Market Allocations under Ambiguity: A Survey *

Antoine Billot[†], Sujoy Mukerji[‡] and Jean-Marc Tallon[§]

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Abstract

We review some of the (theoretical) economic implications of David Schmeidler's models of decision under uncertainty (Choquet expected utility and maxmin expected utility) in competitive market settings. We start with the portfolio inertia result of Dow and Werlang (1992), show how it does or does not generalize in an equilibrium setting. We further explore the equilibrium implications (indeterminacies, non revelation of information) of these decision models. A section is then devoted to the studies of Pareto optimal arrangements under these models. We conclude with a discussion of experimental evidence for these models that relate, in particular, to the implications for market behaviour discussed in the preceding sections.

Allocations des biens et ambiguïté: une revue de la littérature.

Nous passons en revue les implications en termes d'allocation du risque des modèles de décision développés par David Schmeidler. Nous revenons sur le résultat d'inertie des portfeuilles de Dow et Werlang (1992) et discutons de l'extension du résultat dans un cadre d'équilibre. Nous procédons ensuite à une revue des propriétés d'équilibre (indétermination, non révélation d'information) liées à ces modèles. Nous exposons ensuite les propriétés d'optimalité et concluons avec une discussion de la littérature expérimentale sur le sujet.

Mots clé: Espérance d'utilité à la Choquet; Minimum d'espérances d'utilité; absence d'échanges; partage du risque; indétermination; expériences.

Keywords. Choquet Expected Utility; Maxmin Expected Utility; No-trade; Risk Sharing; Indeterminacy; Experimental evidence.

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[†]Lemma, Université Panthéon-Assas, Paris 2, 4 rue Blaise Desgoffe 75006 Paris, France (billot@u-paris2.fr)

[‡]School of Economics and Finance, Queen Mary University of London, Mile End Road London E1 4NS, U.K. (s.mukerji@qmul.ac.uk)

[§]Paris School of Economics, CNRS, 48bd Jourdan, 75014 Paris, France (jean-marc.tallon@psemail.eu)

1 Introduction

David Schmeidler's seminal papers (Schmeidler (1982), Schmeidler (1989), Gilboa and Schmeidler (1989)) started a renewal of the way we think and model decision under uncertainty. The decision theoretical literature that followed these advances is enormous and serves therefore to measure the influence of Schmeidler's ideas on microeconomic thought in the last forty years. They also led to a substantial economic literature applying these new decision criteria to various economic environments. One early, and now classic, application was a short paper by Dow and Werlang (1992) that showed how (uncertainty averse) Choquet Expected Utility (henceforth CEU) leads to the existence of a price interval at which a decision maker does not want to hold a non-zero position on a particular asset whose payoffs are uncertain. This opened the way to explore how populating our abstract economies with CEU maximizers or Maxmin Expected Utility (henceforth MEU—also known as the multiple prior model) agents affect the economic outcomes, with a particular focus on risk sharing arrangements and asset pricing. Around the same time David was coming up with the CEU model, other non linear models (Quiggin (1982), Yaari (1987), Segal (1987), Bewley (1986), Weymark (1981), Chew (1983)) emerged. Some of the economic consequences of CEU and MEU hypotheses are shared by these models but it is fair to say that the main bulk of these applications was primarily motivated by David's work.

In this paper, we review this economic literature, which is mostly theoretical and has provided new insights into the way markets allocate ambiguity. It does not aim at being exhaustive.[1] For instance, the more applied work, in particular in macro-finance, has lately followed mostly another yet related route, namely applying the smooth ambiguity model of Klibanoff et al. (2005) that is not reviewed here.

An important feature of the CEU and MEU models is that uncertainty aversion produces a kink of the agent's indifference curve at the "certainty line". This non-differentiability is, in a way, a surprising outcome of the axiomatization of uncertainty averse behavior in Schmeidler (1982) and Gilboa and Schmeidler (1989). It has a number of economic applications, such as, portfolio inertia, equilibrium price indeterminacy, the absence of betting even under disagreement. Interestingly, the non-differentiability at certainty carries consequences away from certainty as well. Actually, the notion of certainty has to be qualified since Schmeidler's analysis is cast in the Anscombe-Aumann setting where a constant act is a lottery and thus inherently stochastic. The economic literature, notably in a general equilibrium environment, has also progressively distinguished ambiguity from risk (of the endowment allocation in particular) and has shown that this distinction is fruitful to explain, qualitatively, the kind of trading arrangement

[1]For a more detailed survey focussed on ambiguity and asset markets, see Epstein and Schneider (2010), and also Guidolin and Rinaldo (2013).

one can expect under uncertainty aversion.

