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Comment on "A praxis-oriented perspective of streamflow inference from stage observations – the method of Dottori et al. (2009) and the alternative of the Jones Formula, with the kinematic wave celerity computed on the looped rating curve" by Koussis (2009)

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Abstract. The estimation of transient streamflow from stage measurements is indeed important and the study of Dottori, Martina and Todini (2009) (henceforth DMT) is useful, however, DMT seem to miss certain of its *practical aspects*. The goal is *to infer the discharge from measurements of the stage conveniently and with accuracy adequate for practical work*. This comment addresses issues of the applicability of the DMT method in the field. DMT also advocate their method as a replacement of the widely used Jones Formula. The Jones Formula was modified by Thomas (Henderson, 1966) to include the temporal derivative of the depth, instead of the spatial one, to specifically allow discharge estimation from at-a-section stage observations. The outcome of the comparison is not surprising in view of this approximation. However, this discussion intends to show that, properly evaluated, the praxis-oriented Jones Formula, which did well in the tests, can perform better than DMT imply. It will be also documented that the DMT methodology relates to a known method for computing flood depth profiles.

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1 Considerations on the applicability of the DMT method

An engineering method is useful when it is theoretically sound and practically applicable. Practicability dictates that the essential prerequisites of a method should be readily secured. In the case of the DMT method, prerequisite is the existence of two *appropriately* positioned gauging stations, however, this may often not be the case. Clearly, the mathematical calculations will miss their target, unless the necessary gauging stations are positioned such that the recorded stages give a good representation of the slope of the wave profile, allowing to obtain a good estimate of the surface slope.

The proper positioning of the two gauges is not a trivial requirement, because depth is controlled by the local stream geometry, in contrast to the flow rate that varies in space more gradually. We can see this by considering steady, gradually varied, subcritical flow, at constant rate in an open channel of rectangular cross-sections of variable width. By continuity of flow and by flow dynamics, the water surface profile responds locally, by rising or falling when the width decreases or increases, respectively; in the extreme case of a choke, the flow becomes critical in the constriction and rises sharply upstream of it. It follows, then, that, over a reach of variable geometry, good or poor estimates of the mean surface slope are obtained depending on the locations of the gauging points. In a flood, the flow rate varies spatially at any time, which adds to the variability of the surface slope relative to the case of constant discharge. That gauging stations will be at hand where needed is all the more doubtful, if not unlikely, as monitoring networks are shrinking worldwide and are increasingly difficult to maintain. Enhancing existing hydrometric networks with new stations is unlikely for reasons of cost (as anyone who has been involved in fieldwork can attest, the setting up of a flow metering station is a work for experts).

It is also noted that, if the two gauges are not positioned optimally, no correction of the measurements is possible; in contrast, if stage variations at a station are suspect over a time interval, selecting a larger interval can correct the problem. And DMT caution "Please note that the distance between the two adjacent sections must be sufficiently small to allow for the constant flow rate assumption to be realistic, but at the same time it must be sufficiently large to allow the difference in water stage to be greater than the measurement instrument sensitivity and the water elevation fluctuations." The first condition (that the two sections must be closely spaced) is not a stringent one; in floods, the discharge is generally different at different cross-sections, therefore, the estimated flow rate should be interpreted as the average discharge in the reach. The second condition, however, is an essential one, not only from the viewpoint of measurement sensitivity, but mainly from the hydraulic perspective, namely, that the measured slope should be representative of the conditions in the reach.

For all these reasons, measuring at two cross-sections – although principally desirable: "the more data, the better" – is not only inconvenient, but may not be feasible.

