PARALLEL INTERFERENCE CANCELLATION IN CDMA BASED ON FUZZY LOGIC USING MATRIX ALGEBRAIC APPROACH

BY

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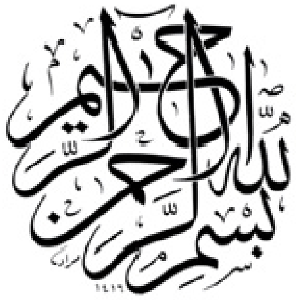

DEDICATED TO

MY PARENTS

AND

MY SIBLINGS

ACKNOWLEDGEMENTS

All praises, all glory and all thanks are due to Allah, The Majestic, The Almighty for bestowing me with knowledge, guidance, patience, courage and health to achieve this work. May peace and blessings be upon prophet Muhammad (PBUH), his family and his companions.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

THESIS ABSTRACT (ENGLISH)

NAME: MOHAMMED ABDUL GHANI

TITLE: PARALLEL INTERFERENCE CANCELLATION IN CDMA BASED ON FUZZY LOGIC USING MATRIX ALGEBRAIC APPROACH

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This thesis extends the previous work on matrix algebraic analysis of parallel interference cancellation. It is shown that the parallel interference cancellation (PIC), whether conventional or weighted, can be seen as a linear matrix filter applied directly to the chip-matched filtered received signal vector. A fuzzy logic system is introduced in the multistage PIC scheme to estimate the interference cancellation weights. The proposed technique is equipped with a set of adaptive weights that are selected through a fuzzy inference system to reduce the poor mutual user interferences estimates in the initial stages that result in reduced performance of the conventional multistage schemes.

We evaluate and compare the proposed scheme's performance with the other known multiuser detectors under AWGN and channel fading. Our objective is to show that, for the proposed scheme, the BER performance approaches that of the decorrelator detector and convergence occurs in fewer stages as compared to other multistage Parallel Interference cancellation schemes.

MASTER OF SCIENCE DEGREE

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THESIS ABSTRACT (ARABIC)

الاسم: محمد عبدالغنى

العنوان: الموازية إلغاء التداخل في سى دى ام ايه تقوم على منطق ضبابي باستخدام نهج جبري مصفوفة

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هذه الأطروحة توسع العمل السابق على تحليل جبري مترابط لإلغاء التقاطع المتوازى .ويظهر أن إلغاء التقاطع المتوازى (PIC)، ما إذا كان تقليدية أو مزونة ، يمكن رؤية على خط مرشح مترابط يتم تطبيقها مباشرة على الرقائق المرشحة متطابقة لإستقبال قائمة الإشارة .نظام المنطق الضبابي معروف بالمراحل المتعددة PIC وهو مشروع لتقدير إلغاء الأوزان .التقنية المقدمة هي مرتبطة مع مجموعة من الأوزان التكيفية التي يتم تحديدها من خلال نظام الاستدلال الضبابي للحد من تدخلات أداء المستخدم الضعيفة في المراحل الأولى التي تؤدي إلى انخفاض الأداء التقليدي لمشروع المراحل المتعددة.

نحن تقييم ونقارن أداء المخطط المقترح مع أداوات كشف متعددة معروفة تحت AWGN وقناة التضائل الجزئي . هدفنا هو إظهار أن لهذا المخطط المقترح ، أداء *BER* يكشف أن ديكلورليتر يلتقط حدوث التقارب في مراحل أقل بالمقارنة مع غيرها من مشاريع إلغاء التقاطع المتوازي متعددة المراحل.

درجة الماجستير في العلوم

جامعة الملك فهد للبترول و المعادن

الظهران المملكة العربية السعودية

CHAPTER 1

INTRODUCTION

1.1 Motivation and Background

Wireless personal communication provides convenient and flexible method to access information to around 3 billion people worldwide [1]. The tremendous increase in demand for wireless services necessitated a search for techniques that improve the capacity of digital cellular systems. Wireless communication has become one of the major areas of research in the world. Efficient sharing of bandwidth among the users becomes of outmost concern as available frequency resources diminish quickly.

Multiple access schemes have been suggested to share the available bandwidth and thereby improve the capacity. These can be put under three main categories:

- Time Division Multiple Access (TDMA): In this scheme each user in the system is assigned dedicated time slots during which they transmit their information using the channel bandwidth entirely.
- Frequency Division Multiple Access (FDMA): The total bandwidth in this scheme is divided into channels and each user is assigned a particular frequency channel for the duration of communication.

- Code Division Multiple Access (CDMA): in this scheme, the total bandwidth is available to each user at the same time. Users are separated by means of unique signature codes or waveforms assigned to each user.

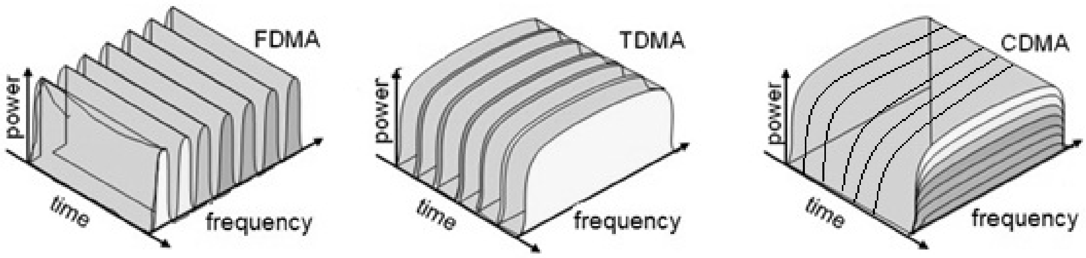

Figure 1-1 Multiple Access Schemes

Figure 1.1 illustrates the sharing of the time - bandwidth space in different multiple access schemes. Among the three multiple access methods, CDMA has become the most popular and finds its implementation in 3rd generation wireless cellular communications systems because it offers higher capacity, higher data rates and better interference rejection than FDMA and TDMA [3, 4]. DS–CDMA (Direct Sequence-CDMA) with BPSK modulation is also implemented in wireless local area networks (WLANs) according to the IEEE 802.11b standard, low-rate wireless personal area networks (WPANs) according to the IEEE 802.15.4 standard and future sensor networks [5].

DS-CDMA involves assigning users unique signature codes that are orthogonal to each other. At the transmitter, user data is spread by a signature code uniquely assigned to the user. The simplest receiver is a matched filter that detects the data by despreading the received signal with the corresponding signature code of users. However, multiple access interference (MAI) and the near-far problem degrade the performance in DS-CDMA system and have been researched extensively [2, 3, 4]. The problem of MAI occurs due to the presence of channel induced de-cross-correlation between signature codes. In addition, the number of perfectly orthogonal codes that can be generated for a given bandwidth is limited. In short some correlation (even if small) does exist. Near-far problem [3] occurs due to the presence of strong signals from users located closer to the receiver that tends to swamp the desired weaker signals from the distant users. Therefore, the matched filter bank receiver in the presence of near-far problem and multiple access interference results in significant degradation and capacity reduction.

Substantial improvement in performance can be achieved by exploiting the structure of signature waveforms to lessen MAI in the matched filter receiver. Multiuser Detection (MUD) makes joint detection of all users' signals by not treating MAI as noise. MUD also is near far resistant. The optimal multiuser detector proposed by Verdu [2] finds the maximum likelihood sequence (MLS) matching the transmitted user's signal. For a K-user N-bit communication system, it requires 2^{NK} times exhaustive searches to find the MLS thus making it impractical as complexity increases exponentially with number of users. Several suboptimal multiuser detection schemes have been proposed in literature to reduce the complexity from that of the optimum maximum likelihood detector [6, 7, 8].

Suboptimal MUDs can be classified as linear and nonlinear. The linear MUDs obtain the output decision statistics by linearly transposing the soft outputs of matched filter detector. In this class, we find the decorrelator detector [9] and the Minimum Mean Square Error (MMSE) detector [10, 11, 12]. Both compute the inverse of the cross correlation matrix of the signature codes. On the other hand, the non-linear MUDs, or interference cancellation (IC) based MUDs estimate the MAI and then remove it from the desired user prior to making decisions. Interference cancellation (IC) schemes can be roughly categorized into three types: serial, parallel and hybrid interference cancellation schemes [14, 15, 16, 17]. The former two are predominantly researched for practical implementation. The main disadvantage of the serial IC proposed by Viterbi [13] is the delay experienced by the users of the system. The parallel interference cancellation (PIC) scheme simultaneously removes the interference from each user's received signal. This procedure treats the users in the IC processing in the same way. As compared with the serial IC (SIC) scheme [17], the PIC operates in parallel on all users' signals, and hence the delay required in completing the interference cancellation is at most a few bits.

PIC scheme has been intensively studied since the original work by Varanasi and Aazhang [6]. For high system load, the conventional PIC approach that attempts to completely cancel the multiuser interference may not be preferred because the erroneous hard decisions made in the previous stage may lead to performance worse than without cancellation. Hence, when the interference estimate is poor as it happens at the earlier stages of PIC, the partial cancellation method proposed by Divsalar et al. [7] performs better than the complete interference cancelling PIC. The partial PIC (PPIC) scheme is implemented simply by assigning cancellation weight to each interference cancellation path. The constant weighted PPIC (W-PPIC) scheme is more suited to perfect power control environments. The weakness of PPIC scheme is that a constant weight assigned to all users at each stage is selected by trial and error rather than using some optimality criterion. In addition, the optimal weight of each interferer depends on reliability of the estimated interference signal. Adaptive parallel interference cancellation (APIC) detectors are developed to update the interference cancellation weights adaptively to reflect the reliability of the estimated interference [11, 14 and 15]. An algorithm e.g. RLS and LMS controls the weight update. These update schemes have been found to be unreliable in highly loaded systems because of slow convergence rate of LMS algorithm and increased mean square error contributes to degradation in performance. In [15], the optimal weight has been related to the noise power and amplitude of interferers. Thus, cancellation weights of interferers should be adapted to improve the reliability of estimated interference signal which in turn depends on the K_{eff} (amplitude of interferers) and *SNR* (signal to noise plus interference ratio).

This dissertation focuses on realizing improvement in the performance of the existing PIC schemes based on matrix algebraic approach with the use of fuzzy inference system (FIS) that updates the weights. Fuzzy inference system developed by Zadeh is based on the principle of fuzzy logic that uses the linguistic concepts [16]. The FIS has drawn a great deal of attention because of its universal approximation ability in the nonlinear problem. The FIS can be used to determine the weight vector of the estimated interferers signal.

We propose a fuzzy-based partial parallel interference cancellation (FPIC) scheme and discuss its performance for CDMA systems operating over wireless fading channels in the presence of AWGN.

1.2 Research Objectives

In the proposed work multiuser DS-CDMA system using weighted PIC are investigated. First, the performances of different Parallel Interference Cancellation methods are studied.

The main objectives are summarized as follows:

1) To develop a multistage parallel interference cancellation multiuser detector.
2) To employ the fuzzy inference system for determining the partial weight to illustrate fuzzy based PPIC.
3) To study the BER performance of different PIC schemes.
4) To study the convergence of the proposed scheme and compare this to different MUDs.
5) Perform computer simulations to show that the proposed scheme delivers improved performance.

1.3 Organization of the Thesis

This thesis is divided into six chapters including this chapter and mainly deals with the problems associated with parallel interference cancellation.

Chapter 2 introduces synchronous CDMA system and channel model. Several common multiuser detectors are also described.

Chapter 3 describes the parallel interference cancellation detectors. Convergence of the PIC detector is also introduced. The Partial PIC scheme is studied and the relation between the weights in the Partial PIC with effective number of users and *SNR* is explored.

Chapter 4 introduces the Fuzzy Logic system, membership functions, and formulation of rules and how FIS is implemented to tackle the issue of weight estimation in partial PIC scheme.

Chapter 5 describes the proposed multistage fuzzy PIC. The estimation of the partial cancellation weights of the PIC detection scheme is performed by a FIS system. Convergence of the proposed scheme and BER performance is studied. Simulation results show that superior performance over the conventional PIC is achieved by the proposed detection scheme and it outperforms the original scheme in both AWGN and Rayleigh fading channels.

Chapter 6 summarizes the objective obtained in this research work. A brief introduction to some future research directions is also given.

CHAPTER 2

CDMA AND MULTIUSER DETECTION

This chapter provides an overview of the wireless CDMA communication system model and multiuser detection techniques. A synchronous CDMA system model is described in Section 2.1. The detection scheme of the conventional receiver is presented therein. Several typical multiuser detectors are summarized in Section 2.2. Optimum and suboptimum multiuser detectors are classified and discussed.

2.1 System Model

In Direct-Sequence (DS) CDMA systems, the information symbols are phase-reversed modulated by the signature sequences (or spreading codes). The spreading codes used to spread the spectrum of the information data are either short or long codes. In the former case, the period of the signature sequence is equal to the symbol period, whereas in the latter case, more than one symbol occur in one period of the signature code. Both of these codes have their advantages and drawbacks. The complexity is lower for short codes but multiuser interference to each user to each user may be different as some users suffer from higher interference than the others. For the long code case, the complexity will be higher and the interference on each user is more randomized.

2.1.1 Transmitter Operation:

A *K*-user discrete model of a synchronous DS-CDMA communication system is shown in figure 2.1. Binary phase shift keying (BPSK) is used for modulating the user information and pseudo random (PN) codes of length *N* chips are used as signature waveforms.

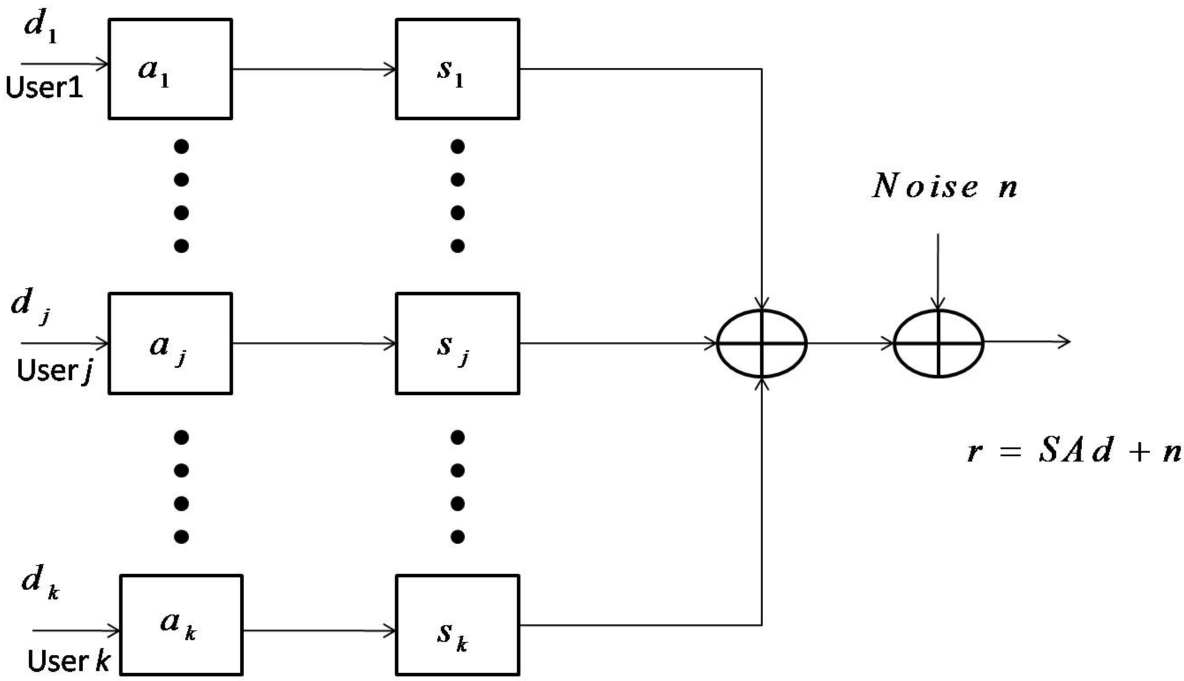

Figure 2-1 CDMA System Model

The received signal $\boldsymbol{r}$ at the base station is expressed in vector form as:

$$\boldsymbol{r} = \boldsymbol{SAd} + \boldsymbol{n} \tag{2.1}$$

where

$$\boldsymbol{S} = [\boldsymbol{s_1}, \boldsymbol{s_2}, \dots \dots, \boldsymbol{s_k}\,] \in \{{}^{-1}/_{N}, {}^{1}/_{N}\}^{N,K}$$

$$\boldsymbol{A} = [a_1, a_2, \dots \dots, a_k\,] \in\ R^{K,K}$$

$\boldsymbol{d} = [d_1, d_2, \dots \dots, d_k\,]^T \in \{-1,1\}^K$

$\boldsymbol{n} = [n_1, n_2, \dots \dots, n_k\,]^T \in R^K$

$\boldsymbol{S}$ is the signature code matrix where $\mathbf{s_k}$ is the signature waveform of user k and $\boldsymbol{s_k}$ is normalized to have unit energy i.e., $\langle \boldsymbol{s_i}, \boldsymbol{s_j} \rangle = 1$, $\boldsymbol{A}$ is diagonal matrix of received amplitudes, $\boldsymbol{d}$ is the vector of binary transmitted symbols and $\boldsymbol{n}$ is vector of independently, identically distributed white Gaussian samples with zero mean and variance $N_0/2$.

The cross correlation of the signature sequences is defined as:

$$\rho_{ij} = \langle \boldsymbol{s_i}, \boldsymbol{s_j} \rangle = \sum_{k=1}^{N} \boldsymbol{s_i}(k)\ \boldsymbol{s_j}(k) \tag{2.2}$$

where N is the length of the signature sequence.

