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EVALUATION OF LEAKAGE THROUGH LABYRINTH SEALS WITH ANALYTICAL MODELS

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Abstract: Secondary flows in turbomachinery highly affect the overall efficiency and rotor stability. A prime example of such a phenomenon are leakage flows. Despite their complexity, they can often be estimated with simple semi-empirical formulae, solved with hand calculations. Such an approach is much more cost and time effective during the design process. The formulae consists of a carry-over coefficient and a discharge coefficient elements. To evaluate the leakage properly, an adequate model of the carry-over coefficient has to be developed. This paper presents how the flow conditions and the cavity geometry changes in a straight through labyrinth seal affect the amount of leakage. The effect of the number of teeth, the gap size, the Reynolds number and the pressure ratio are considered. The data to validate the results was obtained from an in-house experiment, where a vast number of cases was tested. Additionally, the study was supported by a two-dimensional steady-state CFD study. Eleven analytical models, Including both very simple as well as more sophisticated methods, were solved according to the experimental case and compared. Six different seal configurations were examined. They included straight through seals with two and three straight knives for various gap sizes. The comparison highlighted differences in the results for models – a certain group presented underestimated results. However, another group of models – presented an excellent agreement with the experimental data. Based on this study, a group of models representing the results within the 10% uncertainty band was selected.

Keywords: labyrinth seals, analytical methods, experiment, turbomachinery, gas turbine, validation

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Symbols

A surface area (m^2)
b width of the labyrinth seal fin at the tip (m)
c velocity ($\mathrm{m \cdot s^{-1}}$)
C_D discharge coefficient (—)
h fin height (m)/specific enthalpy ($\mathrm{J \cdot kg^{-1}}$)
κ isentropic exponent (—)
m mass rate ($\mathrm{kg \cdot s^{-1}}$)
Ma Mach number (—)
p pressure (Pa)
π pressure ratio (—)
R individual gas constant ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)
s gap size (m)/specific entropy ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)
T temperature (K)
t fin pitch (—)
v specific volume ($\mathrm{m^3 \cdot^{-1} kg^{-1}}$)
z fins number, (—)
xc distance between fins (m)

Index

0 total parameter
in inlet
out outlet
s isentropic parameter

1. Introduction

An accurate prediction of the amount of gas flowing through the labyrinth seals is of utmost importance for the efficiency and rotor dynamics analysis of turbomachinery. Three methods can be used to evaluate the leakage flow in labyrinth seals or narrow ducts. They are experimental tests, CFD calculations and analytical methods. Historically, analytical methods were the first to predict the secondary flows, as early as in the 1920s. In details, the labyrinth seals working principle is as follows: the air expands adiabatically in consecutive gaps, causing a pressure and enthalpy loss, together with a kinetic energy increase (line 0–1′ in Figure 1). In the cavity between fins, the kinetic energy is recovered back to the thermal energy (line 1′–1). These processes repeat in following stages of a sealing. In the case of a labyrinth seal with a constant tip gap cross-section area the dependency of the mass flow on the velocity c and fluid volume v is described with the Fanno curve:

$$\frac{c}{v} = \frac{\dot{m}}{A} = idem \tag{1}$$

Equation (1) describes the so-called Fanno curve which on the h–s diagram (Figure 1) represents endpoints of the expansion line in successive constrictions.