2 The implications of the Jones Formula for flood routing and discharge estimation

First, there seems to be an oversight in the sign of Eq. (4) of DMT for the celerity $c$of the kinematic wave (KW), which should be positive: $c = \mathrm{d}Q/\mathrm{d}A|_{x=\mathrm{const.}} = B^{-1}\mathrm{d}Q/\mathrm{d}y|_{x=\mathrm{const.}}$; Q is the flow rate passing through the cross-section of a channel of area A, with top width B at depth y, at location x along the stream axis. DMT imply that the Jones Formula is evaluated explicitly. This is feasible, if the KW celerity is computed based on the rating relationship for uniform flow, $c(Q_o) = \mathrm{d}Q_o/\mathrm{d}A|_{x=\mathrm{const.}}$, and such results are good as long as the flow conditions are quasi-kinematic. But when the flow departs markedly from the KW status, the Jones Formula should be evaluated *with* $c(Q)$ *computed on the looped rating curve*. An indication of the correctness of computing the KW celerity on the loop-shaped rating curve is that $c(Q_{\max}) = 0$: the maximum discharge does not propagate downstream! However, care must be exercised in the iterative calculation of c, to ensure convergence (Koussis, 1975; Weinmann, 1977; Weinmann and Laurenson, 1979; Ferrick et al., 1984; Perkins and Koussis, 1996). We demonstrate this point below with an example of Weinmann (1977), also reported by Weinmann and Laurenson (1979).

Weinmann considered a rapidly rising flood wave (rate of rise of the inflow wave ∼1 m/h) in a trapezoidal prismatic channel, with a fairly mild slope $S_o = 2\times10^{-4}$ and Manning's $n = 0.04$; the cross-section has base width $b = 50$ m and side slopes 1 V:1.5 H. The slope ratio (Koussis and Chang, 1982) $\mathrm{SR} = -(\partial y/\partial x)/S_o \approx (\partial y/\partial t)/(cS_o)$ is ∼1, well over the limit of 0.5 suggested in DMT. The inflow hydrograph was of the form of Eq. (36) of DMT, with $Q_b = 100\,\mathrm{m^3/s}$, $Q_p = 1000\,\mathrm{m^3/s}$, $t_p = 10$ h and $\gamma = 6.67$, and was routed for 40 km using the St. Venant equations and a diffusive-wave equivalent model developed by Koussis (1975). The latter is a nonlinear KW model corrected for wave diffusion (by matching the routing scheme's numerical to the physical diffusion coefficient); however, in contrast to Muskingum-Cunge and storage-type models, (i) stage-discharge conversions are based on the flow rating formula of Jones, and (ii) the KW celerity $c(Q)$ is computed via the Jones Formula (steady flow rating curves may be used if the flow is quasi-kinematic). The Jones Formula is readily modified to a form that is appropriate when a discharge hydrograph is given, by replacing the term $\partial y/\partial t/(cS_o)$ by $\partial Q/\partial t/(Bc^2 S_o)$. The modified Jones rating relationship is useful when, e.g., the channel inflow is a basin's outflow computed by a watershed model, or a routed flow hydrograph. Figure 1 (top) shows the flood routing results at $x = 40$ km, and (bottom) the rating curves at $x = 20$ km, corresponding to the complete solution, the diffusion-corrected KW solution and steady uniform flow.

Koussis (1975) verified the good accuracy of the KCD model (more appropriately termed, Kinematic wave Corrected for Diffusive effects), *which incorporates the Jones Formula*, by comparing it to a complete routing solution (idealised model of the lower Mississippi) with SR ∼0.25; Bowen et al. (1989) showed the KCD model to be a useful tool for the design of and the simulation of flows in storm drain networks. But the KCD model's performance is remarkable in the case of Weinmann's wave because the slope ratio is ∼1. The wide loop of the rating curve reflects the strongly transient character of the flow even after 20 km (∼2 m-wide at $500\,\mathrm{m^3/s}$); Weinmann (1977) showed that also in this case can the acceleration terms be ignored (his CI = Complete Implicit and ACI = Approximately Complete Implicit models give indistinguishable results). In contrast, when c is computed from steady/uniform-flow rating curves, as DMT apparently did in their tests, the routed outflow hydrograph differs substantially from the reference solution of the St. Venant equations. The Jones Formula would perform very well in the tests of DMT, if $c(Q) = \mathrm{d}\,Q/\mathrm{d}A|_{x=\mathrm{const.}}$ were computed from it iteratively. This assertion rests on the fact that the DMT transients are milder than Weinmann's wave (gauged by the SR values). Therefore, the application range of the Jones Formula should be greater than the DMT tests suggest. Figure 2 shows the discharge variation of the wave

