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Switch actuators in process control: constraint problems and corrections

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The paper deals with switch actuators in process control. The inclusion of a signal modulator required for commanding the actuator in proportional-integral-derivative (PID) controller structures is discussed. In particular, it is analysed how the restrictions of this modulator may worsen the well-known problems of process-input saturation, such as windup or loss of control directionality among others. From this analysis a simple correction methodology is proposed for simultaneously addressing constraints in both modulator and actuator. This methodology is easily implemented from feeding back a modulator signal and takes advantage of the wide knowledge existing in anti-windup techniques. Two examples (SISO and MIMO) are presented to assess the effectiveness of the proposed correction.

Keywords: PID controller; switch actuator; actuator constraint; sliding mode; reach mode

1. Introduction

Proportional-integral-derivative (PID) controllers are widely used in industrial applications, being commonly preferred over controllers obtained from more sophisticated techniques. Their popularity has also encouraged the formulation of a large number of methods for tuning the controller gains (O'Dwyer, 2006). Like other industrial controllers, PIDs must address corrective actions to overcome the constraints of the process and power actuators which cause, among other problems, controller windup, plant windup, loss of control directionality in MIMO systems, loss of decoupling properties, etc. (Åström & Hagglund, 2006; Hippe, 2006).

Depending on the characteristics of the process and the type of control action, actuators can be continuous or switched (relays, electronic devices operating as power switches, etc.). For efficiency reasons, switching actuators are commonly used in processes where the control action is an electrical variable. In these cases, the control action switches between two values at high frequency with respect to the process dynamics in such a way that produces, on average, the same effects that the continuous control action (Garelli, Mantz, & De Battista, 2011). To command this type of actuators is also necessary to modulate the controller output in two levels (Sira-Ramírez & Villeda, 2004). Although the signal modulator can be connected to the output of the continuous controller, it is common to incorporate it into the controller structure which simplifies the implementation (Al-Hosani, Utkin, & Malinin, 2011). In greater or lesser degree, the insertion of the modulator tends to reduce the closed loop performance with respect to the case of continuous actuators. Processes with fast dynamics are more sensitive to this loss of performance (e.g. electrolysis processes for clean hydrogen production, controls of fuel cell converters, energy conversion systems for process excitation, etc.). Although the effects of the restrictions of both actuator and modulator are dependent on each other, they have been treated independently. This is probably due to the fact that in the nonlinearity used in the modulator (relay type) it is not possible to define the saturation error used in most of the methods that address the constrained input problems. From a practical point of view, the problem has been usually addressed with a more conservative controller tuning.

The paper is organized as follows. Section 2 presents how the modulator can be incorporated in the structure of PID controllers and suggests a structure 2DOF/PID to command switching actuators. In Section 3, it is shown how the restrictions of the modulator can degrade the closed loop response with respect to the case of continuous actuator. The following section proposes to use a common framework to solve the problems inherent to the constraints of both the modulator and the actuator, that is, reach mode (RM) and saturation problems, respectively. In this context, well-proven anti-reset-windup (ARW) algorithms can be adapted to improve the RM of the modulator at the same time that the closed loop performance in presence of constrains is increased. In order to clarify the advantages of using a common framework, a simple case of RM correction, based on ARW observer ideas, is considered.

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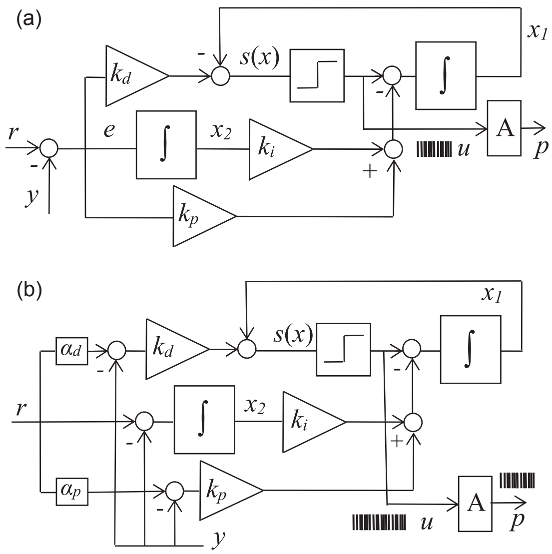

Figure 1. (a) Classical and (b) 2DOF-PID controllers with output to command switch actuators.