The cross correlation matrix is defined as: $\boldsymbol{R} = \{\rho_{ij}\}$

$$\boldsymbol{R} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots\cdots & \rho_{1K} \\ \rho_{21} & \rho_{22} & \cdots\cdots & \rho_{2K} \\ . & . & . & . \\ . & . & . & . \\ \rho_{K1} & \rho_{K2} & \cdots\cdots & \rho_{KK} \end{bmatrix} \tag{2.3}$$

$\boldsymbol{R}$ is symmetric, non-negative definite matrix.

2.2 Conventional Matched Filter Receiver

For a single user digital communication system, the matched filter is used to generate the sufficient decision statistics. The conventional receiver for a multiuser system is a bank of matched filter each matched to the signature waveforms of the individual users.

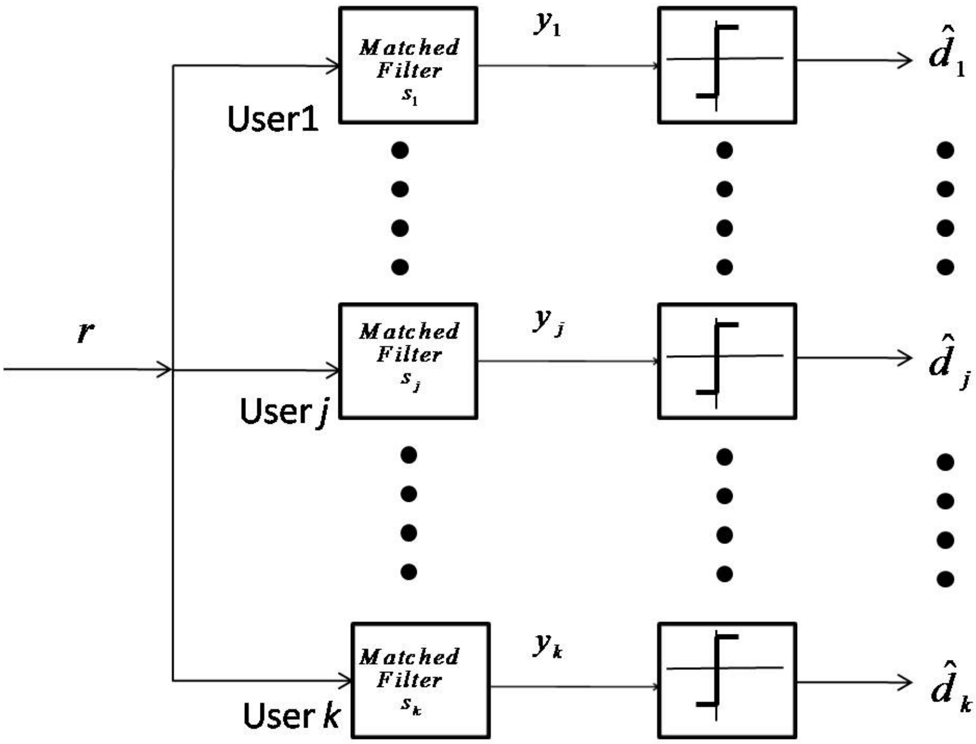

Figure 2-2 Conventional Matched Filter Receiver

Figure 2.2 shows the multiuser matched filter receiver used in CDMA systems. Each receiver filter is matched to the signature sequences assigned to users. Assuming that coherent detection is used in the receiver, the decision statistics at the output of the matched filter is given as:

$$y_k = \int_o^T \boldsymbol{r}(t)\boldsymbol{s_k}(t)dt \tag{2.4}$$

$$y_k = \int_o^T \left\{ \sum_{j=1}^{K} \boldsymbol{s}_j(t) a_j d_j + n(t) \right\} \boldsymbol{s_k}(t)dt$$

$$y_k = \sum_{j=1}^{K} \rho_{jk}\, a_j d_j + w_k \tag{2.5}$$

where $w_k = \int_o^T n(t)\boldsymbol{s_k}(t)dt$

Equation (2.5) is simplified as:

$$y_k = a_k\, d_k \;+\; \sum_{\substack{j=1 \\ j\neq k}}^{K} \rho_{jk}\, a_j d_j \;+\; w_k \tag{2.6}$$

The second term in (2.6) is the MAI that is similar to the term characterized as zero mean Gaussian noise. The power of the noise at the output of matched filter is estimated as:

$$E(w_k^2) = E\left[\int_o^T n(t)\boldsymbol{s_k}(t)dt \int_o^T n(s)\boldsymbol{s_k}(s)ds\right] \tag{2.7}$$

$$= \int_0^T \int_0^T E\,[n(t)n(s)]\;\; \boldsymbol{s_k}(s)\boldsymbol{s_k}(t)\, dt\, ds$$

$$= \int_o^T \int_0^T N_0\, \delta(t-s)\;\; \boldsymbol{s_k}(s)\boldsymbol{s_k}(t)\, dt\, ds$$

$$= \int_o^T N_0\, \boldsymbol{s_k^2}\, (t)dt = N_0$$

Similarly the noise covariance can be defined as: $E(\,n_i n_j\,) = N_0\rho_{ij}$ and in a matrix form it is written as: $E(\boldsymbol{nn^T}\,) = \{N_0\rho_{ij}\}_{ij} = N_0\boldsymbol{R}$ (2.8)

where $\boldsymbol{R}$ is the correlation matrix given by (2.3) and $\boldsymbol{n} = [n_1, n_2, \ldots\ldots, n_k]^T$.

Equation (2.6) is written in matrix form as :

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_K \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots\cdots & \rho_{1K} \\ \rho_{21} & \rho_{22} & \cdots\cdots & \rho_{2K} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \rho_{K1} & \rho_{K2} & \cdots\cdots & \rho_{KK} \end{bmatrix} \begin{bmatrix} a_1 & 0 & \ldots\ldots & 0 \\ 0 & a_2 & \ldots\ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots\ldots & a_K \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ d_K \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ w_K \end{bmatrix} \tag{2.9}$$

The matched filter outputs can be grouped into a K-dimensional vector and express as

$$\boldsymbol{Y} = \boldsymbol{RAd} + \boldsymbol{w} \tag{2.10}$$

where the following notions are used

$\boldsymbol{Y} = [y_1, y_2, \dots \dots, y_k\ ]^T -$ matched filter output vector

$\boldsymbol{R} =$ correlation matrix

$\boldsymbol{A} =$ diagonal matrix of the received amplitudes

$\boldsymbol{d} = [d_1, d_2, \dots \dots, d_k\ ]^T$

$\boldsymbol{w} = [w_1, w_2, \dots \dots, w_k\ ]^T$

Figure 2.3 shows the average bit error rate (BER) performance of the Matched filter receiver for different number of users. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. The BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^4$ bits and using Gold codes of length 31 and having equal received power. It can be observed that the performance degrades with increasing number of users. The poor performance of the matched filter is due to ignoring the MAI term as noise.

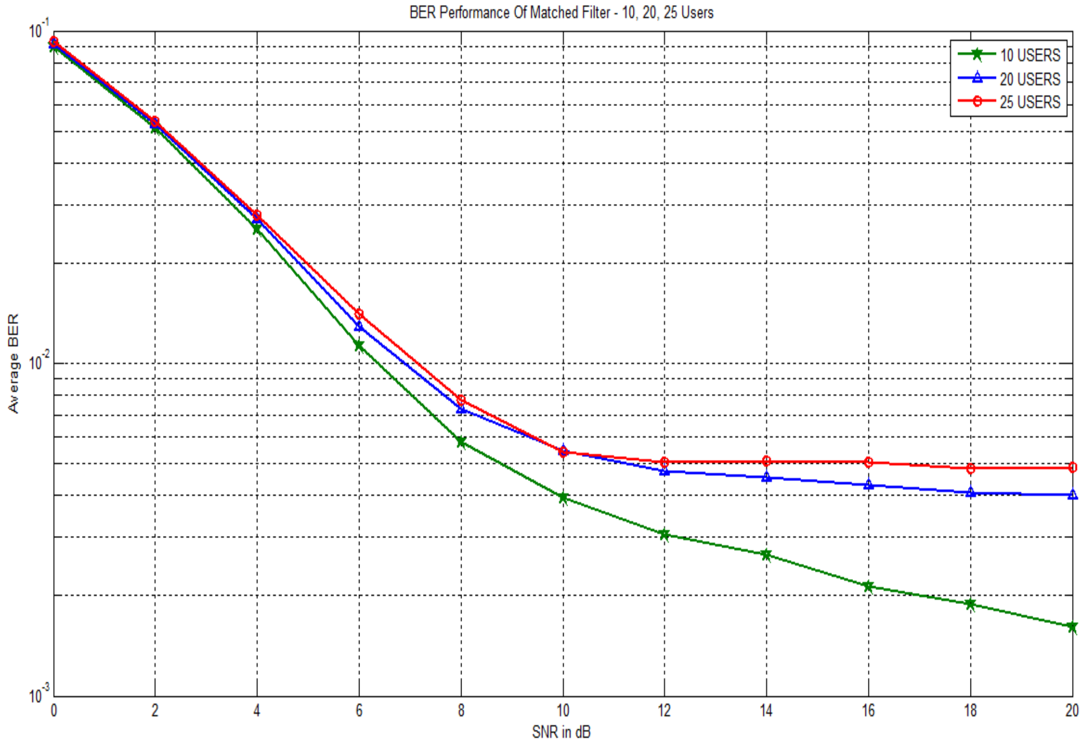

Figure 2-3 BER Performance of Matched Filter

Limitations of conventional detector:

It can be proven that while decision metric y_k is not sufficient for detecting d_k, a set of decision metrics $\{y_1, y_2, \dots \dots, y_k\}$ is sufficient statistics for joint detection of $\{d_1, d_2, \dots \dots, d_k\}$ [2]. The poor performance of matched filter is due to assumption that models MAI as background Gaussian noise. Another serious limitation of the matched filter is its vulnerability near-far effect [2].

2.3 Multiuser Detection

Multiuser detector is a receiver that jointly detects all the user's signals simultaneously. Multiuser detection is also defined as a class of algorithms or methods in a communication receiver that exploits the structure of the multiuser interference in order to increase utilization of the channel resources [2]. Unlike AWGN, MAI has a structure that is quantified by the cross correlation matrix of the signature matrix. MUD is the design of signal processing algorithms that operate in the MUD box show in figure 2.4

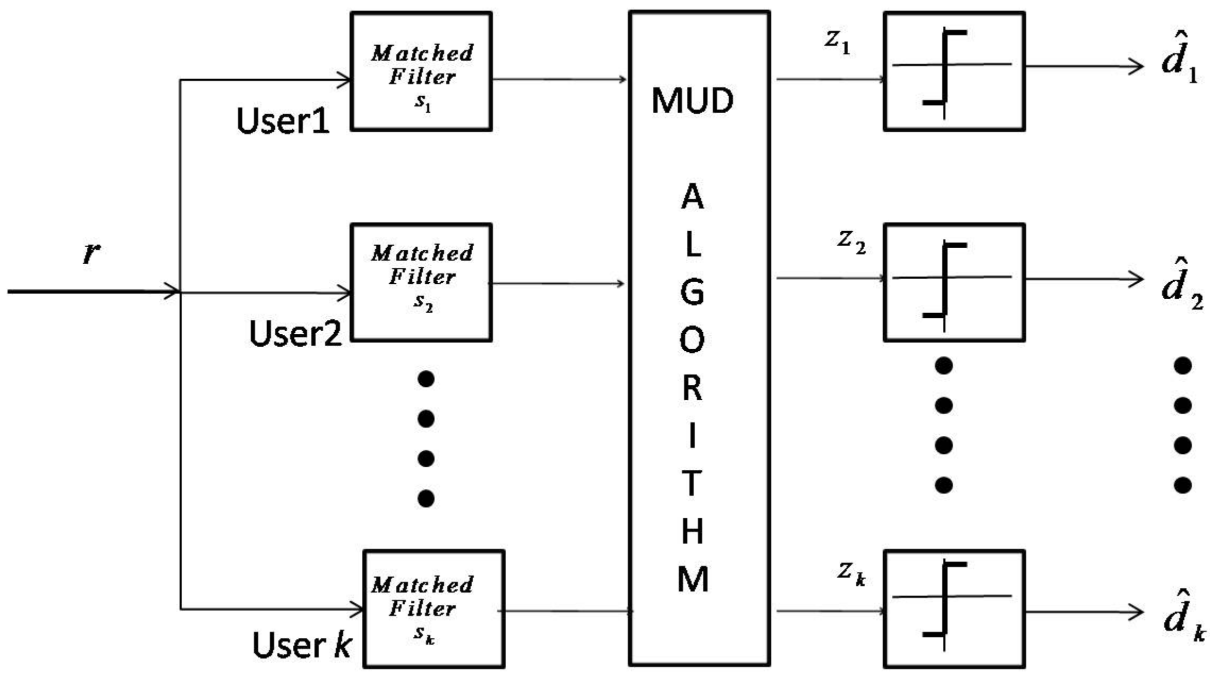

Figure 2-4 General Concept of Multiuser Detection

The matched filter detector when employed as the front-end of multiuser detection scheme sacrifices no information relevant to demodulation [2]. Most MUD's therefore have matched filter as the front end. Decoding decision made on processed signals from multiuser detector generate significantly lowers bit error rates for individual users. The reduction in MAI allows more active users thus boosting the system capacity.

The classification of MUD schemes is illustrated in figure 2.5. MUD's can be optimum and suboptimum. Since the optimum detector complexity is beyond implementation, the suboptimum methods are mostly investigated. Under suboptimum group, there are two main types: linear and interference cancellation-based detectors. The Interference cancellation detectors are broadly classified by the method of MAI removal as successive interference cancellation and parallel interference cancellation detectors. We begin with an overview of the different types of Multiuser detection schemes.

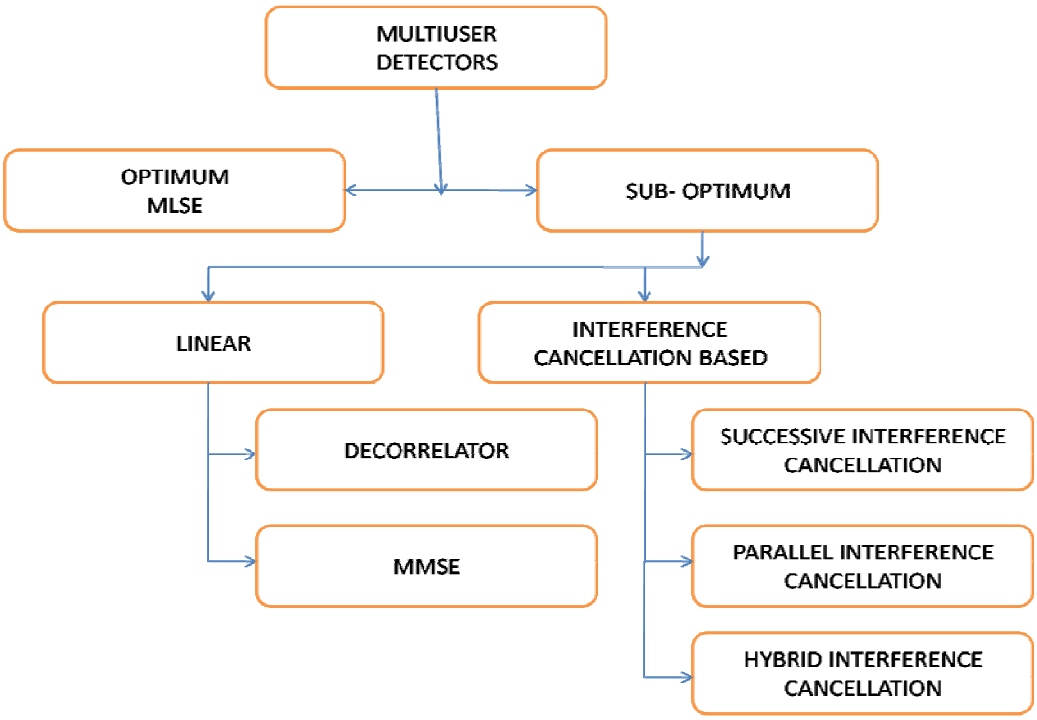

Figure 2-5 Classification of MUD

2.3.1 Optimum Multiuser Detection

In 1986, Sergio Verdu developed an optimum multiuser detector that delivers minimum bit error rate for CDMA communication systems [2]. The optimum multiuser detector is based on the Maximum-Likelihood (ML) detection. The baseband received signal at the output of stage 1 can be written as in (2.1)

In each bit interval, optimal detector selects the most likely bit sequences $\boldsymbol{d} = [d_1, d_2, \dots\dots, d_k\ ]^T$ such that the criterion

$$exp\left(-\frac{1}{2\sigma^2 T}\int_0^T\ [\boldsymbol{y}-\boldsymbol{SAd}]^2\ dt\right) \quad (2.11)$$

is maximum [2].

For this criterion to be maximum, it requires maximization of the function given as:

$$\Omega(\boldsymbol{d}) = 2\boldsymbol{d^T A y} - \boldsymbol{d^T H d} \quad (2.12)$$

where

$\boldsymbol{d} = [d_1, d_2, \dots\dots, d_k\ ]^T$ data vector.

$\boldsymbol{A} =$ diagonal matrix of the received amplitudes.