This review will start with the portfolio inertia phenomenon. We will then show that this result is subject to fragilities when immersed in an equilibrium model. We will show thereafter how the non differentiability typical of the CEU and MEU models do produce new insights for equilibrium and optimal risk sharing. Finally, a section on the experimental support for these models (and more specifically the non-differentiability they induce) ends this review.

2 Portfolio inertia

In this section, we first review the Dow and Werlang (1992) argument at the individual level, how it can be extended and how it produces new insights in representative agent asset pricing models. We also point to fragilities of the result and how the inertia property can be included in a general equilibrium setting with heterogeneous agents.

2.1 Portfolio inertia at the individual level

Following the publication of David's work on CEU and MEU hypotheses, a first economic application of these new decisions criteria was made by Dow and Werlang (1992). They showed that, in a simple portfolio choice problem, ambiguity aversion leads to *portfolio inertia*. Recall that, under expected utility, a decision maker is locally risk neutral and decides to short or long an asset as soon as its price is above or below the expected return (Arrow (1965)). Under MEU (or in the convex CEU case) this property is not satisfied anylonger if the decision maker's initial position is riskless. The reason is that the "minimizing probability", that is, the probability that the decision maker ends up using (among all the distributions in his set of priors) to evaluate the decision under consideration, when contemplating going short is different from the minimizing probability distribution when going long. As it were, the "bad states" when going short (in which the agent has to pay back a lot) become the "good states" if he were to go long. As a result, there is an interval of prices at which it is optimal not to go short nor long, leading to portfolio inertia: at the zero position, the optimal portfolio (i.e., holding zero uncertain assets) is not responsive to price changes as long as they remain within the interval identified. Chateauneuf and Ventura (2010) extend Dow and Werlang's original result within the CEU model, showing it holds with possibly negative outcomes and under a weaker condition than convexity of the capacity. Higashi et al. (2008) explore further this inertia property without assuming a particular decision model, and give an axiomatic foundation for the "kink at certainty" property that underlies portfolio inertia.

The economic intuition behind this property of uncertainty averse behavior is straightforward: an uncertainty averse investor, who is not exposed to uncertainty, will require an extra premium (compared to an uncertainty neutral investor) to move away from that situation and include an asset with ambiguous payoffs in her portfolio. What the CEU and MEU add to this is that the premium is of the first order (does not vanish when the investment becomes small—whereas the risk premium does), reflecting the non-differentiability of the decision criterion.

This prediction of the MEU model leads to formulate a possible explanation to the well documented puzzle of too little participation of individuals to the stock market. Based on the intuition recalled above, non-participation would stem from the fact that uncertainty averse individuals view stocks as ambiguous and thus require an extra premium (with, naturally a higher premium the higher the uncertainty aversion) to hold them; thus, individuals who are sufficiently uncertainty averse will prefer not to hold these stocks. Dimmock et al. (2016a) finds evidence of this phenomenon in a report ran on a representative survey of US households.

Note, the result carries over in the domain of risk, where Rank Dependent Utility (RDU henceforth) functionals might exhibit the same inertia phenomenon (not surprisingly since from a technical point of view, RDU can be seen as a particular case of CEU)—which is thus compatible with probabilistic sophistication. In the RDU model, first order risk aversion is the explanation of the fact that the overall premium required by the decision maker to hold the asset does not vanish when holdings become small.

Portfolio inertia was also identified by Bewley (1986) as a consequence of incomplete preferences. The inertia identified in Dow and Werlang should however be distinguished from the one exhibited by a decision maker with Bewley preferences. In the latter case, the inertia is built in the decision model as a way to solve the conflicting recommendations of different priors. Such an inertia is the result of the incompleteness of the decision maker's preferences and is effective essentially at any (initial) position. By contrast, in the simple Dow and Werlang example, if the decision maker had an initial position in some other assets whose payoffs depend on the same states as the one under consideration, the inertia property (i.e., the fact that there is a non-degenerate interval of prices at which the decision maker does not want to go long nor short) fails. An uncertainty averse agent will, in that situation, use the asset to hedge against the uncertainty present in his initial endowments.