The main features of the proposal are validated through two examples. Finally, conclusions are summarized.

2. PID controller with output for switching actuators

Although the problems addressed in this paper can appear with different kinds of controllers and techniques of modulation, for reasons of clarity, the analysis is restricted to PID controllers (for being the most commonly accepted in industrial applications) and to modulation by a sliding mode (SM) regimens (because SM theory allows a more precise and easier analysis than other techniques). Figure 1(a) shows a PID controller structure, implemented via a sliding regime that is suitable to command switching actuators. Indeed, while the values of the output $u(t)$ enable or not the power flow $p(t)$ into the plant, its mean value preserves the PID controller actions. An attractive property of this structure is that it avoids the implementation of a derivator and therefore the problems associated with the measurement noises. The circuit can be modelled by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = f(x) + g(x)u + d = \begin{bmatrix}-k_i x_2\\ 0\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u + \begin{bmatrix}-k_p e\\ e\end{bmatrix}. \quad (1)$$

In a normal operation, the controller states $x^{\mathrm{T}} = [x_1, x_2]$ evolve inside an invariant control surface defined by:

$$s(x) = k_d e - x_1 = 0, \quad (2)$$

that verifies two necessary conditions for achieving the SM regime (its relative degree with respect to the discontinuous signal $u(t)$ is 1, and the Lie derivative $L_g s(x) < 0$), being the sliding range associated to the limit values u^+ and u^- (Sira-Ramírez, 1988). Once the circuit operates in SM, the invariance conditions are satisfied

$$s(x) = k_d e - x_1 = 0$$
$$\dot{s}(x) = k_d \dot{e} - (u_{\mathrm{eq}} - k_i x_2 - k_p e) = 0 \quad (3)$$

being u_{eq} the equivalent control, that is, a fictitious continuous signal that produces the same effect on the process than the actual discontinuous signal $u(t)$. From Equation (3), it can be verified that contains the PID action

$$u_{\mathrm{eq}} = k_p e + k_i \int e\,\mathrm{d}t + k_d \dot{e}. \quad (4)$$

Here a small modification is suggested to the PID structure of Figure 1(a) which, without modifying the SM surface, allows us to obtain the control action of a 2DOF/PID

$$u_{\mathrm{eq}} = k_p(\alpha_p r - y) + k_i \int e\,\mathrm{d}t + k_d \frac{\mathrm{d}(\alpha_d r - y)}{\mathrm{d}t}. \quad (5)$$

The additional parameters (α_p, α_d) permit to meet simultaneous specifications (basically, set-point tracking and perturbation rejecting) and, even, contribute to reduce windup risks (Åström & Hagglund, 2006; Bianchi, Mantz, & Christiansen, 2008; Puleston & Mantz, 1995).

3. Effects of the modulator constraints

As previously mentioned, the discontinuous signal $u(t)$ can be used to command the switching actuator in such a way that the mean value of the process input preserve the PID actions. However, it is important to keep in mind that the equivalent controls (4) and (5) are only defined on the surface $s(x) = 0$, that is, during the SM. Throughout the time that precedes the SM, that is, during the RM, the modulator output only takes one of the limit values u^+ or u^-. From a practical standpoint, the control loop is opened as the process-input loses the information of the PID action. In greater or lesser degree, this deteriorates the behaviour of the system with respect to the case in which the actuator is continuous. Moreover, this fact can induce integrator windup in the PID, or vice versa, the actuator saturation can produce, besides of integrator windup in the PID, an inadequate charging of the integrator of the modulator and as a consequence the extending of the RM. In extreme cases, the interaction of these effects (RM and windup) could cause system instability.