$\boldsymbol{Y} = [y_1, y_2, \dots\dots, y_k\ ]^{\mathrm{T}}$ matched filter output vector.

$\boldsymbol{H} = \boldsymbol{ARA}$ Unnormalized cross correlation matrix.

For the maximization of (2.12), the solution can be found by exhaustive search, i.e. compute the criterion function for every possible combination of argument and select the one as optimal solution that maximizes the function. The optimum multiuser detector provides the most reliable decision outputs and totally eliminates the effect of MAI. Hence, its performance is superior to all other multiuser detectors. Despite of its great performance, the computational complexity of the optimum detector is subject to an exponential growth in the number of users since it performs a full search using the Viterbi algorithm. Therefore, when the number of active user K is large, this method with the operational complexity of $O(2^K)$ turns out to be too complex to implement in practice. As a result, sub-optimum multiuser detectors with lower complexity, without sacrificing performance are of interest.

2.3.2 Sub-Optimum Multiuser detection

Sub-optimum multiuser detectors can be divided into two major categories: linear detectors and nonlinear detectors.

2.3.2. (a) Linear Multiuser Detection

Linear multiuser detectors apply a linear mapping to the matched filter's outputs to form a more reliable decision metric. Linear multiuser detectors have the advantage of easier implementation and relatively good performance when the interference is low. Some well-known linear detection schemes are Decorrelating Detector (DD) [9, 18] and Minimum Mean Squared Error (MMSE) detector [10, 11, 12].

Decorrelating Detector

The decorrelating detector attempts to completely remove the effect of the multiple access interfering term. As shown in figure 2.6, the decorrelating detector operates by processing the output of matched filter bank with the $\boldsymbol{R}^{-1}$ operator where $\boldsymbol{R}$ is the cross correlation matrix defined by (2.3).

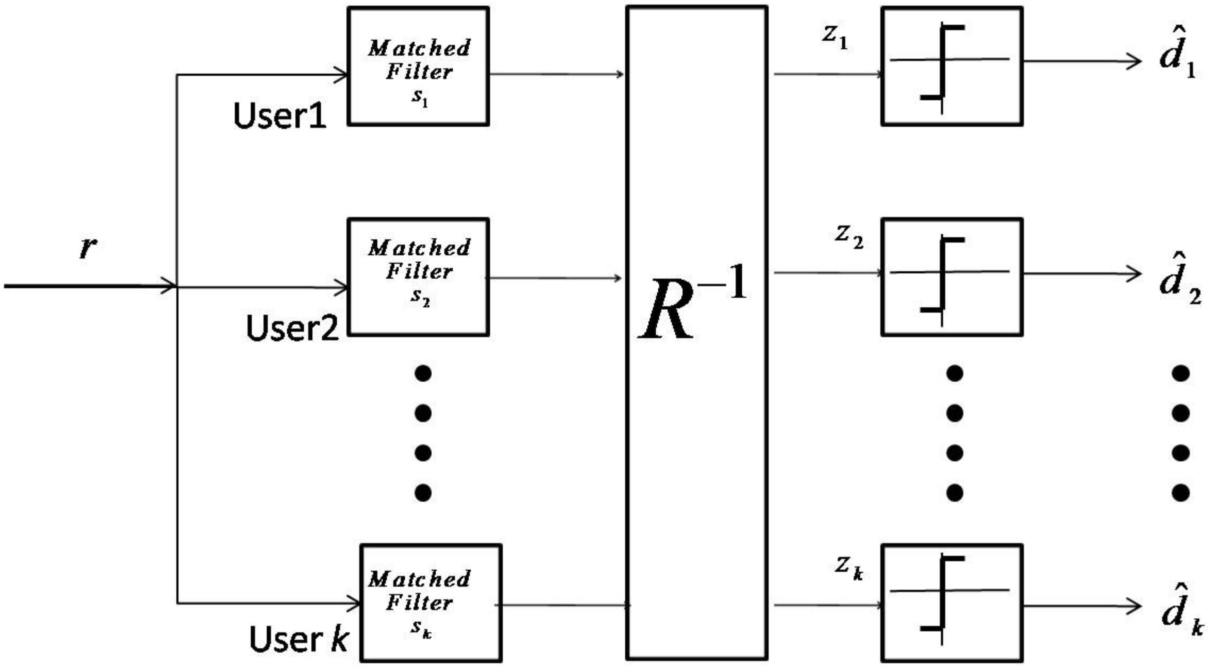

Figure 2-6 Decorrelator Detector

The final decision statistics is made by

$$\begin{aligned}\hat{\boldsymbol{d}} &= sgn(\boldsymbol{R}^{-1}\boldsymbol{y}) \\ &= sgn\,(\boldsymbol{R}^{-1}(\boldsymbol{RAd} + \boldsymbol{w})) \\ &= sgn(\,\boldsymbol{R}^{-1}\boldsymbol{RAd} + \boldsymbol{R}^{-1}\boldsymbol{w}) \\ &= sgn(\,\boldsymbol{Ad} + \boldsymbol{w}') \qquad (2.13)\end{aligned}$$

As can be seen by Eqn. (2.13), in a noiseless environment this approach recovers the original signals, which the matched filter could not do.

The magnitude of the noise enhancement is given by $\boldsymbol{w}' = \boldsymbol{R}^{-1}\boldsymbol{w}$. It can be minimized by choosing spreading codes that are mutually orthogonal. If the codes are perfectly orthogonal then no noise enhancement takes place. However, this ideal situation is difficult to realize. On the other hand, a greater degree of correlation between codes results in more noise enhancement that will result in greater degradation in performance. Generally, decorrelating detector provides a good performance in many scenarios and serves as a benchmark to evaluate other multiuser detection schemes.

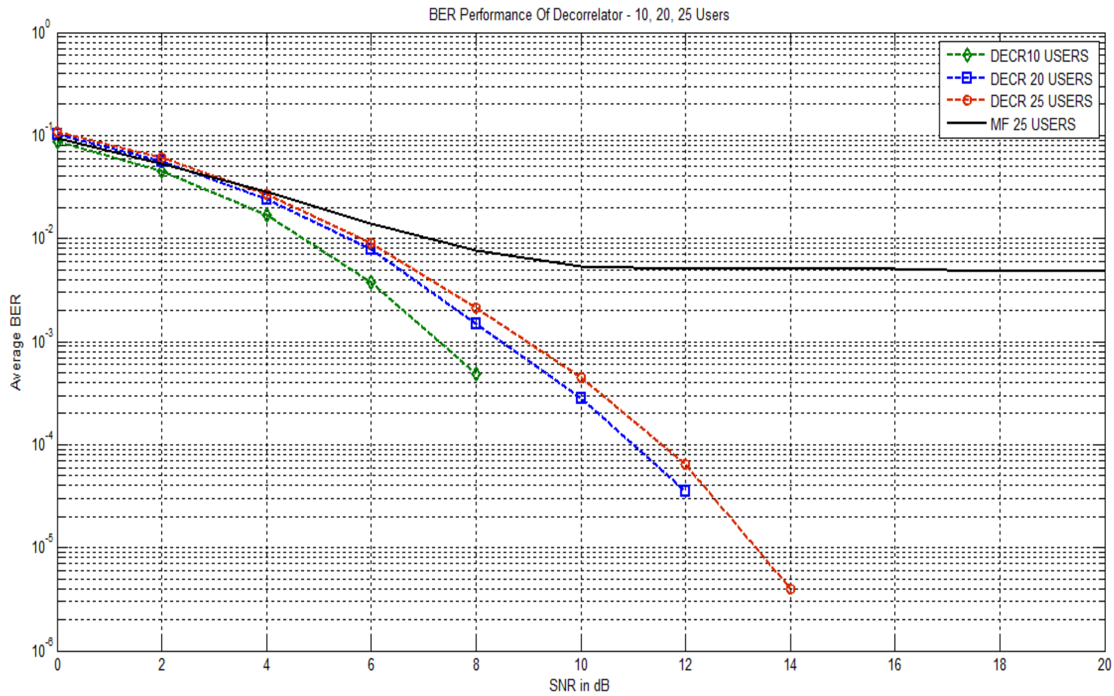

Figure 2-7 BER Performance of Decorrelator

In addition, we see that the decorrelating receiver performs only the linear operation on the received statistics $\boldsymbol{y}$ and hence it is indeed a linear detector. The average bit error rate performance of decorrelator for different number of users is shown in figure 2.7. The simulations are performed for synchronous CDMA system using BPSK modulation in

AWGN channel. The BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^4$ bits and using Gold codes of length 31 and having equal received power. From figure 2.7, we observe that as SNR increases, the performance of the decorrelating detector becomes better. However, it is observed that at low SNR's the matched filter receiver performs slightly better.

Minimum Mean-Squared Error Detector

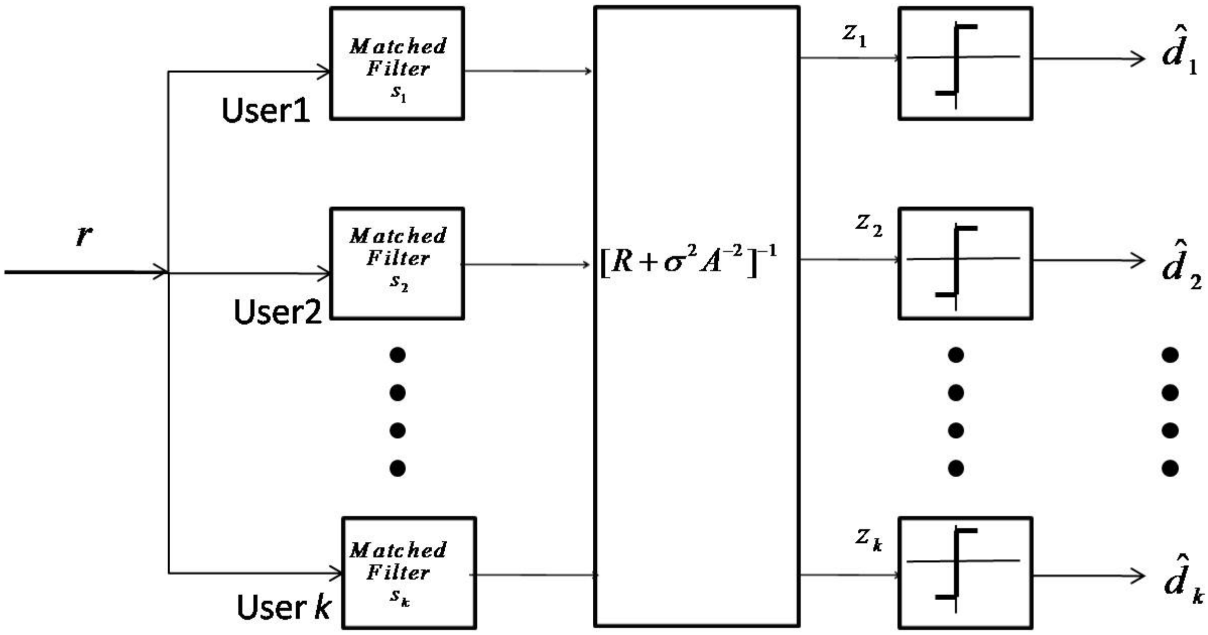

Figure 2-8 MMSE Detector

The Minimum Mean-Squared Error (MMSE) Detector is an improved linear approach by assuming that strength of each user's received signal is known. The MMSE works by applying a linear transformation $[\boldsymbol{R} + \sigma^2 \boldsymbol{A}^{-2}]^{-1}$ as shown figure 2.8. MMSE minimizes the mean-squared error between the outputs and the data, i.e.

$$min|\boldsymbol{d} - f(\boldsymbol{y})|^2 \tag{2.14}$$

where $f(.)$ is the function that maps $\boldsymbol{y}$ to $\widehat{\boldsymbol{d}}$, and is chosen as to minimize the expected mean-squared error. The final decision metric can be written as

$$\widehat{\boldsymbol{d}} = sign([\boldsymbol{R} + \sigma^2 \boldsymbol{A}^{-2}]^{-1} \boldsymbol{y}) \tag{2.15}$$

Here σ is the noise variance.

From (2.15) we notice that MMSE is a compromise between the conventional detector and the decorrelating detector. That is, if the signal to noise ratio goes to infinity, $\sigma \to 0$ and $[\boldsymbol{R} + \sigma^2 \boldsymbol{A}^{-2}]^{-1} \to \boldsymbol{R}^{-1}$, which is the decorrelating detector. Similarly, if signal to noise ratio goes to zero, $\sigma \to \infty$, and $[\boldsymbol{R} + \sigma^2 \boldsymbol{A}^{-2}]^{-1}$ will tend to zero, the MMSE acts as a conventional detector. Figure 2.9 shows the average bit error rate performance of the MMSE linear detector. The simulations are performed for a 20 user synchronous CDMA system using BPSK modulation in AWGN channel. The BER is evaluated in SNR range 0-20 dB for equal received power users transmitting $1*10^4$ bits and using Gold codes of length 31. Comparison of the different MUDs is also given in figure 2.9. We can see that the performance of MMSE detector is slightly better than the decorrelating detector.

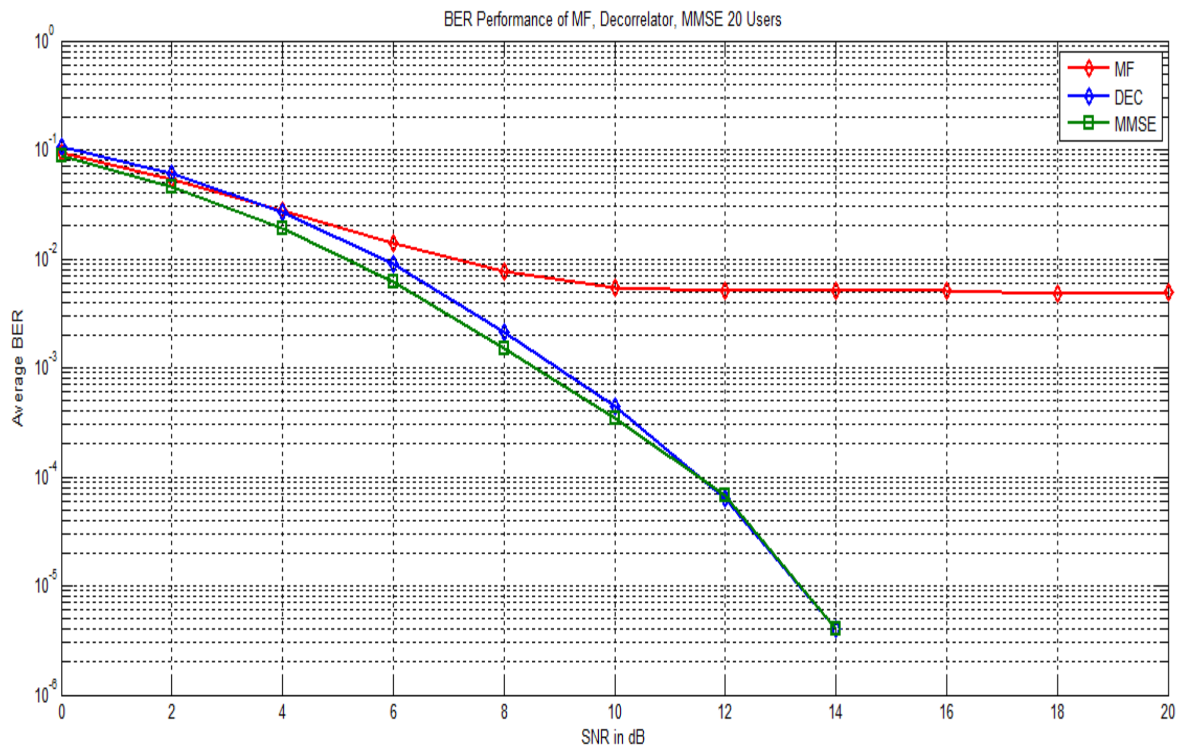

Figure 2-9 BER Performance of MMSE Detector

2.3.2. (b) Interference Cancellation Detectors

As we see that both the decorrelating detector and MMSE detector involves matrix inversion operation. If the number of users becomes large, size of the matrix to be inverted grows, resulting in more computations. In the Interference Cancellation scheme, interference estimates are generated and removed from the received signal before detection. They are referred as interference cancellation based as these detectors aim to cancel the estimated interference from the received signal. The principle involves making tentative decisions to generate an estimate of some or all of the multiple access interference and then the estimated MAI is subtracted from the received signal to form a new output based on which new decision are made. The idea is that if under the assumption that MAI were estimated perfectly, the output of the detector is composed only of the desired signal and the unstructured additive channel noise. The MAI is removed and the fidelity of the modified received signal is improved, which in turn results in improved bit error rates or increased system capacity.

The interference cancellation detectors can be categorized as Successive/Serial Interference Canceller, Parallel Interference Canceller and Hybrid Interference Canceller. The process of interference cancellation can be done through a number of stage (multistage detectors) or iteratively (iteration detectors).

Successive Interference Cancellation

In successive interference cancellation approach, every users signal is decoded successively [9, 18 and 22]. Users are ranked on the basis of their received power to allow detection of the strongest user first, since this user gives most reliable decision. A new modified received signal is created after removal of the strongest user signal. Following this, the second strongest user is detected from the remainder of received signal. The procedure is continued until all users are detected. Although SIC is simple in implementation compared to other type of multiuser detectors, it has several disadvantages.

- The delay is linear with the number of users which makes this scheme less efficient for the heavily loaded systems.
- The detection of all other users depends strongly on the accuracy of detecting strong user as any inaccuracies will propagate through detection of all other users.
- At every stage, a ranking process is needed to reorder the users according to their received powers.

