Comparing Spatial Patterns

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Abstract

The comparison of spatial patterns is a fundamental task in geography and quantitative spatial modelling. With the growth of data being collected with a geospatial element we are witnessing an increased interest in analyses requiring spatial pattern comparisons (e.g., model assessment, change analysis). In this paper we review quantitative techniques for comparing spatial patterns, examining key methodological approaches developed both within and beyond the field of geography. We highlight the key challenges using examples from widely known datasets from the spatial analysis literature. Through these examples we identify a problematic dichotomy between spatial pattern and process – a widespread issue in the age of big geospatial data. Further, we identify the role of complex topology, the interdependence of spatial configuration and composition, and spatial scale as key (research) challenges. Several areas ripe for geographic research are discussed to establish a consolidated research agenda for spatial pattern comparison grounded in quantitative geography. Hierarchical scaling and the modifiable areal unit problem are highlighted as ideas which can be exploited to identify pattern similarities across spatial and temporal scales. Increased use of 'time-aware' comparisons of spatial processes are suggested, which properly account for spatial evolution and pattern formation. Simulation-based inference is identified as particularly promising for integrating spatial pattern comparison into existing modelling frameworks. To date, the literature on spatial pattern comparison has been fragmented and we hope this work will provide a basis for others to build on in future studies.

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1. Introduction

The comparison of maps is a fundamental part of how geographers try to understand the world. Quantifying spatial distributions and patterns, and comparing across regions or over time is central to many types of geographical research and applications. To illustrate, Figure 1 presents two maps from the recent Intergovernmental Panel on Climate Change (IPCC) Synthesis Report (IPCC, 2014) which shows recorded surface temperature change (1986-2005) and projected changes (2081-2100). We are confronted with two simultaneous map comparison tasks. First, we make comparisons locally within a map, noticing spatial differentiation within both observed and projected temperature regimes – noting in particular the rapid warming in polar regions. Second, we compare the maps globally, recognizing large magnitude shifts in temperature across most continents in the projected scenario compared to the period of observed temperature changes. In making these interpretations broadly, we mask uncertainties associated with more precise questions of change, such as which populations are likely to be most impacted by increasing temperatures, where should conservation resources be allocated, are countries in the global North more impacted than the global South, were the data collected equally in all regions, and countless other geographic questions. These maps are included in the IPCC report designed as guidelines for policy and decision-makers. The recognition and quantification of spatial change through comparison of spatial patterns, both globally and locally, represents an important and under-recognized research area for geography which we aim to review, critique, and contextualize.

<Figure 1 here >

Many of the origins for studying changes and differences in spatial patterns arose during geography's *quantitative revolution*. Today the sheer volume of geographically referenced data is providing new opportunities for geographers to compare spatial patterns across space, and time. Recent geographical data-intensive research streams include geocomputation (Openshaw & Abrahart, 1996); geospatial big data (Li et al., 2016), human dynamics (Shaw, Tsou, & Ye, 2016), and geographic data mining (Miller & Han, 2009). Long-term archives of satellite imagery, crowdsourced geospatial databases, and open data portals are now being developed and maintained for a variety of subject areas. Despite the amount of data-intensive geographic research taking place, geographers have not consolidated methods and models for performing spatial pattern comparisons which facilitate replication, identification of broader trends and underlying spatial process dynamics, and local anomalies.

In this paper we review existing quantitative techniques for comparing spatial patterns and discuss commonly encountered issues. We briefly cover basic concepts and terminology associated with spatial patterns before moving on to a review of selected techniques, and provide illustrative examples that highlight strengths and shortcomings of current methods. We conclude with some thoughts on the research needs for spatial pattern comparison (SPC) in geography today. In doing so we hope to provide some coherency and unity to the SPC research which is fractured across fields, and identify research opportunities for geographers interested in quantitative spatial analysis.

2. Characteristics of Spatial Pattern Comparison Problems

2.1 Characteristics of spatial patterns

As geographers, we typically ascribe meaning to spatial patterns as the outcomes of multiple and interacting spatial processes (O'Sullivan & Unwin, 2010). Isolation of processes themselves outside of laboratory or simulation environments is impossible at most geographically-relevant scales, so while detecting change in a spatial pattern can reveal changes in underlying processes, it is not sufficient to reveal those dynamics. Complicating matters, pattern itself acts on and perturbs the processes generating patterns (Turner, 1989). Even the term *spatial pattern* can itself imply multiple and conflicting phenomena. Here we use a definition (see Box 1 for a glossary of terms and definitions that will be used throughout) based on that of Dale (2000), that a spatial pattern is the scale-dependent predictability of the physical arrangement of observations.

< Box 1 (Glossary) Here >

2.2 Spatial representation

Spatial data are abstractions of reality, with 'features' (i.e., the things we demarcate, categorize, and label in the world) represented by *points*, *lines*, *polygons (areas)*, and *continuous spatial lattices (irregular or regular)* in a digital mapped form. These are the core spatial data types available in a geographic information system (GIS) and there exists relatively few other ways to represent spatial phenomenon in a GIS (Roberts & Robertson, 2016). The representation of complex physical and societal characteristics (in spatial data) influences how feature or attribute data can be characterized, visualized and subsequently analysed (Miller & Wentz, 2003). Nearly all examples of SPC that will be discussed in this paper represent what can be termed 'diagonal' comparisons (referring to a matrix of spatial data types; e.g., see O'Sullivan & Unwin, 2010, p. p26), that is, for example, point-point and lattice-lattice comparisons.

2.3 Statistical properties

Methods for SPC can also be characterised as being *entity-based* or *attribute-based*. Entity-based comparison consider only the *locations* of objects in the two maps, and are strictly limited to spatial comparisons of points, lines, and polygons (areas). Attribute-based comparisons simultaneously consider patterns associated with both location and attributes. With attribute-based comparisons, whether the attribute type is *continuous* or *categorical* also impacts how a spatial comparison is framed. Most attribute-based comparisons are associated with fixed spatial arrangements (i.e., the locations are the same in both maps), but this need not be the case.

SPC methods can further be broken down along dimensions of spatial pattern that they compare, whether local or global aspects of pattern, or the abundance (composition) or arrangement (configuration) of mapped values (Figure 2). At the global level, SPC can be undertaken by either computing a single univariate measure, such as Moran's I index of spatial autocorrelation (Cliff & Ord, 1973), individually on each map; or by computing a bivariate measure that simultaneously compares values in two maps. In the first case, only a coarse understanding of spatial pattern change can be inferred, for example a change from complete spatial randomness to spatially clustered. One of the challenges with the former approach, is that both local-to-global scaling and composition vs configuration are highly interdependent concepts, posing challenges for robust statistical significance testing. Some specific measures have been proposed for disentangling global and local spatial structure such as the O_i statistic (Ord & Getis, 2001), however these have not been widely adopted. Similarly, composition and configuration are also interdependent, and several authors have highlight the need to compare measures of spatial configuration only in the context of spatial composition (Cushman, McGarigal, & Neel, 2008; Long, Nelson, & Wulder, 2010; Remmel & Csillag, 2003). In bivariate comparison measures, these issues are partially overcome, as the distributional issues associated with comparison are resolved by reduction of the parameter space to a single metric (e.g., the root mean square error or the Kappa statistic).

2.4 Types of questions

Finally, SPC can be characterised as one of three types of question: *change*, *similarity*, and *association*. Each type of SPC question can be specified in terms of the constraints and variability in space, time, and theme of the patterns being investigated (Sinton 1978). Studies of change involve cases where space and theme are fixed and time varies. The goal of change analysis is often to identify whether change has occurred (globally), where such changes are located (locally), and whether changes represent a significant change (Boots & Csillag, 2006; Remmel & Csillag, 2003). Similarity questions involve situations where space is varied and theme is fixed (time can be fixed or varying). Similarity tasks are prominent in the image retrieval literature, and have been framed as SPC problems in many cases involving satellite imagery (e.g., Yang & Newsam, 2013). Studies of associations involve cases where space is fixed but theme varies (again time can be fixed or varying). Questions concerning spatial associations are common in both the physical and social sciences (e.g., Austin et al., 2005; Jones, Rendell, Pirotta, & Long, 2016).

