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DYNAMIC BEHAVIOR OF PREDATOR-PREY WITH RATIO DEPENDENT, REFUGE IN PREY AND HARVEST FROM PREDATOR

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Abstract In this paper, we discuss a dynamical behavior of Predator-Prey with ratio dependent, refuge in prey, and harvest from predator. Model reconstruction is organized by adding the refuge control in prey with the values $0 < m < 1$, and linear predator harvesting. The aim of analysis is to describe the equilibrium points and their stability. In analysis, the possible fixed points are the prey extinction, the predator extinction, and predator-prey coexists. By using linearization, the stability of predator extinction point is unstable, and the prey extinction point, coexists point becomes stable with certain condition. Finally, the dynamical simulation show that the trajectories of solution convergent to their stability, and the refuge strategy suitable to avoid the extinction of prey.
Key Word: *Dynamic Behavior, Predator-Prey, Predation, Refuges, Harvest*

1. INTRODUCTION

In ecology system, the predation model to be interesting in the depth discussion. One of the popular predation models is predator prey by Lotka (1925)-Volterra (1927). Lotka-Volterra proposes of the predation model is,

$$\begin{aligned} \frac{dN}{dt} &= N(a - bP) \\ \frac{dP}{dt} &= P(-c + dN). \end{aligned} \tag{1}$$

Where N, P represents prey and predator, and a, b, c, d are the growth rates of prey, the predation of predator-prey, the predator mortality rate, and corventions rate of prey biomass. Then, the model (1) modified by Xiao danRuan (2001), that the natural rate of prey hold on logistically model and including the ratio dependent response in prey. Based on the dynamical analysis, the predator-prey population density possible isvanished. Therefore, Kar and Chudori (2001),suggest predator-prey harvesting with the model of Holling type II to prevent population extinction.

By the next investigation, Rayungsariet all.(2014), move of Holling Type II model with the ratio dependent response and harvesting from predator, because of the population density should be based on the predator-prey density and to avoid the extinction, that is,

$$\frac{dN}{dt} = r_1 N\left(1 - \frac{N}{K_1}\right) - \frac{aNP}{P + bN}$$
$$\frac{dP}{dt} = r_2 P\left(1 - \frac{P}{K_2}\right) + \frac{c_1 NP}{P + bN} - c_2 P. \quad (2)$$

Here N, P describe of predator-prey density in time, and the all of parameters must be positive constant. From model analysis (2), we get likely to become extinct of predator-prey hence there is no refuge in prey. Refuge activity was studied by Ilmiyahet all. (2014), and Trisdianiet all. (2014), and the result is refuge process in prey enable to manage the extinction. In the other side, dynamical refuge was analyzed and used to in predator-prey with infection in predator Pusawidjayantiet all. (2015), and analysis of protection behaviour was supported by Abdulghofour and Naji (2018).

In this paper, we reconstruct the model (2) by addingrefuge in prey by constant $(0 < m < 1)$, and the goals of protection is surviving of predator-prey population,then the modified model as follow,

$$\frac{dN}{dt} = r_1 N\left(1 - \frac{N}{K_1}\right) - \frac{a(1-m)NP}{P + bN}$$
$$\frac{dP}{dt} = r_2 P\left(1 - \frac{P}{K_2}\right) + \frac{c_1(1-m)NP}{P + bN} - c_2 P. \quad (3)$$

With N, P denote the prey and predator, the parameters r_1, r_2 represent the intrinsic rate of predator-prey, and K_1, K_2 describe of carryingcapacity of predator-prey, respectively. Parameters a declare predation rate, c_1 denote the intrinsic convertion of biomass to be predator reproduction, and the last c_2 is a linear harvest from predator.

2. Methods

In this analysis, we use the research methodology and theory for describing solution behavior, as follows;

Autonomous System

An autonomous system has a form

$$\frac{dx}{dt} = F(x). \quad (4)$$

Where, $F(x)$ are real functions that do not depend explicitly on the independent variable t (Nagle danSaff, 1993).

