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Development of a two-dimensional dynamic model of the foot-ankle system exposed to vibration

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Development of a two-dimensional dynamic model of the foot-ankle system exposed to vibration

Workers in mining, mills, construction and some types of manufacturing are exposed to vibration that enters the body through the feet. Exposure to foot-transmitted vibration (FTV) is associated with an increased risk of developing vibration-induced white foot (VIWFt). VIWFt is a vascular and neurological condition of the lower limb, leading to blanching in the toes and numbness and tingling in the feet, which can be disabling for the worker. This paper presents a two-dimensional dynamic model describing the response of the foot-ankle system to vibration using four segments and eight Kelvin-Voigt models. The parameters of the model have been obtained by minimizing the quadratic reconstruction error between the experimental and numerical curves of the transmissibility and the apparent mass of participants standing in a neutral position. The average transmissibility at five locations on the foot has been optimized by minimizing the difference between experimental data and the model prediction between 10 and 100 Hz. The same procedure has been repeated to fit the apparent mass measured at the driving point in a frequency range between 2 and 20 Hz. Monte Carlo simulations were used to assess how the variability of the mass, stiffness and damping matrices affect the overall data dispersion. Results showed that the 7 degree-of-freedom model correctly described the transmissibility: the average transmissibility modulus error was 0.1. The error increased when fitting the transmissibility and apparent mass curves: the average modulus error was 0.3. However, the obtained values were reasonable with respect to the average inter-participant variability experimentally estimated at 0.52 for the modulus. Study results can contribute to the development of materials and equipment to attenuate FTV and, consequently, lower the risk of developing VIWFt.

Keywords: whole-body vibration, standing vibration, biomechanical response,
vibration-induced white-foot

1 Introduction

Up to 7 % of the workforce in Canada, the United States and Europe are exposed to vibration (Bovenzi, 1998; NIOSH, 1997). Whole-body vibration (WBV) exposure, experienced when driving mobile equipment, is associated with an increased risk of low-back disorders, neck pain, headaches, and fatigue (Magnusson et al., 1996; Wikström et al., 1994). Workers who operate pneumatic power tools are exposed to hand-arm vibration (HAV) and can develop HAV syndrome (HAVS) (House et al., 2010). HAVS can result in vascular, neurological, and musculoskeletal impairments (Chetter et al., 1998; Griffin & Bovenzi, 2002), eventually leading to upper extremity disability (House et al., 2009). However, many workers with HAVS also experience cold-induced vasospasm in their feet (Sakakibara et al., 1988; House et al., 2010). Exposure to HAV can stimulate both the local and central sympathetic nervous system (Stoyneva et al., 2003) and Hashiguchi et al. (1994) have proposed that a pathological basis for symptoms in the feet is the presence of vascular medial muscle hypertrophy and increased collagen in fingers and toes connective tissue.

Raynaud's phenomenon in the feet has been related to both vibration exposure at the hands (House et al., 2010) and direct exposure at the feet (Thompson et al., 2010; Toibana et al., 1994; Eger et al., 2014). The development of vibration-induced white feet (VIWFt) has been linked to exposure to foot-transmitted vibration (FTV), associated with drilling/bolting off platforms (Eger et al., 2014; Hashiguchi et al., 1994; Hedlund, 1989). Symptoms of VIWFt can include pain and numbness in the toes and feet, increased sensitivity to cold, blanching in the toes, and joint pain (Thompson et al., 2010; Eger et al., 2014), leading to disability of the lower limbs. The smaller anatomy of the peripheral appendages (hands and feet), makes narrowing the exact cause of vibration-induced symptoms in the feet less transparent, and even less is understood about the biomechanical response of the foot to FTV.

International and European Standards have been created for evaluating the health risk to occupational exposure to WBV and HAV (ISO 2631-1:1997, ISO 5349-1:2001, and EU Directive 2002), but FTV exposure has been lumped in with standing WBV exposure. Recently, the transmissibility response to standing vibration exposure at 24 anatomical locations on the feet was captured, and the responses were found to differ between the toes, midfoot and heel regions (Goggins et al., 2019). These findings suggest that in order to model the dynamic response of the foot and understand associated health effects, the foot cannot be treated as a single component and should not be lumped in with the whole body for standing vibration exposure (Subashi et al., 2008).