3.1. *Example*

Consider the system

$$P(s) = \frac{1}{(1 + sT_E)sT_I}, \quad (6)$$

being $T_E = 0.01$ y $T_I = 1$, controlled with a 2DOF/PI where $k_p = 50$, $k_i = 25k_p$, $\alpha_p = 0$ and a continuous actuator (Garelli et al., 2011; Hippe, 2006). Assuming that there

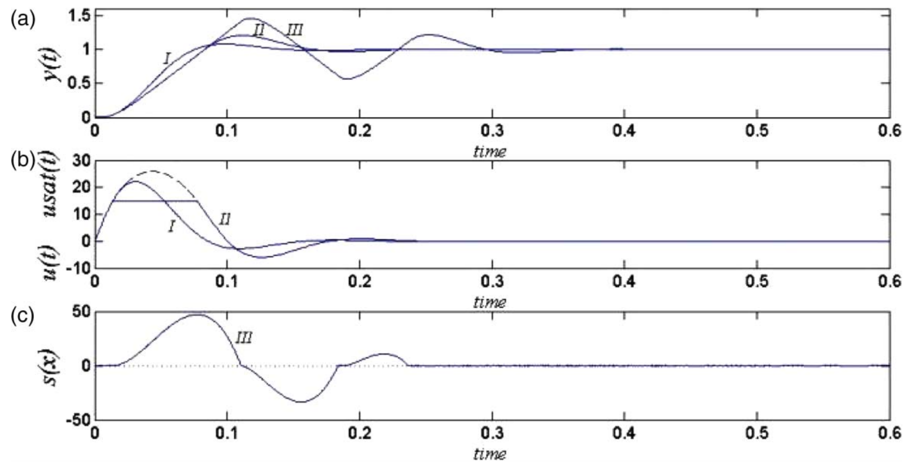

Figure 2. (a) Set-point tracking responses: (I) without constraints, (II) continuous actuator with constrains, (III) switch case with considers both modulator and actuator constraints; (b) control actions; (c) $s(x)$.

are no restrictions, this linear control allows obtaining suitable set-point tracking and disturbance rejection. However, the set-point tracking is affected when the actuator saturates. This fact can be corroborated in Figure 2(a) where the curve I corresponds to the case without restrictions and the curve II to the case where the actuator saturates at ± 15. Figure 2(b) displays the corresponding controller outputs and process inputs.

Consider now that, due to requirements of energy efficiency, the system is controlled using a switch actuator and a 2DOF/PI controller with structure shown in Figure 1(b) and with the same gain values than the continuous 2DOF/PI. The system response is shown in curve III of Figure 2(a), being observed a severe deterioration compared with the two previous cases of continuous actuators. As was mentioned above, this worsening can be attributed to the mutual interaction between two problems: (1) the PI controller windup and (2) the RM preceding the sliding regime during which the PI action is not present in the mean value of $u(t)$.

As can be seen in Figure 2(c), the SM is just reached at $t = 0.237$ (when $s(x) = 0$) which is substantially greater than the time during which the actuator remains saturated in the continuous case (see Figure 2(b)). It is important to note that while the windup and RM are different problems, they manifest in the same way by sending the limit signal at process input, which does not correspond to the ideal PI action.

4. Proposal of a common framework for RM and windup

Conventional methods to reduce the RM are based on the increase of extreme values u^+ and u^-. However, these values are constrained by the DC electrical sources used for implementing the PID circuit. To overcome this shortcoming, and even though that the RM and the actuator

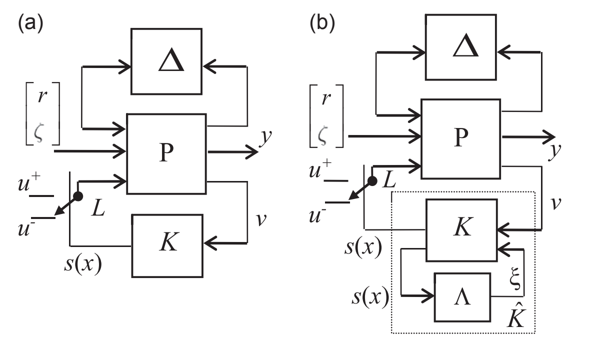

Figure 3. (a) Schematic diagram of generic VSS; (b) RM correction as a problem of windup in a wide sense.

saturation could be addressed independently; it is proposed to use a unique theoretical framework for both problems.