Fixed Point

The point $x^*(t)$ that fulfill $F(x) = 0$ is called the equilibrium point of model

(4), such that the point to be autonomous system solution (Robinson, 2004).

Linearization of the Nonlinear System

Let the system of (4) is nonlinear system and $F(x)$ has continuous partial derivative at the point $x^*(t)$, then the Taylor series of the function $F(x)$ around the point $x^*(t)$ is,

$$F_i(x) = F_i(x^*) + \sum_{j=1}^{n} \frac{\partial F_i(x^*)}{\partial x_j}(x_j - x_j^*), \quad i = 1,2,\ldots,n. \tag{5}$$

By using $F_i(x) = \frac{dx_i}{dt}$ and $\frac{dx_i}{dt} = \frac{d}{dt}(x_i - x_i^*)$, $i = 1,2,\ldots,n,$ the equation (5) can be written,

$$\frac{d}{dt}(x - x^*) = F_i(x^*) + \begin{bmatrix} \frac{\partial F_1(x^*)}{\partial x_1} & \frac{\partial F_1(x^*)}{\partial x_2} & \cdots & \frac{\partial F_1(x^*)}{\partial x_n} \\ \frac{\partial F_2(x^*)}{\partial x_1} & \frac{\partial F_2(x^*)}{\partial x_2} & \cdots & \frac{\partial F_2(x^*)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n(x^*)}{\partial x_1} & \frac{\partial F_n(x^*)}{\partial x_2} & \cdots & \frac{\partial F_n(x^*)}{\partial x_n} \end{bmatrix} (x - x^*). \tag{6}$$

Suppose $w = x - x^*$ and based on the definition of equilibrium point, we have $F(x^*) = 0$, such that the equation of (6) become,

$$\frac{d}{dt}(w) = \begin{bmatrix} \frac{\partial F_1(x^*)}{\partial x_1} & \frac{\partial F_1(x^*)}{\partial x_2} & \cdots & \frac{\partial F_1(x^*)}{\partial x_n} \\ \frac{\partial F_2(x^*)}{\partial x_1} & \frac{\partial F_2(x^*)}{\partial x_2} & \cdots & \frac{\partial F_2(x^*)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n(x^*)}{\partial x_1} & \frac{\partial F_n(x^*)}{\partial x_2} & \cdots & \frac{\partial F_n(x^*)}{\partial x_n} \end{bmatrix} (w). \tag{7}$$

The equation (7) rewrite simple form is,

$$\frac{d}{dt}(w) = J(w),$$

with

$$J(x^*) = \begin{bmatrix} \frac{\partial F_1(x^*)}{\partial x_1} & \frac{\partial F_1(x^*)}{\partial x_2} & \cdots & \frac{\partial F_1(x^*)}{\partial x_n} \\ \frac{\partial F_2(x^*)}{\partial x_1} & \frac{\partial F_2(x^*)}{\partial x_2} & \cdots & \frac{\partial F_2(x^*)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n(x^*)}{\partial x_1} & \frac{\partial F_n(x^*)}{\partial x_2} & \cdots & \frac{\partial F_n(x^*)}{\partial x_n} \end{bmatrix} (w). \tag{8}$$

Here, $J(x^*)$is called JacobianMatrices or partial derivative matrices (Robinson, 2004).

Dynamic Stability of 2Dimensions

Let A is a second ordermatrices,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

Withdeterminant $\Delta = a_{11}a_{22} - a_{12}a_{21}$ and trace $\tau = a_{11} + a_{22}$. The stability analysis by usingmatric determinant and trace hold on as follow,

1. If $\Delta < 0$, then the system unstable.
2. If $\Delta = 0$, then the system unstable.
3. If $\Delta > 0,\ \tau > 0$, then the system unstable.
4. If $\Delta > 0,\ \tau < 0$, then the system asymptotically stable (Robinson, 2004).