Several lumped-parameter linear models have been proposed in the literature to describe the response of the upper limb (Rakheja et al., 2002, Dong et al., 2004, Dong et al., 2007, Adewusi et al., 2012, Dong et al., 2018) and of the whole body (Wei and Griffin, 1998; Wu et al., 1999; Matsumoto and Griffin, 2003; Fritz, 2005; Kim et al., 2005) to vibration. Models can be used to estimate the effectiveness of anti-vibration devices (Dong et al., 2009) or to reproduce the interaction between the vibrating surface and the human body (Tarabini et al., 2013; Busca et al., 2014). Rakheja et al. (2002) suggested that a model based on the estimation of the driving point mechanical impedance (DPMI) was not sufficient to get injury risk insight for hand-arm structures as the anatomical specificities were not considered (Besa et al., 2007; Gurram et al., 1995). However, models based on measured transmissibility which considered the anatomical structures, performed poorly (Cherian et al., 1996; Fritz, 1991). Consequently, methodologies using both the DPMI and the measured transmissibility have been developed (Adewusi et al., 2012; Dong et al., 2015).

To date, there are few models describing the response of the foot to vibration (Gefen, 2003; Kim and Voloshin, 1995; Simkin and Leichter, 1990). FTV models proposed by Gefen (2003) and Simkin and Leichter (1990) used two inclined rigid bodies hinged at the apex of the truss and a spring to model the foot's longitudinal arch and the plantar fascia, respectively. Kim and Voloshin (1995) enhanced the Simkin and Leichter model by introducing viscoelastic

properties of the plantar fascia. The main limitation of these models is that they were originally developed to understand the lower limb response to quasi-static stimuli (i.e. walking and running). Furthermore, the limited number of degrees-of-freedom (DOF) in these models prevent them from describing all the foot resonances evidenced by Goggins et al. (2019). Moreover, Rakheja et al. (2006) and Muksian and Nash (1974), modelled FTV from a seated position which limits comparisons to standing subjects.

In order to design suitable methods to protect workers, a model that describes the biomechanical response of the foot-ankle system (FAS) to FTV is required. Thus, this paper presents a model that reproduces the transmissibility of vibration and the apparent mass of the foot-ankle system with errors that are small with respect to the inter-subject variability. The model could be useful to simulate the effects of different boots, mats, or insoles (Tarabini et al., 2019) as well as to identify how small postural changes affect the energy absorbed by the foot segments.

2 Methodology

The FAS of a standing subject has been modeled with a lumped parameter mechanical system. The model parameters (stiffness and damping) were optimized to fit the experimental transmissibility and apparent mass collected from two different studies. Due to different instrumentation requirements and frequency ranges observed, two different procedures were required to measure the apparent mass and the transmissibility (Appendix A).

The two-dimensional model of the FAS reproduces the response of the foot supporting the lumped parameter model for the whole body proposed by Matsumoto and Griffin, 2003 (Figure 1). The FAS model (connected to the body at the ankle joint) is composed of four segments representing the talus and the calcaneus (i.e. rearfoot); the cuneiforms and the navicular (i.e. midfoot); the metatarsals (i.e. forefoot); and the toes. The four segments were assumed to be uniform rigid bodies of length $L_{I...IV}$, mass $m_{I...IV}$, and moment of inertia $I_{I...IV}$. The inertial and geometrical properties of the segments were derived from (Isman and Inman, 1969; Lee et al., 2011; Zatsiorsky, 2002) and are summarized in Table 1. Kelvin-Voigt models of stiffness and

damping coefficients k_b , k_c , and c_b , c_c , were used to describe the viscoelastic properties of human ligaments and tendon between the four segments. Moreover, the plantar aponeurosis behaviour was expressed by a standard viscoelastic solid material model of stiffness and damping coefficients k_d and c_d . The absorbing capability of the fat pad and soft tissues composing the foot sole were assumed to be viscoelastic materials and described with a Kelvin-Voigt model of properties k_e , k_f , k_g and c_e , c_f , c_g . The DOF of the FAS were four rotations occurring between each segments referred to as $\theta_{1..4}$, as well as the vertical displacements of the ankle $y_A(t)$, and of the vertical displacement of the two masses representing the whole body except the foot, $y_B(t)$ and $y_C(t)$ (Figure 1). The static values of FAS posture for $\theta_{1..4}$, corresponded to 49° , 69° , 82° and 180° respectively. The sole of the foot was driven by an imposed harmonic displacement $y_{in}(t)$. The foot response in the frame of reference (x, y) was estimated at the middle of the rearfoot segment ($x_{g1}(t)$, $y_{g1}(t)$), at the distal end of the midfoot ($x_2(t)$, $y_2(t)$), forefoot ($x_3(t)$, $y_3(t)$), and toes ($x_4(t)$, $y_4(t)$).