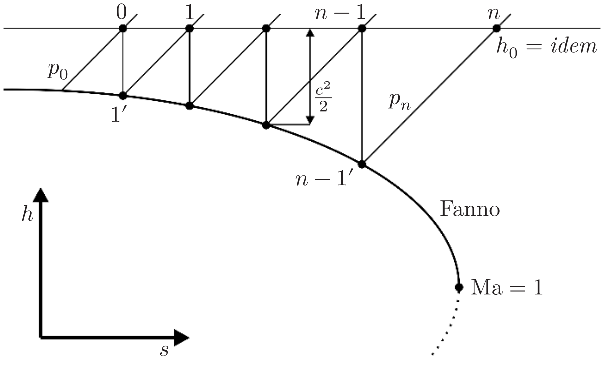


Figure 1. Fanno curve in labyrinth seal, with n number of constrictions

This paper presents a number of selected analytical models used for leakage flow estimation in a straight-through labyrinth seal (Figure 2). Analytical methods were compared with in-house CFD calculations and experimental studies for selected seal configurations and parameters. For the purpose of a comparative study, the analytical models were divided into two types: direct and indirect. In the direct models, the labyrinth seal is treated as a single component, regardless of the number of fins for which the mass flow is determined. In this approach, the result is obtained in one simple step, without iterations. Indirect methods require iterative determination of the pressure in the chambers between the fins and thus the leakage flow over each constriction separately. By definition, some indirect models take into account more details concerning geometry which can be essential in complex structures, as stepped seals, or seals with various cavity geometry. To the knowledge of the authors, none of the analytical models presented in the open literature takes into account the honeycomb structure, the rotational velocity of the rotor or a fin inclination angle. Some of the models take into account a different shape of consecutive cavities what brings up their accuracy.

The models investigated in this paper base on one of the following formulae: St. Venat, Martin or Neumann [1–3]. Each existing model is a combination of any of them, with various carry-over coefficient modeling.

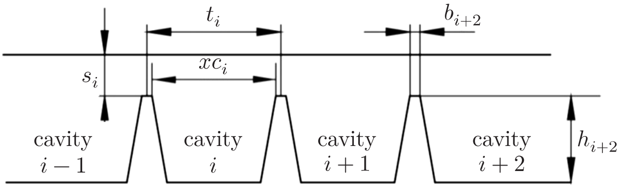


Figure 2. Scheme of straight-through type labyrinth seal used in analysis

2. Analytical models

2.1. Direct models

2.1.1. Stodola equation

The Stodola model [4] is a semi-empirical equation intended for labyrinth seals evaluation. It is one of the first models applied in the design of turbomachinery seals. It originates from the first design considerations of steam turbines in the early 1920s. The carry-over coefficient is defined as μ (by Milne-Thomson [5]):

$$\mu=\frac{\pi}{(\pi+2)} \tag{2}$$

where $\pi=\frac{p_0}{p_s}$ is the pressure ratio defined as a total pressure ahead of the seal over the static pressure behind the seal.

In his approach, the leakage depends on the inlet total parameters, the pressure ratio, the cross-section area and the number of fins – cavities. It does not take into account the geometry. In practice, this model is relevant for straight-through seals with a relatively high number of fins (usually more than 6). In other configurations, it underestimates the results, as presented in [6].

$$\dot{m}=\mu\frac{A}{\sqrt{z}}\cdot\frac{p_{0in}}{\sqrt{T_{0in}}}\cdot\sqrt{\frac{1}{R}\left(1-\frac{1}{\pi^2}\right)} \tag{3}$$

2.1.2. Martin equation

Historically, Martin [2] published the first model intended solely for labyrinth seals. The Martin equation bases on the approach to determine the number of cavities to release a certain pressure drop. The pressure loss is then related to the work done for the pressure change. The work done can be then translated to the kinetic energy of the fluid. The formula assumes the incompressible flow of an ideal gas through the constant gap. The formula to solve the mass flow takes into account only the number of fins – z. The model does not take into account the carry-over coefficient.

$$\dot{m}=\frac{A\cdot p_{0in}}{\sqrt{R\cdot T_{0in}}}\cdot\sqrt{\frac{1-\left(p_{sout}/p_{0in}\right)^2}{z+\ln\left(p_{sout}/p_{0in}\right)}} \tag{4}$$

2.1.3. Egli equation

The Egli model [7] is based on the Martin equation with the empirical factor μ, combining the correction and the carry-over coefficient. With the factor $\mu=1$, the equation produces the same results as the Martin model. In the presented test, the factor is $\mu=0.85$. The μ factor is based on the assumption that the real flow area to be included in the model is lower than the gap size and higher than vena contracta. It is assumed that A is the cross section of the jet of the fluid after the air passes through the constriction. This assumption is based on the fact that shortly after it had passed the gap, air enters the cavity with a certain static pressure value.