3. Results and Discussion

Existences of Equilibrium Points

Repose on the definition of fixed point, the system (3) has possible three equilibrium points, namely the prey extinction $E_1 = \left(0, K_2\left(1 - \frac{c_2}{r_2}\right)\right)$, the predator extinction $E_2 = (K_1, 0)$, and the interior equilibrium $E_3 = (N^*, P^*)$, where

$$N^* = \frac{-B + \sqrt{D}}{2A} \text{ or } N^* = \frac{-B - \sqrt{D}}{2A}, \text{ and}$$

$$P^* = \frac{r_1 b N\left(1 - \frac{N^*}{K_1}\right)}{a(1-m) - r_1\left(1 - \frac{N^*}{K_1}\right)}.$$

With,

$$A = \frac{r_1}{K_1}\left(\frac{r_2 b}{K_2} + \frac{c_1 r_1}{ab(1-m)K_1}\right)$$

$$B = \frac{r_1}{K_1}\left((r_2 - c_2) + \frac{2c_1}{ab(1-m)}(a(1-m) - r_1) - \frac{r_1 r_2 b}{K_2}\right)$$

$$C = (a(1-m) - r_1)\left((r_2 - c_2) + \frac{c_1}{ab(1-m)}(a(1-m) - r_1)\right)$$

$$D = B^2 - 4AC.$$

The existence condition of E_1 is $c_1 < r_2$, the second fixed point always exists without any condition, and the third equilibrium to be able to exist, if one of the any conditions hold in system,

1. $r_1 > a(1-m)$,

2. $r_1 < a(1-m), B < 0,$ and $D > 0,$
3. $r_1 = a(1-m)$ and $r_2 - c_2 < \frac{br_2K_1}{K_2}.$

Local Stability

If the linearization theory applies in system (3), then we get Jacobianmatrices construction as follow,

$$J[N,P] = \begin{bmatrix} r_1\left(1-\frac{2N}{K_1}\right) - \frac{a(1-m)P^2}{(P+bN)^2} & -\frac{ab(1-m)N^2}{(P+bN)^2} \\ \frac{c_1P^2}{(P+bN)^2} & r_2\left(1-\frac{2P}{K_2}\right) + \frac{bc_1(1-m)N^2}{(P+bN)^2} - c_2 \end{bmatrix}. \quad (9)$$

The Jacobian matrices form around each fixed points are,

$$J\left[E_1\left(0,1-\frac{c_2}{r_2}\right)\right] = \begin{bmatrix} r_1 - a(1-m) & 0 \\ c_1 & -r_2 \end{bmatrix},$$

$$J[E_2(K_1,0)] = \begin{bmatrix} -r_1 & -\frac{a(1-m)}{b} \\ 0 & (r_2-c_2)+\frac{c_1(1-m)}{b} \end{bmatrix},$$

$$J[E_3(N^*,P^*)] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix},$$

Where,

$$J_{11} = r_1\left(1-\frac{2N^*}{K_1}\right) - \frac{a(1-m)P^{*2}}{(P^*+bN^*)^2}$$

$$J_{12} = -\frac{ab(1-m)N^{*2}}{(P^*+bN^*)^2}$$

$$J_{21} = \frac{c_1P^{*2}}{(P^*+bN^*)^2}$$

$$J_{22} = r_2\left(1-\frac{2P^*}{K_2}\right) + \frac{bc_1(1-m)bN^{*2}}{(P^*+bN^*)^2} - c_2.$$

From $J(E_1)$ obviously E_1 comes to local asymptotically stable, if the requirement $r_1 < a(1-m)$ holds, and unstable if $r_1 > a(1-m)$. The second Jacobian matrix has one positive eigen value is $(r_2-c_2)+\frac{c_1(1-m)}{b} > 0,$ and it is clear that E_2 always unstable.

Theorem: The equilibrium E_3 is local asymptotically stable if

$\Delta = J_{11}J_{22} - J_{12}J_{21} > 0$, and $\tau = J_{11} + J_{22} < 0$.