4.1. *Similarities between the RM in variable structure systems and the windup problems. Correction*

The purpose of this section is to show that the effects of RM in variable structure systems (VSS) can be analysed as problems of windup in a wide sense. Figure 3(a) shows a diagram of a generic VSS, where P is the system to be controlled and L represents a switching device which is governed by the output of the block K. Controller K is often conformed by static gains, but in same cases it also has its own dynamics (e.g. it is common practice to expand the states to ensure disturbance rejection at steady state). Then, considering that K is linear time invariant (LTI) system, it can be expressed as $K(s) = C(sI - A)^{-1}B + D$ or schematically:

$$K = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c|c|c} A & B_r & B_y & B_{x_s} \\ \hline C & D & D_y & D_{x_s} \end{array}\right]$$

being its input $v = [r, y, x_s]^{\mathrm{T}}$. Obviously, in the static case, only D must be contemplated. The goal of an SM control is to force the states to evolve on the surface $s(x) = 0$ where the system has the required dynamic properties and the necessary robustness to parameter uncertainties and state disturbances $\zeta(t)$. When this target is met, namely:

$$s(x) = 0 \quad \dot{s}(x) = 0 \tag{7}$$

it is possible to define the equivalent continuous control action $u_{\text{eq}}(x_s, x_k)$ that produces the same effect on the states than the discontinuous signal $u(t)$ which switches ideally with infinite frequency.

It is necessary and sufficient condition for the existence of SM that:

$$u^- \le u_{\text{eq}} \le u^+ \tag{8}$$

being u^- and u^+ the extreme values of the available control action (Sira-Ramírez, 1988).

Since all the benefits of robustness of the VSS reside in the SM, it is important that this operation mode predominates over the RM. To this end, the states should reach the surface $s(x) = 0$ in minimum time and, once it is achieved, verify the condition (8). In this paper, the delay in reaching the surface and in establishing of the SM is attributed to lack of correspondence between the states of K and the P input, which is due to the opening of the loop (if K is static, we can speak of no correspondence between the $s(x) = 0$ and the input of P).

In order to recover the lost correspondence, we propose adding a correction Λ on K for conditioning its states and/or its input (Figure 3(b)). For the new controller to perform as an LTI system is established that:

$$\Lambda : v \to \xi = \begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}^{\mathrm{T}}, \tag{9}$$

being ξ_1 and ξ_2 the correction terms of the states and input of K, respectively; it must be casual, lineal and time invariant. Furthermore, assuming that the correction only has to act during the RM,

$$\text{if} \quad s(x) = 0 \Rightarrow \xi(t) = 0. \tag{10}$$

Note: the previous reasoning has led us to a statement of the RM problem that keeps extreme similarity with the theoretical framework proposed by Kothare, Campo, Morari, and Nett (1994) to address the problem of control with input constraints and on which significant contributions have been made in recent years (Garelli et al., 2011). To overcome the main difference, referred to the discontinuous action of sliding regimes, the switching device can assimilated to an actuator with saturation and gain tending to infinity in the linear region. In this context, the error of saturation commonly used in anti-windup algorithms can be modified to:

$$e = \lim_{k\to\infty} \left(s(x) - \frac{u}{k} \right) = s(x) \tag{11}$$

and applied for conditioning K according to

$$\xi = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} \lim_{k\to\infty} \left(s(x) - \frac{u}{k} \right) = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} s(x), \tag{12}$$

where, to verify Equation (10), the elements Λ_1 and Λ_2 of Λ must be static gains.

Note that in order to select Λ for correcting the RM, we can take advantage of the discussed similarity. Then, proposing small modifications to the well-proven ARW methods it is possible to introduce different algorithms to RM improvement. Next, we evaluate a RM correction based on one among many possible ARW algorithms.