Masses m_b and m_c were computed according to the study of Matsumoto and Griffin, 2003, considering half of the whole-body mass, as reported in Goggins et al., 2019. Two Kelvin-Voigt elements connected masses m_b and m_c . Mass m_b was connected to the ankle through the Kelvin-Voigt properties k_a , and c_a . The two masses account for the flexibility of the upper body and allow the replication of the main whole-body resonance around 5 Hz. The equations of motion of the model are reported in Appendix B.

The model parameters were identified by minimizing the difference between the transmissibility and apparent mass predicted by the model and the experimental data. Note that, as it is impossible to directly measure the force at the foot bone junctions, the transmissibility was computed as the ratio between the velocity of the vibrating plate and the velocity of five foot locations (the middle of the rearfoot segment ($x_{g1}(t)$, $y_{g1}(t)$), the distal end of the midfoot ($x_2(t)$, $y_2(t)$), the forefoot ($x_3(t)$, $y_3(t)$), the toes ($x_4(t)$, $y_4(t)$), and the ankle ($x_A(t)$, $y_A(t)$)). Matrices $M_{FF,FC,CF,CC}$ (Eq. 28), $C_{FF,FC,CF,CC}$ (Eq. 29), and $K_{FF,FC,CF,CC}$ (Eq. 30) were defined using geometrical and inertial characteristics reported in Table 1. The unknown dynamical properties

were stiffness $k_{a..h}$ and damping $c_{a..h}$ of each element of the FAS model. The minimization, implemented in *Matlab R2017b* software, consisted of:

1. a genetic algorithm (GA) used to find the first set of parameters by exploring a wide range of values. The initial population of the GA was based on previously reported stiffness and damping (Wee, 2012);
2. a least-squares minimization approach used to refine the solution and identify the optimal set of parameters; the initial set of data was the output of the GA.

The objective functions for the previous steps, included the experimental complex normalized apparent mass (between 2 and 20 Hz) and the five complex transmissibility functions $T_{1..5}$ measured at five foot locations (between 10 and 100 Hz). The error ε to be minimized was defined starting from the apparent mass reconstruction error ε_{am} and the transmissibility reconstruction error ε_T as

$$\varepsilon_{am} = \sqrt{\frac{1}{19} \sum_{f=2}^{20} |(\widetilde{am}(f) - am(f))^2|}, \quad (1)$$

and

$$\varepsilon_T = \sqrt{\frac{1}{455} \sum_{i=1}^5 \sum_{f=10}^{100} |(\widetilde{T}_i(f) - T_i(f))^2|}, \quad (2)$$

where f is the frequency, \widetilde{am} and am are the modelled and the measured (average) apparent masses, \widetilde{T}_i and T_i are the modelled and the measured transmissibility at the locations i . The error ε to be minimized was computed as

$$\varepsilon = \sqrt{w_{am}\varepsilon_{am} + w_T\varepsilon_T}, \quad (3)$$

where w_{am} and w_T were the weights of the apparent mass and transmissibility functions. In order to focus the optimization process primarily on the vibration transmissibility or on the apparent mass, two sets of weights were used:

- **Set α :** optimization of the transmissibility functions: $w_{am}=0$, $w_T=1$.

- **Set β** : optimization of five transmissibility and apparent mass functions: $w_{am}=0.5$, $w_T=0.5$

Hereinafter, the apparent mass and transmissibility functions evaluated with each set will be referred to as $\widehat{am}(f)|_\alpha$, $\widehat{am}(f)|_\beta$, $\widehat{T}(f)|_\alpha$ and $\widehat{T}(f)|_\beta$. With each set of weights, the reconstruction errors of the transmissibility modulus (mod) and for the phases (arg) were computed for each position i as:

$$\varepsilon_{T,i}^{mod}|_\alpha = \sqrt{\frac{1}{91} \sum_{f=10}^{100} (|\widehat{T}_i(f)|_\alpha - |T_i(f)|)^2}, \quad (4)$$

$$\varepsilon_{T,i}^{mod}|_\beta = \sqrt{\frac{1}{91} \sum_{f=10}^{100} (|\widehat{T}_i(f)|_\beta - |T_i(f)|)^2}, \quad (5)$$

$$\varepsilon_{T,i}^{arg}|_\alpha = \sqrt{\frac{1}{91} \sum_{f=10}^{100} (\arg(\widehat{T}_i(f)|_\alpha) - \arg(T_i(f)))^2}, \quad (6)$$

and

$$\varepsilon_{T,i}^{arg}|_\beta = \sqrt{\frac{1}{91} \sum_{f=10}^{100} (\arg(\widehat{T}_i(f)|_\beta) - \arg(T_i(f)))^2}. \quad (7)$$