- It may become difficult to sort out users having nearly the same power.

Parallel Interference Cancellation

To solve the problem of the delay in SIC, the PIC was suggested and was more researched as it is more amenable to practical implementation. PIC effectively estimates and subtracts out the MAI for each user in parallel. All the users are decoded and cancelled from the received signal simultaneously. The tradeoff between delay and complexity is well balanced in PIC detector. Moreover, the performance of parallel IC can approach successive IC with less number of stages.

The research work in this thesis focuses on Parallel Interference Cancellation detectors. The first PIC detector for CDMA communication system was proposed Varanasi and Aazhang in [6, 19] with multistage implementation. The detector was demonstrated to have close relations to Verdu's optimum detector and possesses several desirable properties including the potential of near optimum performance, low decision delay, and lower computational complexity. Parallel Interference Cancellation detector is discussed in detail in the next chapter.

Hybrid Interference Cancellation

The main idea behind Hybrid interference cancellation (HIC) is that instead of cancelling all K users either in series or in parallel, they are cancelled in parallel and partially in series. Hence a mix of SIC and PIC will yield optimal result in terms of computational time and BER. In HIC, the active users are split into several groups. Within each group, parallel interference is performed and serial interference cancellation is carried out between the groups.

CHAPTER 3

PARALLEL INTERFERENCE CANCELLATION

As observed in the previous chapter, the decorrelating detector and MMSE detector perform the code correlation matrix inversion which can become prohibitively complex for a large number of users. In the light of these difficulties, suboptimal methods like interference cancellation have gained a lot of interest [20]. The interference cancellation detectors are constructed around the matched filter receivers. Conceptually, the interference cancellers make use of the tentative decisions to estimate all interferences for a particular user and subtract them from the received signal. Parallel Interference Cancellation (PIC) technique first suggested by Varanasi and Aazhang [6] assumed that all amplitude estimates were completely unbiased. The linear version of this has been shown by Elders-Boll et al. to be equivalent to the Jacobi iteration for solving a set of linear equations [21]. However, direct implementation of PIC results in a biased statistics for the desired signal due to accumulation of residual correlation between the desired signal and the interference [7]. Significant improvement in mitigating this bias was achieved by Divsalar et al [7] by weighting the estimated interference. They studied a weighted (or partial) cancellation scheme for both the linear and nonlinear decision functions based on maximum likelihood principle [7, 18, 19 and 23]. Linear interference schemes were further exploited by several authors after Guo and Rasmussen proposed linear PIC schemes using matrix algebra. Convergence and conditions to ensure convergence have also been presented in [24, 25, 26 and 41]

In section 3.1, multistage approach for parallel interference cancellation is described. The weighted parallel interference cancellation technique is discussed in section 3.2 and the need for an adaptive weight for each user at different stages is illustrated in section 3.3.

3.1 Conventional Multistage Parallel Interference Cancellation

The multistage Parallel interference cancellation detector is one of effective ways to cancel the MAI. At each stage, the PIC detector estimates and subtracts in parallel all the MAI for each user [2]. At 1^{st} stage of cancellation, the detector uses the tentative decision statistics from the output of matched filter to regenerate the MAI, and then subtracts it from the received signal to isolate the user of interest. The modified received signal is then fed through the matched filter bank and another set of decision statistics is obtained. This forms the first stage of PIC. The new set of decision statistics are used to regenerate a more accurate version of the user signals which are then cancelled from the received signal to decode the user of interest.

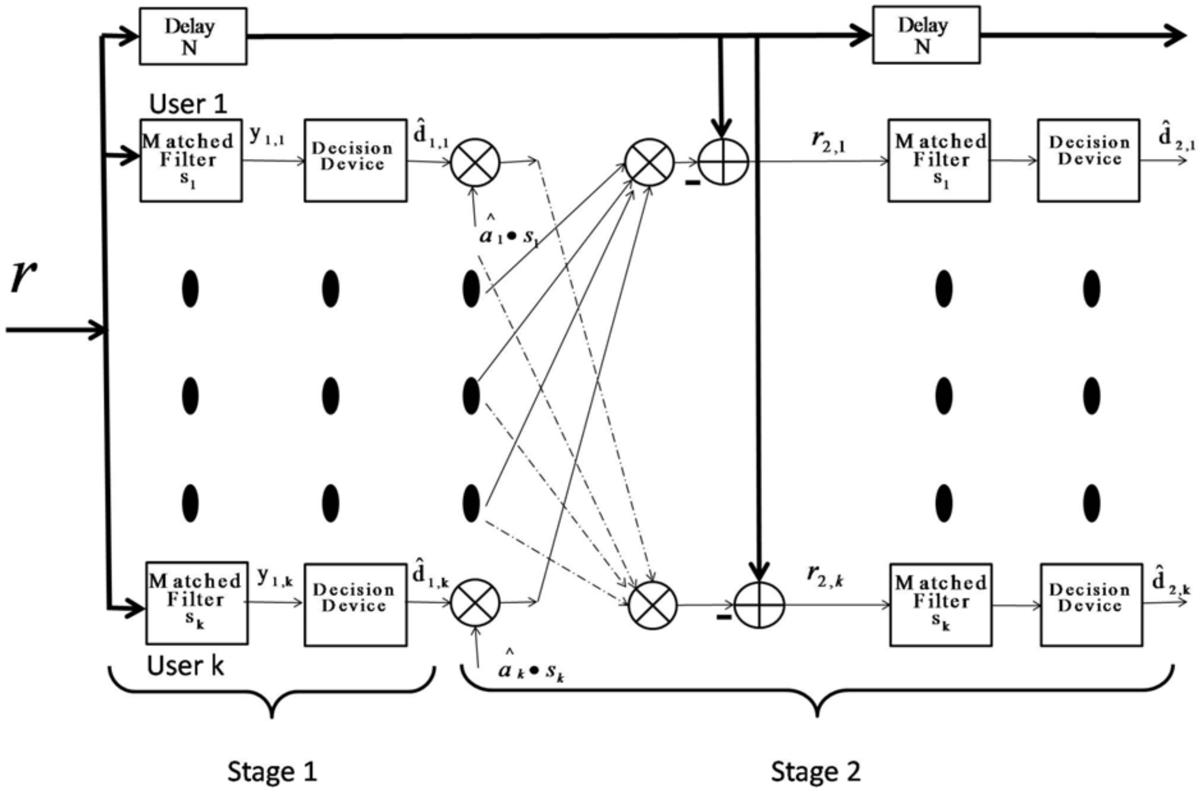

Figure 3-1 Multistage Parallel Interference Cancellation

The structure for first two stages of a multistage PIC detector is shown in figure 3.1. The chip matched filtered received vector $\boldsymbol{r}$ is given as input to the detector. Here input vector $\boldsymbol{r}$ is as denoted by equation 2.1:

$$\boldsymbol{r} = \boldsymbol{SAd} + \boldsymbol{n} \tag{3.1}$$

In conventional multistage approach, previous tentative decisions are used to estimate the interference for full cancellation at each stage [2]. The detector first reconstructs the interference for each user from all the other users based on their decision statistics (either soft or hard) and then cancels it out from the received signal.

The estimated interference for user k at stage i is given as: $\sum_{\substack{j=1 \\ j\neq k}}^{K} \boldsymbol{s_j}\ \hat{d}_{i-1,j}$, where $\hat{d}_{i-1,k}$ is a tentative decision for user k from the previous stage (i-1) based on decision statistics $y_{i-1,k}$, which can be represented as:

$$\hat{d}_{i-1,k} = TD(y_{i-1,k}) \tag{3.2}$$

T.D. represents tentative decision.

The procedure to regenerate and subtract interferers from desired user is given below:

The modified received signal for user k at stage (i-1) is given as:

$$\boldsymbol{r_{i-1,k}} = \boldsymbol{r} - \sum_{\substack{j=1 \\ j\neq k}}^{K} \boldsymbol{s_j}\ \hat{d}_{i-1,j} \tag{3.3}$$

$$= \boldsymbol{r} - \sum_{j=1}^{K} \boldsymbol{s_j}\ \hat{d}_{i-1,j} + \boldsymbol{s_k}\ \hat{d}_{i-1,k}$$

where $\boldsymbol{r}$ is the received signal $\hat{d}_{i-1,k}$ is a tentative decision from the previous stage (i-1) and $\boldsymbol{s_k}$ is the signature code for the user k. The modified received signal is assumed to be interference free so that code matched filtering can be performed to yield a current stage decision statistic for user k, which is expressed as:

$$y_{i,k} = \boldsymbol{s_k^H}\ \boldsymbol{r_{i-1,k}}$$

$$= \boldsymbol{s_k^H}\left(\boldsymbol{r} - \sum_{j=1}^{K} \boldsymbol{s_j}\ \hat{d}_{i-1,j} + \boldsymbol{s_k}\ \hat{d}_{i-1,k}\right) \tag{3.4}$$

$$= \boldsymbol{s_k^H}\left(\boldsymbol{r} - \sum_{j=1}^{K} \boldsymbol{s_j}\ \hat{d}_{i-1,j}\right) + \hat{d}_{i-1,k}$$

where $\boldsymbol{s_k^H}$ represents the hermitian transpose of the signature code $\boldsymbol{s_k}$ of user k.

Assuming that soft decisions are made as given in equation (3.2), we rewrite the above equation as:

$$y_{i,k} = \boldsymbol{s_k^H} \left(\boldsymbol{r} - \sum_{j=1}^{K} \boldsymbol{s_j} \ \hat{d}_{i-1,j}\right) + y_{i-1,k} \tag{3.5}$$

In vector notation:

$$\begin{aligned} \boldsymbol{y_i} &= \boldsymbol{S^H} \left(\boldsymbol{r} - \boldsymbol{S} \ \boldsymbol{y_{i-1}}\right) + \boldsymbol{y_{i-1}} \\ &= \boldsymbol{S^H} \boldsymbol{r} - \boldsymbol{S^H S} \boldsymbol{y_{i-1}} + \boldsymbol{y_{i-1}} \\ &= \boldsymbol{S^H} \boldsymbol{r} + \left(\boldsymbol{I} - \boldsymbol{S^H S}\right) \boldsymbol{y_{i-1}} \end{aligned} \tag{3.6}$$

where $\boldsymbol{y_i} = \left(y_{i,1}, y_{i,2}, y_{i,3}, \dots, y_{i,k}\right)^T$ is defined as vector of decision statistics at stage *i*, $\boldsymbol{r}$ is the received signal vector and $\boldsymbol{I}$ is (*K* x *K*) identity matrix and $\boldsymbol{S}$ is (*N* x *K*) matrix of signature codes.

Considering, $\boldsymbol{y_0} = \boldsymbol{0}$, we can express the soft output at the m^{th} stage recursively as:

$$\begin{aligned} \boldsymbol{y_m} &= \boldsymbol{S^H} \boldsymbol{r} + \left(\boldsymbol{I} - \boldsymbol{S^H S}\right) \boldsymbol{y_{m-1}} \\ &= \boldsymbol{S^H} \boldsymbol{r} + \left(\boldsymbol{I} - \boldsymbol{S^H S}\right) \boldsymbol{S^H} \boldsymbol{r} + \left(\boldsymbol{I} - \boldsymbol{S^H S}\right) \boldsymbol{y_{m-2}} \\ &= \sum_{i=1}^{m} \left(\boldsymbol{I} - \boldsymbol{S^H S}\right)^{i-1} \boldsymbol{S^H r} \end{aligned} \tag{3.7}$$

From the above we can see that linear PIC scheme corresponds to linear matrix filtering that can be performed directly on the chip matched filtered received vector $\boldsymbol{r}$ as:

$$\boldsymbol{y_m} = \boldsymbol{G_m^H} \boldsymbol{r} \tag{3.8}$$

where $\boldsymbol{G_m^H} = \sum_{i=1}^{m} \left(\boldsymbol{I} - \boldsymbol{S^H S}\right)^{i-1} \boldsymbol{S^H}$

The matrix filter $\boldsymbol{G}_{\boldsymbol{m}}^{\boldsymbol{H}}$ is then referred to as the equivalent one shot cancellation filter for an m stage conventional linear PIC.

The final bit decision is made for the desired user by hard limiter applied on the decision statistics at the last stage. Using this procedure, an arbitrary number of stages of the PIC may be employed to obtain the data bits of each user. When the estimate from the previous stage become more accurate, the performance of the multistage PIC will improve and $\left(\lim_{m\to\infty}\right)$, its performance is expected to converge to the ideal decorrelator.

3.1.1 Convergence of the Parallel Interference Cancellation detector

In this section, we study the convergence of the conventional PIC i.e. the behavior of the PIC as the number of stages increases to infinity.

Considering (3.8), we have

$$\boldsymbol{G}_{\boldsymbol{m}}^{\boldsymbol{H}} = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{m}} (\boldsymbol{I} - \boldsymbol{S}^{\boldsymbol{H}}\boldsymbol{S})^{\boldsymbol{i}-\boldsymbol{1}}\, \boldsymbol{S}^{\boldsymbol{H}}$$

Assuming that $N \geq K$, the signature code matrix $\boldsymbol{S}$ can be decomposed as:

$$\boldsymbol{S} = \boldsymbol{U}\Sigma\boldsymbol{V}^{\boldsymbol{H}} \tag{3.9}$$

where $\boldsymbol{U}_{N\,X\,N}$ and $\boldsymbol{V}_{K\,X\,K}$ are unitary matrices and Σ is a N x K matrix of the form:

$$\Sigma = \begin{bmatrix} \sqrt{\boldsymbol{\Lambda}} \\ \boldsymbol{0} \end{bmatrix} \tag{3.10}$$

where $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_K,)$ is a K x K diagonal matrix of eigenvalues of $\boldsymbol{S}^{\boldsymbol{H}}\boldsymbol{S}$ and the $\boldsymbol{0}$ is zero matrix is of appropriate dimension. The code correlation matrix $\boldsymbol{S}^{\boldsymbol{H}}\boldsymbol{S}$ can thus be represented as:

$$\boldsymbol{S^H S} = \boldsymbol{V \Sigma^H U^H U \Sigma V^H}$$

$$= \boldsymbol{V \Sigma^H \Sigma V^H} \qquad (3.11)$$

$$= \boldsymbol{V \Lambda V^H}$$

The code correlation matrix $\boldsymbol{R} = \boldsymbol{S^H S}$, is hermitian and hence $\boldsymbol{\Lambda}$ formed by the eigenvalues of $\boldsymbol{S^H S}$ are all non-negative. Equation (3.8) can now be rewritten as:

$$\boldsymbol{G_m^H} = \sum_{i=1}^{m} (\boldsymbol{I} - \boldsymbol{V \Sigma^H U^H U \Sigma V^H})^{i-1} \boldsymbol{V \Sigma U^H}$$

$$= \sum_{i=1}^{m} \boldsymbol{V} (\boldsymbol{I} - \boldsymbol{\Sigma^H \Sigma})^{i-1} \boldsymbol{\Sigma U^H} \qquad (3.12)$$

$$= \boldsymbol{V} \begin{bmatrix} \sum_{i=1}^{m} (\boldsymbol{I} - \boldsymbol{\Lambda})^{i-1} \sqrt{\boldsymbol{\Lambda}} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{U^H}$$

A sufficient and necessary condition for $\boldsymbol{G_m^H}$ to converge is that $\sum_{i=1}^{\infty} (\boldsymbol{I} - \boldsymbol{\Lambda})^{i-1} \sqrt{\boldsymbol{\Lambda}}$ converges, i.e, $\sum_{i=1}^{\infty} (\boldsymbol{I} - \lambda_k)^{i-1} \sqrt{\lambda_k}$ converges for k=1, 2, …, K. The solution is $0 \leq \lambda_k < 2$, or in other words, all the eigenvalues of $\boldsymbol{S^H S}$ are less than 2.

Clearly when the above eigenvalue condition is satisfied,

$$\sum_{i=1}^{\infty} (I - \lambda_k)^{i-1} \sqrt{\lambda_k} = \begin{cases} \lambda_k^{-1} \sqrt{\lambda_k} & \text{if } \lambda_k > 0, \\ 0 & \text{if } \lambda_k = 0. \end{cases} \qquad (3.13)$$

Hence $\sum_{i=1}^{\infty} (\boldsymbol{I} - \boldsymbol{\Lambda})^{i-1} \sqrt{\boldsymbol{\Lambda}} = \begin{bmatrix} \sqrt{\boldsymbol{\Lambda_1}} & 0 \\ 0 & 0 \end{bmatrix}$

where $\boldsymbol{\Lambda_1}$ is a diagonal matrix containing all the non-zero eigenvalues of $\boldsymbol{S^H S}$. Therefore

$$\lim_{m \to \infty} \boldsymbol{G_m^H} = \boldsymbol{V} \begin{bmatrix} \boldsymbol{\Lambda_1}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{V^H V} \begin{bmatrix} \sqrt{\boldsymbol{\Lambda_1}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{U^H}$$

$$= \boldsymbol{S^H S} + \boldsymbol{S^H} \tag{3.14}$$

In conclusion, the conventional PIC converges if and only if all the eigenvalues of $\boldsymbol{S^H S}$ are less than 2. When this eigenvalue constraint is satisfied, the PIC detector converges to the decorrelating detector i.e.

$$\lim_{m\to\infty} \boldsymbol{G_m^H} = \boldsymbol{G_{dec}^H} = \boldsymbol{S^H S} + \boldsymbol{S^H} \tag{3.15}$$

3.2 Weighted (Partial) Parallel Interference Cancellation (WPIC)

The eigenvalue constraint in the Conventional PIC is too rigid to be reasonable. For example nearly all the code sets have largest eigenvalue greater than 2 for a 15 user system with a processing gain of 31. Moreover conventional multistage PIC may not guarantee that performance improves with more stages when hard decisions are used at intermediate stages, because any inaccurate decision can lead to a performance even worse than that without cancellation. The inaccurate decisions can occur in the direct implementation of PIC because of the biased statistics for the desired signal component caused by accumulation of correlations between the desired signal and interference [7, 42, 43]. If a wrong decision is subtracted, the increased interference power would be four-fold [42].