A distinct and popular application for SPC is in spatial model assessment. Models that produce mapped outputs can be compared to reference data to assess model fit, or across model frameworks and parameterizations. Early examples of using SPC for model assessment include Cliff (1970) and Sokal et al. (1983), who both characterized model outputs with spatial autocorrelation statistics. Spatially explicit model assessment is critical as it can reveal patterns in error structures not evident in error statistics (e.g., Plouffe, Robertson, & Chandrapala, 2015). SPC for model assessment, has seen more recent interest in the area of categorical spatial data (e.g., land cover maps; Hagen-Zanker & Martens, 2008; Visser & de Nijs, 2006). Examining the spatial pattern of model outputs as a complementary measure of model quality underscores the importance of spatial pattern/process in environmental modelling (see Bennett et al., 2013).

<Figure 2 here>

3. General approaches for Quantitative Spatial Pattern Comparison

3.1 Visual spatial pattern comparison

The human visual system excels at recognizing shapes and patterns. Our brains are able to process information on shapes and patterns independently from context (Marr, 1985). Thus, to date a large body of work on SPC has involved visual comparisons, and most commonly this involves the presentation of maps side-by-side as a tool for visual SPC (see, for example, Figure 1 above). Comparing patterns in maps has led to some significant geographical insights, for instance, Wegener's work on continental drift theory was largely initiated by identifying similar patterns in maps of the coastlines of Africa and South America (Wegener 1966). However, comparing spatial patterns is visually challenging, as the human visual system is not well adapted to judging spatial correspondence between two variables on side-by-side maps and is more sensitive to the color classification scheme (e.g., Lloyd & Steinke, 1977; Steinke & Lloyd, 1983). Moreover, MacEachren (1995, p. 403) suggests that we would expect visual comparisons of maps to be more successful when changes are compositional (e.g., symbol size, color) then with configuration changes (e.g., shape, and orientation).

Perception of patterns in maps is a function of our perceptual attention (i.e., where, and for how long we look), also termed *saliency*. When comparing maps, *cosaliency* represents the importance of locations in side-by-side comparisons, and cosalient features in map pairs may not correspond to salient features in individual maps (Jacobs, Goldman, & Shechtman, 2010). Within the cartographic literature a number of techniques for enhancing visual map comparison tasks have been developed (See Ch. 9 in MacEachren, 1995 for example). Many newer techniques have abandoned side-by-side comparisons in favor of overlay techniques, and implement tools such as translucents, swiping, and lenses (Lobo, Pietriga, & Appert, 2015). However, it is widely acknowledged that visual SPC is challenging, most notably in assessing changes in spatial configuration, a problem that continues to hinder both visual and quantitative assessments of SPC.

3.2 Comparing spatial point patterns

Tobler (1965) studied the correspondence between pairwise point patterns of the locations of birth for a sample of married couples in Japan using a comparison measure which he termed the affine correlation statistic. The approach was innovative in terms of its attempt to draw on Pearsons correlation coefficient, but was limited to cases where two patterns had the same number of points and the points are naturally paired. Ecologists were early adopters of statistical methodologies for comparing bivariate point patterns, notably the work of Anderson (1992) who drew on seminal methods from Diggle (2003), to compare one point pattern to another with the bivariate extension of the K-function.

In biology, tight coupling of spatial and physical factors that drive the control and function of cellular functions and processes demands rigorous methods to detect differences in pattern. Myers (2012) highlights the increasing need for quantitative approaches for SPC in microscopy resulting from new forms of medical imaging data that are often used to derive spatial point patterns. Many proposed approaches have bene tailored to specific biological applications (e.g., Bell & Grunwald, 2004; Burguet & Andrey, 2014) however opportunities exists for generalization to other, more complex, classes of spatial data.

3.3 Comparing line and polygon data

There are fewer methods available for comparative analysis of line and polygonal pattern data. Within the geographical literature, line and polygonal spatial representations tend to be treated as features rather than patterns, with emphasis of methods on the proximity and orientation relations between pairs of objects, and summarizing such metrics over the dataset provide a measure of global similarity. Maruca and Jacquez (2002) provide polygon comparison statistics called area-based association measures, which essentially quantify the degree of correspondence based on area overlap between two of polygon pattern datasets. Proximity relations for linear data have been explored in the context of spatial data accuracy assessment, such as buffering a reference line and comparing the proportional overlap of a test line (e.g., Goodchild & Hunter, 1997).

Graph analysis provides a large body of theory and methods for characterizing networks, which is a common way to represent spatial line data in geographic applications. These network representations can be compared through measures of including centrality, connectivity, degree and others. However, often the geographical context is ignored to focus on topological properties. The uptake of these methods in a spatial context has been greatest in landscape research, examining structural (e.g. configurational) properties of a matrix of habitat patches (i.e., nodes) and their spatial connectivity (i.e., edges) (Bunn, Urban, & Keitt, 2000; Urban & Keitt, 2001; Urban, Minor, Treml, & Schick, 2009). Similarly, landscape ecologists are interested in SPC of maps of land cover types, using indices of diversity, fractal dimension, and shape (e.g., Mladenoff, White, Pastor, & Crow, 1993). However, these comparisons are focused on comparing the measures of pattern in one map to another, rather than on bivariate methods. Robertson et al. (2007) provide a comparison framework adapted from Sadahiro and Umemura (2001) for studying temporal changes in spatial polygons which focuses on problems where the spatial locations of polygons move through time (e.g., a forest fire). Here, changes are characterized as events derived from topological and proximity relations of two polygon patterns.

3.4 Comparing patterns in spatial lattices

There are many more methods for comparing patterns on spatial lattices, and here we refer to the case where the spatial structure of the lattice does not differ between the two maps (e.g., the spatial units are the same). One of the first applied quantitative analyses comparing spatial patterns was that of Robinson and Bryson (1957) who looked at the spatial correlation between precipitation and population in Nebraska by mapping regression residuals to describe the spatial correspondence – a technique which has since been employed for assessing spatial models (e.g., Hengl, Heuvelink, & Stein, 2004). Cliff (1970) looked at the correspondence of Hägerstrand's (1967) innovation diffusion data, comparing empirical data to theoretical simulations, which represents the first example where the spatial autocorrelation of the difference between maps (tested using joint counts based on whether the difference was positive or negative) was used as a measure of spatial correspondence. Spatial autocorrelation analysis of residual differences as proposed by Cliff (1970) remains influential today (e.g., Wulder, Boots, Seemann, & White, 2004).

Several contemporary authors have proposed varied approaches for quantifying spatial associations, predominantly for use with continuous-valued lattice datasets (e.g., attributes in counties). Sokal and Wartenberg (1983) used spatial correlograms (from Moran's I and Geary's C) to characterize similarity in gene-frequency surfaces between simulated populations under an isolation-by-distance model. Hubert et al. (1985) propose a spatial cross-product statistic in an attempt to distinguish spatial pattern similarity from attribute similarity. Specifically, Hubert et al., demonstrate how various map patterns can arise when holding Pearson's correlation coefficient constant, confounding spatial comparison problems. Global statistics (like the correlation coefficient) are insensitive to variations in local spatial patterning. Haining (1991) proposed spatial adjustments for the Pearson and Spearman correlation coefficients by adjusting the significance test of the statistic to account for spatial structure (measured as spatial autocorrelation) present in the data.

Cumulative distribution functions have been proposed for SPC problems because they are able to consider the shape of the underlying empirical distributions (Syrjala, 1996). Wong (2001) proposes a local cumulative distribution function as a means to use the widely employed cumulative distribution function in a spatially-local comparison. In analysis of neighbourhoods and their social characteristics, comparisons of both geographic and multivariate demographic characteristics has led to use of self-organizing maps to link social factors and spatial patterns (Spielman & Thill, 2008), and approach which decomposes spatial and thematic properties into separate 'map' spaces, which can then be visualized and explored for patterns.

Additional methods for SPC have focused on the development of local forms of spatial analysis (Boots & Okabe, 2007; Fotheringham & Brunsdon, 1999). Fotheringham et al. (2002) propose a geographically weighted correlation coefficient as a spatially-local tool for studying bivariate associations extending Pearsons correlation coefficient to local analysis. A further extension of the locally weighted correlation coefficient was presented by Lee (2001) which, combines Pearson's R with a bivariate Moran's I into a single statistic that simultaneously considers correlation and autocorrelation. A spatially-local version of the statistic is also presented along with a formal statistical testing framework (Lee, 2001). Robertson et al. (2014) extend an image comparison metric - the structural similarity index (SSIM; Z. Wang, Bovik, Sheikh, & Simoncelli, 2004) - for comparing spatial patterns within a spatial model assessment framework. Further, separation of local patterns into the first order, second order and pattern components provides significant opportunity for studying differences in local spatial patterns.