4.2. A RM correction based on the conventional ARW observer method

From the previous analysis and accepting the interpretation that the RM is consequence of the mismatch between K controller output and the plant input, in this section is introduced a RM correction deduced from the observer-based ARW method. For this purpose it is assumed that during the open-loop operation, $\widehat{K}$ acts as a state observer of a fictitious controller that provides the actual input to the plant

$$\dot{\widehat{x}}_K = A\widehat{x}_K + B_r r + B_y y + B_{x_s} x_s + L \left(\lim_{k\to\infty} \frac{u}{k} - s(x) \right), \tag{13}$$

being the controller output

$$s(x) = C\widehat{x}_k + Dv = C\widehat{x}_k + D_r r + D_y y + D_{x_s} x_s. \tag{14}$$

Then, the state equations of $\widehat{K}$, that is, of the controller with RM correction from the conditioning of the state vector x_k, results:

$$\begin{aligned} \dot{\widehat{x}}_K &= (A - LC)\widehat{x}_K + (B_r - LD_r)r + (B_y - LD_y)y \\ &\quad + (B_{x_s} - LD_{x_s})x_s, \\ s(x) &= C\widehat{x}_k + Dv = C\widehat{x}_k + D_r r + D_y y + D_{x_s} x_s. \end{aligned} \tag{15}$$

Note that in the context of the previous analysis, the correction of RM by the method of the observer corresponds to:

$$\xi = \begin{bmatrix} L \\ 0 \end{bmatrix} s(x). \tag{16}$$

For the purposes of assessing the effects of the proposed correction, we can compare the rates of change of $s(x)$ in both the conventional RM and the RM with correction from the observer ideas, Equations (17) and (18) respectively:

$$\dot{s}(x) = C(A\widehat{x}_k + Bv) + D\dot{v}, \tag{17}$$

$$\dot{s}(x) = -CLs(x) + C(A\widehat{x}_k + Bv) + D\dot{v}, \tag{18}$$

where it can be seen that during the RM the proposed correction has introduced a stabilization term $(-CLs(x))$ in the differential equation $s(x)$.

Observation. It is important to note that $s(x)$, beside of containing information about the operation mode of the

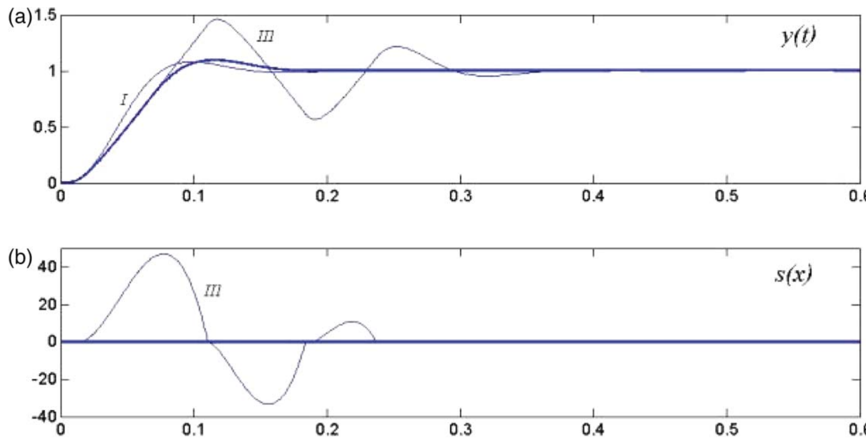

Figure 4. (a) Set-point tracking responses: (I) ideal case, (II) with constraints, and (thick line) with constraints and the propose correction; (b) $s(x)$ without and with correction.

modulator, also has information about the process-input saturation, which happens when the equivalent control exceeds the limit values. Then the signal $s(x)$ can also be employed to correct the state of the controller in order to overcome constraint problems such as windup or loss of control directionality. This is an interesting advantage of addressing in a common way both RM and input saturation. Effectively, the use of $s(x)$ avoids filtering $u(t)$ for calculating the saturation error (most of the ARW algorithms evade dynamics in the correction loop because this fact degrades their performance).

Consider again the example in Section 3.1, where now the correction proportional to $s(x)$ based on concepts of ARW observer method is applied. The simulation has been obtained from adding correction terms $-s(x)$ in the weighted errors of the 2DOF/PI that affects both modulator and PI states with $L = [L_1 \;\; L_2]^{\mathrm{T}} = [k_p \;\; 1]^{\mathrm{T}}$. The effects of the correction are presented in the thick trace curves of Figure 4 and compared with the curves I and III of the Figure 2(a) that correspond to the ideal case (without considering restrictions) and the case that considers both modulator and actuator constraints. The response and the settling time of the controlled variable greatly improve with respect to the non-compensated case. The effectiveness of the proposed correction is also revealed in Figure 4(b), which shows a drastic reduction of the RM time (thick trace). From a practical point of view the RM has been eliminated.