The inertia phenomenon that Dow and Werlang first pointed out has been an important feature of subsequent application of David's models of decision making under uncertainty. Soon after Dow and Werlang's contribution, Epstein and Wang (1994) generalized this idea of portfolio inertia beyond the simple static framework contemplated by Dow and Werlang. In their paper, portfolio inertia induces volatility of asset prices and the possibility of sunspot equilibria. The context is that of a Lucas tree model of asset pricing with a representative agent. At a kink of the indifference curve, there exist multiple prices that support the initial endowment as a market equilibrium. Thus, prices can change with no quantity change for instance; more precisely, two states that have the same endowment can have different price associated to them. This was the first step towards an equilibrium analysis of the consequences of uncertainty averse behavior, with the caveat that in a single-agent economy, there is no notion of trade.

Still in a representative agent framework, Epstein and Schneider (2008) explore how the portfolio inertia exhibited by a MEU investor affects the way arrival of information induces portfolio changes. They introduce two kinds of information. One is tangible (consists of past dividends etc) and the other is intangible (consisting of news that is hard to quantify, such as news reports for instance). The latter is thus ambiguous. Epstein and Schneider then show that investors behave as if they overreact to bad intangible signals. This asymmetric response to ambiguous information leads to skewness in returns. Furthermore, shocks to information quality can have persistent negative effects on prices even if fundamentals do not change.

Illeditsch (2011) builds on Epstein and Schneider (2008) along two dimensions: (i) investors are risk-averse and (ii) investors receive stochastic labor income. The paper shows that the interaction between risk and ambiguity leads to portfolio inertia for risky portfolios when investors process ambiguous news (public information). When news is disappointing, investors can find risky stock allocations that hedge against ambiguous news. It is thus optimal to stick to these allocations even if prices change. That paper thus helps explain why many investors who own stocks do not show much trading activity.

Recently, Greinecker and Kuzmics (2019) showed that, if agents have to take decision in the form of limit orders (i.e., deciding on how much to invest contingent on the realized price of the asset), then ambiguity aversion does not produce results that can be distinguished from standard expected utility maximization. The result rests on the observation that ambiguity averse agents have a preference for randomization (built in the convex CEU and MEU criterion) because the randomization allows them to hedge the ambiguity. A limit order is akin to a mixed strategy since it is specifying an action contingent on a price that is ex ante stochastic. Thus, an order buying below a certain price and selling above a different price, will be dominated by a "mixture" which ends up specifying a single price above which one sells and below which one buys. Thus, the (implicit) market structure in Dow and Werlang—that the agent observes the price prior to making his portfolio decision—is also important for the inertia property.

While the portfolio inertia property attracted a lot of attention, moving from a single agent analysis to an analysis of ambiguity sharing and equilibrium with different uncertainty averse agents leads to new insights.

2.2 Portfolio inertia, market freeze and trade

The portfolio inertia property identified in the previous section could lead one to think that uncertainty averse agents will end up trading very little. This intuition however has to be refined, as market clearing conditions have some bite on what is feasible or not in terms of trading and, in particular, whether agents can all achieve an allocation without ambiguity (i.e., a full insurance allocation). In a two-state, two-agent economy with MEU agents, it is easy to see that, unless the endowments of the agents are both certain—which implies that there cannot be aggregate uncertainty—they will engage in some trade at equilibrium.

Chateauneuf et al. (2000) showed that the set of equilibrium allocations in an economy consisting CEU agents (with identical convex capacities) is the same as that of an economy populated by expected utility agents with identical beliefs. This, seemingly, limits the potential for uncertainty aversion to account for and explain phenomena such as market freeze that have been intuitively associated with rises in uncertainty.

Easley and O'Hara (2009) provide such an equilibrium model of market non-participation, or more precisely, an equilibrium model in which uncertainty averse agents (called *naive* agents) decide not to hold any risky asset (which are all held by sophisticated investors). As such, and as they recognize, this is different from a reduced amount of trading or "market freeze". Note, regardless of their endowments, what ambiguity averse agents aim for is to have a final asset position that is as free of uncertainty as possible. So, in an equilibrium in which ambiguity-averse investors choose not to hold a risky asset, they will trade, if necessary, in order to achieve a zero asset position.

