VIBRATIONS OF THE BICARDANIC TRANSMISSIONS WITH ELASTIC SUPPORTS

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Abstract: *In order to study the lateral vibrations of the policardanic transmissions, they assimilate with the elastic systems of bars articulated between them and suspended by elastic cantilivers, the bars having in their component different sections.In this paper some bases of computation for own pulsations are elaborated and numerical calculations are performed for this purpose.*

Keywords: cardan, peak revolutions.

1. FIELD MATRIX.STATUS VECTOR

The partial differential equation of the opened transverse vibration of an average fiber bar (see Figure 1) is [1], [6], [10] :

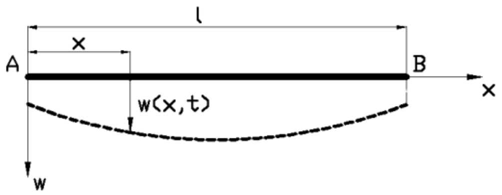

Fig.1.

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where: w - is the deflection;

A- normal section area;
ρ - the density;
E - longitudinal elasticity moment;
I- is the geometrical inertial moment of a bar in regard to the normal main central axis on the AXY plane.

A particular solution of the equation (1) is [1], [6], [10] :

$$w(x,t) = f(x)\cos(pt - \varphi) \tag{2}$$

where *f(x)* is the harmonic vibrations amplitude, and this function fulfils the conditions:

$$\frac{df}{dx} = \theta(x)\ ;\ \frac{d^2 f}{dx^2} = -\frac{M(x)}{EI}\quad ;\ \frac{d^3 f}{dx^3} = -\frac{F(x)}{EI} \tag{3}$$

θ, M, F being the amplitudes, respectively of the section rotation, of the bending moment and cutting force.

One considers the notations:

- $f_A = F(0)$; $\theta_A = \theta(0)$; $M_A = M(0)$; $F_A = F(0)$; (4)

$f_B = F(l)$; $\theta_B = \theta(l)$; $M_B = M(l)$; $F_B = F(l)$

for the amplitudes of the displacements from the end sections A and B;

- $[\Delta]$; $[\Delta_A]$; $[\Delta_B]$ for the status vectors, defined by the relations:

$$[\Delta]=[f,\theta,M,F]^T \ ; \ [\Delta_A]=[f_A,\theta_A,M_A,F_A]^T \ ; \ [\Delta_B]=[f_B,\theta_B,M_B,F_B]^T \tag{5}$$

- $f_i(z)$, i=1,2,3,4 for the Krâlov functions, defined by the relations:

$$f_1(z)=\frac{ch(z)+\cos(z)}{2} \ ; \ f_2(z)=\frac{sh(z)+\sin(z)}{2} \ ; \tag{6}$$

$$f_3(z)=\frac{ch(z)-\cos(z)}{2} \ ; \ f_4(z)=\frac{sh(z)-\sin(z)}{2}$$

$$\text{where : } ch(z)=\frac{e^z+e^{-z}}{2} \ ; \ sh(z)=\frac{e^z-e^{-z}}{2} \ ;$$

-$[F(z)]$– for the Krâlov matrix defined by the relation :

$$[F(z)]=\begin{bmatrix} f_1(z) & f_2(z) & f_3(z) & f_4(z) \\ f_4(z) & f_1(z) & f_2(z) & f_3(z) \\ f_3(z) & f_4(z) & f_1(z) & f_2(z) \\ f_4(z) & f_3(z) & f_2(z) & f_1(z) \end{bmatrix} \tag{7}$$

- α – for the parameter defined by the relation:

$$\alpha=\sqrt[4]{p^2\frac{\rho A}{EI}} \tag{8}$$

- $[\alpha]$, $[\alpha]^{-1}$– for the diagonal matrix:

$$[\alpha]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\frac{1}{\alpha^2 EI} & 0 \\ 0 & 0 & 0 & -\frac{1}{\alpha^3 EI} \end{bmatrix} \ ; \ [\alpha]^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & -\alpha^2 EI & 0 \\ 0 & 0 & 0 & -\alpha^3 EI \end{bmatrix} \tag{9}$$