For each reconstruction error, the average and the standard deviation upon varying the measurement location i $\overline{\varepsilon_T^{mod}}|_\alpha$, $SD(\varepsilon_T^{mod}|_\alpha)$, $\overline{\varepsilon_T^{mod}}|_\beta$, $SD(\varepsilon_T^{mod}|_\beta)$, $\overline{\varepsilon_T^{arg}}|_\alpha$, $SD(\varepsilon_T^{arg}|_\alpha)$, $\overline{\varepsilon_T^{arg}}|_\beta$, $SD(\varepsilon_T^{arg}|_\beta)$ were computed. Similarly, the reconstruction errors of the apparent mass modulus and phase were obtained as

$$\varepsilon_{am}^{mod}|_\alpha = \sqrt{\frac{1}{19} \sum_{f=2}^{20} (|\widehat{am}(f)|_\alpha - |am(f)|)^2}, \quad (8)$$

$$\varepsilon_{am}^{mod}|_\beta = \sqrt{\frac{1}{19} \sum_{f=2}^{20} (|\widehat{am}(f)|_\beta - |am(f)|)^2}, \quad (9)$$

$$\varepsilon_{am}^{arg}|_\alpha = \sqrt{\frac{1}{19} \sum_{f=2}^{20} (\arg(\widehat{am}(f)|_\alpha) - \arg(am(f)))^2}, \quad (10)$$

and

$$\varepsilon_{am}^{arg}|_{\beta} = \sqrt{\frac{1}{19} \sum_{f=2}^{20} (\arg(\widetilde{am}(f)|_{\beta}) - \arg(am(f)))^2}. \quad (11)$$

Once the model parameters were determined with the optimization, a Monte Carlo simulation was performed to estimate the effect of the model parameters uncertainties on the model response. The five transmissibility functions (\tilde{T}_i) and the normalized apparent mass function (\widetilde{am}) were evaluated with 100 randomized combinations of $k_{a..h}$ and $c_{a..h}$, obtaining $\tilde{T}_{i,j}$ and \widetilde{am}_j ; the simulation index j varies between 1 and 100. Stiffness and damping of the Kelvin-Voigt elements were assumed to be normally distributed with standard deviation of 20%. The variability of the transmissibility function (modulus and argument) was summarized by their coefficients of variation (COV):

$$COV_{T,i}^{mod}(f)|_{\alpha} = \frac{\sum_{j=1}^{100} (|\tilde{T}_{i,j}(f)|_{\alpha} - |\overline{\tilde{T}_{i,j}(f)}|_{\alpha})^2}{\sum_{j=1}^{100} |\tilde{T}_{i,j}(f)|_{\alpha}}, \quad (12)$$

$$COV_{T,i}^{mod}(f)|_{\beta} = \frac{\sum_{j=1}^{100} (|\tilde{T}_{i,j}(f)|_{\beta} - |\overline{\tilde{T}_{i,j}(f)}|_{\beta})^2}{\sum_{j=1}^{100} |\tilde{T}_{i,j}(f)|_{\beta}}, \quad (13)$$

$$COV_{T,i}^{arg}(f)|_{\alpha} = \frac{\sum_{j=1}^{100} (\arg(\tilde{T}_{i,j}(f)|_{\alpha}) - \arg(\overline{\tilde{T}_{i,j}(f)}|_{\alpha}))^2}{\sum_{j=1}^{100} \arg(\tilde{T}_{i,j}(f)|_{\alpha})} \quad (14)$$

and

$$COV_{T,i}^{arg}(f)|_{\beta} = \frac{\sum_{j=1}^{100} (\arg(\tilde{T}_{i,j}(f)|_{\beta}) - \arg(\overline{\tilde{T}_{i,j}(f)}|_{\beta}))^2}{\sum_{j=1}^{100} \arg(\tilde{T}_{i,j}(f)|_{\beta})}. \quad (15)$$

Similarly, the variability of the am (modulus and argument) was summarized by the following COV:

$$COV_{am}^{mod}(f)|_{\alpha} = \frac{\sum_{j=1}^{100} (|\widetilde{am}_j(f)|_{\alpha} - |\overline{\widetilde{am}_j(f)}|_{\alpha})^2}{\sum_{j=1}^{100} |\widetilde{am}_j(f)|_{\alpha}}, \quad (16)$$

$$COV_{am}^{mod}(f)|_{\beta} = \frac{\sum_{j=1}^{100} (|\widetilde{am}_j(f)|_{\beta} - |\overline{\widetilde{am}_j(f)}|_{\beta})^2}{\sum_{j=1}^{100} |\widetilde{am}_j(f)|_{\beta}}, \quad (17)$$

$$COV_{am}^{arg}(f)|_{\alpha} = \frac{\sum_{j=1}^{100} (\arg(\overline{am}_j(f)|_{\alpha}) - \arg(\overline{am}_j(f)|_{\alpha}))^2}{\sum_{j=1}^{100} \arg(\overline{am}_j(f)|_{\alpha})} \quad (18)$$

and

$$COV_{am}^{arg}(f)|_{\beta} = \frac{\sum_{j=1}^{100} (\arg(\overline{am}_j(f)|_{\beta}) - \arg(\overline{am}_j(f)|_{\beta}))^2}{\sum_{j=1}^{100} \arg(\overline{am}_j(f)|_{\beta})}. \quad (19)$$