4.3. *Example*

Consider the benchmark MIMO problem (Campo & Morari, 1990; Mantz & De Battista, 2002) with strong coupling between variables

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{10s+1}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \qquad (19)$$

and a PI-MIMO centralized controller that give nominal decoupling under close loop operation

$$\mathrm{PI}(s) = \frac{10s+1}{s}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix},$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{10s+1}{s}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}. \qquad (20)$$

Figures 5–8 show the closed loop MIMO tracking responses when set points $r_1 = 0.6$ and $r_2 = 0.8$ are applied, considering different cases. Simulations in Figure 5 consider continuous and ideal actuators. Curves in Figure 6 correspond to actuators with saturation at $u_i^+ = +40$ and $u_i^- = -40$. As discussed by Campo and Morari (1990) the deterioration in the system response is not due to a problem of windup but to the loss of control directionality when actuators saturate. Figure 7 shows the response of the same system but considering a PI controller whose output is modulated to command a switching actuator. A deterioration in the responses (which is attributable to the RM of the modulator and to its interaction with the pre-existing problem of control directionality) is clearly observed. Part b of the figure displays the signals $s_1(x)$ and $s_2(x)$ which contain information on the status of the actuator and the operating mode of the modulator (i.e. if the modulator has reached the sliding regime where it can be ensured that the control contains, in average, the actions PI/MIMO (20)). In the box at the top right of this figure it can be seen that the period during which the system inputs are saturated is greater than the period corresponding to the case with continuous controller and actuator.

According to the previous discussion, it is now added a correction $-s_1(x) - s_2(x)$ in each error in order to correct simultaneously the constraint problems of both modulator and actuator. In terms of the observer method this correction is equivalent to $L = [L_{1\mathrm{M}} \; L_{1\mathrm{PI}} \; L_{2\mathrm{M}} \; L_{2\mathrm{PI}}]^{\mathrm{T}} = [k_{p1} \;\; 1 \;\; k_{p2} \;\; 1]^{\mathrm{T}}$ being k_{pi} the PI proportional gains and the subscripts $_{\mathrm{M}}$ and $_{\mathrm{PI}}$ corresponding to the state of the modulator

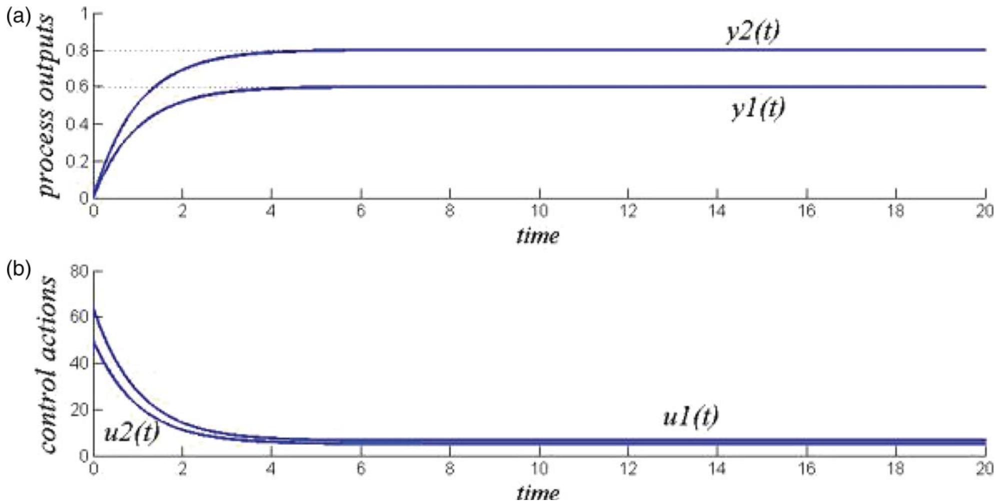

Figure 5. Ideal case without constrains. (a) Set-points tracking responses; (b) unconstrained control actions.