The definition of the flow physics in labyrinth seals assumes the need for using the carry-over coefficient. The stream entering a labyrinth seal expands in every cavity. After each throttling, a small part of the kinetic energy of the stream jet will be reconverted into the pressure energy, the majority of it will be dissipated into heat and the remaining amount will enter the following section of the seal. The carry-over coefficient itself represents the portion of energy carried from one cavity to the next one. The jet increases with the increasing axial distance, the amount of kinetic energy carried forward drops with an increase in the spacing between fins or with a decrease in the gap. The Egli model defines a global correlation between the number of fins and the mass flow and can be defined as $\dot{m} \approx c \cdot z^{1/2}$.

$$\dot{m} = \mu \cdot \frac{A \cdot p_{0in}}{\sqrt{R \cdot T_{0in}}} \cdot \sqrt{\frac{1-(p_{sout}/p_{0in})^2}{z+\ln(p_{sout}/p_{0in})}} \tag{5}$$

2.1.4. Hodkinson equation

The Hodkinson equation [8] is a modified Egli equation, with a developed carry-over coefficient. While Egli assumes a constant carry-over coefficient, defined experimentally, it is a semi-empirical function of the seal geometry in the Hodkinson model. Hodkinson developed this expression assuming the gas jet geometry – the jet expands conically at a small angle from the tip of the fin, and only some portion of air carries on undisturbed into the next cavity. The Hodkinson model bases on Egli's experimental data. However, the Egli equation in its method did not take into account the high velocity above the last fin. Egli derived a carry-over coefficient based on a linear increase in the pressure drop with each constriction, ignoring the vena contracta effects. Hodkinson has pointed out that the carry-over coefficient plays a more important role at pressure ratios significantly lower than critical.

Experimental data [8] shows that the expansion angle of the jet with a tangent of 0.02 best fits the data, therefore, this was assumed as a constant in the model. The model takes into account the number of fins, the gap size and the cavity width. The model does not predict pressure distribution in cavities between fins.

$$\dot{m} = \mu_i \cdot \frac{A \cdot p_{0in}}{\sqrt{R \cdot T_{0in}}} \cdot \sqrt{\frac{1-(p_{sout}/p_{0in})^2}{z+\ln(p_{sout}/p_{0in})}} \tag{6}$$

$$\mu_i = \sqrt{\frac{1}{1-\left(\frac{z-1}{z}\right)\cdot\left(\frac{(s_i/(xc_i)}{(s_i/(xc_i))+0.02}\right)}} \tag{7}$$

2.1.5. Vermes equation

The Vermes model origins from the Martin equation [9], although it proposes a new carry-over coefficient, based on the boundary layer theory. It takes into account the number of fins, the gap and the cavity size.

$$\dot{m} = \mu_i \cdot \frac{A \cdot p_{0in}}{\sqrt{R \cdot T_{0in}}} \cdot \sqrt{\frac{1-(p_{sout}/p_{0in})^2}{z+\ln(p_{sout}/p_{0in})}} \tag{8}$$

$$\mu_i = \sqrt{\frac{1}{(1-\alpha_i)}} \tag{9}$$

$$\alpha_i = \frac{8.52}{\frac{t_i-b_i}{s_i}+7.23} \tag{10}$$

2.2. Indirect models (iterative models)

2.2.1. St. Venant equation

The St. Venant equation [1] is derived from an energy balance on a one-dimensional flow of a portion of fluid obtained from the Euler equation. Assuming the isentropic and compressible flow, the following equation was obtained. The full derivation of the presented equation can be found in [10] and [11]. Shultz [11] was the first to use this equation to evaluate the flow leakage through a turbine shaft. Similarly to the Martin model, the geometry is not taken into account.

$$\dot{m}_i = \frac{A \cdot p_{0in}}{\sqrt{\kappa \cdot R \cdot T_{0in}}} \cdot \sqrt{\frac{2\cdot\kappa^2}{\kappa-1}\left[\left(\frac{p_{i+1}}{p_i}\right)^{2/\kappa} - \left(\frac{p_{i+1}}{p_i}\right)^{(\kappa+1)/\kappa}\right]} \tag{11}$$