Mukerji and Tallon (2001) provide a simple general equilibrium model in which agents, in order to share risk need to exchange ambiguous assets. More precisely, the asset payoffs depend on the same states of nature as the agent's endowments. But they also carry some idiosyncratic uncertainty which, crucially for the result, is ambiguous. If agents are sufficiently averse to that uncertainty (modelled as having sets of beliefs—in their model the core of a convex capacity—with higher (smaller) upper (lower) bound) then, they choose not to trade in these assets and prefer to stay with their initial endowment. Therefore, assets that would be traded (for risk sharing purposes) when agents are uncertainty neutral are not traded when agents are sufficiently uncertainty averse. Mukerji and Tallon (2001) also show that the usual trick to get rid of asset idiosyncratic risk, that is simple diversification strategies, does not work here. Indeed, replicating these uncertain assets, each with its own idiosyncratic uncertainty, is not effective if agents are sufficiently uncertainty averse. Related to this analysis, Mukerji and Tallon (2004a) provide an argument explaining the fact that uncertainty averse agents might prefer to trade non-indexed contracts rather than indexed assets. Quite intuitively, if relative prices are ambiguous in the economy, indexed assets introduce some extra uncertainty into agents' portfolio and the ones that are sufficiently uncertainty averse will shy away from this type of asset.[2]

Chateauneuf and de Castro (2011) provide the conditions under which more ambiguity aversion implies less trade (in the sense of a smaller set of Pareto improving trades at any endowment), for a class of preferences that includes CEU and MEU. The condition is that endowment be unambiguous. The reduction in trade caused by ambiguity aversion can be as severe as to lead to no-trade. In an economy with MEU decision makers, they show that if the aggregate endowment is unanimously unambiguous then every Pareto optimal allocation is also unambiguous.

These analysis can, as in the portfolio choice example of Section 2, be contrasted with what happens in economies populated with agents with Bewley preferences. As Bewley (1986) already noticed, the type of inertia stemming from his model of incomplete preferences and that coming from Gilboa and Schmeidler's approach have different market implications: "Uncertainty aversion [*à la* Bewley] could discourage insurance. [...] Even if endowments were very asymmetric and preferences were the same, there might be no-trade in [equilibrium]. Nevertheless, the equilibrium would be Pareto optimal. [...] [On the other hand], people with Gilboa-Schmeidler preferences would be very apt to buy insurance."

Bewley's argument has been generalized by Rigotti and Shannon (2005) who study Pareto optimal allocations and define an equilibrium notion (equilibrium with inertia) applicable to incomplete preferences. In a spirit similar to Mukerji and Tallon (2001), Rigotti and Shannon (2005) also show that when there is uncertainty only about some events, there may be equilibria in which securities contingent on these events are not traded, while securities contingent on the remaining (risky) events are traded. In this case, a more limited degree of market incompleteness is possible in equilibrium, in that risky securities are traded while uncertain securities are not.

Thus, whereas with incomplete preferences, absence of trade and insurance is somewhat built in the model, it is not the case for ambiguity averse preferences *à la* (convex) CEU or MEU. Conditions on how the endowments are perceived by the individuals are necessary to explain absence of trade.

3 Equilibrium properties of economies populated with uncertainty averse agents

In the previous section we reviewed some implications of the non-differentiability of the CEU and MEU preferences. In this section, we go further in this direction by studying the possibility of equilibrium indeterminacies brought about by such non-differentiability and review how this

[2] For a similar argument explaining the absence of wage indexation, see Mukerji and Tallon (2004b).

can affect information revelation. We conclude the section by reviewing answers to what is, essentially, a converse question: if the price functional is a Choquet functional, what can we infer of the underlying market structure?