Each iteration stage, PIC detectors try to eliminate the interference caused by all the other users. This is not necessarily the best philosophy, rather, when the interference estimate is poor (as in the early stages of interference cancellation), it is preferable not to cancel the entirely the estimated multiuser interference. Divsalar proposed a partial cancellation of the MAI by weighting the amount of cancellation [7]. As the IC operation progresses, the estimates of the multiuser interference improve and, thus, in the later stages of the iterative scheme, it becomes desirable to increase the weight of the estimated interference being removed. The WPIC scheme adds a weight to the interference cancellation path and all weights in a stage are identical and fixed. The partial cancellation weights for the weighted PIC are a fraction of one, and their values reflect the reliability of the tentative decision of the estimated interference. The block diagram for WPIC is given in figure 3.2. As it can be seen, the only difference between the PIC and WPIC is the weight introduced on each of the regenerated interferers signal.

In the Weighted PIC, the relationship between the current decision statistics and the previous set of decision statistic are obtained as shown:

The decision statistics for user k at stage i is a weighted sum of the statistics from previous stage (i-1) and the statistics based on current cancellation i.e.,

$$y_{i,k} = (1 - \alpha\, p_i) y_{i-1,k} + p_i\, \boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{H}} \left(\boldsymbol{r} - \sum_{j=1}^{K} y_{i-1,j} \boldsymbol{s}_{\boldsymbol{j}}\right) \tag{3.16}$$

where $p_i = p_1, p_{2,} \dots \dots, p_m$ denote the weighting factors and α is a non negative parameter defined later in this section. The remaining parameters are as defined in previous section.

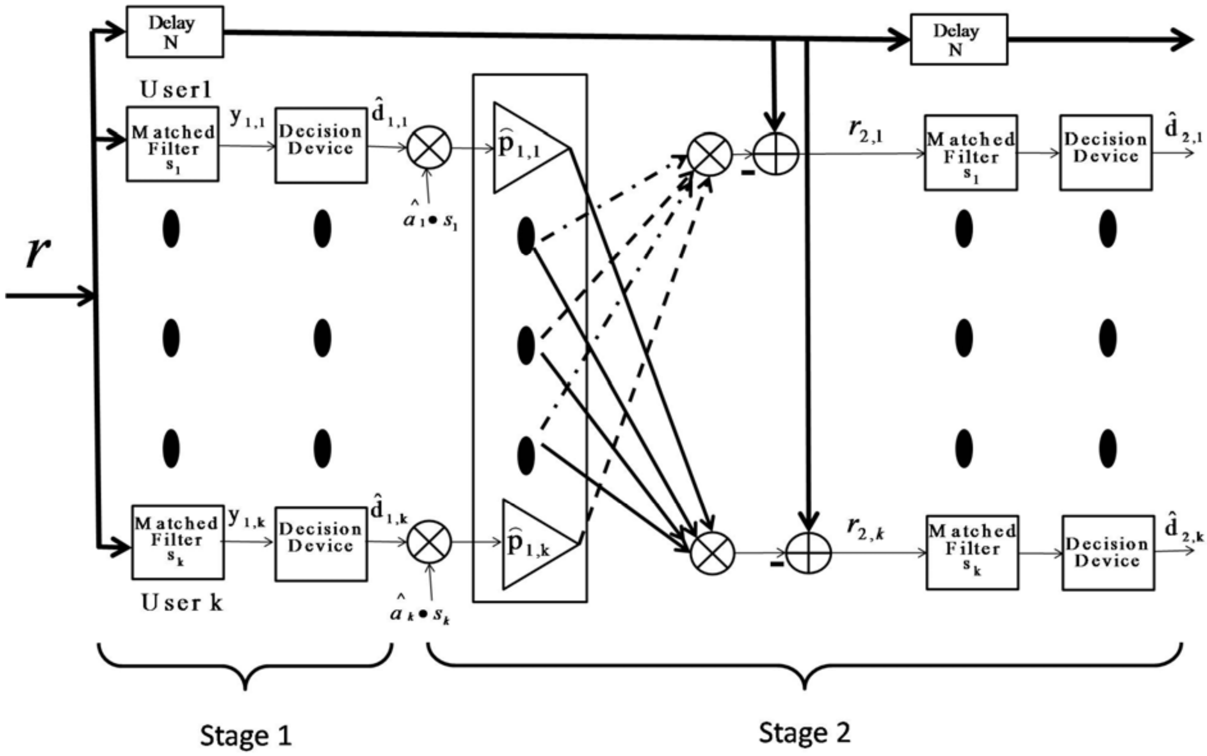

Figure 3-2 Weighted Parallel Interference Cancellation

In vector representation, we have the recursive formula of decision statistics as

$$\begin{aligned} \boldsymbol{y_i} &= (1-\alpha\, p_i)\boldsymbol{y_i} + \ p_i\, \boldsymbol{S^H}\,(\boldsymbol{r} - \ \boldsymbol{S}\ \ \boldsymbol{y_{i-1}}) \\ &= \ [\boldsymbol{I} - p_i(\boldsymbol{S^H S} + \alpha \boldsymbol{I})]\ \boldsymbol{y_{i-1}} + p_i \boldsymbol{S^H r} \qquad (3.17) \\ &= \ [\boldsymbol{I} - p_i(\boldsymbol{R} + \alpha \boldsymbol{I})]\ \boldsymbol{y_{i-1}} + p_i \boldsymbol{S^H r} \end{aligned}$$

The weighted PIC given in (3.17) resembles the concept of Steepest Descent Method (SDM) for updating the adaptive filter weights to minimize the Mean Square Error (MSE) [24]. The close relationship helps in finding the optimal weighting factors or equivalently the step sizes for SDM with respect to the MSE. Studying the relationship that exists between the WPIC and SDM, we see that WPIC is a realization of steepest descent method using variable step sizes. The step sizes are analogous to weighting factors in the PIC structure. The derivation for optimum step size of the SDM is given in appendix A. We can see that SDM is equivalent to equation (3.17) for WPIC where $\alpha = \sigma^2$. Therefore the weighted PIC can be considered as realization of SDM for implementing MMSE detector with $\alpha = \sigma^2$ and decorrelating detector when $\alpha = 0$.

The mean square error of the weighted PIC designed to converge toward the decorrelating detector is determined as:

$$J_{MSE} = \sum_{k=1}^{K} \frac{\sigma^2}{\lambda_k+\sigma^2} + \frac{\lambda_k+\sigma^2}{\lambda_k}\left((1-p_i\lambda_k)^m - \frac{\sigma^2}{\lambda_k+\sigma^2} \right)^2 \tag{3.18}$$

The minimization of the above function gives us the optimal weighting factors for WPIC.

Equation (3.16) can be rearranged as:

$$\begin{aligned} y_{i,k} &= (1-\alpha\, p_i)y_{i-1,k} + p_i \boldsymbol{s_k^H}\left(\boldsymbol{r} - \sum_{j=1}^{K} y_{i-1,j}\boldsymbol{s_j}\right) \\ &= (1-\alpha\, p_i)y_{i-1,k} + p_i s_k^H\, r - \sum_{j=1}^{K} p_i\; y_{i-1,j}\, s_k^H s_j \\ &= (1-\alpha\, p_i)y_{i-1,k} + p_i(\, y_{i,k} - \hat{I}_{i,k}) \end{aligned} \tag{3.19}$$

where $\hat{I}_{i,k} = \sum_{\substack{j=1 \\ j\neq k}}^{K} y_{i-1,j}\, \boldsymbol{s_j^H s_k}$ is the estimated MAI from the previous stage.

From the above we can see that the soft output $y_{i,k}$ consists of two items: the output of the conventional PIC with the weight and the soft output from the previous stage with the complementary weight. p_i is the weight for the i^{th} stage of the cancellation and $0 \leq p_i < 1$ .The final bit decision is made based on the soft metric as:

$$d_{i,k} = sgn\{Re(y_{i,k})\}$$

The weakness of the WPIC scheme is that the weights assigned to all users regenerated signal in each stage is uniform, which are not optimal. The weight reflects the reliability of data estimates. The reliability of data estimates varies from one user to another and from bit to bit depends on the MAI levels, it is more reasonable for each user to have its own weight per bit interval, rather than constant weight for all users in each stage throughout the cancellation.

3.3 Adaptive Parallel Interference Cancellation:

Based on this discussion, adaptive parallel interference cancellation techniques are developed to update the interference cancellation weights adaptively to improve the bit decision. Different criterions have been used in the development of various adaptive PIC detectors. The optimal solution for adaptive PIC detectors can be obtained by using the least square criterion. In [11] a recursive least square algorithm is employed to obtain the solution to the criterion:

$$\min_{d}(n) \sum_{m=1}^{n} P^{n-m} \mid \boldsymbol{r}(m) - \hat{\boldsymbol{r}}(m) \mid^2 \qquad (3.20)$$

and the tentative decision is updated through iteration as:

$$\widehat{\boldsymbol{d}}\,(n) = \boldsymbol{Ad} + \boldsymbol{R}^{-1}(n) \sum_{m=1}^{n} \boldsymbol{p}^{n-m}\ \boldsymbol{c}(m)\, \boldsymbol{s}(m) \qquad (3.21)$$

where $\boldsymbol{p}$ is the weight vector, $\boldsymbol{c}(m)$ denotes the coefficients of the adaptive filter and $\boldsymbol{s}$(m) denotes the signature codes of users. The drawback of *RLS* based detector is in its complexity which is $O(NK^2)$ per bit. Since $N>K$ in a CDMA system, the complexity can be considered as $O(K^3)$ which is same as decorrelating detector. A modification of the weighted PIC was derived by Xue *et al.* [8], where the weighs are updated based upon the Least Mean Square (LMS) algorithm at each stage.

The error between received signal and its estimate is given as:

$$e^{i}(m) = \boldsymbol{r}(m) - \sum_{m=1}^{n} \boldsymbol{p_i}\,(m)\,\boldsymbol{s_k}\boldsymbol{d_k^{i-1}} \tag{3.22}$$

The adaptation scheme is intended to minimize the LMS error $E\left\{\left|e^{i}(m)\right|^{2}\right\}$.

This adaptive PIC becomes unreliable however, with the increased system load, due to the slow convergence property of LMS algorithm and the increased mean square error also degrades the performance. Moreover, the sensitivity of the LMS algorithm to the initial state may lower the convergence rate and hence further degrade the performance.

3.4 Simulation results

Simulations evaluate BER performance for PIC using different signature codes. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. In figure 3.3, BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^5$ bits and using Gold codes of length 31 and having equal received power. Here the maximum eigenvalue of the code correlation matrix is less than 2 as we are using gold codes.

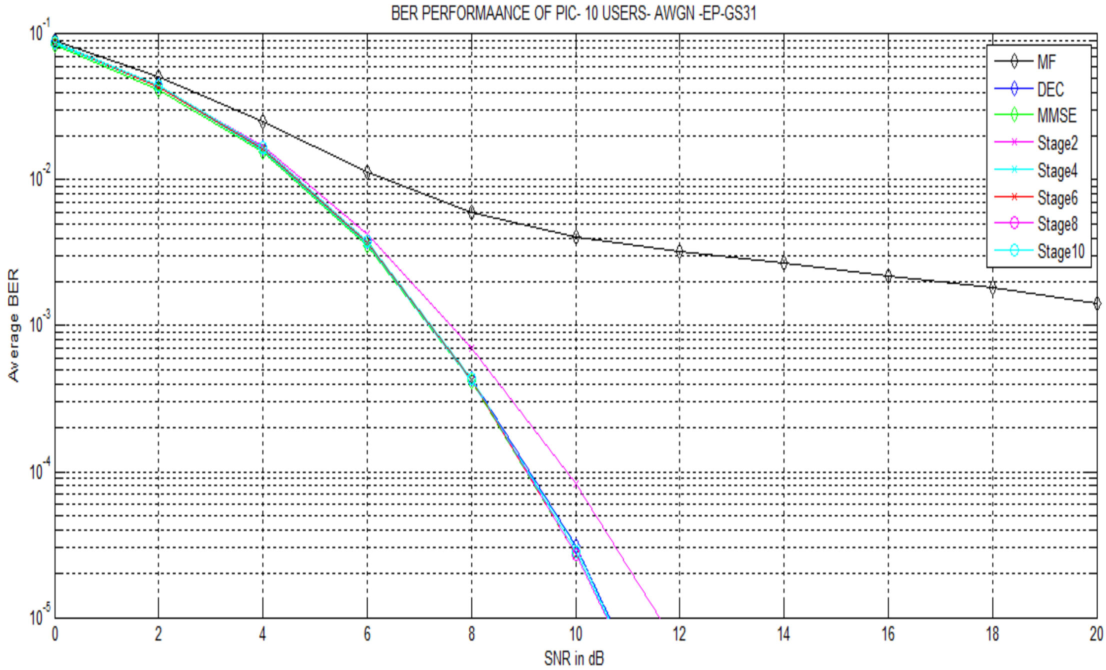

Figure 3-3 BER Performance of PIC using Gold Codes- 10 equal power users

As seen in the above plot, the BER performance of the PIC is close to decorrelator when 10 equal power users are considered in AWGN channel. The convergence behavior for this PIC is given in figure 3.4. We can see from the figure that PIC converges to decorrelator.

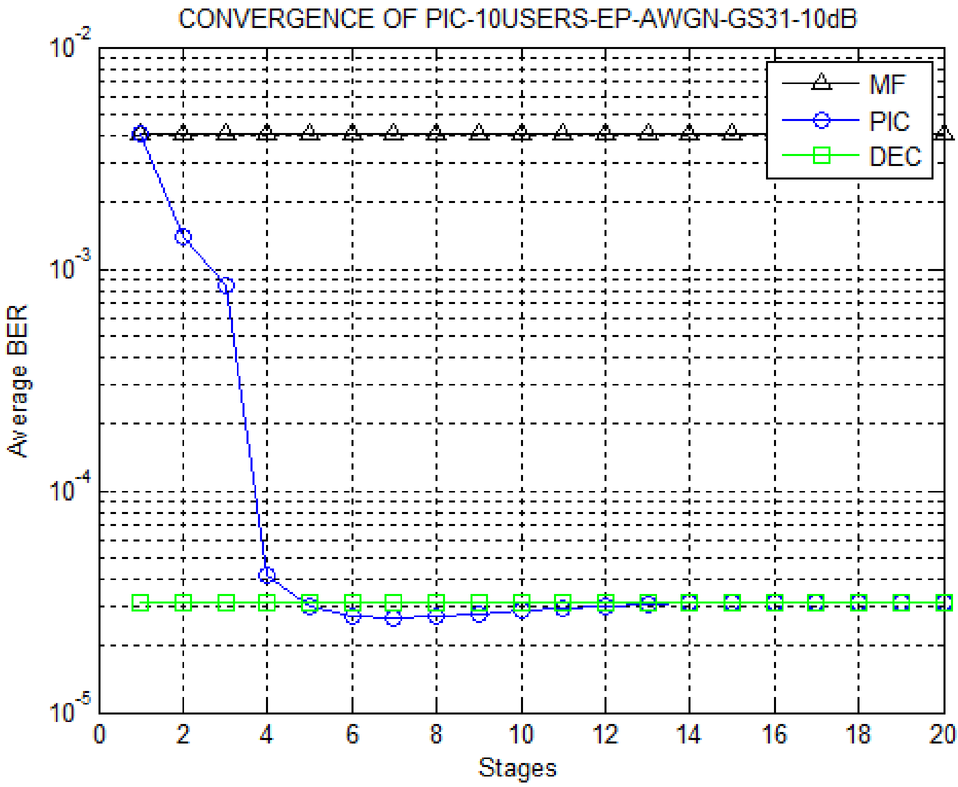

Figure 3-4 Convergence of PIC using Gold Codes – 10 equal power users

BER Performance of PIC using random PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10-user system is shown in figure 3.5. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting 1e4 bits and using PN codes of length 31 and having equal received power.

As we can see from the plot, the performance degrades with increased number of stages as maximum eigenvalue of the cross correlation matrix of signature sequence is greater than 2.