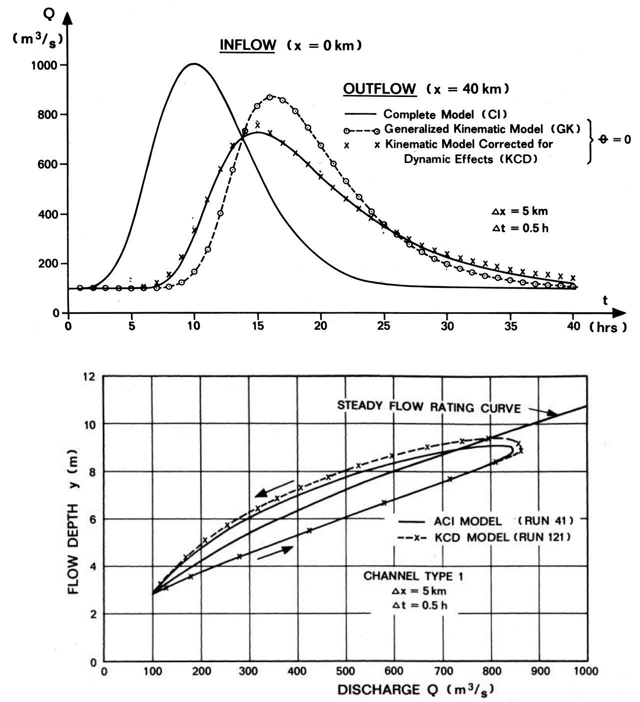

Fig. 1. Comparison of routing solutions, of the St. Venant equations and of two diffusive-wave equivalent models, for a rapidly rising wave through a prismatic channel of trapezoidal cross-section: (top) in- and outflow ($x=40$ km) hydrographs; (bottom) rating curves at $x=20$ km. KCD = Kinematic wave Corrected for Dynamic effects: $y-Q$ conversions and c computed with the Jones Formula; GK = Generalised Kinematic: diffusive-wave equivalent, but $y-Q$ conversions and c computed with the steady-flow rating curve (from Weinmann, 1977).

celerity, computed from the loop-shaped rating curve for transient flow by the KW formula $c(Q) = \mathrm{d}\,Q/\mathrm{d}\,A|_{x=\text{const.}} = B^{-1}\mathrm{d}Q/\mathrm{d}y|_{x=\text{const.}}$. The graph shows that $c(Q)$ has two branches, one for the rising flood and one for the flood recession, so c takes on two different values for the same Q, a higher one on the rising-flood limb and a lower one on the falling-flood limb. It is also worthy of note that: (i) the wave celerity vanishes at the discharge maximum, $c(Q_{\max})=0$, indicating that the peak of wave does not propagate, (ii) that $c \to \infty$ at the depth maximum, $y_{\max}$, indicating quasi-steady flow ($\partial y/\partial t = 0$), and (iii) the Jones Formula ignores the c-region between $y_{\max}$ and $Q_{\max}$. It is thus clear that only for mild transients – in the sense of the slope ratio SR – is the error of computing $c(Q)$ from the steady-flow (kinematic) rating curve small.