Computer scientists have also been intensively developing methods for image matching and comparison, which is analogous to the comparison of raster data. For example, Scharstein (1994) used an image shifting approach based on localized gradient field to assess how well two image patterns align. Comparative histogram-binning methods such as the Earth-mover's

Distance (Rubner, Tomasi, & Guibas, 2000) use multi-dimensional histograms that describe colour and texture and define distance measures in these spaces to characterize similarity of images. Applications of these methods in geographic contexts, most notably for retrieval and characterization of satellite imagery, are increasing (Jasiewicz, Netzel, & Stepinski, 2014; Kranstauber, Smolla, & Safi, 2016; Shao, Zhou, Zhang, & Hou, 2014).

The above methods focus predominantly on comparisons of continuous value attribute data on a lattice, but there is a great deal of work on comparing categorical lattices as well. The Kappa statistic tests agreement between lattices relative to what would be expected by chance, and its widespread use in remote sensing is thought to be due to its familiar interpretation, even when its mathematical underpinnings are poorly understood or erroneous (Pontius Jr & Millones, 2011). Hagen-Zanker (2009) extended the statistic to use fuzzy relations between categories and spatial location similarities as a bivariate SPC tool for categorical lattices. Pontius Jr and Millones (2011) argue for a new type of comparison measure that considers both *quantity* and *allocation* disagreement in mapped categories. Pontius Jr and Millones emphasize that a valid metric for spatial comparison should a) avoid compressing the two dimensions of pattern into one metric, and b) characterize disagreement rather than agreement.

4. Issues in SPC analysis

4.1 Examples

To demonstrate the challenges associated with SPC we have hand-picked a set of five examples (see Table 1 and Figure 3). For each example, we have chosen a representative and current technique for quantitative SPC associated with each data type. In all cases we have selected a single global statistic for comparison. Through the use of basic comparison statistics as a starting point, we highlight some the challenges associated with SPC analysis.

< Table 1 Here>

<Figure 3 here>

4.2. Highlight problems in spatial pattern comparisons

Problem 1: Pattern vs Process

Perhaps the biggest challenge emerging from the 'big data' revolution is that of connecting the analysis of patterns in the data with the underlying processes that we are interested in studying (Miller & Goodchild, 2015). As an example, consider the comparison of the red oak and white oak patterns. The results suggest there is evidence of a relationship between the spatial patterns of the two species, but we do not have any theory to support this at the process level. Perhaps there is inter-species attraction due to seed dispersal, shading characteristics, or interactions with forest disturbance agents (e.g., wildfire, insects). Note also that this is really of question of spatial *interaction* at the process level, which manifests in spatial *similarity* at the pattern level. It is unclear how likely this similarity is under independent spatial processes governing the distribution of red and white oaks. Static spatial patterns are inherently limited in their ability to describe dynamic processes. More data does not necessarily improve this limitation and may in fact add additional noise.

<u>Problem 2: Topological Complexity</u>

Comparison of topological characteristics of spatial data is typically reduced to comparison of connectivity matrices or graphs. In the example comparing node degree for OSM street networks for Waterloo, Canada and St. Andrews, Scotland. Spatial non-stationarity in road network density in Waterloo was present, whereby node degree of the dense parts of the network in the downtown area more difficult to observe in contrast with the less dense parts of the network in rural outlying areas, which tended to have four-node intersections. The similarity in node degree in the two networks was masked partially by the dis-similarity in network densities. While computing the node-degree values is straightforward, the results here highlight the difficulty in isolating one component of pattern to compare. Typically, the overall comparison of pattern similarity for the HVS is a composite of several dimensions of spatial pattern. Developing metrics or aggregate indicators of similarity of spatial patterns therefore hinges on identifying the key dimensions of pattern for a specific comparison task. Comparing topological properties may be an example where 'spatial intuition' and computed values are misaligned, as slight spatial changes can have large impacts on topology (e.g, undershoots in routing problems).

Topology is also confounded by spatial representation decisions in maps when visualizing comparisons. The visual assessment of pattern similarity between the mountain pine beetle polygons is certainly impacted by a number of classical cartographic pitfalls, such as a failure to include a reference basemap, map graticule, grid lines, or even a scale bar. However, a much more challenging problem arises when comparing objects that exhibit such a highly complex topology (such as the infestation polygons with irregularly shaped borders, holes, and multiple polygon parts). Had these two infestation polygons exhibited regularly shaped boundaries the comparison process would be easier (both visually, but also computationally). But complex topological shapes, including less binary gradients and boundaries, are the norm in environmental applications (Gustafson, 1998), and are salient in many anthropogenic examples (Batty & Xie, 1994). Thus, characterizing the similarities between complex shapes and patterns in a single (or multiple) index remains an ongoing challenge in SPC.

Problem 3: Composition vs Configuration

The description of spatial patterns can be decomposed into two unique but interdependent components: *composition* and *configuration* (Boots, 1982, 2003). To generalize, composition refers strictly to the aspatial properties of the elements of a spatial pattern (e.g., the type, number, and statistical properties of *what* is being mapped), while configuration refers to the strictly spatial arrangement of these elements (i.e., the *where*). Consider the Plum Island Ecosystem maps in Figure 2 (g-h). The most basic description of configuration refers to homogeneous (no variation exists) vs. heterogeneous (i.e., the observed pattern varies across space) spatial patterns. In practice, assessing configuration involves quantifying the level and nature of heterogeneity in mapped data and a wide set of terminology and techniques are available. These terms are typically both data and application specific; and can be used differently depending on the context of the analysis.

Dependency between composition and configuration of spatial patterns is demonstrated in Figure 4. Previous research has demonstrated that the potential for different spatial configurations to arise is largely dependent on the composition of elements in the map (Remmel & Csillag, 2003; X. Wang & Cumming, 2011). Thus, quantifying SPC is complex due to the potential for changes in configuration to arise solely due to changes in composition (i.e., Figure 4), confounding inferences into SPC (Long et al., 2010; Remmel & Csillag, 2003; X. Wang & Cumming, 2011). Indices for SPC must be able to simultaneously consider and disentangle the level of compositional and configurational change to be effective.

<Figure 4 here>

Problem 4: Spatially Global Indices

To date, most approaches for SPC are spatially global, producing a single statistic for the entire study area (indeed all five of the indices we employed fall into this category). With large-area and 'big' sources of spatial data, this can be misleading as global statistics fail to adequately capture spatial non-stationarities in observed patterns. However, spatially local analysis of big data also poses challenges since outputs require some interpretation, a non-trivial task with increasingly large datasets. With spatial-temporal local models, more sophisticated geovisual analytics may be required to understand the complex output stemming from local analysis of large mapped datasets (Foley & Demšar, 2012). But relying on visual interpretations can be challenging given the characteristics of many modern large datasets (i.e., coverage over broad-scales, with fine spatial resolution). Visualization as a tool for SPC (see Section 2.2) can be challenging with large datasets, due to maps being portrayed at a minimum resolution that is beyond our perceptual limits. Geographic knowledge discovery (Miller & Han, 2009) approaches may be suitable for performing SPC in large geographic databases.

A further challenge commonly encountered is that a single output statistic may not be sufficient for performing SPC with complex spatial patterns. For example, the negative correlation identified in the Georgia data may not be consistent across the entire state, and a spatially sensitive correlation measure (e.g., see p. 172 in Fotheringham et al., 2002) would shed further insight into the spatial variation in correlation. With increasingly large datasets (big data), moving the analysis scale from the global to the spatially local scale is necessary to capture how spatial pattern comparisons vary across space.

4. Moving the Spatial Pattern Comparison Research Agenda Forward

4.1 Comparing maps as spatial processes

Csillag and Boots (2005) advocate a process-based framework for comparing spatial patterns and identify two underlying questions that we, as geographers, should be seeking to answer in all SPC related-tasks: 1) Could the observed differences in spatial patterns have arisen purely by chance? and 2) Could the observed spatial patterns have been generated by the same process?. Pearl (2009) makes the case for a clear discrimination between *associative* and *causative* statistical analysis where associative analysis considers any relationship that can be defined by joint distribution of two variables and a causative relationship is one that cannot be defined by the joint distribution alone. With respect to SPC nearly all methods would fall into the former category, whilst Csillag and Boots (2005) make the emphatic case for models that fit squarely into the latter. One of the potential areas where new models are providing avenues for new insight along this causative line of thinking is through the development of complex simulations which can be used to test spatially explicit hypotheses (O'Sullivan & Perry, 2013).

