ is the kinetic energy after and $\mathrm{E}_{k(a)}$ prior to impact on a flat surface.

Whilst there have been countless experimental and theoretical investigations, refinements made to experimental techniques and the addition of empirically derived closures (Briggs, 1945; Garwin, 1969; Goldsmith, 1960; Kuwabara and Kono, 1987; Weir and Tallon, 2005) most previous reported values, for similar materials, vary considerably (Cross, 2002; Walsh et al., 2008; Antonyuk et al., 2010). As the CoR is related to the amount of energy dissipation, the energy dissipation depends mainly on the materials' properties (Crüger et al., 2016). Previous works on spherical particles have also suggested that even the smallest of imperfections at the micro-scale can lead to complex macroscopic tangential and rotational velocity components (Raman, 1918; Campbell and Brennen, 1985; Johnson and Johnson, 1987). As suggested by many theoretical and even experimental works (Brach, 1984; Li et al., 2004; Yao et al., 2005; Hastie, 2013) to accurately determine the CoR, the energy relating to all six degrees of freedom (DoF) (three translational and three rotational) must be fully resolved.

To these authors' knowledge, whilst a few studies have managed to capture the three translational DoF (Li et al., 2004; Ammi et al., 2009; Hastie, 2013; Crüger et al., 2016), and others have managed to determine one additional rotational DoF (Briggs, 1945; Li et al., 2004, 2016), there have been no studies which have experimentally determined all six DoF to fully close the rotational and translational energy budget. In the present study, using state-of-the-art digital image processing and stereo high-speed cameras, all six DoF and the total energy budget (see Eq. 1) is closed to determine the CoR via:

$$\mathrm{E}_k = \mathrm{E}_{k,v} + \mathrm{E}_{k,\omega} \tag{2}$$

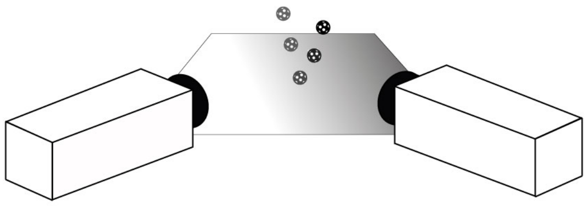

Fig. 1 Sketch of experimental setup (not to scale).

where, $E_{k,v}$, is the sum of kinetic energy associated to the three translation degrees of freedom defined by, $\frac{1}{2}m\mathbf{v}_i^2$, where, $\mathbf{v}_i$, contains the three velocity components and m, is the mass of the sphere. $E_{k,\omega}$, is the sum of the energy associated to the three rotational degrees of freedom defined by $\frac{2}{5}mr^2\omega_i^2$, where ω contains the three angular velocity components, and r is the radius of the sphere. In short, the purpose of this note is to draw attention to the apparent effects of all six DoF on the coefficient of restitution that might offer a method to ensure the consistency of computations based on more variable normal and tangential coefficients of restitution currently required to determine a body's velocity post-collision.

2 Experimental setup

An experimental investigation was undertaken in the MFAL Labs at the National Energy Technology Laboratory, Department of Energy, USA. The experiment aimed to determine the three dimensional path and angular rotations of a single spherical Nylon particle of diameter (D=5mm) and mass (m=1.23g) dropped from a height (h = 8cm) on to a glass surface. To determine the three dimensional (3D) spatial location of the particle two stereo high-speed Phantom v2640 cameras were used to capture two overlapping images from different orthogonal views (see Fig. 1). The cameras were time synced using the Phantom capture software internal clocks of the cameras, the images were acquired at 1000fps with a shutter speed of 200μs. Using an in-house code for 3D multi-camera reconstruction, based on the method of Maas et al. (1993), the 3D spatial locations were determined. Any image deformations were removed using a second order polynomial registration as fully described in Higham and Brevis (In Press) and the images were denoised using the PODDEM algorithm (Higham et al., 2016). To determine the (3D) rotational motion of the particle, a matrix of regular spaced circular points was printed on its surface (see Fig. 2). Using the same procedure as Bradley and Roth (2005) the three dimensional angular rotations were determined using six DoF.