3.1 Indeterminacies

As recalled in Section 2.1, Epstein and Wang (1994) showed, in a representative agent framework, that uncertainty averse behavior generates asset price indeterminacy at equilibrium. Chateauneuf et al. (2000) and Dana (2004) compare equilibria in a convex CEU economy (with identical capacities) and those of a vNM economy with identical beliefs. The equilibrium allocations in the vNM economy do depend on beliefs, and it is not trivial to assess the relationship between the equilibrium set of a vNM economy with identical beliefs and the equilibrium set of the CEU economy. If aggregate endowments are different in all states of the world, then, equilibria of the CEU economy are the equilibria of the vNM economy with beliefs equal to that probability distribution in the core of the capacity that is used to evaluate the aggregate endowment. On the other hand, if there are some states with the same aggregate endowment, it is a priori not possible to assimilate all the equilibria of the CEU economy with equilibria of a given vNM economy. In particular, there might be a multiplicity of supporting prices. More precisely, Dana (2004) shows that whenever there are several probabilities in the core of the capacity that minimize the expected value of aggregate endowment and not all agents have the same expected endowments under those probabilities, then equilibrium is indeterminate. As a consequence, small changes in aggregate endowments might have drastic welfare implications. Dana (2004) extends these results in infinite dimensional economies.

These indeterminacies of equilibrium prices might thus appear to be non-robust to small perturbations in endowments, since equality of endowments across (some) states is needed. Actually, Rigotti and Shannon (2012) show that generic determinacy is a robust feature of economies with ambiguity sensitive agents (they prove this in the variational preferences setting, which encompass the convex CEU model and the MEU model). However, Mandler (2013) argues that if agents are ambiguity-averse and can invest in productive assets, asset prices can robustly exhibit indeterminacy in the markets that open after the productive investment has been launched. Intuitively, if we leave the possibility to ambiguity averse agents to affect through production the endowment they have in the second period, the technology that allows to equate these endowments across states will have a premium, since such full insurance is highly valued. They will thus invest in these assets and the endowment configuration in the second period (the timing is more subtle: one needs to introduce an intermediate period at which production is realized and agents can trade assets contingent on states in the second period) will be precisely the one that produces price indeterminacies. Thus, as Mandler states "For indeterminacy to occur, the aggregate supply of goods must appear in precise configurations but the investment levels that generate endogenously these supplies arise systematically." Note that the fact that indeterminacy arises only at a knife-edge set of aggregate supplies (that lead the economy second period endowment to allow for full insurance of the agents) allows for a simple explanation of the volatility of asset prices: small changes in supplies in this neighborhood necessarily lead to a big price response and thus extra volatility.

These results can again be contrasted with those obtained in a Bewley economy: Rigotti and Shannon (2005) find robust indeterminacies, for every initial endowment vector. Provided there is sufficient overlap in agents' beliefs, there is a continuum of equilibrium allocations and prices, regardless of other features of agents' beliefs, initial endowments, or aggregate endowments. They show, on the other hand, that despite such robust indeterminacies, the set of equilibria varies continuously with the amount of uncertainty agents perceive. In particular, as uncertainty goes to zero (that is, agents perceive only risk), the equilibrium correspondence converges to an equilibrium of the economy in which there is only risk. Dana and Riedel (2013) generalize Rigotti and Shannon's static results to a dynamic economy.

3.2 Non-revelation of information

As uncertainty refers to situations where information is scarce, it is natural to investigate if and how uncertainty aversion may interfere with the way privately held information spreads in the economy. Tallon (1998) is an early investigation of this issue that shows that ambiguity averse investors might buy "redundant" information even if the equilibrium is fully revealing. This is possible if the investor has less faith in the information revealed by prices (possibly because of model mis-specification) than in information privately acquired. Condie and Ganguli (2011a) show that non-smooth ambiguity aversion, i.e., convex CEU or MEU, may lead to informational inefficiency: even in the absence of noise traders, private information might not be fully revealed at a rational expectations equilibrium. The mechanism relies on the fact that non-smooth uncertainty aversion implies that investor demand does not change with information (i.e., beliefs) for some range of parameters. This feature of preferences then can be used to construct a non-revealing equilibrium. Intuitively, if a privately informed investor is uncertainty averse, an allocation that fully insures him might be optimal for him (at a given price) for different beliefs, due to the non-differentiability in his preferences. Hence, no matter what signal he received, that allocation and associated price is an equilibrium; the information received does not get to be revealed. Note that, while based on the non-smoothness of indifference curves, the mechanism is not exactly the same as the one involved in the portfolio inertia of Dow and