Spatial analysis theory considers a map as a single realization of a stochastic spatial process, and thus inference regarding two static maps, if treated independently, yields a sample size of two. Spatial inferences pertain to the underlying process, though the particulars of what a mapped pattern represents have been debated (e.g., Summerfield, 1983). Cressie (1993) cites two basic contexts for doing spatial modelling; when a spatial process has reached temporal equilibrium and its spatial properties describe causative components of that process, and when short-term causal effects are aggregated over a fixed time period and expressed spatially. Comparing spatial patterns disconnected from their generative (temporal) processes incurs a high risk of finding differences resulting from natural variability. Explicit incorporation of *time* into an SPC framework may provide a way to both handle big spatial data and still reason about generating processes. Two ways we may be able to develop this are to 1) develop comparative tools for continuous spatial evolution, and 2) undertake spatial multi-pattern comparisons; both which imply a more explicit treatment of spatial processes.

Simulations provide an attractive framework for SPC as they allow experimentation with model parameters, incorporation of nonlinear dynamics and feedbacks, and flexibility in the types of model output (e.g., maps) that are generated. Two dominant approaches to simulating spatial patterns and processes are widely used; individual-based models (IBMs), and spatial-covariance models (SVMs). IBMs provide complete flexibility to specify all important dynamics of the geographical system under investigation, which can then be used to draw patterns from the model. Generating reference distributions for SPC metrics can be part of model sensitivity testing. For example, evaluating the model's sensitivity to parameter uncertainty from the perspective of spatial pattern is an interesting application area for SPC ., Emergent spatial patterns play a central role in developing, parameterizing, and extracting knowledge from IBMs (Grimm et al., 2005), and exemplar spatial patterns for specific processes can be used to find model parameter values through inverse fitting procedures that depend on a pattern comparison metric (Burnham & Anderson, 2002; Wiegand, Revilla, & Knauer, 2004). Such an approach has been recently tested in a more formal framework that provides model selection of parameter values *and* structure by 'approximate Bayes' methods (van der Vaart, Beaumont, Johnston, & Sibly, 2015). The extension of these new approaches for constructing, fitting, and assessing IBMs to incorporate explicitly spatial metrics is an exciting research opportunity for SPC.

SVMs instead require specification of the form of spatial pattern that results from the model (or process that generates it), which might more accurately reflect observed data, but tend to have less mechanistic meaning. Remmel et al. (2002) used a conditional autoregressive (CAR) model to simulate three types of landscapes and to compare landscape pattern indices under each landscape-type scenario. The resulting distributions provide reference for interpreting differences between two LPI values when performing landscape. Long et al. (2012) used simulations from a space-time model that incorporated a similar spatial covariance structure (CAR prior) to model the probability of spread of a binary infection process on a lattice. These types of spatial simulations are now widely employed in model testing, comparison, and evaluation where simulated data is used to compare spatial parameter estimates from different model specifications to a known underlying spatial process (e.g., Fotheringham & Oshan, 2016). Currently, visual comparisons and aspatial metrics are the de facto standard for SPC in this context (e.g., Wheeler, 2010) however the specification of SPC metrics are equally important in the SVM approach, and could perhaps serve as a point of reference for comparing inferences obtained from different modelling frameworks .

4.2 Spatially local analysis

There is a clear need for robust spatially sensitive metrics, which seems like a surprising thing to be championing given widely available tools for local spatial analysis. However, these tools are largely appropriate only with spatial lattices (regular and irregular) and have failed to be adopted more broadly. In a growing number of applications and decision-making contexts, rigorous definitions of pattern similarity need to be adopted (e.g., Churchill et al., 2013; Sakieh, Amiri, Danekar, Feghhi, & Dezhkam, 2015). When numeric or categorical data are obtained over comparable spatial units and the SPC task pertains to how that data are spatially configured across those units, measures of spatial pattern such as Moran's I, Geary's C, or local variants can be employed. Waller (2014) provides a convincing argument for the need for explicitly *spatial statistical* thinking in approaching analysis of geographical data, citing common research motivations such as assessing fit of spatial models or spatial assessment of statistical performance. Methods for SPC reviewed here can directly contribute to development of a spatial statistical approach to science by providing tools for the robust comparison of spatial patterns.

Yet the methods needed to answer comparative questions are often lacking. To demonstrate this, a linear regression performed between '% rural' and '% with a college degree' from the Georgia dataset, and the residuals were retained and shuffled across the counties randomly (Figure 5). Two very different spatial patterns emerge which have identical error statistics (MAE 0, RMSE 4.46). A reasonable question might be to ask whether the differences are due to chance or the result of different underlying spatial processes, or rather, what are the chances of obtaining I= -0.169 and I = 0.274 from this configuration of spatial units and values, if the underlying processes are the same. Given the exact distribution of Moran's I (Tiefelsdorf & Boots, 1995) we can compute probabilities of observed patterns based on a null hypothesis of no spatial structure, but cannot use these results to compare two patterns directly. Tiefelsdorf (1998) gives a conditional expectation of Moran's which allows comparison of competing spatial process hypotheses as expressed through the spatial weights matrix. Clifford et al. (1989) provide a t-test for comparing spatial structure in the context of a correlation coefficient, yet do not give us a tool to understand if the spatial process giving rise to the two patterns is the same or not.

<Figure 5 here>

4.3 Scale and MAUP

Spatial scale is often determined either arbitrarily or by a fixed set of intervals – both in terms of grain and extent. This is critical for SPC because scale is intimately tied to the definition of and observation of spatial patterns (Levin 1992; Dale 2000). The infinite number of scales available for both making observations (i.e. grain) as well as observing patterns (i.e., extent), and the inter-relatedness of these constructs, makes comparison tasks challenging. Not all scales are created equal for a given problem, and a set of *characteristic scales* are optimal for analysis (Wiens, 1989). Big data provides opportunities for linking spatial processes across scales, especially if data evolve over time. Previously, issues of scale in geographical analysis tended to focus on the modifiable areal unit problem, whereby 'scale effects' are assessed by varying aggregation units (e.g., Jelinski, Wu, & Wu, 1996). For SPC problems, variances due to scale may be a critical aspect of pattern-observation and thus comparison. Sémécurbe et al. (2016) provide an example of using multifractal analysis that quantifies MAUP to better understand spatial heterogeneities in population density in France, developing a typology of settlement patterns.

Yan and Li (2015) stress the need for both mathematical and psychological justifications in the definition of spatial similarity measures (i.e., linking similarity to the HVS). For the case of automated map generalization, a hierarchical scheme of maps, layers, groups, and objects (i.e., points, lines, areas) is presented which define the fundamental units for which spatial similarity relations are sought. The relations for comparing spatial objects at different scales are distinct from the comparison of patterns. In the Yan and Li system, object properties (size, shape, area etc.) and object group properties (topology, distance, correction etc.) may be a way to integrate dimensions of similarity at the pattern or regional scale.

4.4 Guidance for performing SPC

We provide six simple guidelines for researchers wishing to compare spatial patterns in their own applications.

1. Visual comparisons are useful – comparing two maps visually is a crucial first step in the exploratory spatial data analysis process.
2. Quantitative measures are necessary – the subjectivity of the visual comparison process means that any visual comparison should be further explored using a quantitative comparison metric.
3. Local SPC measures are preferred – global SPC measures are subject to all the issues associated with global spatial analysis procedures (Fotheringham & Brunsdon, 1999).

4. Quantifying dissimilarity is better – indices that focus on characterizing differences in patterns, over similarities in pattern are more likely to provide meaningful inferences (Pontius Jr & Millones, 2011).
5. Consider multiple elements of spatial pattern – spatial patterns have a variety of characteristic components. Comparison measures capable of disentangling different elements of spatial pattern within the SPC context are more informative than summary measures.
6. Don't forget processes – understanding the linkages between processes and patterns is the most challenging part of spatial analysis. Quantified pattern (dis)similarities may be related to unknown confounding processes.