- $[R]$- the field matrix defined by the relation:

$$[R]=[\alpha]^{-1}[F(\alpha x)][\alpha] \tag{10}$$

where was noted with: $z=\alpha x$

With the relations [1], [6], [10] are obtained the relations between the status vectors:

$$[\Delta]=[\alpha]^{-1}[F(\alpha x)][\alpha][\Delta_A] \quad ; \quad [\Delta_B]=[R][\Delta_A] \tag{11}$$

In the case where the bar is made out of segments with different sections (see Figure 2), then the field matrices of the segments are given by the relations:

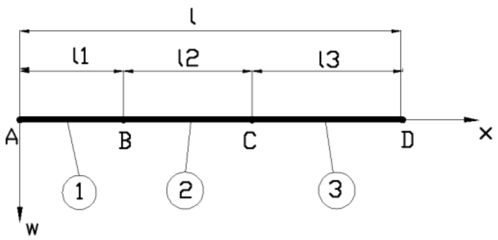

Fig.2.

$[R_i] = [\alpha_i]^{-1}[F(\alpha_i x_i)][\alpha_i]$, in the presented case, i=1,2,3 (12)

and between the status vectors are the relations:

$$[\Delta_B] = [R_1][\Delta_A]\ ;\ \ [\Delta_C] = [R_2][\Delta_B]\ ;\ [\Delta_D] = [R_3][\Delta_C] \tag{13}$$

from which results:

$$[\Delta_D] = [R_3][R_2][R_1][\Delta_A] \tag{14}$$

and the field matrix for the entire bar is:

$$[R] = [R_3][R_2][R_1] \tag{15}$$

2. THE DISTRIBUTED MASS MODEL OF THE OF THE DOUBLE DRIVE SHAFT TRANSMISSION

One associates the equivalent model from to the constructive model of the double drive shaft transmission (see Figure 3.) and in the sections A and D are placed the elastic bearers that are connected to the mounting frame, that have the elastically constants:

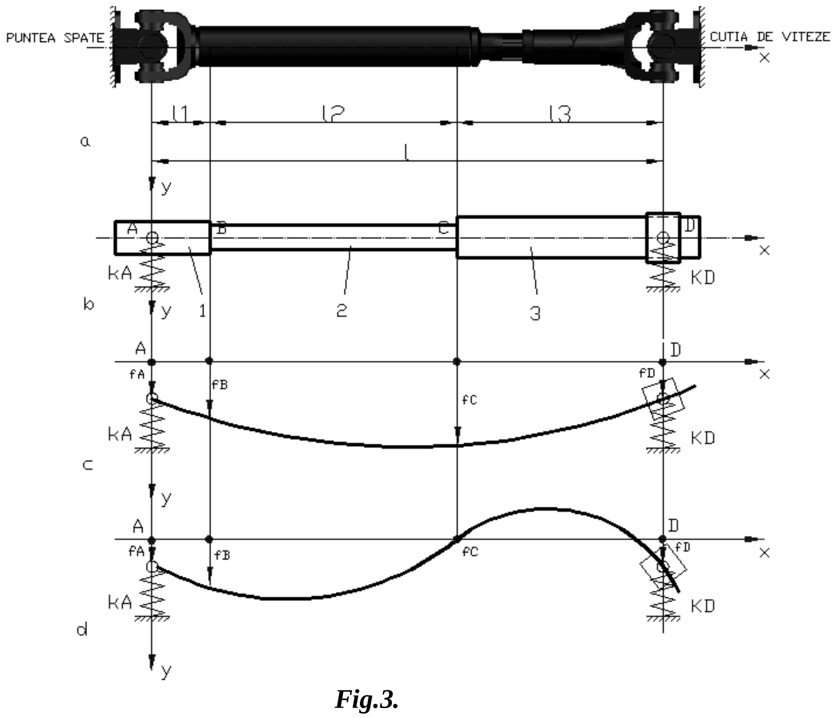

Fig.3.