Werlang (1992). Condie and Ganguli (2011b) complement this finding by showing that fully revealing equilibria also exist in these economies and Condie and Ganguli (2017) further explore the pricing implication of this informational inefficiency. In a similar vein, Condie et al. (2019) study how aversion to ambiguity about the predictability of future asset values and cash flows affects optimal portfolios and asset prices. They show that investors' portfolios do not always react to new information, even away from full insurance. The equilibrium price of the market portfolio does not always incorporate all available public information, in particular it might fail to incorporate bad news. This informational inefficiency leads to price underreaction. The economic mechanism that leads to this "information inertia" does not occur at the kink in investors' utility in contrast to the portfolio inertia previously discussed.

The asset pricing implication of ambiguous information have also been explored by Ozsoylev and Werner (2011). They show that ambiguous information gives rise to the possibility of illiquid market where arbitrageurs choose not to trade in a rational expectations equilibrium. As a consequence of this illiquidity, small informational or supply shocks have relatively large effects on asset prices. Mele and Sangiorgi (2015) analyze costly information acquisition in asset markets characterized by ambiguity. They show how uncertainty aversion affects the incentives to acquire information and can lead to the existence of multiple equilibria which in turn can account for large price swings event after small changes in ambiguity.[3] These investors prefer to trade on aggregate signals if those reduce ambiguity, even if it is at the cost of a loss in information. This feature of ambiguity averse investors might explain both under-reaction to overall news and, concurrently, overreaction to specific components of the overall news.

3.3 CEU as a pricing functional

The fundamental theorem of asset pricing for frictionless complete markets enforces a linear pricing rule: the cost of replication of any security is given by the mathematical expectation of its payoffs stream under the unique state contingent price or risk-neutral probability obtained by the no-arbitrage principle. In a financial economy where agents can trade a finite and potential limited number of frictionless securities, the pricing rule gives the minimum cost of getting a payoff equal to (or larger than) a given contingent claim in any state of nature, which is also known as the super-replication price. Importantly, by no-arbitrage and assuming the presence of a fair risk-free security, the super-replication price of any security can be determined by its supremum expected value with respect to all risk-neutral probabilities. Frictions including bid-ask spreads and indeterminacies of the kind discussed in the previous sections may imply that we

[3] Other form of informational inefficiencies might arise with smooth preferences. Caskey (2009) for instance shows how asset mis-pricing is consistent with the presence of ambiguity-averse investors of the smooth ambiguity type.

have one more underlying risk-neutral probability and the pricing rule is given by the supremum of expected values with respect to all these risk-neutral probabilities. As a consequence, the pricing rule is non-linear and maybe characterized in terms of a capacity.

Subadditive Choquet pricing rules were first studied and characterized by Chateauneuf et al. (1996), see also Castagnoli et al. (2002) and Araujo et al. (2012). The main insight of this approach is that the super-replication price functional derived from a particular arbitrage-free financial market can be viewed as a pricing rule represented by a maximum of expected values over the closure of the set of risk-neutral probabilities. Cerreia-Vioglio et al. (2015) extend the fundamental theorem of finance to markets with frictions. Assuming that the Put-Call Parity condition holds as well as the absence of arbitrage opportunities they obtain a representation of the pricing rule as a discounted expectation with respect to a nonadditive risk neutral probability, i.e., a Choquet capacity. They provide testable conditions under which transaction costs generate this sublinear pricing rule which is also a Choquet expectation. Araujo et al. (2012) ask the opposite question: what type of two-period market structure emerges from an arbitrary set of probabilities characterizing a pricing rule? They show that finitely generated pricing rules reveal an efficient complete securities market.

Going beyond the characterization of arbitrage free prices, Beisner and Riedel (2019) study an equilibrium concept with sublinear prices that they call Knight-Walras equilibrium. They interpret this sublinear pricing as reflecting cautiousness from a market maker who would have an imprecise probabilistic information about the states of the world, and thus computes the maximal expected present value over a set of models, so as to hedge uncertainty. They study this notion of equilibrium and compare it with the more standard notion of Walrasian equilibrium (based on linear pricing). They prove that Knight-Walras equilibria are generically inefficient. In the particular case of no-aggregate uncertainty, they show that even a small amount of uncertainty leads to no-trade at a Knight-Walras equilibrium, contrary to what happens at the Walrasian equilibrium which entails full insurance.