5 Conclusions

SPC is a complex task, which is difficult to automate, has a mixture of computational and psychological components, and is increasingly required as geography and other fields exploit bigger and more varied spatial datasets. Here we have reviewed the literature on SPC that comes from a wide array of disciplines where applied problems have developed specific comparison methods, lacking any coherent conceptual or theoretical framework. Our review has focused on comparing spatial patterns of similar spatial representations (e.g., point-point, lattice-lattice), there are however significant prospects for developing new methods for 'off-diagonal' comparisons (e.g., point-polygon, line-lattice etc.). Many of the classical problems of geography such as pattern vs process, scale, MAUP, and topology become exacerbated in SPC. The spatial patterns we observe in maps are determined partially by spatial representation, aspatial characteristics, data collection components, the truly spatial component, and some element of randomness. More research into how these various components interact to create spatial distributions we observe, through simulation and empirical data catalogs, would bolster our ability to develop spatial modeling tools that support SPC.

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References

Andersen, M. (1992). Spatial analysis of two-species interactions. *Oecologia*, *91*, 134–140.

Austin, S. B., Melly, S. J., Sanchez, B. N., Patel, A., Buka, S., & Gortmaker, S. L. (2005). Clustering of fast-food restaurants around schools: A novel application of spatial statistics to the study of food environments. *American Journal of Public Health*, *95*, 1575–1581.

Baddeley, A., & Turner, R. (2005). Spatstat: an R package for analyzing spatial point patterns. *Journal of Statistical Software*, *12*, 1–42.

Batty, M., & Xie, Y. (1994). Modelling inside GIS: Part 1. Models structures, exploratory spatial data analysis and aggregation. *International Journal of Geographical Information Systems*, *8*, 291–307.

Bell, M. L., & Grunwald, G. K. (2004). Mixed models for the analysis of replicated spatial point patterns. *Biostatistics*, *5*, 633–648.

Bennett, N. D., Croke, B. F. W., Guariso, G., Guillaume, J. H. a, Hamilton, S. H., Jakeman, A. J., … Andreassian, V. (2013). Characterising performance of environmental models. *Environmental Modelling & Software*, *40*, 1–20.

Bivand, R. S., Yu, D., Nakaya, T., & Garcia-Lopez, M. A. (2015). Package “spgwr.”

Boots, B. (1982). Comments on the use of eigenfunctions to measure structural properties of geographic networks. *Environment and Planning A*, *14*, 1063–1072.

Boots, B. (2003). Developing local measure of spatial ssociation for categorical data. *Journal of Geographical Systems*, *5*, 139–160.

Boots, B., & Csillag, F. (2006). Categorical maps, comparisons, and confidence. *Journal of Geographical Systems*, *8*, 109–118.

Boots, B., & Okabe, A. (2007). Local statistical spatial analysis: Inventory and prospect. *International Journal of Geographical Information Science*, *21*, 355–375.

Bunn, A. G., Urban, D. L., & Keitt, T. H. (2000). Landscape connectivity: A conservation application of graph theory. *Journal of Environmental Management*, *59*, 265–278.

Burguet, J., & Andrey, P. (2014). Statistical comparison of spatial point patterns in biological imaging. *PLOS ONE*, *9*, e87759.

Burnham, K. P., & Anderson, D. R. (2002). *Model selection and multi-model inference: a practical information-theoretic approach*. Springer.

Churchill, D. J., Larson, A. J., Dahlgreen, M. C., Franklin, J. F., Hessburg, P. F., & Lutz, J. A. (2013). Restoring forest resilience: From reference spatial patterns to silvicultural prescriptions and monitoring. *Forest Ecology and Management*, *291*, 442–457.

Cliff, A. (1970). Computing the spatial correspondence between geographical patterns. *Transaction of the Institute of British Geographers*, *50*, 143–154.

Cliff, A., & Ord, J. K. (1973). *Spatial Autocorrelation*. London: Pion Limited.

Clifford, P., Richardson, S., & Hemon, D. (1989). Assessing the significance of the correlation between two spatial processes. *Biometrics*, *45*, 123–134.

Cressie, N. (1993). *Statistics for Spatial Data*. New York: John Wiley & Sons Inc.

Csillag, F., & Boots, B. (2005). A framework for statistical inferential decisions in spatial pattern analysis. *The Canadian Geographer*, *49*, 172–179.

Csillag, F., & Boots, B. (2005). Toward comparing maps as spatial processes. In *Developments in Spatial Data Handling* (pp. 641–652).

Cushman, S. A., McGarigal, K., & Neel, M. C. (2008). Parsimony in landscape metrics: Strength, universality, and consistency. *Ecological Indicators*, *8*, 691–703.

Dale, M. (2000). *Spatial Pattern Analysis in Plant Ecology*. Cambridge: Cambridge University Press.

Diggle, P. J. (2003). *Statistical Analysis of Spatial Point Patterns* (2nd ed.). New York, NY: Arnold.

Eugster, M. J. A., & Schlesinger, T. (2013). osmar: OpenStreetMap and R. *The R Journal*, *5*, 53–63.

Foley, P., & Demšar, U. (2012). Using geovisual analytics to compare the performance of geographically weighted discriminant analysis versus its global counterpart, linear discriminant analysis. *International Journal of Geographical Information Science*, *27*, 633–661.

Fotheringham, A. S., & Brunsdon, C. (1999). Local forms of spatial analysis. *Geographical Analysis*, *31*, 340–358.

Fotheringham, A. S., Brunsdon, C., & Charlton, M. (2002). *Geographically Weighted Regression: The Analysis of Spatially Varying Relationships*. Chichester: Wiley.

Fotheringham, A. S., & Oshan, T. M. (2016). Geographically weighted regression and multicollinearity: dispelling the myth. *Journal of Geographical Systems*, *18*, 303–329.

Goodchild, M. F., & Hunter, G. J. (1997). A simple positional accuracy measure for linear features. *International Journal of Geographical Information Science*, *11*, 299–306.

Grimm, V., Revilla, E., Berger, U., Jeltsch, F., Mooij, W. M., Railsback, S. F., … DeAngelis, D. L. (2005). Pattern-oriented modeling of agent-based complex systems: lessons from ecology. *Science*, *310*, 987–991.

Gustafson, E. (1998). Quantifying landscape spatial pattern: what is the state of the art? *Ecosystems*, *1*, 143–156.

Hagen-Zanker, A. (2009). An improved Fuzzy Kappa statistic that accounts for spatial autocorrelation. *International Journal of Geographical Information Science*, *23*, 61–73.

Hagen-Zanker, A., & Martens, P. (2008). Map comparison methods for comprehensive assessment of geosimulation models. *Computational Science and Its Applications–ICCSA 2008*, 194–209.

Hägerstrand, T. (1967). On Monte Carlo simulation of diffusion. *Quantitative Geography, Part*, *1*, 1–32.

Haining, R. (1991). Bivariate correlation with spatial data. *Geographical Analysis*, *23*, 210–227.

Hengl, T., Heuvelink, G. B. M., & Stein, A. (2004). A generic framework for spatial prediction of soil variables based on regression-kriging. *Geoderma*, *120*, 75–93.

Hubert, L. J., Golledge, R. G., Costanzo, C. M., Gale, N., Costanzo, M., & Gale, N. (1985). Measuring association between spatially defined variables: An alternative procedure. *Geographical Analysis*, *17*, 37–46.

IPCC. (2014). Climate Change 2014 Synthesis Report. In I. Core Writing Team, R. Pachauri, & L. Meyer (Eds.), *Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* (p. 151). Geneva: IPCC.

Jacobs, D. E., Goldman, D. B., & Shechtman, E. (2010). Cosaliency: where people look when comparing images. *Proceedings of the 23nd annual ACM symposium on User* *interface software and technology*, 219–228.

Jasiewicz, J., Netzel, P., & Stepinski, T. F. (2014). Landscape similarity, retrieval, and machine mapping of physiographic units. *Geomorphology*, *221*, 104–112.

Jelinski, D. E., Wu, & Wu, J. (1996). The modifiable areal unit problem and implications for landscape ecology. *Landscape Ecology*, *11*, 129–140.

Jones, E. L., Rendell, L., Pirotta, E., & Long, J. A. (2016). Novel application of a quantitative spatial comparison tool to species distribution data. *Ecological Indicators*, *70*, 67–76.