Proof: Obviouslydecided that the $J_{12} = -\dfrac{ab(1-m)N^{*2}}{(P^* + bN^*)^2} < 0$, and $J_{21} = \dfrac{c_1 P^{*2}}{(P^* + bN^*)^2} > 0$. While, we analyze the sign of J_{11} by manipulating algebra is,

$$J_{11} = r_1\left(1 - \frac{2N^*}{K_1}\right) - \frac{a(1-m)P^{*2}}{(P^* + bN^*)^2}$$

$$= r_1\left(1 - \frac{2N^*}{K_1}\right) - a(1-m)\left(\frac{r_1\left(1 - \frac{N^*}{K_1}\right)}{a(1-m)}\right)^2$$

$$= r_1\left(1 - \frac{2N^*}{K_1}\right) - \frac{r_1^{\,2}}{a(1-m)}\left(1 - \frac{N^*}{K_1}\right)^2$$

$$= r_1\left(1 - \frac{N^*}{K_1}\right)\left(1 - \frac{r_1}{a(1-m)}\left(1 - \frac{N^*}{K_1}\right)\right) - \frac{r_1 N^*}{K_1}$$

By condition $r_1 > a(1-m)$, we have

$$J_{11} = r_1\left(1 - \frac{N^*}{K_1}\right)\left(1 - \frac{r_1}{a(1-m)}\left(1 - \frac{N^*}{K_1}\right)\right) - \frac{r_1 N^*}{K_1}$$

$$< r_1\left(1 - \frac{N^*}{K_1}\right)\left(1 - \frac{a(1-m)}{a(1-m)}\left(1 - \frac{N^*}{K_1}\right)\right) - \frac{r_1 N^*}{K_1}$$

$$= r_1\left(1 - \frac{N^*}{K_1}\right)\left(\frac{N^*}{K_1}\right) - \frac{r_1 N^*}{K_1}$$

$$= r_1\left(1 - \frac{N^*}{K_1} - 1\right)\left(\frac{N^*}{K_1}\right) = -r_1\left(\frac{N^*}{K_1}\right)^2 < 0.$$

Such that, it is clear that $J_{11} < 0$, and then we will show that $J_{22} < 0$ with under condition of the existence of E_3, namely;

$$J_{22} = r_2\left(1 - \frac{2P^*}{K_2}\right) + \frac{bc_1(1-m)P^{*2}}{(P^* + bN^*)^2} - c_2$$

$$= -\frac{c_1(1-m)N^*}{P^*+bN^*} + \frac{bc_1(1-m)N^{*2}}{\left(P^*+bN^*\right)^2} - \frac{r_2P^*}{K_2}$$

$$= \frac{c_1(1-m)N^*}{P^*+bN^*}\left(-1+\frac{bN^*}{P^*+bN^*}\right) - \frac{r_2P^*}{K_2} < 0.$$

Obviously, it is given $J_{22} < 0$, respectively we have the determinant value is positive and the trace value is negative, and the theorem is proven completely.

Numerical Simulation

This part, we share about numerical simulation of model (3) to describe the existences of fixed points and their stability, and the aim of numerical simulation for supporting the dynamical analysis based on theory. By taking the parameters in this table is,

Table 1. Parameter of the simulation

Parameter	Simulation 1	Simulation 2
r_1	0.2	0.6
r_2	0.5	0.8
K_1	15	15
K_2	10	10
a	1	1
b	1	1
m	0.6	0.6
c_1	0.6	0.6
c_2	0.4	0.4

By taking the value on the table 1 and based on the simulation 1 above, we have the behavior solution figure of the system (3) is,