4 Optimal ambiguity sharing

In this section we review optimal ambiguity sharing in CEU and MEU economies. Does the non-differentiability in the decision criterion ultimately lead to optimal ambiguity sharing arrangements of a different nature than the ones under expected utility? In particular, can we say that, at the aggregate level, optimal allocations of economies with ambiguity averse agents are somehow less prone to ambiguity than optimal allocations of economies populated with expected utility agents?

Chateauneuf et al. (2000) explore the Pareto optimal allocations of a single good economy populated by CEU maximizers that have the same convex capacity. In this setting, the set of Pareto optimal allocations is independent of the capacity and, furthermore, is identical to the set of optima of an economy in which agents are expected utility maximizers and have the same probability. Hence, optimal allocations are comonotone: optimality dictates that each agent's allocation is increasing with the aggregate endowment. This in turn "fixes" the decision weights agents use to evaluate their allocation and implies that they are all equal. Thus, the aggregate implication is not different under CEU (with same capacity) and expected utility (with same probabilistic beliefs). While somewhat surprisingly at first sight, this result echoes the classical finding that the Pareto optimal allocations in an expected utility economy do not depend on the beliefs of the agents as long as they are the same across agents. And indeed, as for the heterogeneous beliefs in an expected utility economy, things are much more difficult to assess and characterize when agents have different capacities.[4]

Pareto optimal allocations in a MEU economy has not been fully characterized to the best of our knowledge. Matters are more complicated since comonotonicity of the optimal allocations, even if it were true under multiple prior hypothesis, does not imply that all agents will have the same decision weights, except in rather contrived environments (e.g., with only two states of the world). Epstein (2001) provides an example of risk sharing with different ambiguous beliefs in a two-country example under MEU.

One could wonder why the difference spotted in the first section between the optimality of non exposure to uncertainty that CEU delivers and the "local uncertainty neutrality" of expected utility agents is not relevant when we look at Pareto optimal allocations. The reason lies in the simple observation that while a single agent can always choose to shy away from uncertainty, at the aggregate level, uncertainty must be borne, thus fixing decision weights. A simple Edgeworth box diagram makes the point.

A particular case emerges though, i.e., when it is actually feasible that all agents be fully insured. This happens in an economy without aggregate uncertainty. In this setting, Billot et al. (2000), assuming MEU agents (and thus including the convex Choquet case) show that the set of Pareto optimal allocations consists of the set of full insurance allocations if and only if agents *share* at least one prior. This generalizes the expected utility case for which full insurance is Pareto optimal if and only if agents all have the *same* probabilistic beliefs.

Rigotti et al. (2008) provide a generalization of this result using a definition of subjective beliefs (at a given allocation) that applies to any model of convex preferences, based on the willingness to take small bets when at this allocation. The reasoning that underlies the result in

[4] The relevance of the Pareto criterion in theses cases have been questioned altogether, see Mongin (2016) and Gilboa et al. (2014).

Billot et al. (2000), based on the MEU model, is thus shown to extend to other models of decision under uncertainty when there is a multiplicity of "beliefs" supporting an allocation. Strzalecki and Werner (2011) also extend these results to more general preferences, through the concept of conditional beliefs. These are the probabilistic beliefs revealed by agents' unwillingness to take fair bets conditional on an event. They thus show that a necessary and sufficient condition for measurability of Pareto optimal allocations with respect to the aggregate endowment is that agents have at least one conditional belief in common for every event in the partition induced by the aggregate endowment. The comonotonicity of consumption plans with the aggregate endowment requires a stronger condition.

In the CEU case (and still in absence of aggregate risk), considering not necessarily convex capacities, Billot et al. (2002) provide a characterization of capacities whose cores have a non-empty intersection and show that if there is a prior that belongs to that intersection, then all optimal allocations provide full insurance. It may be the case that the cores of the capacities do not intersect, yet some and even all optimal allocations provide full insurance. Yet, if the economy is "replicated", i.e., if we consider a continuum of agents of each type, the equivalence result is reinstated. Thus, Billot et al. (2002) establish that for large economies populated by CEU maximizers with possibly non convex capacities, commonality of "beliefs" (in the sense of the intersection of the cores of the capacities being non empty) is still necessary and sufficient for some, or all Pareto optimal allocations to entail full insurance. Ghirardato and Siniscalchi (2018) provide the most general analysis so far of the conditions on beliefs and preferences under which the optimality of full insurance holds in an economy without aggregate uncertainty, that can in particular accommodate non-convex preferences. Their approach builds on Rigotti et al. (2008) and identifies a notion of an individual's set of local beliefs from his preferences, that does not require preferences to be overall convex. If these sets have a non-empty intersection, and in the absence of aggregate uncertainty, Pareto optimal allocations are the full insurance allocations.