Kranstauber, B., Smolla, M., & Safi, K. (2016). Similarity in spatial utilization distributions measured by the earth mover's distance. *Methods in Ecology and Evolution*. doi:10.1111/2041-210X.12649

Lee, S.-I. (2001). Developing a bivariate spatial association measure: An integration of Pearson's r and Moran's I. *Journal of Geographical Systems*, *3*, 369–385.

Levin, S. (1992). The problem of pattern and scale in ecology. *Ecology*, *73*, 1943–1967.

Li, S., Dragicevic, S., Castro, F. A., Sester, M., Winter, S., Coltekin, A., … Cheng, T. (2016). Geospatial big data handling theory and methods: A review and research challenges. *ISPRS Journal of Photogrammetry and Remote Sensing*, *115*, 119–133.

Lloyd, R., & Steinke, T. (1977). Visual and Statistical Comparison of Choropleth Maps. *Annals of the Association of American Geographers*, *67*, 429–436.

Lobo, M.-J., Pietriga, E., & Appert, C. (2015). An Evaluation of Interactive Map Comparison Techniques. In *CHI'15 Proceedings of the 33rd Annual ACM Conference on Human Factors in Computing Systems* (pp. 3573–3582). Seoul, South Korea: ACM.

Long, J. A., Nelson, T. A., & Wulder, M. A. (2010). Characterizing forest fragmentation: Distinguishing change in composition from configuration. *Applied Geography*, *30*, 426–435.

Long, J. A., Robertson, C., Nathoo, F. S., & Nelson, T. A. (2012). A Bayesian space-time model for discrete spread processes on a lattice. *Spatial and Spatio-Temporal Epidemiology*, *3*, 151–62.

Long, J. A., Robertson, C., & Nelson, T. A. (n.d.). stampr: An R package for spatial-temporal analysis of moving polygons. *Journal of Statistical Software*, *In Review*, 18p.

MacEachren, A. M. (1995). *How Maps Work: Representation, Visualization, and Design*.

New York: The Guilford Press.

Marr, D. (1985). Vision: The philosophy an the approach. In A. M. Aitkenhead & J. M. Slack (Eds.), *Issues in Cognitive Modeling* (pp. 103–126). London: Erlbaum.

Maruca, S. L., & Jacquez, G. M. (2002). Area-based tests for association between spatial patterns. *Journal of Geographical Systems*, *4*, 69–83.

Miller, H. J., & Goodchild, M. F. (2015). Data-driven geography. *GeoJournal*, *80*, 449–461.

Miller, H. J., & Han, J. (2009). Geographic data mining and knowledge discovery An Overview. In *Geographic data mining and knowledge discovery* (pp. 1–26).

Miller, H. J., & Wentz, E. a. (2003). Representation and Spatial Analysis in Geographic Information Systems. *Annals of the Association of American Geographers*, *93*, 574–594.

Mladenoff, D. J., White, M. A., Pastor, J., & Crow, T. R. (1993). Comparing spatial pattern in unaltered old-growth and disturbed forest landscapes. *Ecological Applications*, *3*, 294–306.

Moulds, S., Buytaert, W., & Mijic, A. (2015). An open and extensible framework for spatially explicit land use change modelling: the lulcc R package. *Geosci. Model Dev.*, *8*, 3215–3229.

Myers, G. (2012). Why bioimage informatics matters. *Nature Methods*, *9*, 659–660.

O'Sullivan, D., & Perry, G. L. W. (2013). *Spatial Simulation: Exploring Pattern and Process*. John Wiley & sons.

O'Sullivan, D., & Unwin, D. (2010). *Geographic Information Analysis* (2nd ed.). Hoboken, NJ: John Wiley & Sons.

Openshaw, S., & Abrahart, R. J. (1996). Geocomputation. In R. J. Abrahart (Ed.), *1st International Conference on GeoComputation* (pp. 665–666). University of Leeds, UK.

Ord, K., & Getis, A. (2001). Testing for local spatial autocorrelation in the presence of global autocorrelation. *Journal of Regional Science*, *41*, 411–432.

Pearl, J. (2009). *Causality: Models, Reasoning and Inference* (2nd ed.). Cambridge: Cambridge University Press.

Plouffe, C. C. F., Robertson, C., & Chandrapala, L. (2015). Comparing interpolation techniques for monthly rainfall mapping using multiple evaluation criteria and auxiliary data sources: A case study of Sri Lanka. *Environmental Modelling & Software*, *67*, 57–71.

Pontius Jr, R. G., & Millones, M. (2011). Death to Kappa: birth of quantity disagreement and allocation disagreement for accuracy assessment. *International Journal of Remote Sensing*, *32*, 4407–4429.

Remmel, T., & Csillag, F. (2003). When are two landscape pattern indexes significantly different. *Journal of Geographical Systems*, *5*, 331–351.

Remmel, T., Csillag, F., Mitchell, S. W., & Boots, B. (2002). Empirical distributions of landscape pattern indices as functions of classified image composition and spatial structure. Ottawa, Ontario, Canada.

Roberts, S., & Robertson, C. (2016). *Geographic Information Systems and Science: A Concise Handbook of Spatial Data Handling, Representation, and Computation*. Don Mills, Ontario: Oxford University Press.

Robertson, C., Long, J. A., Nathoo, F. S., Nelson, T. A., & Plouffe, C. C. F. C. F. (2014). Assessing quality of spatial models using the structural similarity index and posterior predictive checks. *Geographical Analysis*, *46*, 53–74.

Robertson, C., Nelson, T. A., Boots, B., & Wulder, M. A. (2007). STAMP: spatial–temporal analysis of moving polygons. *Journal of Geographical Systems*, *9*, 207–227.

Robinson, A. H. H., & Bryson, R. A. (1957). A Method for Describing Quantitivatively the Correspondence of Geographic Distributions. *Annals of the Association of American Geographers*, *47*, 379–391.

Rubner, Y., Tomasi, C., & Guibas, L. J. (2000). The earth mover's distance as a metric for image retrieval. *International Journal of Computer Vision*, *40*, 99–121.

Sadahiro, Y., & Umemura, M. (2001). A computational approach for the analysis of changes in polygon distributions. *Journal of Geographical Systems*, *3*, 137–154.

Sakieh, Y., Amiri, B. J., Danekar, A., Feghhi, J., & Dezhkam, S. (2015). Scenario-based evaluation of urban development sustainability: an integrative modeling approach to compromise between urbanization suitability index and landscape pattern. *Environment, Development and Sustainability*, *17*, 1343–1365.

Scharstein, D. (1994). Matching images by comparing their gradient fields. *Proceedings of*

12th International Conference on Pattern Recognition, *1*. doi:10.1109/ICPR.1994.576363

Sémécurbe, F., Tannier, C., & Roux, S. G. (2016). Spatial distribution of human population in France: Exploring the modifiable areal unit problem using multifractal analysis. *Geographical Analysis*, *48*, 292–313.

Shao, Z., Zhou, W., Zhang, L., & Hou, J. (2014). Improved color texture descriptors for remote sensing image retrieval. *Journal of Applied Remote Sensing*, *8*, 83584.

Shaw, S.-L., Tsou, M.-H., & Ye, X. (2016). Editorial: human dynamics in the mobile and big data era. *International Journal of Geographical Information Science*, *30*, 1687–1693.

Sokal, R. R., & Wartenberg, D. E. (1983). A test of spatial autocorrelation analysis using an isolation-by-distance model. *Genetics*, *105*, 219–237.

Spielman, S. E., & Thill, J. C. (2008). Social area analysis, data mining, and GIS. *Computers, Environment and Urban Systems*, *32*, 110–122.

Steinke, T., & Lloyd, R. (1983). Judging the similarity of choropleth map images. *Cartographica*, *20*, 35–24.

Stoyan, D., & Stoyan, H. (1994). *Fractals, Random Shapes and Point Fields: Methods of Geometrical Statistics*. (J. W. & Sons, Ed.). Chichester, New York.

Summerfield, M. (1983). Populations, samples, and statistical inference in geography. *The Professional Geographer*, *35*, 143–149.

Syrjala, S. E. (1996). A Statistical Test for a Difference between the Spatial Distributions of Two Populations. *Ecology*, *77*, 75–80.

Tiefelsdorf, M. (1998). Some practical applications of Moran's I's exact conditional distribution. *Papers in Regional Science*, *77*, 101–129.