2.2.2. Neumann equation

Neumann developed an empirical flow equation proposed by Childs [3]. The formula uses a semi-empirical flow coefficient Cf and a carry-over coefficient μ. The former is defined with the Chaplygin equation (15), defined by Gurevich [12]. According to this approach, the most important parameter is the relative gap (s_i/t_i) size of consecutive cavities.

$$\dot{m}_i = Cf_i \cdot \mu_i \cdot A \cdot \sqrt{\frac{p_i^2 - p_{i+1}^2}{R \cdot T_{0in}}} \tag{12}$$

$$Cf_i = \frac{3.1415}{3.1415+2-5\cdot\beta_i+2\cdot\beta_i^2} \tag{13}$$

$$\beta_i = \left(\frac{p_i}{p_{i+1}}\right)^{(\kappa-1)/\kappa} - 1 \tag{14}$$

$$\mu_i = \sqrt{\frac{z}{z\cdot(1-\alpha_i)+\alpha_i}} \tag{15}$$

$$\alpha_i = 1 - \frac{1}{\left(1+16.6\cdot\frac{s_i}{t_i}\right)^2} \tag{16}$$

2.2.3. Scharrer equation

Childs and Scharrer [13, 14] developed their model treating the cavities between fins as a control volume, using the Neumann equation (16). Furthermore, Scharrer presented his model using the Neumann and Vermes equations to evaluate the carry-over coefficient.

$$\dot{m}_i = Cf_i \cdot \mu_i \cdot A \cdot \sqrt{\frac{p_i^2 - p_{i+1}^2}{R \cdot T_{0in}}} \tag{17}$$

$$Cf_i = \frac{3.1415}{3.1415 + 2 - 5 \cdot \beta_i + 2 \cdot \beta_i^2} \tag{18}$$

$$\mu_i = \sqrt{\frac{1}{(1-\alpha_i)}} \tag{19}$$

$$\alpha_i = \frac{8.52}{\frac{t_i - b_i}{s_i} + 7.23} \tag{20}$$

2.2.4. Esser and Kazakia equation

Esser and Kazakia [15] also deployed the Neumann equation (12) as a base of their model. Instead of the Chaplygin Cf coefficient, they assumed a constant value, $Cf = 0.716$, based on the detailed CFD study of the steam flow through the planar constriction-like shaped fin of a labyrinth seal. As a result of the calculation, they assumed new constant Cf value is more accurate than the Chaplygin formula [15].

$$\dot{m}_i = Cf_i \cdot \mu_i \cdot A \cdot \sqrt{\frac{p_i^2 - p_{i+1}^2}{R \cdot T_{0in}}} \tag{21}$$

$$Cf_i = 0.716 \tag{22}$$

$$\mu_i = \sqrt{\frac{z}{z \cdot (1-\alpha_i) + \alpha_i}} \tag{23}$$

$$\alpha_i = 1 - \frac{1}{\left(1 + 16.6 \cdot \frac{s_i}{t_i}\right)^2} \tag{24}$$

2.3. Kurohashi equation

The motivation behind the Kurohashi model [16] development was the calculation of the circumferential and axial pressure distribution in the seal, in case of the journal displacement. It bases on the Neumann equation with a modified carry-over coefficient μ. Coefficient α, related to the flow coefficient Cf_i, assumes the jet expansion angle (optically measured as 6°) and geometrical parameters of sealing.

$$\dot{m}_i = Cf_i \cdot \mu_i \cdot A \cdot \sqrt{\frac{p_i^2 - p_{i+1}^2}{R \cdot T_{0in}}} \tag{25}$$