The above COV were evaluated at 4, 8, 12, 16 and 20 Hz (*am*) and at 20, 40, 60, 80 and 100 Hz (*T_i*). Similarly to what was previously done for the reconstruction error, the average and the standard deviation of $COV_{T,i}$ were estimated upon varying the measurement location *i*.

3 Results

The proposed model of the foot-ankle system well reproduced the measured apparent mass and transmissibility with errors that were smaller than the inter-subject variability (Figures 2 and 3). Using the apparent mass in the optimization function led to a generalized increase of stiffness (Table 2). As for the stiffness, the damping obtained by fitting simultaneously the apparent mass and the transmissibility (**Set β**) were higher than those obtained by fitting the transmissibility (**Set α**), but for the rearfoot.

The average reconstruction quadratic error of the transmissibility modulus among all the locations points, evaluated with the **Set α** , $\overline{\varepsilon_T^{mod}}|_{\alpha}$, was 0.1, $SD(\varepsilon_T^{mod}|_{\alpha})$ was 0.1 (Figure 2). The average reconstruction quadratic error of the phase among all the locations points $\overline{\varepsilon_T^{arg}}|_{\alpha}$, was 0.2, $SD(\varepsilon_T^{arg}|_{\alpha})$ was 0.1 rad (Figure 2). The modelled transmissibility was always included in the standard deviation estimated experimentally based on both the inter-participant repeatability and the two-dimensional reduction of the model. However, the model did not correctly reproduce the apparent mass at the driving point: the body's main resonance was estimated below 1 Hz while it was expected around 5 Hz.

The average reconstruction quadratic error of the transmissibility modulus among all the locations points, evaluated with the **Set β** , $\overline{\varepsilon_T^{mod}}|_{\beta}$, was 0.3; $SD(\varepsilon_T^{mod}|_{\beta})$ was 0.1 (Figure 3). The

average reconstruction quadratic error of the phase among all the locations points $\overline{\varepsilon_T^{arg}}|_{\beta}$, was 0.3; $SD(\varepsilon_T^{arg}|_{\beta})$ was 0.3 rad (Figure 3). Although the reconstruction errors increased compared to the **Set α** , most of the modelled transmissibility was included in the admissible interval based on the experimental standard deviation. Further, accordingly to the measurements, the apparent mass modelling reproduces the main body resonance at 5 Hz, although the resonance peak amplitude is underestimated.

Results of the sensitivity analysis showed that a variation of 20 % of the stiffness and damping led to a variation of the modelled transmissibility lower than the experimental variability (Figures 4 and 5). COV are provided Figure 6. The transmissibility modulus **was more** affected by the model parameters variations than the phase: $COV_T^{mod}(f)|_{\alpha} = 15 \pm 9 \%$, and $COV_T^{arg}(f)|_{\alpha} = 11 \pm 2 \%$. Considering the **Set β** , similar results were obtained: $COV_T^{arg}(f)|_{\beta} = 23 \pm 14 \%$, and $COV_T^{mod}(f)|_{\beta} = 10 \pm 8 \%$. Finally, no noteworthy result was outlined regarding how the apparent mass was affected by variations in the model parameters.

4 Discussion

The proposed model reproduced the transmissibility and the apparent mass of the FAS exposed to FTV while minimizing the error with respect to inter-subject variability. Monte Carlo simulations showed that a variability of 20 % of the model stiffness and damping leads to a variability of results lower than the experimental one. The model is thus relevant and consistent with the expectations **and allows** a discussion on the implications.