Assimilating the bonds from A and D with the articulations, one will obtain:

$$M_A = M_D = 0 \quad (16)$$

The cutting forces from A and D are given by the relations:

$$F_A = k_A f_A \ ; \ F_D = -k_D f_D \quad (17)$$

Next are obtained the status vectors:

$$[\Delta_A] = [F_A / k_A, \theta_A, 0, F_A]^T \quad ; [\Delta_D] = [f_D, \theta_D, 0, -k_D f_D]^T \quad (18)$$

If one uses the notations:

$$[T_A] = \begin{bmatrix} 0 & \frac{1}{k_A} \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \ ; \ [R^*] = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ R_{31} & R_{32} \\ R_{41} & R_{42} \end{bmatrix} = [R_3][R_2][R_1][T_A] \quad (19)$$

then the status vectors from points A and D can be written under:

$$[\Delta_A] = [T_A][\theta_A, F_A]^T \quad (20)$$

and the equality :

$$[\Delta_D] = [R_3][R_2][R_1][\Delta_A] \quad (21)$$

becomes:

$$\begin{bmatrix} f_D \\ \theta_D \\ 0 \\ -k_D f_D \end{bmatrix} = [R^*] \begin{bmatrix} \theta_A \\ F_A \end{bmatrix} \quad (22)$$

and from here is obtained the homogenous equation system in θ_A , F_A , f_D :

$$f_D = R_{11}\theta_A + R_{12}F_A \ ; \ 0 = R_{31}\theta_A + R_{32}F_A \ ; \ -k_D f_D = R_{41}\theta_A + R_{42}F_A \quad (23)$$

In order that the system (23) to have a solution different then zero is needed that the determinant to be equal with zero.

$$\Psi(p) = 0 \quad (24)$$

where :

$$\Psi(p) = \begin{bmatrix} R_{11} & R_{12} & 1 \\ R_{31} & R_{32} & 0 \\ R_{41} & R_{42} & -k_D \end{bmatrix} \quad (25)$$

3. THE REPRESENTATION OF THE SPECIFIC WAYS OF VIBRATION

In the graphical representation of a vibration mode there are calculated the amplitudes *f* in different sections of the defections for the pulse *p* that corresponds to this mode of vibration. So it is assigned to the deflection f_A the numerical value equal with the unity and then from the system () results:

$$F_A = k_A \ ; \ \theta_A = -\frac{R_{32}}{R_{31}} \ ; \ f_D = k_A(R_{12} - R_{11}\frac{R_{32}}{R_{31}}) \quad (26)$$

The deflections from sections B and C are the first elements of the column matrices $[\Delta_B]$, $[\Delta_C]$, that are obtained from the relations:

$$[\Delta_B]=[R_1][T_A]\begin{bmatrix}\theta_A\\ F_A\end{bmatrix}\ ;\quad [\Delta_C]=[R_2][R_1][T_A]\begin{bmatrix}\theta_A\\ F_A\end{bmatrix} \qquad (27)$$

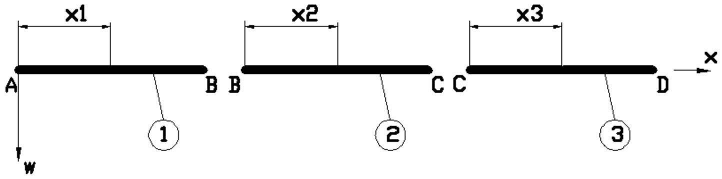

Fig.4.