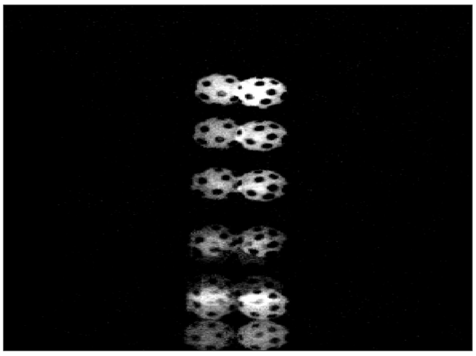

Fig. 2 Super imposed images and temporal displacements from stereo high-speed cameras. Where the diameter of the particle (D=5mm).

3 Results

Figure 3 shows the 3D trajectories and the respective rotational motion of a single Nylon particle dropped from H=8cm on a levelled glass plate. This approach gave a very small range of impact velocities and therefore any velocity dependency on the coefficient of restitution (as observed by Müller and Pöschel (2011)) was deemed negligible during the analysis, especially given the rigidity of the Nylon particle and glass plate. It is clear that in each case in Figure 3 the 3D trajectories differ due to the different contributions from the rotational velocity components during each impact. From the images, the trajectories created after the initial bounce and the onset of the rotation highlight how even the smallest of imperfections in the particles can have a significant effect on particle motion. In Fig. 4 the energy contribution from the impact and rotational components are plotted to illustrate the different contributions to the total energy budget over the collision.

The coefficient of restitution calculated using the individual translational and rotational energy component, as well as for the total translational and rotational energy and the total energy budget are detailed in Table 1. Each coefficient of restitution is given for the three collisions, averaged over the three cases shown in Figure 3, and the standard deviation is defined by the average of the variations. Note that coefficients for $E_{k,\omega(y)}$, $E_{k,\omega(x)}$ & $E_{k,\omega(z)}$ are zero due to particle having no rotation upon the first collision, and that negative coefficients indicate energy increasing in that rotational mode due to transfer from the translational energy prior to the collision.

It is possible to draw several conclusions from these results. If all three translational components ($E_{k,v}$) are taken into consideration, the coefficient of restitution differs by $\sim$40% between subsequent bounces. Between the three cases at each bounce there is a standard deviation of $\sim$30% with an average coefficient of restitution of $\sim$0.5. If only the vertical velocity component

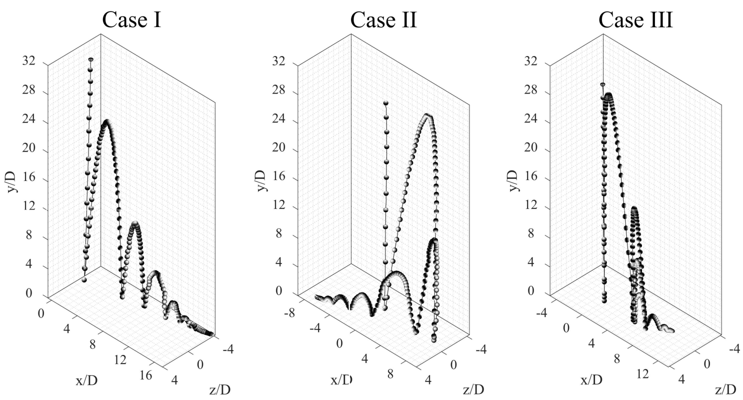

Fig. 3 The paths of cases I-III, where the origin of the axes is the point of first impact. (Size of particle is not to scale and all capture temporal locations not plotted for presentation purposes).

$(E_{k,v(y)})$ is taken into consideration between each bounce the results appear to improve, varying by only ∼8%, with a standard deviation of ≤5% between the cases. This would suggest why the vertical velocity approach to estimating the coefficient of restitution has been widely used to date. However, by only taking into consideration the vertical velocity component the coefficient of restitution is much greater; ∼0.72 on average. In comparison, the tangential components demonstrate a significantly higher variation and standard deviation, illustrated by the values for $\mathrm{E}_{k,v(x)}$ and $\mathrm{E}_{k,v(z)}$ in Table 1. This highlights the inherent inconsistency when attempting to utilise the tangential coefficient of restitution to determine repeatable information about velocity changes during a collision. Both results demonstrate that the normal and tangential coefficients of restitution cannot be classified as material constants, and that particle spin must be contributing to the energy dissipation within the collision.