Tiefelsdorf, M., & Boots, B. (1995). The exact distribution of Moran's I. *Environment and Planning A*, *27*, 985–999.

Tobler, W. (1965). Computation of the correspondence of geographical patterns. *Papers of the Regional Science Association*, *15*, 131–139.

Turner, M. G. (1989). Landscape ecology: The effect of pattern on process. *Annual Review of Ecology and Systematics*, *20*, 171–197.

Urban, D. L., & Keitt, T. (2001). Landscape connectivity: A graph-theoretic perspective. *Ecology*, *82*, 1205–1218.

Urban, D. L., Minor, E. S., Treml, E. A., & Schick, R. S. (2009). Graph models of habitat mosaics. *Ecology Letters*, *12*, 260–273.

van der Vaart, E., Beaumont, M. A., Johnston, A. S. A., & Sibly, R. M. (2015). Calibration and evaluation of individual-based models using approximate Bayesian computation. *Ecological Modelling*, *312*, 182–190.

Visser, H., & de Nijs, T. (2006). The Map Comparison Kit. *Environmental Modelling & Software*, *21*, 346–358.

Waller, L. A. (2014). Putting spatial statistics (back) on the map. *Spatial Statistics*, *9*, 4–19.

Wang, X., & Cumming, S. G. (2011). Measuring landscape configuration with normalized metrics. *Landscape Ecology*, *26*, 723–736.

Wang, Z., Bovik, A. C., Sheikh, H. R., & Simoncelli, E. P. (2004). Image Quality Assessment : From Error Visibility to Structural Similarity. *IEEE Transactions on Image Processing*, *13*, 600–612.

Wheeler, D. C. (2010). Visualizing and Diagnosing Coefficients from Geographically Weighted Regression Models. In B. Jiang & X. Yao (Eds.), *Geospatial Analysis and Modelling of Urban Structure and Dynamics* (Vol. 99, pp. 415–436). Berlin: Springer Science and Business Media.

Wiegand, T., Revilla, E., & Knauer, F. (2004). Dealing with uncertainty in spatially explicit population models. *Biodiversity & Conservation*, *13*, 53–78.

Wiens. (1989). Spatial scaling in ecology. *Functional Ecology*, *3*, 385–397.

Wong, D. (2001). Location specific cumulative distribution function (LSCDF): An alternative to spatial correlation analysis. *Geographical Analysis*, *33*, 76–93.

Wulder, M. A., Boots, B., Seemann, D., & White, J. C. (2004). Map comparison using spatial autocorrelation : an example using AVHRR derived land cover of Canada. *The Canadian Geographer*, *30*, 573–592.

Yan, H., & Li, J. (2015). *Spatial Similarity Relations in Multi-scale Map Spaces*. Heidelberg: Springer. doi:10.1007/978-3-319-09743-5

Yang, Y., & Newsam, S. (2013). Geographic image retrieval using local invariant features.

IEEE Transactions on Geoscience and Remote Sensing, *51*, 818–832.

Box 1: Glossary of terms.

Spatial Pattern Comparison - a numerical assessment of the (dis)similarity between two (or more) mapped datasets.
Spatial Pattern - scale-dependent predictability of the physical arrangement of observations
Spatial Process – model that produces spatial patterns with a known probabilistic function
Global Statistic – summary statistic that quantifies a property of spatial distribution with a single value
Local Statistic - summary statistic that quantifies a property of a spatial distribution at each location and sums to a global statistic
Composition – dimension of a spatial pattern that relates to the abundance of mapped values
Configuration – dimension of a spatial pattern that relates to the arrangement of mapped values

Table 1: Example datasets and methods for exploring issues in spatial pattern comparison analysis.

Data	**Source (R Package)**	**Method (Reference)**	**Range**	**Interpretation**	**Result**
Point	Lansing Woods (spatstat; Baddeley & Turner, 2005)	NN Correlation (Stoyan & Stoyan, 1994)	0 – 1	Proportion of NN in other group. 0 = all NN from same group 1 = all NN from other group	0.64
Polyline	Waterloo vs St Andrews (osmar; Eugster & Schlesinger, 2013)	Average Deg. Of Intersections	0 – n	Average number of roads connecting at each intersection (for each of W and StA)	W =2.56 SA = 2.48
Polygon	MPB Infestation (stampr; Long, Robertson, & Nelson, n.d.)	Area of intersection Index (Maruca & Jacquez, 2002)	0 – 1	Proportion of overlap 0 = no overlap 1 = perfect overlap/alignment	0.10
Lattice (categorical)	Plum Island Ecosystem (lulcc; Moulds, Buytaert, & Mijic, 2015)	Kappa Coefficient (K_hat) statistic	0 – 1	% of agreement in categories relative to chance. 0 = same as chance 1 = perfect agreement	0.88
Lattice (continuous)	Georgia Degree vs Rural (spgwr; Bivand, Yu, Nakaya, & Garcia-Lopez, 2015)	Pearson Correlation coefficient	-1 – 1	-1 = perfect negative correlation 0 = no correlation 1 = perfect positive correlation	-0.62

Figures

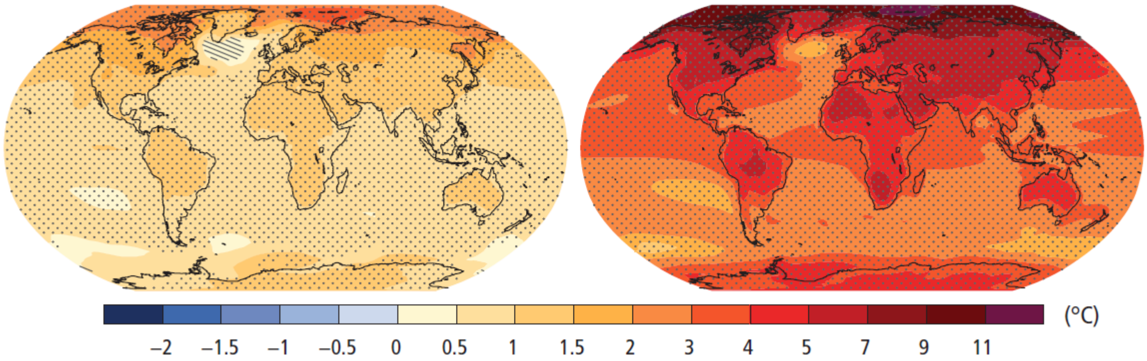


Figure 1: IPCC temperature changes globally, a) recorded observations and b) projections.