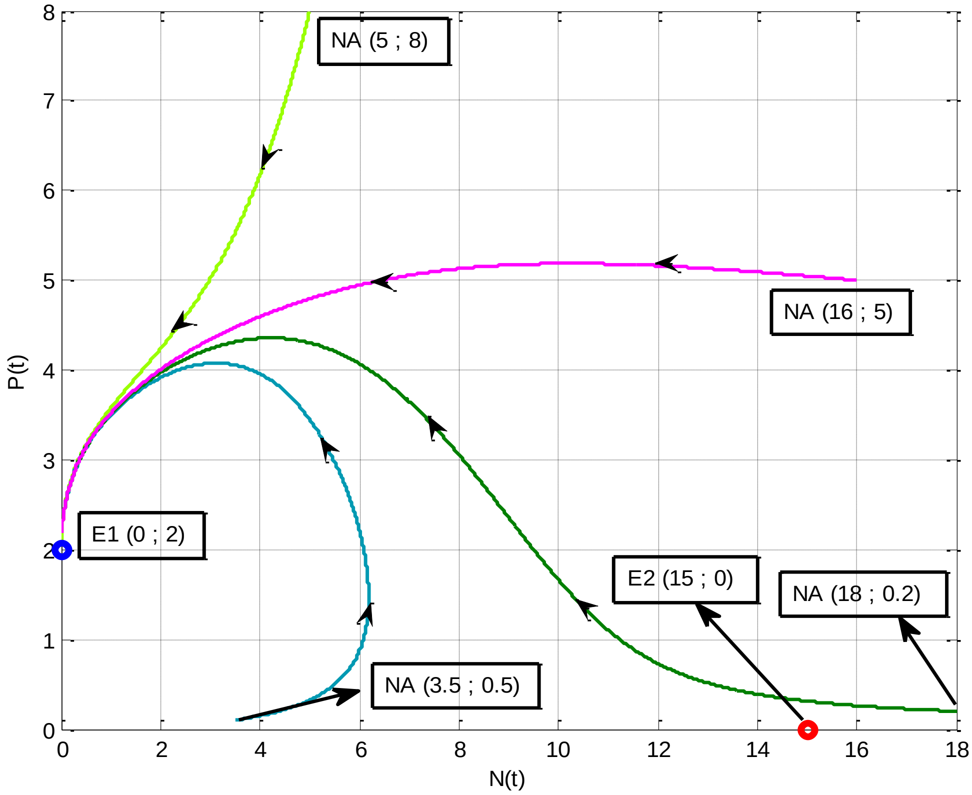

Figure 1. The Dynamical Solution of Simulation 1

The simulation figure (1) above, we read the result giving information that there are two fixed points exist, namely $E_1 = (0,2)$ by under condition $c_2 = 0,4 < 0,5 = r_2$. Then, the fixed points of $E_2 = (15,0)$ always be exist without any condition. Based on the initial values $NA_1 = (3.5,0.5)$, $NA_2 = (18,0.2)$, $NA_3 = (16,5)$, $NA_4 = (5,8)$ are describing the several dynamical solution. It is clear that the all solution by using the initial value comes to E_1 or E_1 to be sink point, so the obviously consequence is stable by local stability condition $r_1 = 0,2 < 0,4 = a(1-m)$. In ecology, it is mean that the predator population will be surviving although the prey density is ruined. Whereas, the equilibriums E_2 always be seen in the figure (1) that the all phase portrait ondynamical solution never tend to the second equilibrium or the other name side is E_2 always be unstable ever. While, the interior equilibrium does not exists since the condition is not fulfilled.

Next simulation by submitting the different values and initial condition (table 1 and simulations 2), the behavior phase portrait and the stability can be dedicated on the next figure by running the program is,