The deflections from the intermediary points (see Figure 4.) are the first elements of the column matrices $[\Delta_1]$, $[\Delta_2]$, $[\Delta_3]$and are calculated with the relations:

$$[\Delta_1]=[\alpha_1]^{-1}[F(\alpha_1 x_1)][\alpha_1][T_A][\theta_A, F_A]^T \qquad (28)$$

$$[\Delta_2]=[\alpha_2]^{-1}[F(\alpha_2 x_2)][\alpha_2][\Delta_B] \qquad (29)$$

$$[\Delta_3]=[\alpha_3]^{-1}[F(\alpha_3 x_3)][\alpha_3][\Delta_C] \qquad (30)$$

4. NUMERICAL APPLICATION

One considers the double drive shaft transmission of an off-road vehicle for which: $k_A = 85\cdot 10^6\, N/m$; $k_D = 20\cdot 10^6\, N/m$; $l_1 = 0{,}07m$; $l_2 = 0{,}59m$; $l_3 = 0{,}25m$; $A_1 = 19{,}6\cdot 10^{-4} m^2$; $A_2 = 3{,}6\cdot 10^{-4} m^2$; $A_3 = 7{,}06\cdot 10^{-4} m^2$; $\rho_1 = \rho_2 = \rho_3 = 7800 kg/m^3$

For the first approximation the double drive shaft transmission is replaced with a constant section bar, yielding seats at both ends, made out of three same length parts.

In this case the following values for the characteristic pulses are obtained:

$p_1 = 748{,}52 s^{-1}$; $n_{1crt} = 7151{,}51 rot/\min$

$p_2 = 2519{,}79 s^{-1}$; $n_{2crt} = 24074{,}42 rot/\min$

considering the bar with the complete section with $\phi_{ext} = 50mm$

$p_1 = 1062{,}53 s^{-1}$; $n_{1crt} = 10151{,}56 rot/\min$

$p_2 = 4004{,}91 s^{-1}$; $n_{2crt} = 38263{,}47 rot/\min$

considering the bar with the circular section with $\phi_{ext} = 50mm$ and $\phi_{\text{int}} = 45mm$

For the real double drive shaft transmission with the data presented above, one obtains these values for the characteristic pulses:

$p_1 = 876{,}66 s^{-1}$; $n_{1crt} = 8375{,}78 rot/\min$

$p_2 = 2827{,}37 s^{-1}$; $n_{2crt} = 27013{,}15 rot/\min$

Accordingly to the characteristic pulses the characteristic modes of vibration are obtained as in Figure 3.c,d

5. CONCLUSIONS

1. Calculating the peak revolutions with this method, even if it is more complicated and hard, it is closest to reality. By comparing the value of the first critical calculated revolution with the relation :

$$n = n_{crt} \approx 1{,}21 \cdot 10^7 \frac{\sqrt{D^2 + d^2}}{L^2} \text{[rot/min]} \tag{31}$$

with the value obtained with this method, for the same double drive shaft transmission, a difference of 17% is obtained between the two values.

2. In automotive construction, the most frequent case is the one where the bearer from A is rigid (the cardan shaft with the front axle) and the one from D is elastic (the connection of the cardan with the engine-gearbox ensemble). Considering those two, the calculation program elaborated on this method can determine the influence of the elastic constants on the peak revolutions.

3. In a matter that concerns the characteristic modes of vibration, the amplitude of the medium fiber bar calculated in the characteristic points drops as the rigidity from D increases. For an increase in the elastically constant from $20 \cdot 10^6$ N/m to $85 \cdot 10^6$ N/m , the points D amplitude drops four times for the first characteristic pulsation. The values of the peak revolutions decrease as the rigidity for bearer D decreases.

4. To avoid the disturbing effects of the peak revolutions it is wanted that the first characteristic frequency n_1 to be as lowest as it can be and the second characteristic frequency n_2 to be as highest as it can be, so that the transition to be smooth.

5. The increase of the second peak revolution can be done by using the cardan shafts with high rigidity that are obtained by reducing the length or by increasing the exterior diameter. In the construction of automotive is usually used the first constructive method, through which the long bio-cardan transmissions are replaced with multi-cardan transmissions with short shafts.

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