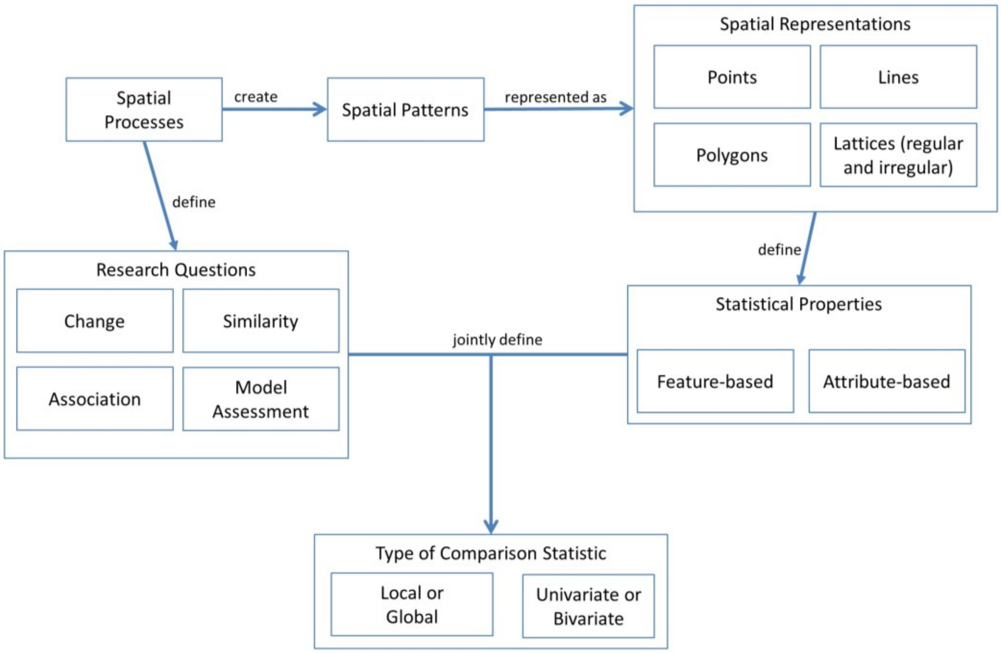
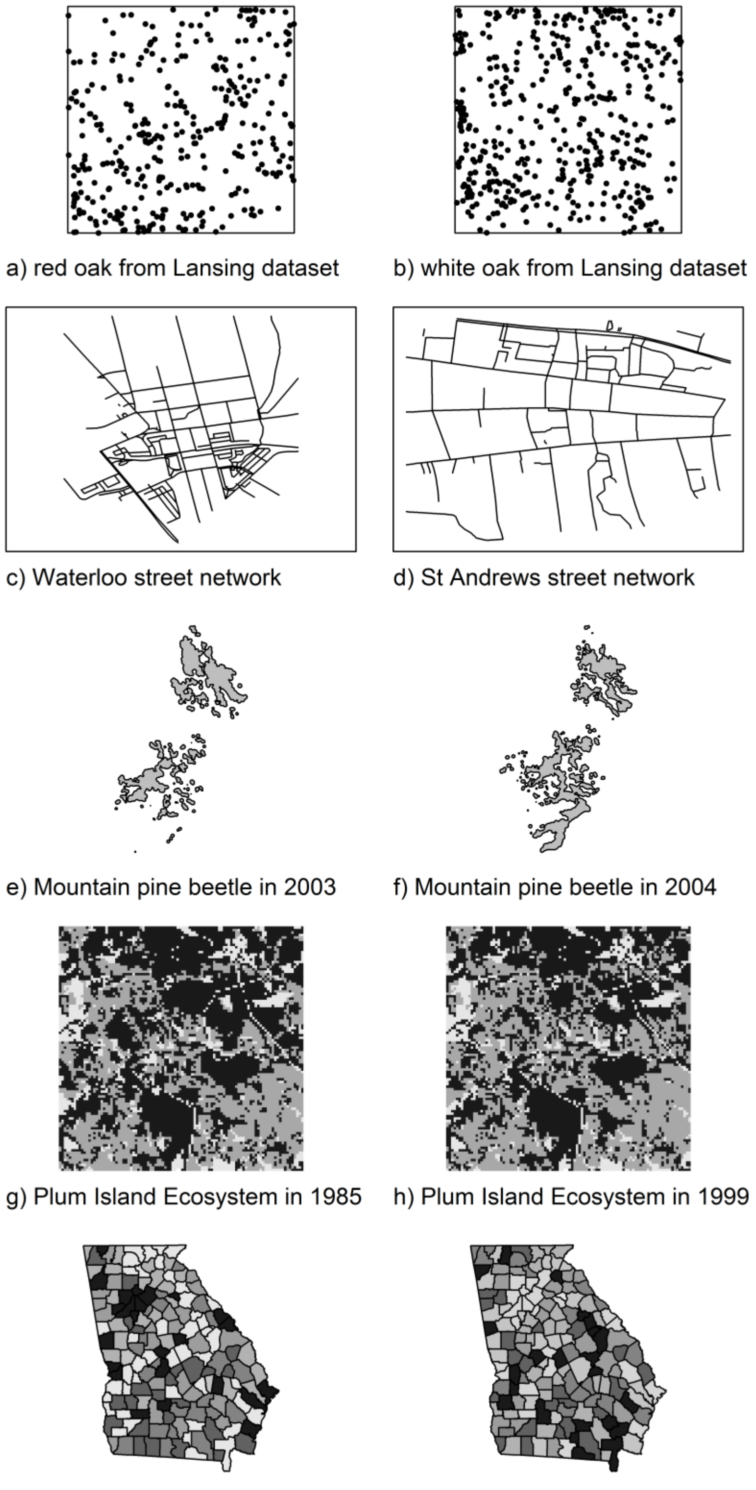


Figure 2: Four dimensions of spatial pattern important for spatial pattern comparison.



a) red oak from Lansing dataset

b) white oak from Lansing dataset

c) Waterloo street network

d) St Andrews street network

e) Mountain pine beetle in 2003

f) Mountain pine beetle in 2004

g) Plum Island Ecosystem in 1985

h) Plum Island Ecosystem in 1999

i) % rural in Georgia 1990

j) % with degree in Georgia 1990

Figure 3: Example datasets used to explore different issues in spatial pattern comparison analysis. All data was sourced from spatial packages in the statistical software R. Data in a) and b) were sourced from the 'spatstat' package; c) and d) were sourced from OpenStreetMap using the 'osmar' package; e) and f) were sourced from the 'stampr' package; g) and h) were sourced from the 'lulcc' package; and i) and j) were sourced from the 'spgwr' package. Please see the text for appropriate references.

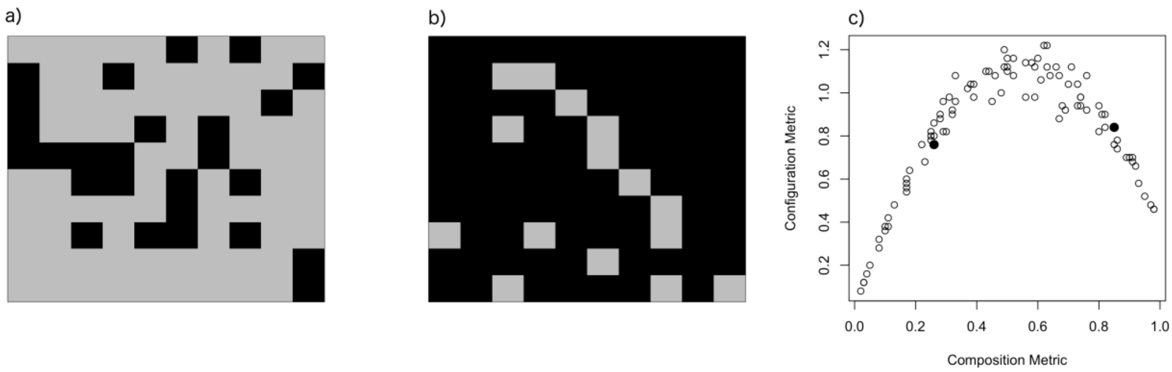


Figure 4: Example of composition and configuration metric dependency on random (uniform) landscapes with a) 30% composition and b) 80% composition. Edge density values for 100 random landscapes are given in c) with the sample landscapes highlighted.

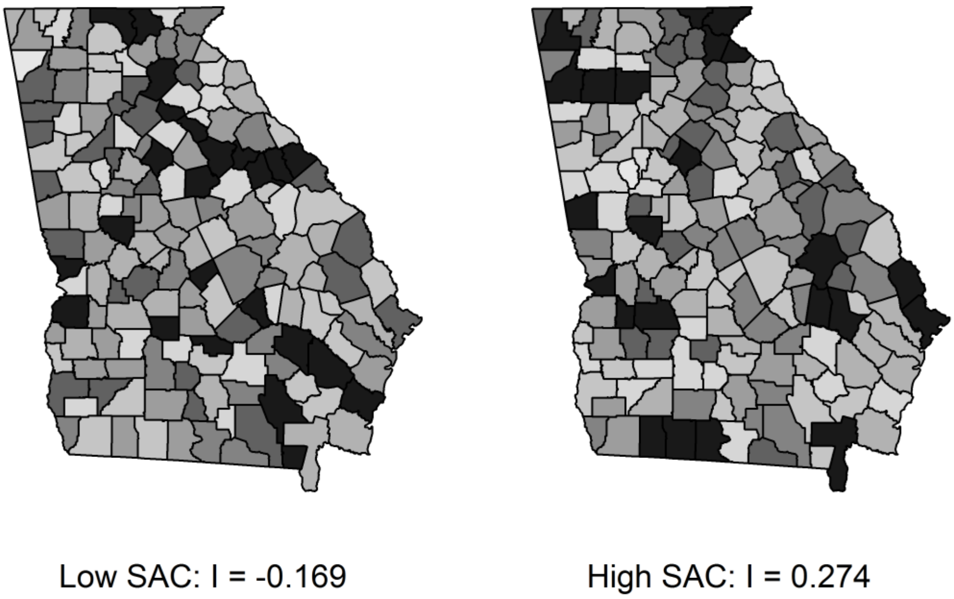


Figure 5: Low and high spatial autocorrelations of model residuals with identical values of root-mean squared error.