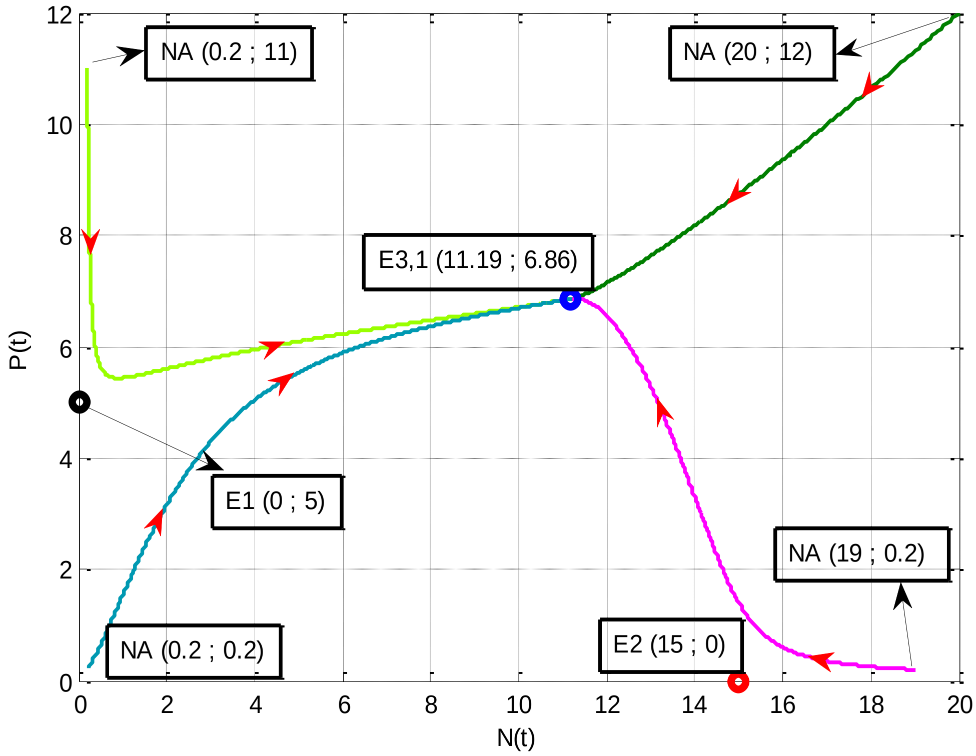

Figure 2. The Dynamical Solution of Simulation 2

In addition of simulation analysis, figure (2) represent aboutthree equilibrium pointsbecomeexist, specifically $E_1 = (0,5)$ is the equilibrium of prey extinction, $E_2 = (15,0)$is the equilibrium of predator extinction, and $E_3 = (11.19,6.86)$ is the interior equilibrium or the prey-predator to be survive of both. On the figure (2) adduce the equilibrium $E_1 = (0,5)$ to be unstable since the analytical stability condition does not fulfilled, with the value $r_1 = 0,6$ greaterthan the value of $a(1-m) = 0,4$. At the same time, the second equilibrium $E_2 = (15,0)$always occurs unstable, with the isocline of initial value never tend to fixed point E_2.Last numerical simulation indicated that the all of trajectorieswith different initial value convergent to interior equilibrium point, this is correspondingtoanalytical stability condition for interior point, namely positive determinant value is $\Delta = 0.9155$ and the negative trace value is $\tau = -4.448$ into corresponding of matrix $J(N^*, P^*)$.Interior pointexistence condition represent that the predator-prey density does not close to be ruined, and the predation activity always being of them. From the simulation, the refuge effect in prey is seemed that with the initial value of $NA_1 = (0.5,0.5)$ condition, the growth of prey density greater than the growth of predator density, respectively for the other initial values. Therefore, the refuges in prey correspond to decrease the extinction.

4. CONCLUSION

In the last performed, weare discussed the behavior stability of the dynamical predator-prey system within the refuge in prey and harvest from predator. By investigation system, the modelhas three equilibrium points by under the existence condition. There are E_1, E_2, and E_3 where E_1 to be stable if the condition $r_1 < a(1-m)$ holds, respectively, and the point of E_2 always be unstable equilibrium. While, the interior equilibrium or E_3 become stable if the determinant of jacobian matrix $J(E_3)$ is positive and the trace of jacobian matrix $J(E_3)$ is negative. Numerical simulation iscarrying the illustration that the predator-prey still becomes extinction, although the refuge in prey and harvesting from predator has applied in models. But, the effect of refuge can be shown with the simulation (2) that refuges in prey afford to decrease prey extinction.

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