In computing $c(Q)$ by the Jones Formula, it should be realised that the rising and falling limbs of $Q(y)$ intersect at ($Q_{\max}$, $y_{\max}$) and that $c = B^{-1}\ \mathrm{d}Q/\mathrm{d}y|_{x=\text{const.}}$ is *numerically discontinuous* there. For this reason the iteration of the looped rating curve near ($Q_{\max}$, $y_{\max}$) must be executed with care. At ($Q_{\max}$, $y_{\max}$), the KW theory gives the finite celerity

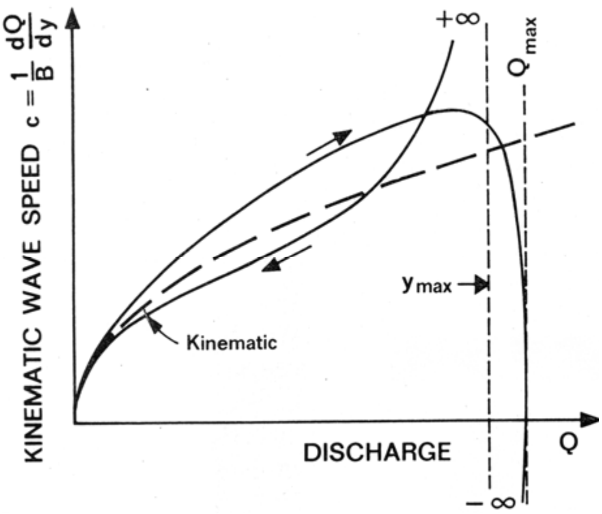

Fig. 2. Wave celerity from looped rating curve (Weinmann, 1977, adapted from Koussis, 1975).

$c(Q_{\max}) = B^{-1}\mathrm{d}(Q_{\text{steady}}/\mathrm{d}y)|_{\max}$, indicating that the maximum flow propagates downstream. In a proper routing procedure, however, the presence of wave diffusion guarantees peak attenuation. This wave diffusion originates in the free-surface slope $\partial y/\partial x$ and appears either directly, as in the actual diffusion wave model, or indirectly, as in flood routing schemes of the matched artificial diffusion type or diffusion-wave- equivalent type, such as, e.g., the Muskingum-Cunge routing scheme (Koussis, 2009).

3 On the correction/extension of the Jones Formula

Henderson (1963, 1966) wanted to improve the not strictly correct basis of the Jones Formula, yet maintain that formula's basic practical format (the shortcoming of the Jones Formula derive from using the KW approximation in converting the spatial to the temporal depth derivative, thus ignoring attenuation of the wave). Displaying a magnificent understanding of flood hydraulics, Henderson corrected the rating formula of Jones for wave subsidence (wave crest region) by adding the *fixed* term $2r^{-2}/3$, i.e., $Q/Q_o = [1+(1/cS_o)\partial y/\partial t + 2r^{-2}/3]^{1/2}$, where r the ratio of the bed slope to the "wave slope" $S_w = 2y_{\text{crest}}/L$, with L the wave length, $r = S_o/S_w = S_o/(2y_{\text{crest}}/L)$. A judicious estimation of S_w is neither difficult (e.g., L can be estimated as $c \times$(time of rise)) nor has strong implications, since typically $r>10$ and thus $2r^{-2}/3$ is small. Indeed, Henderson had already derived the simplified formula of Fenton and Keller (2001), with the second temporal derivative of stage (Eqs. 9–57, p. 379), also including approximations for the inertial terms at Froude numbers less than ~0.7 (Eqs. 9–64 and 9–65, p. 381), but insisted on applying the *fixed-term* correction *only to the crest region* (see Eqs. 9–92 and 9–94, p. 393). Note also that

Henderson was careful not to adopt generally the form with the second derivative, despite considering prismatic channels. Given that the routing scheme ensures wave attenuation, it is argued here that attempting to correct the Jones Formula, by introducing higher-order derivatives (e.g., formulae of Fenton and Perumal 2) while incurring numerical oscillations, does not seem advisable, especially when considering the morphologic variability of natural streams. In contrast, incorporating in the Jones Formula Henderson's (1963, 1966) *fixed* correction $2r^{-2}/3$ improves the estimate of *the flood peak* without oscillations.