As suggested in this note, the coefficient of restitution determination is greatly improved when using the total energy budget. Similarly to using the different components of translational energy, using only the individual components of rotational energy to determine the coefficient of restitution give high variances between different collisions and standard deviations between cases; and summing the rotational energies offers no improvement. Yet despite the small contribution of rotational energy towards the total energy budget, illustrated in Figure. 4, it must be incorporated into the determination of the final coefficient of restitution. The final column in Table 1 shows the best consistency in the coefficient of restitution over multiple collisions ($\epsilon^2 \pm 0.8\%$) and the smallest variance ($\sigma_{\epsilon^2} \pm 2\%$) over the three cases of interest, only achievable when using the total energy during the collision ($\mathrm{E}_{k,v+\omega}$). Therefore, despite the random effects of imperfections on collisions, it is possible to calculate a consistent coefficient of restitution as a material constant only by using the total energy budget approach.

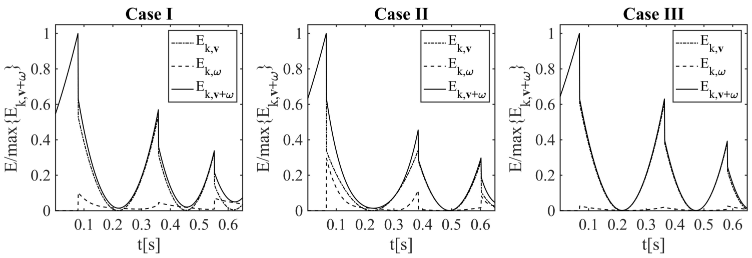

Fig. 4 Impact kinetic energy of a function of time $E_{k,\mathbf{v}}$, rotational kinetic energy $E_{k,\omega}$, and sum of impact and rotational kinetic energy $E_{k,\mathbf{v}+\omega}$

Collision	$E_{k,v(y)}$	$E_{k,v(x)}$	$E_{k,v(z)}$	$E_{k,\mathbf{v}}$	$E_{k,\omega(y)}$	$E_{k,\omega(x)}$	$E_{k,\omega(z)}$	$E_{k,\omega}$	$E_{k,\mathbf{v}+\omega}$
1	0.77±0.04	8.91±12.77	0.36±0.15	0.48±0.14	0	0	0	0	0.62±0.01
2	0.67±0.04	6.15±6.22	2.24±1.35	0.67±0.13	0.02±0.85	-3.67±6.96	-1.61±1.05	0.77±0.65	0.61±0.03
3	0.70±0.04	1.43±1.39	2.34±1.48	0.46±0.10	1.24±2.87	1.47±1.40	1.19±3.03	5.4±2.31	0.62±0.02

Table 1 Coefficient of restitution squared (ϵ^2) for each component, translational ($E_{k,\mathbf{v}}$), rotational ($E_{k,\omega}$) and their combination ($E_{k,\mathbf{v}+\omega}$) for three consecutive collisions averaged over three cases.

4 Conclusions

From the brief experimental results presented, it is clearly shown that to obtain a consistent material constant coefficient of restitution, all six degrees of freedom need to be determined. It is highlighted that upon collision even a spherical particle on a flat surface shifts energy into its different translation and rotational components. This energy shift could easily be mistaken for a dissipation in energy leading to poorly estimated normal and tangential coefficients of restitution and would explain their apparent variability. Whilst this note is brief, and only shows the results for one material, it fully meets its aim; to show that technical capabilities exist to determine all six degrees of freedom of a single particle, and how they could pave the way for accurate measures of the coefficient of restitution in future.

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6 Ethical Statement

This submission is fully compliant with the Ethical Standards of the journal. This work is a novel study not presented or published by these authors' previously.

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