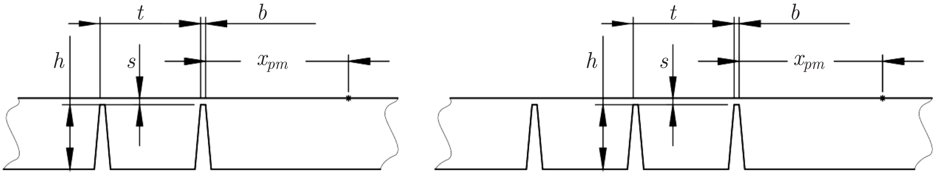
$$Cf_i = \frac{3.1415}{3.1415 + 2 - 5 \cdot \beta_i + 2 \cdot \beta_i^2} \tag{26}$$

$$\mu_i = \begin{cases} \sqrt{\frac{1}{1-\alpha_i+\alpha_i^2}} & \text{for } i = 1 \\ \sqrt{\frac{1}{1-2\cdot\alpha_i+\alpha_i^2}} & \text{for } i > 1 \end{cases} \tag{27}$$

$$\alpha_i = \frac{\frac{s}{t-b}}{\left(\frac{s}{t-b}\right) \cdot Cf_i + \tan 6^\circ} \tag{28}$$

3. Geometries of investigated seals

The subject of the research were two labyrinth seal configurations: with two and three straight fins. The cross sections and dimensions are presented in Figure 3 and Table 1. For clarity, we also show relative dimensions, helping to place given results among those seen in the literature, where specific parameters are often neglected. The adopted fin height and spacing dimensions are typical for low-pressure gas turbine/aero-engine expander stages (where the diameter is roughly 1000–1400 mm). The relative clearance size in this study is between s/b = 0.625–1.875, while the most common range for turbomachinery applications is 0.5–2 [17].



(a) Two-finned labyrinth seal (b) Three-finned labyrinth seal

Figure 3. Cross sections of investigated labyrinths

Table 1. Dimensions of investigated specimens

h	10 mm	h/b	12.5
s	0.5–1.5 mm	s/b	0.625–1.875
t	15 mm	t/b	18.75
b	0.8 mm		
x_{pm}	30 mm		

The parameter x_{pm} describes the distance between the last fin and the point of the static pressure measurement. It was constant for every configuration. In the case of CFD calculations, the pressure ratio was determined in the same manner – the stagnation pressure at the inlet was related to the pressure at the point 30 mm downstream of the last fin. More details on the experimental setup and the CFD approach can be found in the previous works [18, 19].

4. Comparative study

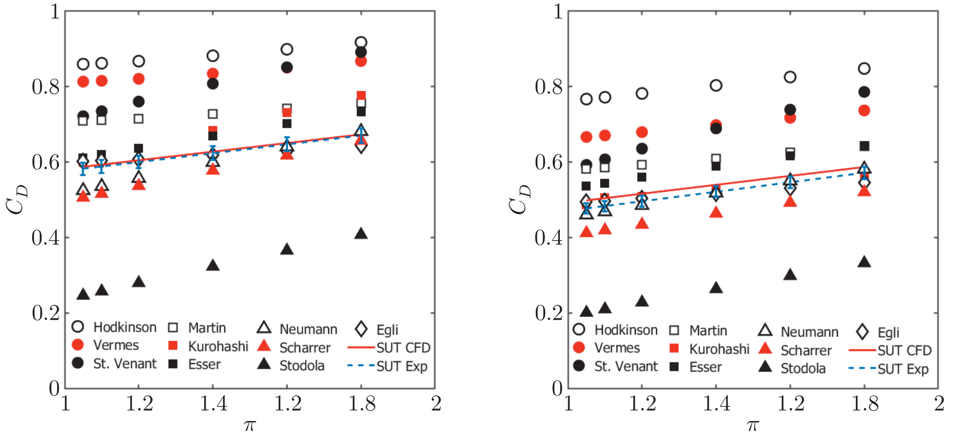
The main contribution of this paper is a comparison of the experimental and CFD results with analytical methods. To this end, the characteristics of the discharge coefficient C_D versus the pressure ratio π were employed. The discharge coefficient is a ratio of the mass rate through a sealing (measured or calculated by means of *e.g.* empirical relations) over an ideal mass rate resulting from the isentropic expansion in the nozzle with the same cross-section.