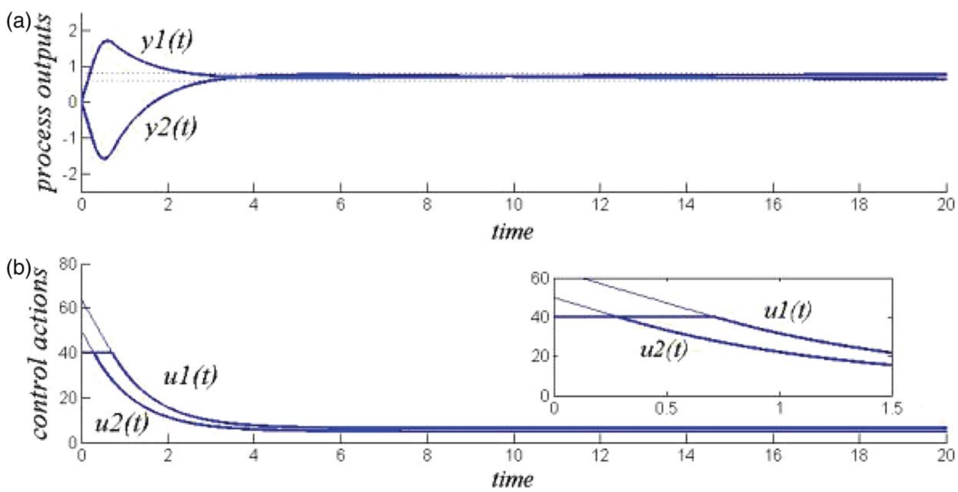

Figure 6. Loss of control directionality due to actuator saturation. (a) Set-points tracking responses; (b) constrained control actions. Box: detail of the actuator saturation.

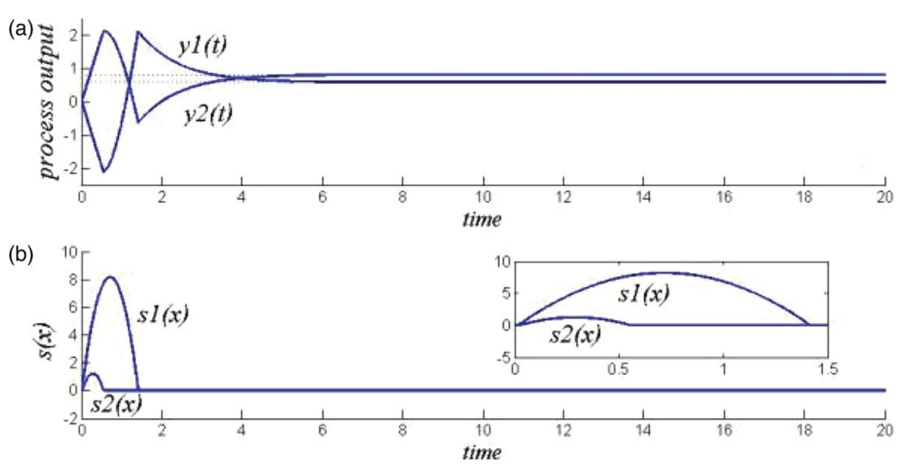

Figure 7. Loss of control directionality with modulator constraints. (a) Set-points tracking responses; (b) $s(x) = [s_1(x)\ s_1(x)]^{\mathrm{T}}$. Box: detail of the period in which $s(x) \neq 0$.