The coefficient numerical value $k_e = 9.6 \text{ kN}\cdot\text{m}^{-1}$ while standing upright, describing the rearfoot sole stiffness of the foot obtained with the **Set α** , was about 100 times lower than the ones reported in the literature by Jorgensen and Bojsen-Moller (1989), that was obtained to reproduce the FAS behaviour at low frequencies while walking. Most likely, these difference is related to the participant posture that greatly affects the foot parameters. Subashi et al. (2008), reported a foot stiffness of $2.4 \cdot 10^5 \text{ N/m}$, comparable to the values reported in this study. Material and

structural differences between the foot segments have also been reported (Teoh et al., 2015), suggesting that heel pad was stiffer and had higher absorbing capability than the second metatarsal head. However, the present study provided opposite results stating that the midfoot was stiffer and had lower absorbing capability than the rearfoot and the forefoot (Table 2). Consequently, the present study is in accordance with HTV models where the stiffness values are higher and the damping values are lower at the skin directly in contact with the vibrating sources (Dong et al., 2004; Reynolds and Falkenberg, 1982).

The parameters describing the connection between the ankle, mass m_b and m_c were comparable to ones in Matsumoto and Griffin (2003). **Set α** , had lower stiffness k_a and damping c_a values, compared to values in Matsumoto and Griffin (2003) since the apparent mass was not reconstructed. However, using the **Set β** , k_h was of the same order of magnitude than in Matsumoto and Griffin (2003), while k_a , c_a and c_h were higher. Model indications were also consistent with values reported by Tarabini et al. (2014), as the apparent mass is mainly concentrated on the talus.

Comparing the two sets of weights defined to simultaneously reproduce the transmissibility and the apparent mass, the stiffness and damping of the ligaments and tendons increased; in line with studies performed on the hand-arm system (Dong et al. 2018). In **Set α** , the FAS model appropriately reconstructed transmissibility at five locations on the foot, while the reconstruction of the apparent mass is an approximation of the main resonance of the human body. This behaviour is due to the high **connection stiffness** between masses m_c and m_b . Using the coefficient **Set β** , the FAS model was able to reproduce better the apparent mass of a standing human than with the coefficient **Set α** . In both cases, errors were smaller than the experimental data variability. Transmissibility curves obtained with **Set β** were biased both at the rearfoot (where the transmissibility is underestimated at low frequencies) and at the forefoot and toes (that are rigidly connected to the supporting surface, as shown by the stiffnesses k_c and k_g). A further investigation showed that the substitution of k_c and k_g derived from **Set α** (by keeping all other parameters of **Set β**) worsen the reconstructed transmissibility at the midfoot and the apparent mass.

The difference between the results obtained with Sets α and β showed that the values of the stiffness and damping coefficients must be intended as general indications and that the model can only be used to predict resonances occurring when the foot is exposed to vertical FTV. The reconstruction errors increased with frequency; this observation can be explained by the importance of bones and tendons in the FAS dynamical behaviour at low frequency, while the human skin and tissues govern the FAS dynamical behaviour at higher frequencies (Lundström, 1985).

The use of the model and the numerical values of the Kelvin-Voigt elements is limited to reproduce the average transmissibility of the FAS exposed to vertical vibration; numerical values of the coefficients must be intended as generic indications of their order of magnitude. In order to simultaneously reproduce the apparent mass at the driving point and the transmissibility, it is necessary to adopt more complex models of the upper body part. For example, Subashi et al. (2008) proposed a lumped parameter mathematical model that includes 6 masses connected by 7 elastic Kelvin-Voigt elements.

Additional limitations arise from the fact that the model ignores the third dimension of the foot. Sanchis-Sales et al. (2018) stated that the dynamic of foot joints during walking is affected by the pronation/supination angle, suggesting that a 3D foot-ankle model would be valuable to better assess gait pathologies or design shoes. Second, human response to vibration is dependent on body posture. Consequently, the parameters of our model are expected to vary with changes in standing posture. Future work should incorporate nonlinear (posture dependent) Kelvin-Voigt elements to describe the vibration response in different body postures.

5 Conclusion

A 2D model of the FAS has been proposed, and this model describes the dynamic response of the FAS from 10 to 100 Hz and the apparent mass in the frequency range of 2 – 20 Hz for participants standing in a neutral position. Resulting transmissibility functions, for Set α , showed a good similarity with the measured transmissibility functions as the reconstructed errors were smaller

than experimental variabilities. The stiffness and damping parameters of the model were in accordance with literature values and were correlated to the biomechanical function of the described FAS elements. However, to reconstruct the apparent mass between 2 – 20 Hz, a different set of parameters was required, as shown in **Set β** . This contribution opens new perspectives in modeling FTV and will be of great interest to address the phenomena occurring in the FAS when altering the standing posture or when using different boots, mats, or insoles.

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Conflict of interest

The results of the study are presented clearly, honestly, and without fabrication, falsification, or inappropriate data manipulation. The authors have no conflict of interest to report.

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Appendices

A. Experimental data used for model construction

In order to develop a FAS model under exposure to FTV, two different data sets were used, collected in different times and with different subjects.