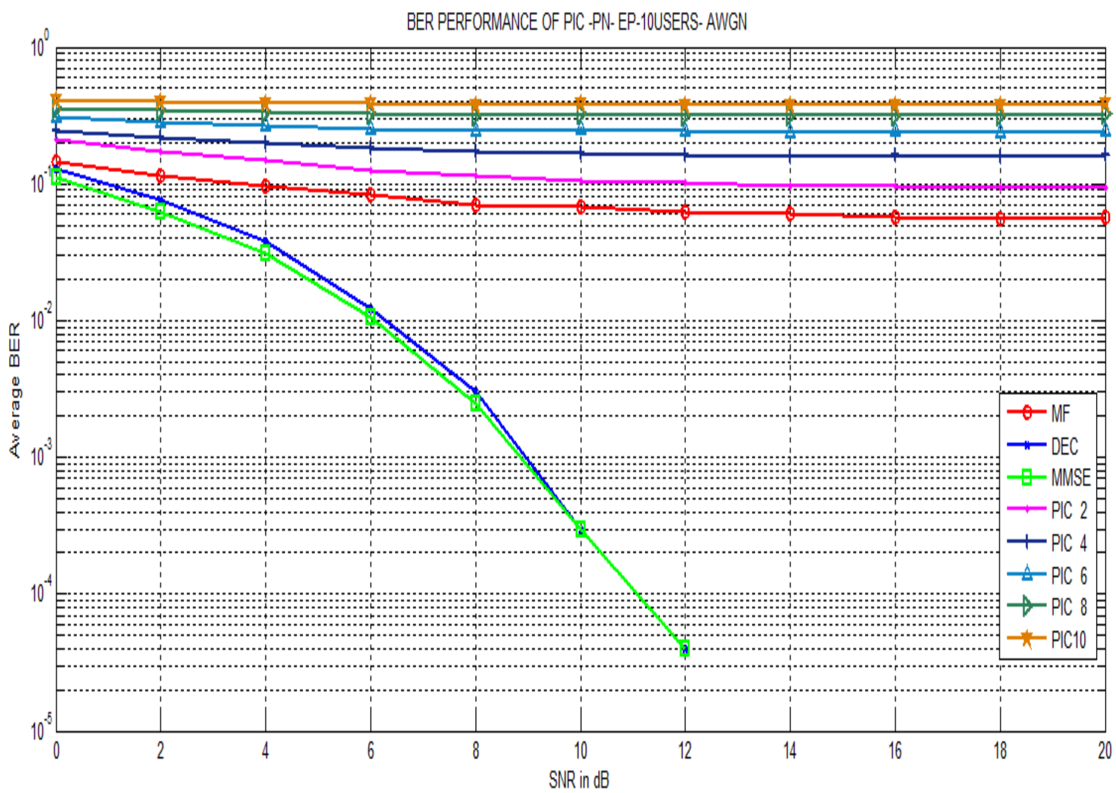

Figure 3-5 BER Performance of PIC using PN Codes – 10 equal power users

The convergence of PIC using random PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10-user system is shown in figure 3.6. It can be seen from the figure that the PIC is diverging instead of converging towards decorrelator and the performance is poor even than the matched filter. Here we can observe the phenomenon of ping pong effect in the convergence where the BER tends to alternate between two state processes as a function of iteration index.

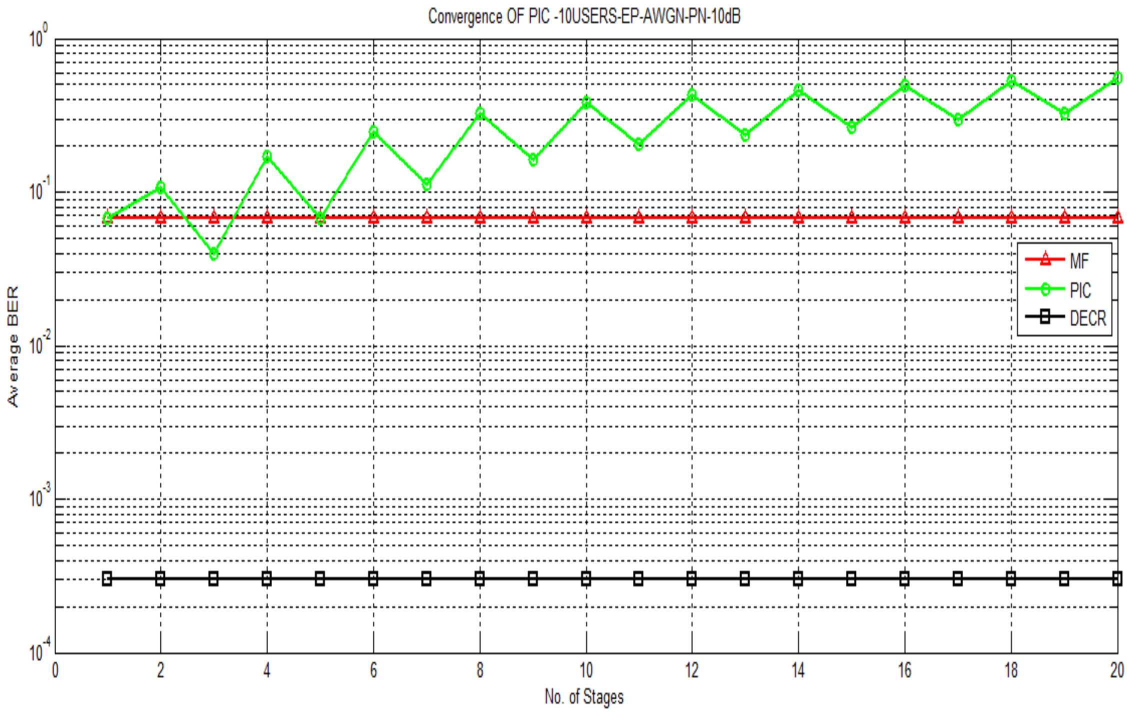

Figure 3-6 Convergence of PIC using PN Codes – 10 equal power users

Ping-pong behavior is explained by Rasmussen in [41]. The eigenvalues of the iteration matrix are closely related to the eigenvalues of the correlation matrix. The decision statistics in most cases exhibits an oscillating behavior around some point due to the negative eigenvalues of the iteration matrix which in turn originates from the extreme eigenvalues if the correlation matrix. A weight factor is suggested which can overcome the oscillating behavior at the expense of convergence rate.

The BER performance of the of Weighted PIC using random PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10 user system is shown in figure 3.7. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^4$ bits, using PN codes of length 31 and having equal received power. From this plot, we can see that the BER initially improves stage by stage.

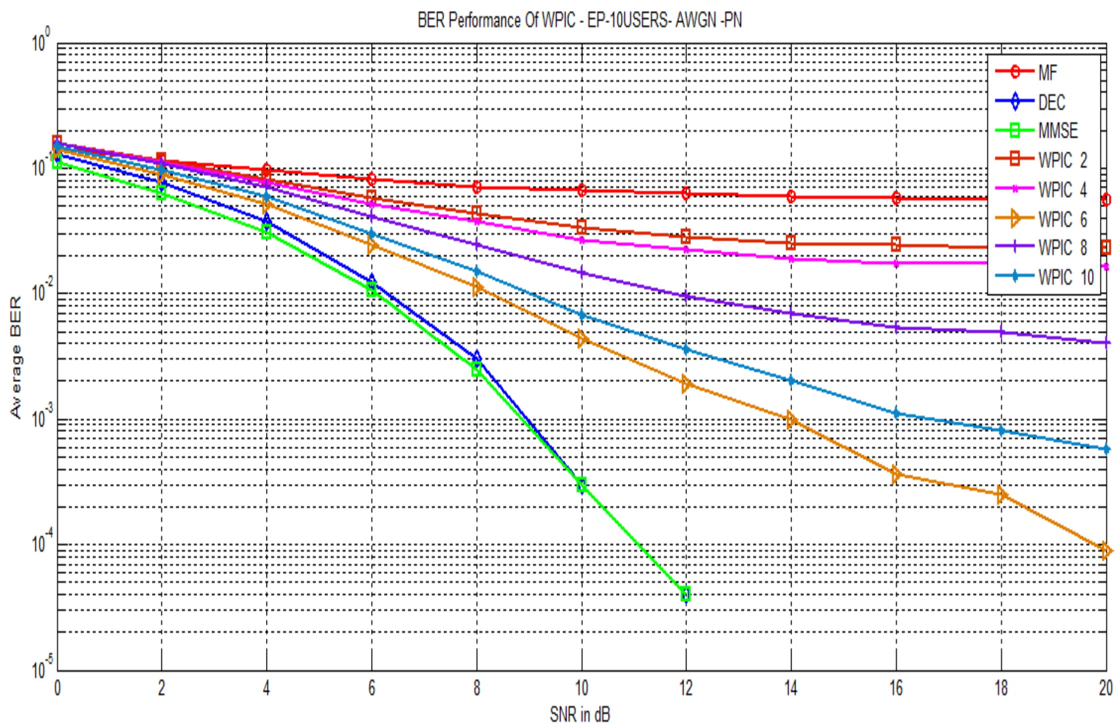

Figure 3-7 BER Performance of Weighted PIC using PN Codes – 10 equal power users

Convergence of Weighted PIC using random PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10 user system is shown in figure 3.8:

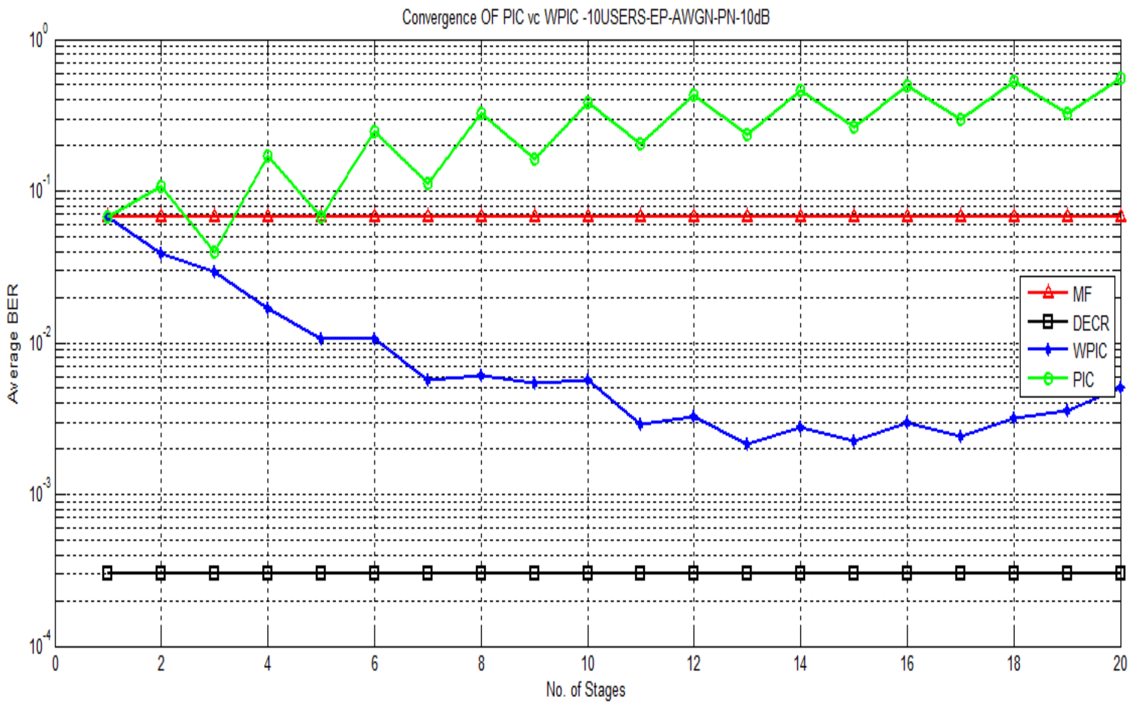

Figure 3-8 Convergence of Weighted PIC using PN Codes – 10 equal power users

As compared to figure 3.6 where the BER is diverging and exhibiting ping-pong behavior, the BER in WPIC scheme in figure 3.8 improves with number of stages and converges towards the decorrelator even when the eigenvalue of cross correlation is more than 2. We selected incremental weights stage wise in the weighted PIC scheme but these weighting factors are not optimal. The performance of the weighted PIC scheme can be further improved by selecting the optimal weights by using some adaptive technique like

Recursive least square (RLS), least mean square (LMS) algorithms or the proposed fuzzy weighted PIC scheme.

The BER performance of the of Weighted PIC using random PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10 user system is shown in figure 3.9. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting 1e4 bits, using PN codes of length 31 and having unequal received power.

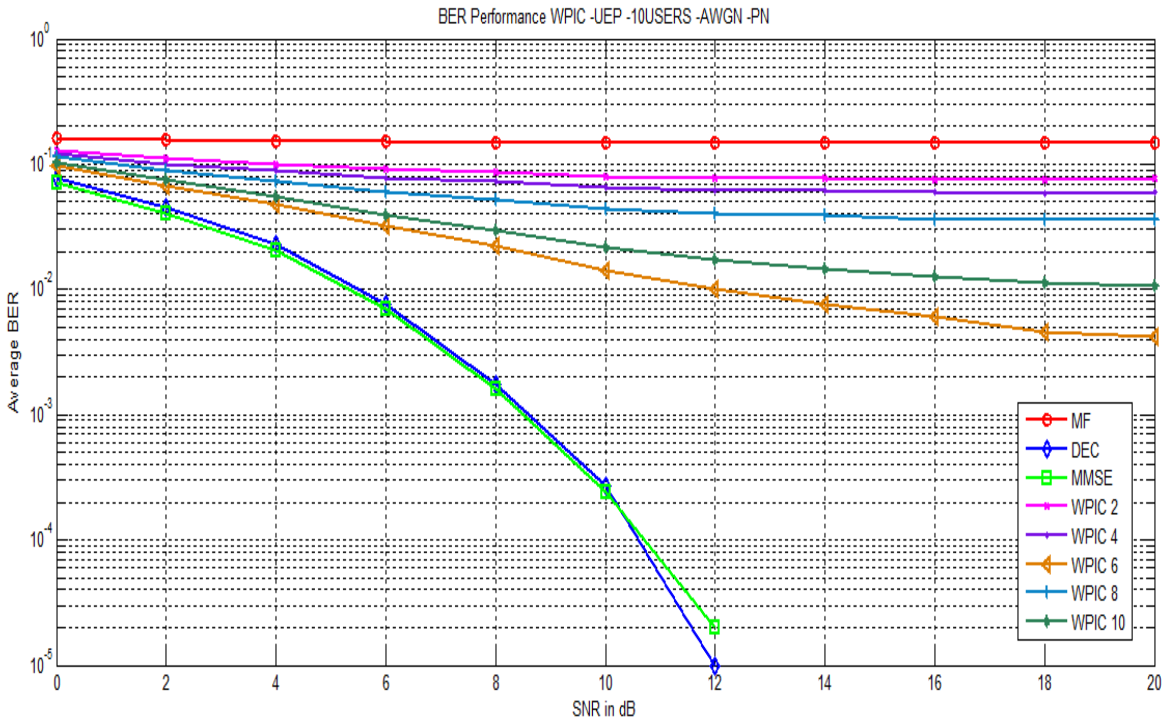

Figure 3-9 BER Performance of Weighted PIC using PN Codes – 10 unequal power users

Convergence of Weighted PIC using random PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10 user unequal power system is shown in figure 3.10:

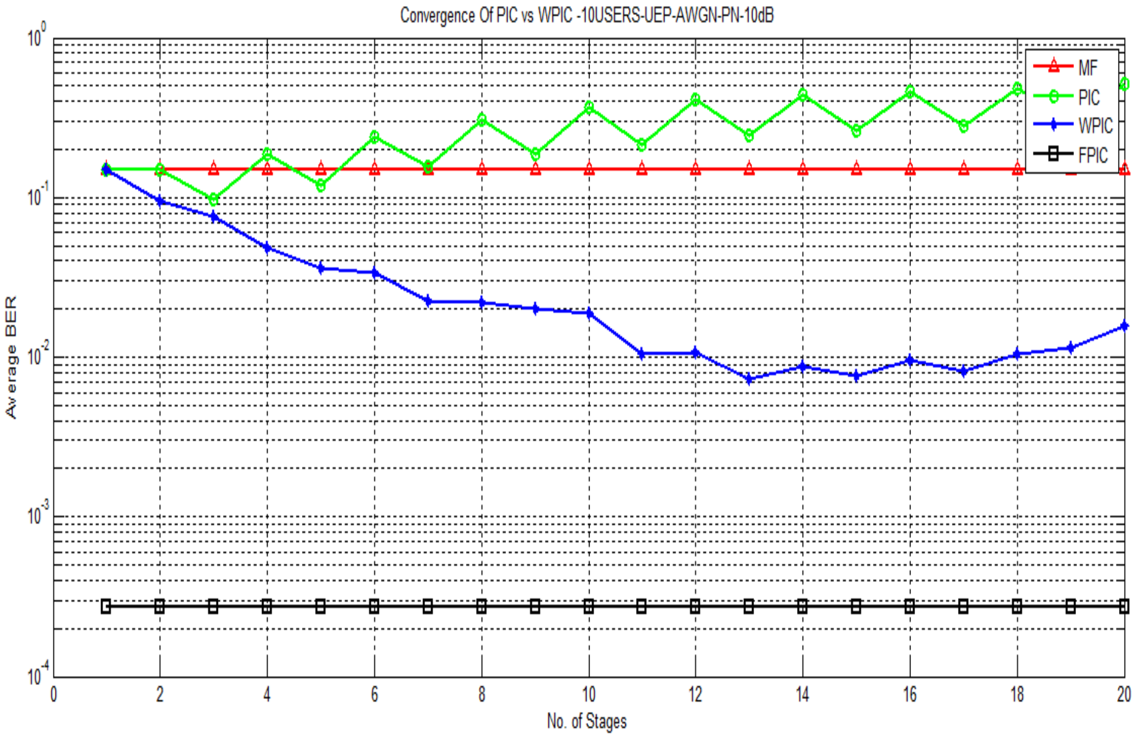

Figure 3-10 Convergence of Weighted PIC using PN Codes – 10 unequal power users

From the above convergence analysis in figure 3.10, we can see that the Weighted PIC is not diverging and the performance is slightly less compared to the equal power user case in figure 3.8.