$$C_D = \frac{\dot{m}}{\dot{m}_{\mathrm{id}}} \tag{29}$$

Where:

$$\dot{m}_{\mathrm{id}} = \frac{p_0 A}{\sqrt{T_0}} \sqrt{\frac{2\kappa}{R(\kappa-1)} \left[\left(\frac{1}{\pi}\right)^{2/\kappa} - \left(\frac{1}{\pi}\right)^{(\kappa+1)/\kappa} \right]} \tag{30}$$

For a validation process of the given analytical models, the results for six different geometrical cases were compiled – labyrinth seals with two and three fins, for gaps $s/b = 0.625$, 1.25 and 1.875. In addition, the results of the analytical models were compared with the in-house CFD calculations and experiment. All the obtained characteristics were linearly increasing as a function of the pressure ratio – which is not entirely evident in the case of the results observed in the literature [20]. Figure 4 summarizes the obtained characteristics for the case with two and three fins, with the gap $s/b = 0.625$. Importantly, none of the models provides for taking into account the rotational speed of the rotor (the seal structure movement is omitted). Few of them allowed taking into account the geometrical parameters of the seal in calculations, which are of great importance for the flow structure. As expected, the most effective models were indirect models, requiring the use of iterative calculations, as well as models including sealing dimensions in their relations. Apart from the three relatively simple models (Stodola, Hodkinson and Vermes), for which the results significantly differed from the others, all the results were in the range of $C_D = 0.5$–0.8.



(a) Two straight fins (b) Three straight fins

Figure 4. Comparison of results from analytical models, CFD calculations and experiment for aseal in a configuration with a smooth wall, gap $s/b = 0.625$

It is also worth noting that the results of the CFD calculations best reflected the flow through the seal, much better than any of the presented models ($\Delta C_D < 3\%$).

Six cases were selected for further analysis, described in more details in Table 2. For each of them, the relative average mass flow rate difference – δ, between the result obtained on the basis of a given model and the measurement is presented. (Figure 5).

Table 2. Description of analyzed geometries

Geometry	gap s, (mm)	Relative gap s/b, (—)	Case name
2 fins	0.5	0.625	(a)
	1	0.9375	(b)
	1.5	1.875	(c)
3 fins	0.5	0.625	(d)
	1	0.9375	(e)
	1.5	1.875	(f)

Different trends in the characteristics of different conditions were observed. Based on the performed study, the four most representative analytical models were selected, which showed the best mapping of the flow characteristics (Table 3). These were the Esser and Kazakia, Neumann, Scharrer and Martin models. On average they presented a relatively low error margin. However, some methods presented individual discrepancies much higher than the average, for instance – the Esser and Kazakia models for the labyrinth with three fins or the Martin model for the low gap sizes.

Table 3. List of most efficient analytical models, with indication of average error

	Gap s, mm	Relative gap, s/b	Esser and Kazakia, %	Neumann %	Scharrer %	Martin %
2 fins	0.5	0.625	6.9	4.9	8.3	17.7
	1	0.9375	5.4	6.2	5.4	7.9
	1.5	1.875	3.4	7.9	0.6	2.1
	Average for 2 fins		5.2	6.3	4.8	9.2
3 fins	0.5	0.625	12.7	1.2	11.5	17.6
	1	0.9375	15.1	1	9.5	6.9
	1.5	1.875	8.4	4.9	11.1	5.4
	Average for 3 fins		12	2.4	10.7	10
	Average		8.6	4.4	7.8	9.6

5. Conclusions

The testing of labyrinth seals with a relatively low number of fins showed that the results obtained on the basis of most models differed significantly from the actual values. According to the dependencies presented by Egli [7], the relationship between the mass flow and the number of fins has the nature of