4 On the prospects of successful extrapolation of the rating curve in real streams

It appears reasonable that simply structured models, such as the properly evaluated Jones Formula amended with Henderson's fixed-term correction for the wave crest region, should be more adept in handling the complications of real streams, which often test the limits of one-dimensional hydraulics, especially flood plains yielding flow rating curves with distinct branches for in-bank and out-of-bank flows (Price, 1973; Natural Environment Research Council, 1975; Wong and Laurenson, 1983). I contend that no procedure based on the 1-D hydraulic equations, no matter how mathematically elaborate, can eliminate a judgment-based "required extrapolation of the rating curve beyond the range of actual measurements used for its derivation", as DMT state in their Abstract; and DMT's claim (Conclusions) for the DyRaC approach that "its calibration procedure only requires the evaluation of roughness coefficient, thus eliminating the extrapolation errors" seems overly optimistic. My reservations stem from the realisation that the variable channel morphology makes transient flow in natural streams a very complex phenomenon.

The stream morphology encompasses the macro-geometry (cross-sectional geometry, bed slope and thalweg tortuosity) and the micro-geometry, i.e., the channel roughness. DMT base their confidence on the roughness coefficient being more or less constant at high-flow regimes, yet claim that "the DyRaC approach allows for an accurate calibration even when using stage-discharge measurements taken at low and medium flow conditions." As long as the flow stays inside the banks of the stream, the 1-D hydraulic equations are a good basis for its description, provided enough data are available to calibrate roughness; "enough" is the operative word, because wall roughness (i.e., bank material and vegetation) varies with depth (in fact, also temporal variations of the bed and of the bank conditions (erosion and deposition) also occur in natural streams). Therefore, calibrating a 1-D transient open-channel flow model with data up to, say, one half the bank-full depth does not guarantee its good performance at bank-full flow. A case in point is the example mentioned by Anonymous Referee #2 (2010) "Worse results were obtained in the Tiber river, where the location of the peak water depth was subject to large uncertainty and major roughness heterogeneity occurs." The situation gets more difficult still, when the water spills out of the banks and flows over to the flood plain, as under such conditions the 1-D model reaches, or exceeds its limits. For all these reasons I find DMT's expectation of eliminating the extrapolation errors overly optimistic.

5 The calculation of flood level profiles by a standard-step method for quasi-steady flow and its relation to the DTM method

A recent review of storage routing methods (Koussis, 2009) lists among the methods for the computation of depth profiles, after the flow routing step, the option "to calculate, over a Δt, quasi-steady flood level profiles by standard-step methods (Henderson, 1966), with the mean of in- and outflow over Δt as discharge, starting at a section with known conditions (BGS, 2000)". It thus follows that the novel idea of DMT of using stage observations at two cross-sections to estimate flood flows and the BGS procedure of flood depth estimation from known flows are inversions of each other; indeed, both methods treat the flow as quasi-steady over the time interval for which the calculations are carried out. This standard procedure of BGS (Darmstadt, Germany) was adopted in the modelling of flood flows in the heavily modified Kiphissos (Kephisos) River, in Athens, Attica Region, Greece (Koussis et al., 2003; Mazi and Koussis, 2006).

6 Summary

This Comment aspired to put the DMT methodology in perspective, mainly from the praxis point of view, and also to offer a theoretical-computational alternative. The praxis viewpoint emphasised the variable channel morphology, which makes transient flow in real streams a very complex phenomenon, and draws attention to potential difficulties in applying the DMT method in natural streams: the use of appropriately located gauging stations was discussed and reservations were explained about extrapolating a rating curve derived from measurements in the low to medium flow range even to out-of-bank flows, given the roughness heterogeneity of real streams. This Comment also contributed to the problem's computing aspects: (i) it pointed out the ability of the Jones Formula, with Henderson's correction, to infer transient flow in open channels with good accuracy from at-a-section stage observations, when the KW celerity is computed on the looped rating curve; and (ii) it showed that the DMT concept is an inversion of an earlier method.

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