The transmissibility data set included experiments that were carried out with 21 participants (Goggins et al., 2019), which were exposed to vertical vibration, while standing barefoot in a natural position, on a rigid plate fixed to the head of an electrodynamic shaker. Participants were 15 males and 6 females with an average (\pm standard deviation) age of 24 (\pm 7.8) years, height of 175.6 (\pm 9.1) cm, mass of 70.1 (\pm 14.0) kg, and total foot length of 25.8 (\pm 2.0) cm. The data acquisition protocol, experimental setup and main limitations are described by Goggins et al. (2019) and briefly summarized here. The stimulus consisted of sine sweep from 10-200 Hz lasting 51 seconds and the series of tests were performed at constant velocity. Vibration transmissibility was measured at 24 anatomical locations on the right foot using a laser Doppler vibrometer. For modelling purposes, data were considered in the frequency range 10-100 Hz. Simplification from 3D data to a 2D foot model was obtained by averaging vibration measured at different locations of the forefoot, midfoot, ankle, and rearfoot segments. Biodynamic responses from 24 anatomical locations were reduced to five average transmissibility functions (Figure 1) based on similarities in transmissibility responses (Goggins et al., 2019).

Apparent mass data were collected according to the experimental setup described in (Tarabini et al., 2013). Ten male participants had an average (\pm standard deviation) age of 26 (\pm 0.9) years, height of 174.7 (\pm 5.0) cm and a mass of 73.5 (\pm 9.9) kg. The vibration stimulus (along the vertical axis) was a sine sweep in the frequency range of 1-30 Hz, with a root-mean square (RMS) acceleration value of 1 m/s². The pressure distribution at the feet was measured through the Pedar-X insoles (Novel, Munich, Germany). The apparent mass obtained was normalized by the static mass value.

The t-test (null hypothesis $H_0: \mu_1 - \mu_2 = 0$) showed no significant difference between the means of heights (P-Value 0.73), body masses (P-Value of 0.50) and ages (P-Value 0.26) of the participants that took part in the two studies.

B. Equations of motions

Under the hypothesis that the transmitted vibrations induced only small perturbations around the equilibrium position, geometric nonlinearities were simplified to the first two terms of the Taylor series expansion. This linearization procedure (justified by the limited nonlinear effects in the biodynamic response of standing participants reported by Tarabini et al. (2014)) led to a simplification of the problem and hence a reduction of the computation time for dynamic simulation. The generalized coordinate was expressed as

$$\underline{r} = \begin{bmatrix} \underline{r}_{dof} \\ \underline{r}_{in} \end{bmatrix} \quad (20)$$

with

$$\underline{r}_{dof} = \begin{bmatrix} y_A \\ y_B \\ y_C \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}, \quad (21)$$

$$\underline{r}_{in} = [y_{in}].. \quad (22)$$

The kinetic energy (E), potential energy (U), and dissipation energy (D) were then derived from the position vectors as

$$E = \frac{1}{2} m_B \dot{y}_B^2 + \frac{1}{2} m_C \dot{y}_C^2 + \frac{1}{2} \sum_{j=I}^{IV} m_j (\dot{x}_j^2 + \dot{y}_j^2) + \frac{1}{2} \sum_{j=I}^{IV} I_j \dot{\theta}_j^2, \quad (23)$$

$$U = \frac{1}{2} \sum_{j=a}^h k_j \Delta_j^2, \quad (24)$$

$$D = \frac{1}{2} \sum_{j=a}^h c_j \dot{\Delta}_j^2 \quad (25)$$

where Δ_j and $\dot{\Delta}_j$ are the displacement and the velocity of the j-th stiffness and damper element, respectively. Δ_j and $\dot{\Delta}_j$ were derived from the elements of r_{dof} and have units of m and m/s respectively. The equations of motion were finally obtained from the Lagrange equation, with respect to the generalized coordinate r

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{r}} \right) - \frac{\partial E}{\partial r} + \frac{\partial U}{\partial r} + \frac{\partial D}{\partial \dot{r}} = \underline{0}, \quad (26)$$

where the conservative generalized force was null. The equations of motion were rewritten according to a matrix form (where matrices were 8x8 sized) as

$$[M]\ddot{r} + [C]\dot{r} + [K]r = \underline{0} \quad (27)$$

where

$$[M] = \begin{bmatrix} [M_{FF}] & [M_{FC}] \\ [M_{CF}] & [M_{CC}] \end{bmatrix}, \quad (28)$$

is the mass matrix,

$$[C] = \begin{bmatrix} [C_{FF}] & [C_{FC}] \\ [C_{CF}] & [C_{CC}] \end{bmatrix}, \quad (29)$$

is the damping matrix, and

$$[K] = \begin{bmatrix} [K_{FF}] & [K_{FC}] \\ [K_{CF}] & [K_{CC}] \end{bmatrix} \quad (30)$$

is the stiffness matrix. Equations (27) to (30) were combined to obtain

$$[M_{FF}]\ddot{r}_{dof} + [C_{FF}]\dot{r}_{dof} + [K_{FF}]r_{dof} = -([M_{FC}]\ddot{r}_{in} + [C_{FC}]\dot{r}_{in} + [K_{FC}]r_{in}). \quad (31)$$