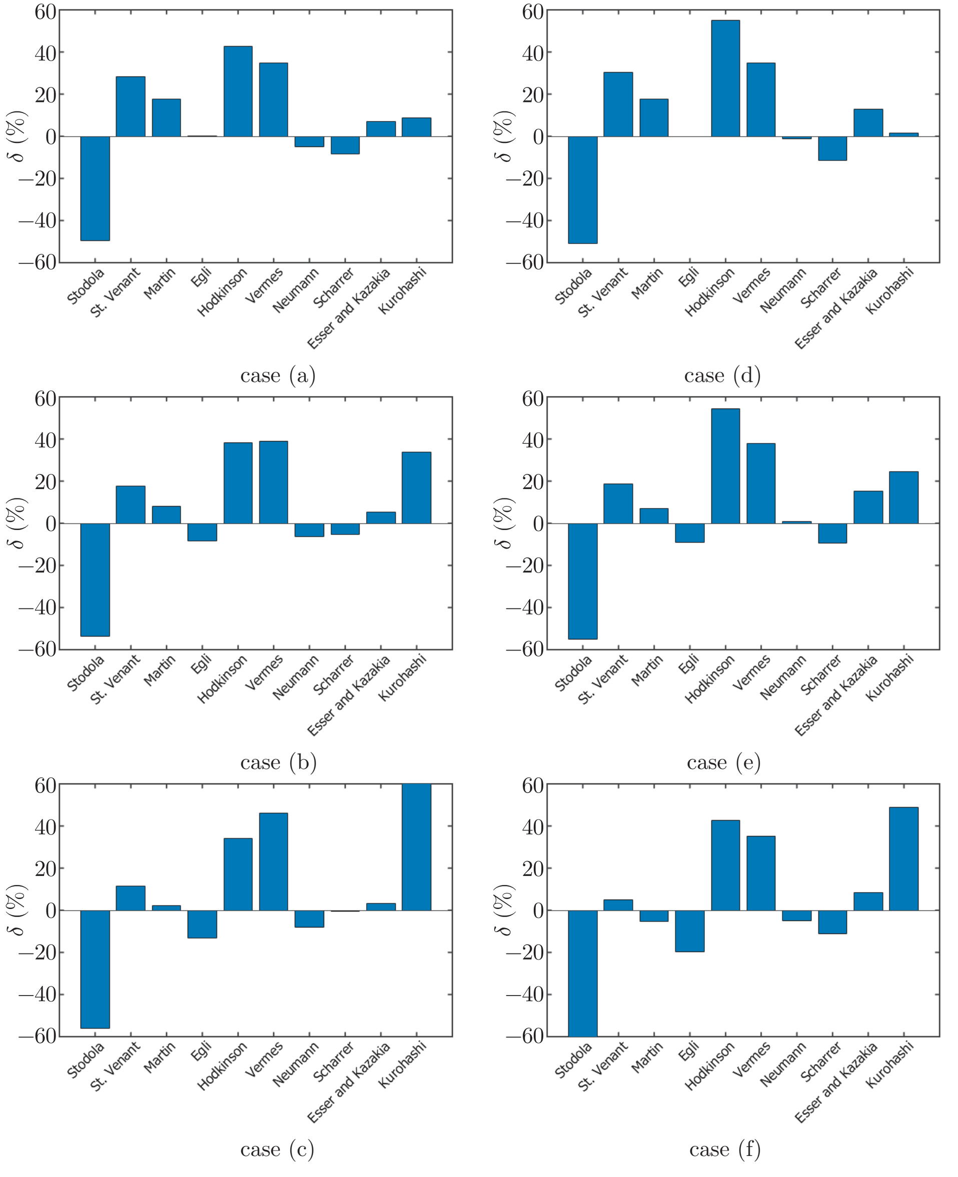


Figure 5. Relative mass flow rate difference – in-house measurement and analytical models

a square root function, and the greatest differences are noted with a small number of fins (2)–(4). For example, the leakage flow obtained on the basis of the Stodola equation is strongly underestimated, although according to other authors, is often sufficient for investigating seals with more fins [6]. Very promising results were obtained using the Egli model, especially for low gap sizes – with discrepancies up to 10%. However, in the case of larger gaps (in the order of $s/b \approx 2$), the error increases – this is due to the assumption of a constant value of the carry-over coefficient which is, in fact, a function of the sealing geometry. For the Hodkinson and Vermes models, the results in each case were highly overestimated (an error of 30–50%). The Kurohashi model was sufficient for low gaps ($s/b = 0.625$), but with larger values, it overestimated the leakage by about 20–40%. The only indirect model, which is not among the top four of the best models, is the St. Venant model. The results from this model were significantly overstated, particularly for the small gap (difference of 20–30%). Interestingly, the relatively simple direct Martin model, despite the overestimation of the result in the range of low gap sizes (on the order of a dozen or so per cent), was found in the four most accurate analytical models, with a satisfactory leakage flow value at high sizes of the gap.