5 Experimental evidence

The individual decision mechanism behind the economic phenomenon we reviewed rests on some form of non-responsiveness of behavior to a change in prices, which can be traced back to non-differentiabilities or kinks in the indifference curves. The experimental literature has provided some evidence that models including some non-differentiability like (α)-MEU and CEU are helpful to explain behavior. As could be expected, heterogeneity is the norm at the individual level.

Ahn et al. (2014) report the result of an experiment in which subjects had a budget to split between three Arrow securities whose returns have an Ellsberg three color urn payoff structure. They show that some individuals have a tendency to bunch the two ambiguous securities, even if they have different prices, a prediction consistent with MEU and CEU models. They however show that this is not the only mode of decision and that some behaviors are more in line with expected utility or non expected utility smooth models predictions. Baillon and Bleichrodt (2015) develop an experiment using naturally occurring ambiguous performances of stock markets, that include gains and losses. They find that propsect theory and α-maxmin models can account for the pattern they observe in the data. Cubitt et al. (2018) elaborate a design specifically aimed at discriminating between the MEU and α-MEU family of models on the one hand and the smooth ambiguity model on the other hand, arguably the most popular models in applications. They find clear and statistically significant patterns in the behavior of the subjects coded as ambiguity averse that conform more closely to the predictions of the smooth ambiguity model than to those of the α-MEU model.

Going outside of the lab, Dimmock et al. (2016a) show, on a US representative household survey, that ambiguity aversion, measured through Ellbserg-type questions is a factor explaining non participation in the stock market, in line with the intuition developed by Dow and Werlang (1992). They also find a negative relation between the degree of ambiguity aversion and the fraction of financial assets allocated to equity. Dimmock et al. (2016b) on the other hand, do not find, on a Dutch household survey, that, for the entire sample, ambiguity aversion and participation are correlated. They do find that ambiguity aversion is negatively related to stock market participation, but only for subjects who perceive stock returns as highly ambiguous. Bianchi and Tallon (2019) provides field evidence on the relation between ambiguity aversion and portfolio choices. They show that ambiguity averse investors tend to keep their risk exposure relatively constant over time. These investors tend to rebalance their portfolio in a contrarian direction relative to the market. This is in accordance to the phenomenon of portfolio inertia consistent with the maxmin type of behavior that has been discussed in this note.

Bryan (2019) tests in the field (through randomized controlled trials in Malawi and Kenya) the relationship between ambiguity aversion and technology adoption. To raise adoption rate, it has been suggested to provide insurance to the farmers who adopt these new technologies. However, theory suggests that insurance will be more effective in areas where the production technology is well known and will be ineffective in promoting take-up of novel technologies among the ambiguity averse. The reason for this is that partial insurance makes payment conditional on a specific state of the world, for which objective information that would help to determine the relevant probabilities is often unavailable, especially when income comes from a new technology.

Thus the value of insurance is ambiguous, and insurance is less useful to those that do not tolerate ambiguity, that is, the ambiguity averse. As the paper explains, the intuition and mechanisms of these results is very similar to that of Mukerji and Tallon (2004a) and Mukerji and Tallon (2004b) showing an endogenous breakdown of trade in markets involving contracts whose payoffs are subject to ambiguity. Hence the paper can be seen as an empirical test of these mechanisms and the model presented a translation of these mechanisms to the particular setting of agricultural production.

Finally, Bossaerts et al. (2010) go beyond the single agent decision making setting and present a market experiment in which the assets traded have ambiguous returns. They find that ambiguity averse agents will not hold the ambiguous securities at equilibrium, but still have an impact on their prices. Overall their findings are in line with predictions from a general equilibrium model with heterogeneous α-maxmin expected utility agents.

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