Vibration transmissibility was computed as the ratio between vibration measured at two locations: the response was measured at anatomical locations on the feet, while the stimulus was the vibration imposed to the plate supporting the participants. Based on equation (29) and on the harmonic motion, the analytical transmissibility (T_{dof}) of the FAS was derived as

$$[T_{dof}] = \frac{r_{dof}}{r_{in}} = \frac{-(-\Omega^2[M_{FC}] + i\Omega[C_{FC}] + [K_{FC}])}{(-\Omega^2[M_{FF}] + i\Omega[C_{FF}] + [K_{FF}])} \quad (32)$$

with $i^2 = -1$ and Ω is the angular frequency. The transmissibility functions were computed between the vibrating ground velocity and the DOF used to describe the model. More specifically, the transmissibility functions (T_1, T_2, T_3, T_4 and T_5) were computed between the vibrating ground velocity ($\dot{y}_{in}(t)$), and the rearfoot ($\dot{y}_1(t)$), the midfoot ($\dot{y}_2(t)$), the forefoot ($\dot{y}_3(t)$), the toes ($\dot{y}_4(t)$), and the ankle ($\dot{y}_A(t)$), and can be computed applying the linearized equations of motions of the system.

The apparent mass at the driving point was computed as the ratio between the sum of the forces exerted at the interface and the imposed acceleration \ddot{y}_{in} . The force F_p is due to spring and dampers at locations e, f and g. The apparent mass (AM) can be computed as:

$$\begin{aligned}
 [AM] &= \sum_p \frac{F_p}{a} \\
 &= \sum_p \frac{(y_1 - y_{in})K_e + (\dot{y}_1 - \dot{y}_{in})c_e + (y_3 - y_{in})K_f + (\dot{y}_3 - \dot{y}_{in})c_f + (y_4 - y_{in})K_g + (\dot{y}_4 - \dot{y}_{in})c_g}{\ddot{y}_{in}}
 \end{aligned}
 \tag{33}$$

For each participant, the apparent mass was divided by the static mass, to obtain the normalized apparent mass, which was compared with data reported by Tarabini et al. (2013).

Figure Captions:

Figure 1: Biodynamic model of the FAS. The four segments are representing the rearfoot (I), the midfoot (II), the forefoot (III), and the toes (IV). The equivalent dynamical properties $k_{a..h}$ and $c_{a..h}$ are describing each joint behavior. $\theta_{1..4}$, and $y_{A, B, C}$ correspond to the system's degrees of freedom.

Figure 2: **Set α** , modelled (black curves) and measured (grey curves) amplitude and phase of the vibration transmissibility functions computed at five locations of the FAS. The reported uncertainty represents a 95% confidence interval. The reconstruction quadratic error ε is reported for each modelled curve.

Figure 3: **Set β** , modelled (black curves) and measured (grey curves) amplitude and phase of the vibration transmissibility functions computed at five locations of the FAS. The reported uncertainty represents a 95% confidence interval. The reconstruction quadratic error ε is reported for each modelled curve.

Figure 4: **Set α** , sensitivity of the model (black curves) around the natural standing position (dotted curves) and measured variability (grey areas) amplitude and phase of the vibration transmissibility functions computed at five locations of the FAS.

Figure 5: **Set β** , sensitivity of the model (black curves) around the natural standing position (dotted curves) and measured variability (grey areas) amplitude and phase of the vibration transmissibility functions computed at five locations of the FAS.

Figure 6: Variability of the transmissibility and the apparent mass functions expressed by $\mathcal{E}(f)$ at 20, 40, 60, 80 and 100 Hz for the transmissibility and at 4, 8, 12, 16 and 20 Hz for the apparent mass, for the sets of weight α (a) and β (b). Each value has been computed according to equations 15 and 16.

Table Captions:

Table 1: Geometrical and inertial characteristics of the four segments composing the foot (Isman and Inman, 1969; Lee et al., 2011; Zatsiorsky, 2002) and masses m_b and m_c values according to the model of Matsumoto and Griffin, 2003.

Table 2: Estimated stiffness and damping coefficients of each model segment.