The BER performance of the of Weighted PIC using Gold sequence having maximum eigenvalue of code correlation matrix less than 2 for a 10 user system is shown in figure 3.11. The simulations are performed for synchronous CDMA system using BPSK modulation under Rayleigh flat fading channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^5$ bits, using PN codes of length 31 and having equal received power.

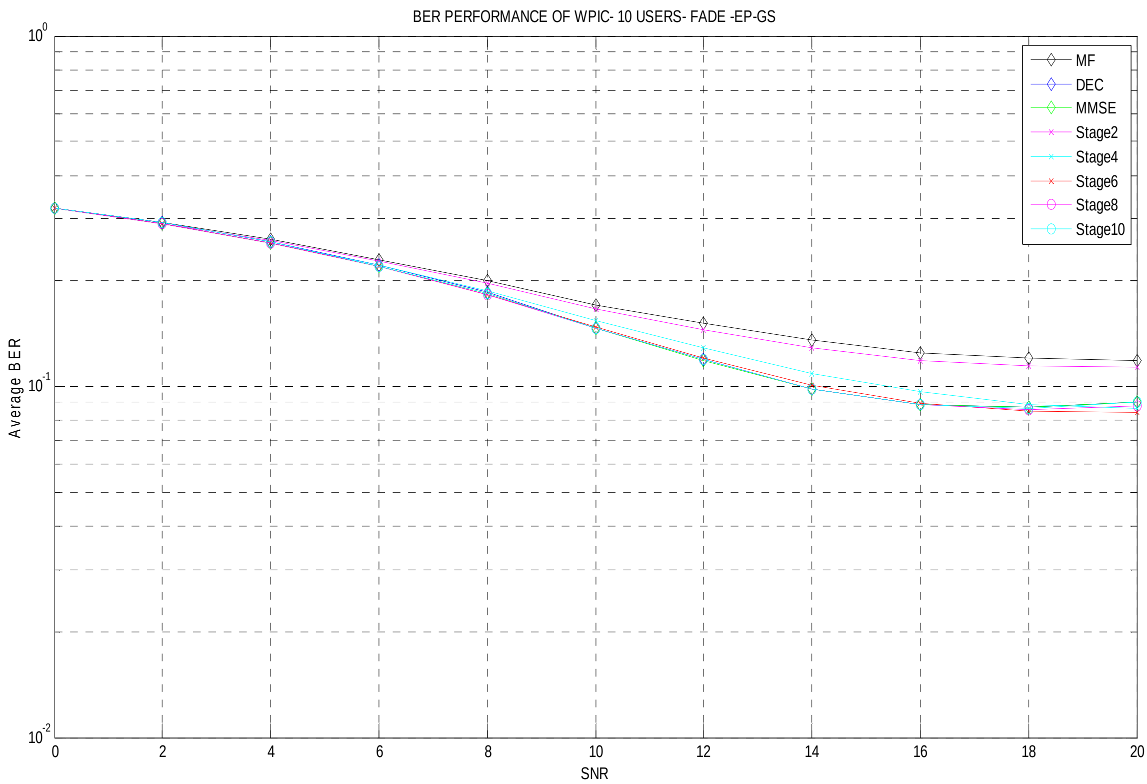

Figure 3-11 BER Performance of Weighted PIC using Gold Codes – 10 equal power users under fading channel

Convergence of Weighted PIC using random Gold sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10-user unequal power system under Rayleigh flat fading channel is shown in figure 3.12:

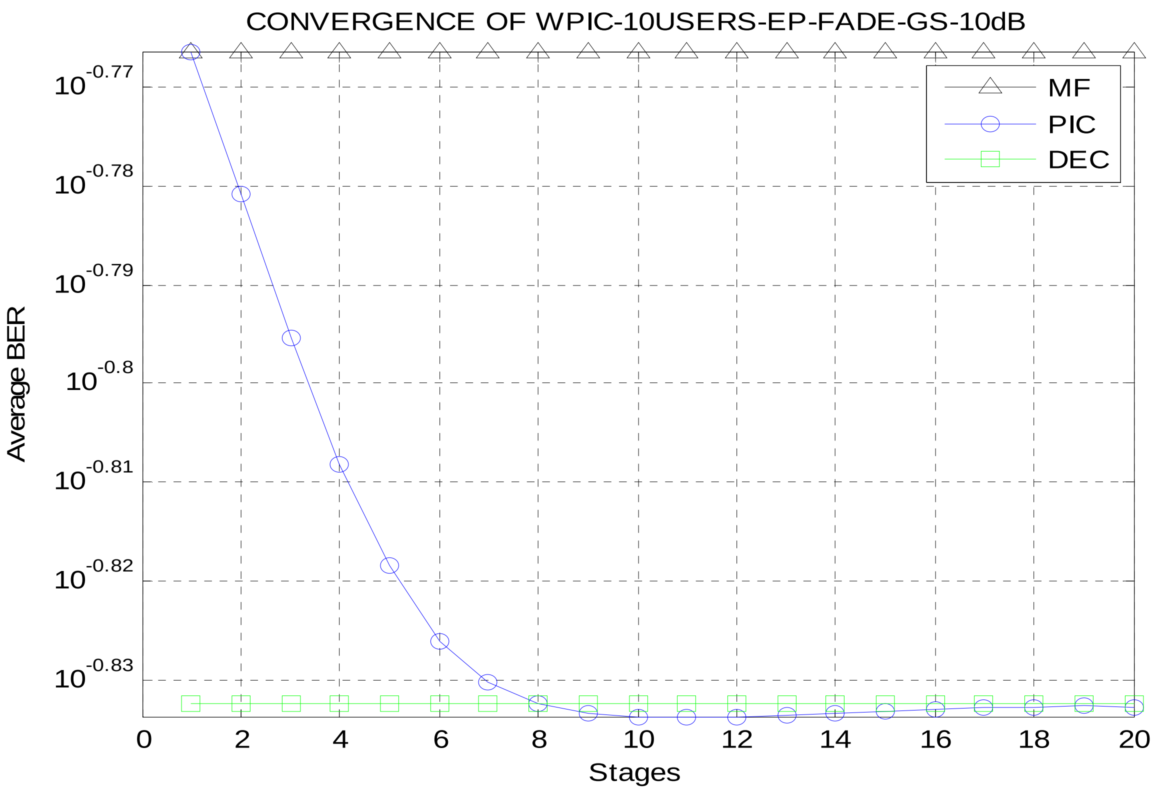

Figure 3-12 Convergence of Weighted PIC using Gold Codes – 10 equal power users under fading channel

From the above convergence analysis of Weighted PIC under Rayleigh flat fading channel, we see that it converges towards the decorrelator around stage 20.

From the above results, we can say that when eigenvalues of correlation matrix is greater than 2 the performance degrades as the number of stages are increased. To improve the convergence and BER performance of the PIC schemes, a fuzzy based Adaptive Multistage Parallel Interference Cancellation detector is proposed in this thesis. The improved adaptive PIC scheme uses a fuzzy inference system to determine the weighting factor in matrix approach. By making the weighting factors fuzzy adaptive we can achieve better stabilization and convergence of PIC. As the weights reflect the reliability of the decision statistics, we study the relation between the strength of weight on each path and the estimated interference statistics to determine the amount of interference to be removed. The determination of weight is achieved by employing a fuzzy inference system produces unique adaptive weights for each bit at every stage.

CHAPTER 4

FUZZY INFERENCE SYSTEM

Fuzzy logic has been recently used in many ways to enhance the performance of multiuser detection techniques. In [44], a fuzzy detector was proposed to estimate directly the bits of users without training process. Fuzzy logic has also been incorporated with linear decorrelating detector to combat impulsive noise [45]. Application of fuzzy logic in Parallel interference Cancellation was explored in detail first by Huang in [46]. This chapter presents an introduction to fuzzy logic and fuzzy inference systems. The main concepts in this field are introduced to facilitate understanding in fuzzy systems. Section 4.1 introduces fuzzy logic, membership functions, fuzzy sets and operations on fuzzy sets. Fuzzy inference system and its implementation are described in section 4.2.

4.1 Introduction to Fuzzy Logic

Fuzzy Logic was introduced in 1965 by Lotfi A. Zadeh, Professor of Computer Science at the University of California in Berkeley [27]. The word fuzzy means dull, misty, infinite, and vague. These synonyms match the content of fuzzy theory. Fuzzy Logic (FL) is a multi-valued logic, that allows intermediate values to be defined between conventional evaluations like true/false, yes/no, high/low, etc. Notions like rather tall or very fast can be formulated mathematically and processed by computers, in order to apply a more human-like way of thinking in the programming of computers [28]. The fuzzy theory provides a mechanism for representing linguistic variables such as “many,” “low,” “medium,” “often,” “few.” In general, the fuzzy logic provides an inference structure that enables appropriate human reasoning capabilities. On the contrary, the traditional binary set theory describes crisp events, events that either do or do not occur. It uses probability theory to explain if an event will occur, measuring the chance with which a given event is expected to occur. The theory of fuzzy logic is based upon the notion of relative grade of membership [29]. Figure 4.1 depicts the basic working system of fuzzy logic.

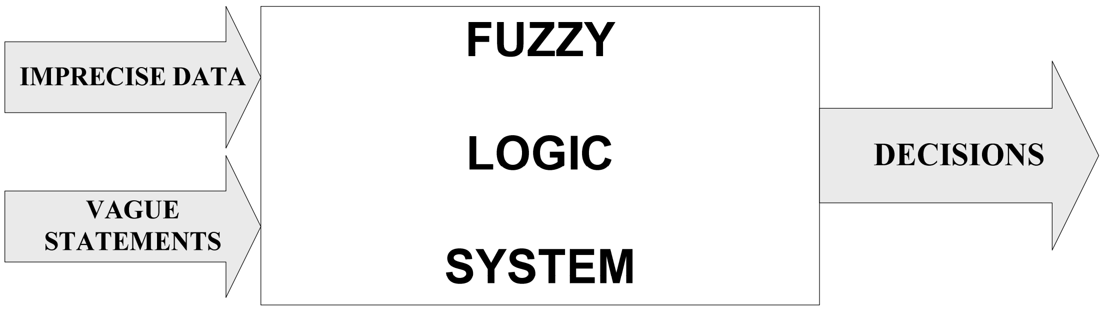

Figure 4-1: A Basic Fuzzy Logic Working System

There are many observations which make fuzzy logic an important tool for solving many problems:

- Fuzzy logic is conceptually easy to understand.
- The basic and mathematical concepts behind fuzzy reasoning are very simple. The “naturalness” of its approach makes fuzzy logic an “easy to understand” theory.
- Fuzzy logic is flexible.

- With any given system, it is easy to manage fuzzy logic and layer more functionality on top of it, without starting again from scratch.

- Fuzzy logic is tolerant of imprecise data. Everything is imprecise, if give a careful observation, but more than that, most things are imprecise even on careful inspection. Fuzzy reasoning builds this understanding into the process rather than tacking it onto the end.

- Fuzzy systems don't necessarily replace conventional methods. In many cases fuzzy systems augment them and simplify their implementation.

Fuzzy Logic has emerged as a profitable tool for the controlling and steering of systems and complex industrial processes, as well as for household and entertainment electronics, as well as for other expert systems and applications.

4.1.1 Fuzzy Sets and Crisp Sets

The very basic notion of fuzzy systems is a fuzzy set. In classical set theory we are familiar with what we call crisp sets. For example a subset A of the universe X is defined by its binary (0 or 1) characteristic function $\mu_A(x): x \rightarrow [0,1]$ such that $\mu_A(x)$ = 1 if $x \in A$ and $\mu_A(x)$= 0 if $x \notin A$. A fuzzy set is thus a set containing elements that have varying degrees of membership in the set. This idea is in contrast with classical or crisp set, because members of a crisp set would not be members unless their membership was full or complete, in that set (i.e., their membership is assigned a value of 1). In contrast, elements in a fuzzy set, because their membership need not be complete, can also be members of other fuzzy set on the same universe. Unlike conventional sets, the characteristic function of a fuzzy set is allowed to have values between 0 and 1, where $\underset{\sim}{A}$ is called a fuzzy set and $\mu_{\underset{\sim}{A}}$ is called the membership function of A [29].

If an element of universe, say *x*, is a member of fuzzy set $\underset{\sim}{A}$, then the mapping is given by $\mu_{\underset{\sim}{A}}(x) \in [0,1]$. This is the membership mapping and is shown in the figure 4.2.

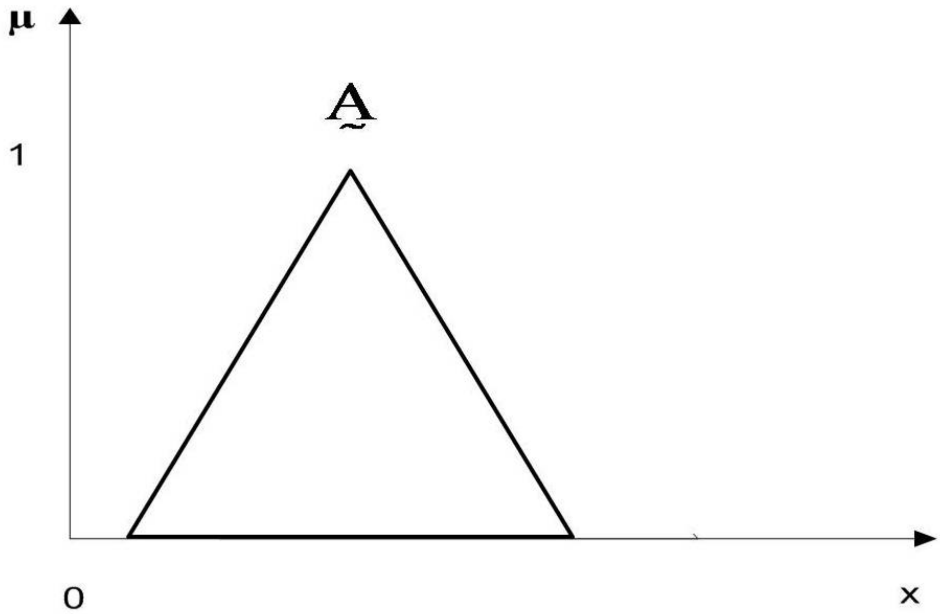

Figure 4-2 A typical Membership Function of Fuzzy Set

For example, the set of slow driving cars can be defined as cars that are driving less than or equal to 40 kilometers per hour. That can be defined with a characteristic function:

$$m_{slow}(x) \begin{cases} 1 : speed(x) \leq 40 \\ 0 : speed(x) > 40 \end{cases} \tag{4.1}$$

Let us assume that there exists a car, which is driving at a speed of 41 km per hour. The driver might think that he or she is still driving pretty slowly, but the function in (4.1) indicates that he or she is not driving slowly. It is very restricting to define a set of slow driving cars like above. In fuzzy set theory, elements belong to fuzzy sets to a certain degree. Degree of belonging to the set of slow driving cars can be defined with a membership function:

$$\mu_{slow}(x)\begin{cases} 1 & : speed(x) \leq 40 \\ \dfrac{1-(speed(x-40)}{20} & : 40 < speed(x) \leq 60 \\ 0 & : speed(x) > 60 \end{cases} \tag{4.2}$$

When the set of slow driving cars is defined as above, the car driving 41 km per hour would have a degree of 0.95 belonging to the set of slow driving cars. In function (4.2) there is a linguistic variable driving speed and three fuzzy sets. For every driving speed, degree of belonging to those fuzzy sets can be calculated and one driving speed can belong to many fuzzy sets with certain degrees [30]. In this way a fuzzy set is uniquely characterized by its membership function. The basic feature about membership functions and a brief description of it is explained in the next section.

4.1.2 Membership Functions (Fuzzy Sets)

Fuzziness in a fuzzy set is characterized by its membership functions. It classifies the element in the set, whether it is discrete or continuous. A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The input space is sometimes referred to as the universe of discourse. The membership functions can also be formed by graphical representations. The graphical representations may include different shapes. There are certain restrictions regarding the shapes used. The "shape" of the membership function is an important criterion that has to be considered. There are different methods to form membership functions. This section discusses some basic features of membership functions.

Features of Membership Function

The feature of the membership function is defined by three properties. They are:

(1) Core (2) Support (3) Boundary

The membership can take value between 0 and 1 (Sivanandam, Sumathi, and Deepa, 2007). Figure 4.3 briefly depicts the properties of membership function.