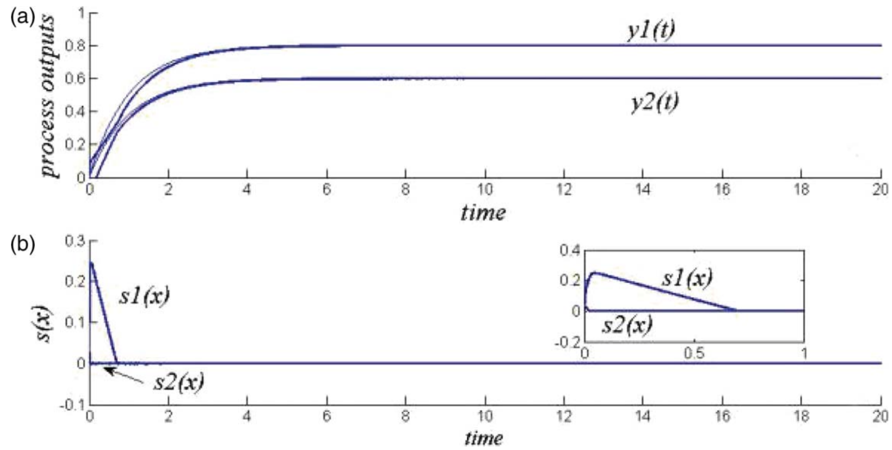

Figure 8. (a) Set-points tracking responses. Thin line: ideal case without constraint. Thick trace: case considering constraints and with the proposed correction. (b) $s(x) = [s_1(x)\, s_1(x)]^{\mathrm{T}}$.

and controller, respectively. The effectiveness of the proposed correction is exhibited in Figure 8. From a practical standpoint the MIMO system outputs remain dynamically decoupled and with a performance that is close to the performance of the ideal unconstrained case. Figure 8(b) shows the drastic reduction in the RM time.

5. Conclusions

The switch actuators, frequently used in industrial applications, require the modulation of the output of the controller. The paper analysed how the insertion of the modulator can reduce the closed loop performance with respect to the case of continuous actuators. Modulator restrictions are manifested through its operation mode known as RM, during which the modulator output does not contain information from the controller output. This can emphasize problems such as windup and loss of control directionality and decoupling among others. Conventional correction methods for constrained systems cannot be directly applied to the modulator because of its type of nonlinearity (relay type) where it is not possible to define a saturation error. Additionally, the measurement of the saturation error in the switching actuators requires to filter the switched control action, fact that introduces dynamics in the correction loop, which loses its effectiveness as a consequence. In this paper these shortcomings are avoided by feeding back a continuous signal of the modulator into the states and/or input of the controller. The paper addresses the constraints of both modulator and switching actuator in a common framework that allows to adapt conventional well-proven ARW techniques to the analysed problem. The effectiveness of the proposal is shown through two examples.

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References

Al-Hosani, K., Utkin, V., & Malinin, A. (2011). Chapter 3: Sliding mode control for industrial controllers. In A. Bartoszewicz (Ed.), *Sliding mode control* (pp. 45–76). Rijeka: InTech.

Åström, K., & Hagglund, T. (2006). *Advanced PID control.* Research Triangle Park: ISA.

Bianchi, F., Mantz, R., & Christiansen, C. (2008). Multivariable PID control with set-point weighting via BMI optimisation. *Automatica*, *44*, 472–478.

Campo, P., & Morari, M. (1990). Robust control of processes subject to saturation nonlinearities. *Computers & Chemical Engineering*, *14*, 343–358.

Garelli, F., Mantz, R. J., & De Battista, H. (2011). *Advanced control for constrained processes and systems. A unified and practical approach* (Chapter 3). London: IET, Control Engineering Series.

Hippe, P. (2006). *Windup in control. Its effects and their prevention* (Chapter 3). Erlangen: Springer.

Kothare, M., Campo, P., Morari, M., & Nett, C. N. (1994). A unified framework for the study of anti-windup designs. *Automatica*, *30*, 1869–1883.

Mantz, R. J., & De Battista, H. (2002). Sliding mode compensation for windup and direction of control problems in two-input-two-output PI controllers. *Industrial and Engineering Chemistry Research*, *41*, 3179–3185.

O'Dwyer, A. (2006). *Handbook of PI and PID controller tuning rules*. London: Imperial College Press.

Puleston, P., & Mantz, R. J. (1995). An anti-windup proportional integral structure for controlling time-delayed multiinput-multioutput processes. *Industrial Engineering Chemistry Research*, *34*, 2993–3000.

Sira-Ramírez, H. (1988). Differential geometric methods in variable structure control. *International Journal of Control*, *48*, 1359–1390.

Sira-Ramírez, H., & Lugo Villeda, L. I. (2004). Sliding modes, delta-modulation and output feedback control of dynamic systems. In A. Sabanovic, L. Fridman, & S. Spurgeon (Eds.), *Variable structure systems from principles to implementation* (Chapter 7, pp. 157–175). London: IET.