The indirect Esser & Kazakia and Scharrer equations, based on the Neumann equation, also proved to be accurate, in particular for geometries with two fins, but in the remaining cases, they showed an error of 10–12%.

The Neumann model proved to be the most accurate for the case with three fins (mean error of 2.4%), while for the geometry with two fins it showed results with an average error of 6.3%, which was a result similar to the other considered methods (Esser and Kazakia – 5.2%, Scharrer – 4.8%). This model showed a satisfactory accuracy for a wide range of gaps, but for a larger gap, the error increased up to 7.9% with two fins and up to 4.9% with three fins. Further, a detailed analysis of this model showed that some discrepancy could be observed at low-pressure ratios, whereas with $\pi > 1.3$ this model was almost identical with the measurements.

A final conclusion is that for the labyrinth configurations with a relatively low number of fins (two and three) it is possible to find an analytical model to adequately predict the leakage. However, if some additional features are considered, such as the fin inclination towards the flow, or introduction of a step in the geometry, additional tests and validation would be necessary.

References

[1] Vennard J K and Street R L 1982 *Elementary Fluid Mechanics*, John Wiley & Sons, New York

[2] Martin H 1908 *Engineering* **85** 33

[3] Childs D W 1993 *Turbomachinery Rotordynamics – Phenomena, Modeling, and Analysis*, John Wiley & Sons, New York

[4] Stodola A 1924 *Dampf- und Gasturbinen*, Springer-Verlag

[5] Milne-Thomson L 1974 *Theoretical Hydrodynamics, 5th ed.*, MacMillan, London

[6] Matthias A and Willinger R 2009 *Influence of Rotation and Eccentricity on Labyrinth Seal Leakage*, Paper 084, ETC 8, 8th European Turbomachinery Conference, Graz, Austria

[7] Egli A 1935 *Trans. ASME* **57** 115

[8] Hodkinson B 1939 *Proceedings of the Institution of Mechanical Engineers* **141** 283

[9] Vermes G 1961 *ASME Transactions – Journal of Engineering for Power* **2** (83) 161

[10] Eldin A 2007 *Leakage and Rotordynamic Effects of Pocket Damper Seals and See-Through Labyrinth Seals*, Dissertation, Texas A&M University

[11] Shultz R R 1996 *Analytical and Experimental Investigation of a Labyrinth Seal Test Rig and Damper Seals for Turbomachinery*, M. S. Thesis, Texas A&M University

[12] Gurevich M I 1965 *Theory of Jets in an Ideal Fluid*, Academic Press, New York

[13] Childs D W and Scharrer J 1988 *Journal of Vibration, Acoustics, Stress and Reliability in Design* **110** 281

[14] Scharrer J 1988 *Journal of Vibration, Acoustics, Stress and Reliability in Design* **110** 270
[15] Esser D and Kazakia J Y 1995 *International Journal of Engineering Science* **15** (33) 2309
[16] Kurohashi M, Inoue Y, Abe T and Fujikawa T 1980 *Vibrations in Rotating Machinery* 215
[17] Chupp R E, Hendricks R C, Lattime S B, and Steinetz B M 2006 *Sealing in Turbomachinery*, NASA/TM-2006-214341, Glenn Research Centre, Cleveland
[18] Szymanski A, Bochon K, Wróblewski W, Marugi K, Dykas S and Fraczek D 2018 *Journal of Engineering for Gas Turbines and Power* **140** 122503 doi: 10.1115/1.4040767
[19] Szymański A, Dykas S, Majkut M and Strozik M 2016 *Transactions of the Institute of Fluid-flow Machinery* **134** 87
[20] Braun E, Dullenkopf K and Bauer H J 2012 *ASME Paper GT2012* 68077