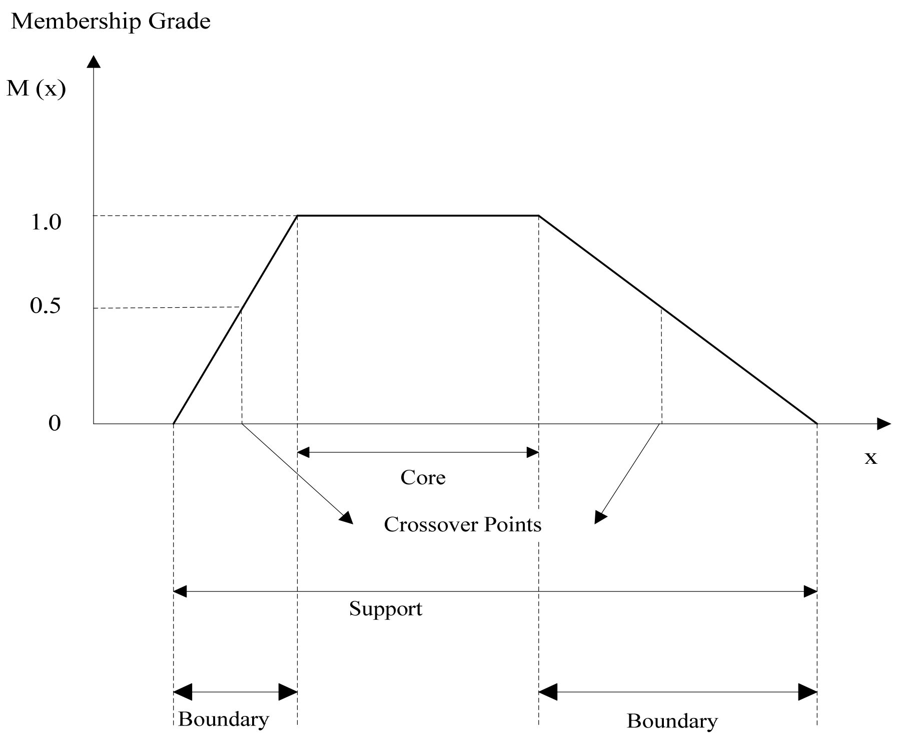

Figure 4-3 Features of Membership Function [31]

The various features of membership function are briefly described as follows:

(1) Core:

If the region of universe is characterized by full membership (1) in the set $\tilde{A}$ then this gives the core of the membership function of fuzzy at $\tilde{A}$. The elements, which have the membership function as 1, are the elements of the core, i.e., here $\mu_{\tilde{A}}(x) = 1$

(2) Support:

If the region of universe is characterized by nonzero membership in the set $\tilde{A}$, this defines the support of a membership function for fuzzy set $\tilde{A}$. The support has the elements whose membership is greater than 0 i.e. ($\mu_{\tilde{A}}(x) > 1$).

(3) Boundary:

If the region of universe has a nonzero membership but not full membership, this defines the boundary of a membership; this defines the boundary of a membership function for fuzzy set $\tilde{A}$. The boundary has the elements, whose membership is between 0 and 1, ($0 < \mu_{\tilde{A}}(x) < 1$).

(4) Cross-over Point:

The crossover point of a membership function is the elements in universe, whose membership value is equal to 0.5, ($\mu_{\tilde{A}}(x) = 0.5$)

(5) Height:

The height of the fuzzy set $\tilde{A}$ is the maximum value of the membership function, max $(\mu_{\tilde{A}}(x))$.

The membership functions can be symmetrical or asymmetrical. Membership value is between 0 and 1. The membership functions can have different shapes like triangle, trapezoidal, Gaussian, etc. Figure 4.6 indicates the different shapes of membership functions.

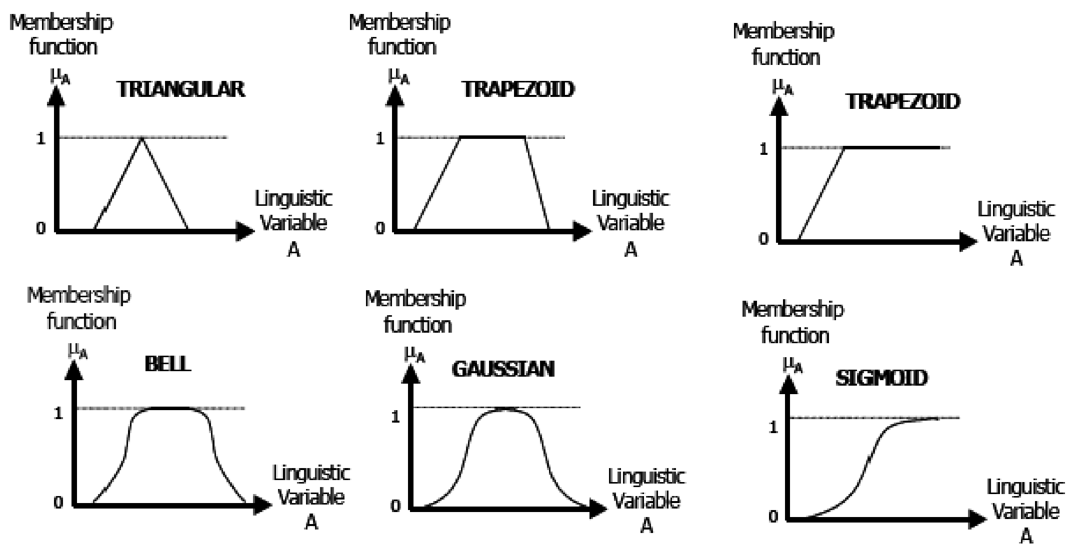

Figure 4-4 Common Shapes for Membership Functions [32]

4.1.3 Fuzzification

Fuzzification is an important concept in the fuzzy logic theory. Fuzzification in simple terms can be defined as the process where the crisp quantities are converted to fuzzy (crisp to fuzzy).

In any practical applications, in industries, in the field of construction etc., measurement of voltage, current, temperature, selection of a contractor etc., there might be a negligible error. This causes imprecision in the data. This imprecision can be represented by the membership functions. Hence fuzzification is performed.

Zadeh [27] says that rather than regarding fuzzy theory as a single theory, we should regard the process of "fuzzification" as a methodology to generalize any specific theory from a crisp (discrete) to a continuous fuzzy) form.

4.1.4 Fuzzy Rule-base

Rules form the basis for fuzzy logic to obtain the fuzzy output. The fuzzy rule-based system uses IF–THEN rules.

A single fuzzy IF-THEN rule assumes the form

IF *x* is *A* THEN *y* is *B* ,

where *A* and *B* are linguistic values defined by fuzzy sets on the ranges (universe of discourse) X and Y, respectively. The IF-part of the rule "*x* is *A*" is called the antecedent or premise, while the THEN-part of the rule "*y* is *B*" is called the consequent or conclusion.

As discussed before, fuzzy logic is a methodology that allows computing with words and no other modeling method offers such flexibility. The basic concept upon which "computing with words" is based is the "granule" that groups points that have similar features; in other words we can say a granule is a fuzzy set. A granule can be atomic (e.g., safe) or composite (e.g., very safe) and is represented by a word which is a fuzzy constraint on the variable. For example, for the proposition “The boy is young” the word “young” represents a granule that groups certain ranges of ages and act as a fuzzy constraint (i.e., fuzzy set) on the linguistic variable “age” [32].

Types of Statements in Fuzzy Rules

The fuzzy logic in the development of fuzzy rules uses three types of statements [29]: Assignment statements, Conditional statements, Unconditional statements.

The detailed descriptions of these statements are as follows:

1. **Assignment statements**

Assignment statements are those in which the variable is assigned a value. The variable and the value assigned are combined by the assignment operator “=.” The assignment statements are necessary in forming fuzzy rules. The value to be assigned may be a linguistic term. The assignment statement restricts the value of a variable to a specific equality. The examples of this type of statements are:

y = high,

Climate = cold

$a = 6$

$p = q + r$

2. Conditional statements

Conditional Statements are those in which, some specific conditions are mentioned. The examples of Conditional Statements are as follows,

IF x = *y* THEN both are equal,

IF Mark > 50 THEN pass, IF Speed > 1, 500 THEN stop.

3. Unconditional statements

Unconditional statements are those in which there is no specific condition that has to be satisfied. Some of the unconditional statements are:

Stop

Push the value

Aggregation of Fuzzy Rules

The fuzzy rule-based system may involve more than one rule. The process of obtaining the overall conclusion from the individually mentioned consequents contributed by each rule in the fuzzy rule is called as aggregation of rule. There are two methods for determining the aggregation of rules [33]:

1. Conjunctive system of rules

The rules that are connected by “AND” connectives satisfy the connective system of rules. In this case, the aggregated output may be found by the fuzzy intersection of all individual rule consequents.

2. **Disjunctive system of rule**

The rules that are connected by “OR” connectives satisfies the disjunctive system of rules. In this case, the aggregated output may be found by the fuzzy union of all individual rule consequents. To discuss how fuzzy rules are used for a real problem, let us reconsider the example of car speed discussed in section 4.1.5. It is a well known fact that the consumption of fuel can be reasoned based on the driving speed. For example, we can assume that: the rise in the driving speed indicates the rise in the consumption of fuel. Based on that knowledge we could build rules describing the relation between driving speed and consumption of fuel. An example of such a rule can be the following:

If Speed(x) ≤ 50 km per hour, then Consumption ≤ 7liters per 100km.

To make the rule more interpretable linguistic values are used:

If Speed(x) is slow, then Consumption is small.

The above rule stands for an example of a Mamdani rule system, which we will discuss in the upcoming sections of this thesis. Usage of the linguistic values "slow" and "small" makes the rule fuzzy and enables the computation with words [30].

4.1.5 Defuzzification

Defuzzification in simple words means conversion of fuzzy values to crisp values. The fuzzy results obtained or generated cannot be used in its original form for applications; hence it is necessary to covert the fuzzy quantities into crisp quantities for further processing. This can be achieved by using the defuzzification process. The defuzzification converts the fuzzy quantity to a crisp single-valued quantity or a set, or to the same form in which fuzzy quantity existed. The defuzzification process is also called as "rounding off" method [29].

There are many defuzzification methods mentioned in the literature. Sivanandam et.al. mentioned seven defuzzification methods [29]. Mizumoto [34] compared about ten defuzzification methods. Two of the more common techniques are the Centroid and Maximum methods. In the Centroid method, the crisp value of the output variable is computed by finding the variable value of the center of gravity of the membership function for the fuzzy value. In the Maximum method, one of the variable values at which the fuzzy subset has its maximum truth value is chosen as the crisp value for the output variable. There are several variations of the Maximum method that differ only in what they do when there is more than one variable value at which this maximum truth value occurs. One of these, the Average-of-Maxima method, returns the average of the variable values at which the maximum truth value occurs. There are five built in methods in the fuzzy logic tool box of MATLAB such as: Centroid, bisector, middle of maximum, largest of maximum, and smallest of maximum [35].

4.2 Fuzzy Inference Systems (Fuzzy Expert Systems)

4.2.1 Expert Systems

The Expert systems are computer programs that emulate the reasoning of a human expert or perform in an expert manner in a domain for which no human expert exists. Any expert system in general is made up of at least three parts: an inference engine, a knowledge base and a global memory. The knowledge contains the expert domain language for use in problem solving. The working memory, which acts as a store house, is used as a scratch pad and to store information gained from the user to the system. The inference engine uses the domain knowledge for use in problem solving. The inference engine uses the domain knowledge together with acquired information about a problem to provide an expert solution. There is plethora of expert systems developed in diverse fields and many are available commercially. However, practical expert systems typically reason with uncertain and imprecise information. There is no limit to sources of imprecision and uncertainty. The knowledge that they embody is often not exact in the same way that a human's knowledge is imperfect. The facts and the supplied information are drastically uncertain. Fuzzy expert systems developed through fuzzy reasoning can provide the basis for representing the imprecision inherent in an expert's knowledge. The Fuzzy Inference System or Fuzzy Expert System uses fuzzy sets and fuzzy logic reasoning process or knowledge representation scheme [36]. This section deals with a detailed study of Fuzzy Expert System.

The Fuzzy Inference System is an expert system that uses a collection of fuzzy membership functions and rules, instead of Boolean logic, to reason about data [37].

Fuzzy inference systems (FIS) are also known as fuzzy rule-based systems, fuzzy model, fuzzy expert systems, fuzzy associative memory, fuzzy logic controllers, and simply (and ambiguously) fuzzy systems, because of their multidisciplinary nature. This is a major unit of a fuzzy logic system. The decision-making is an important part in the entire system. The FIS formulates suitable rules and based upon the rules the decision is made. This is mainly based on the concepts of the fuzzy set theory, fuzzy IF–THEN rules, and fuzzy reasoning. FIS uses "IF. . . THEN. . . " statements, and the connectors present in the rule statement are "OR" or "AND" to make the necessary decision rules. When the

FIS is used as a controller, it gives a crisp output by using the defuzzification method. The whole FIS is discussed in detail in the following subsections [29]

4.2.2 Working of Fuzzy Inference System

Fuzzy inference system consists of a fuzzification interface, a rule base, a database, a decision-making unit, and finally a defuzzification interface. A Fuzzy Inference System with five functional units is described in the figure 4.7. The function of each unit is as follows:

A **rule base** containing a number of fuzzy IF–THEN rules;

A **database** which defines the membership functions of the fuzzy sets used in the fuzzy rules;

A **decision-making unit** which performs the inference operations on the rules;

A **fuzzification interface** which transforms the crisp inputs into degrees of match with linguistic values; and

A **defuzzification interface** which transforms the fuzzy results of the inference into a crisp output.

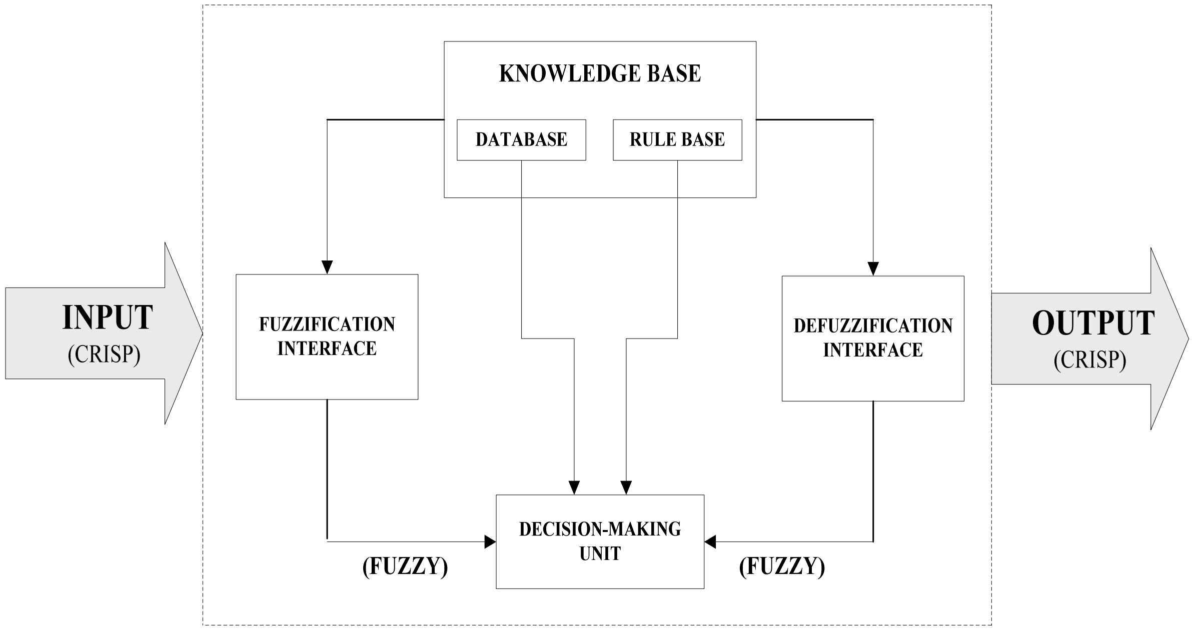

Figure 4-5 Working of Fuzzy Inference System [29]

From figure, 4.5 the working of fuzzy inference system in brief can be understood as follows:

The crisp input is converted into fuzzy quantities by using fuzzification method. After fuzzification the rule base is formed. The rule base and the database are jointly referred to as the knowledge base. Defuzzification is used to convert fuzzy value to the real world value, which is the needed output for use in practical system. In this thesis, Fuzzy controller of Fuzzy Tool Box of MATLAB is used for the fuzzification and defuzzification process.

4.2.3 Fuzzy Inference Methods

The most important two types of fuzzy inference method are Mamdani's fuzzy inference method, which is the most commonly used inference method. This method was introduced by Mamdani and Assilian [38]. Another well-known inference method is the so-called Sugeno or Takagi–Sugeno–Kang method of fuzzy inference process. This method was introduced by Sugeno [39]. This method is also called as TS or TSK method. We will use the short-form TSK in this thesis.

The main difference between the two methods lies in the consequent of fuzzy rules. Mamdani fuzzy systems use fuzzy sets as rule consequent whereas TS fuzzy systems employ linear functions of input variables as rule consequent.

The two methods are explained in detail as follows:

4.2.3. (a) Mamdani's Fuzzy Inference Method

Mamdani's fuzzy inference method is the most commonly used fuzzy methodology. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed by Mamdani [38] as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Mamdani's effort was based on Zadeh's [27] paper on fuzzy algorithms for complex systems and decision processes [31].

Mamdani type inference method, expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. It is possible, and in many cases much more efficient, to use a single spike as the output memberships function rather than a distributed fuzzy set. This is sometimes known as a singleton output membership function, and it can be thought of as a pre-defuzzified fuzzy set.

To compute the output of the Mamdani FIS given the inputs, six steps has to be followed:

- Determining a set of fuzzy rules
- Fuzzifying the inputs using the input membership functions
- Combining the fuzzified inputs according to the fuzzy rules to establish a rule strength.
- Finding the consequence of the rule by combining the rule strength and the output membership function.
- Combining the consequences to get an output distribution.
- Defuzzifying the output distribution

In this thesis for developing the "Fuzzy Inference System Model" we have used the Mamdani FIS method. The advantages of this method are explained in upcoming subsections of this research.

4.2.3. (b) Takagi-Sugeno's Fuzzy Inference Method

Sugeno fuzzy model also known as Sugeno–Takagi (or TSK) model was proposed by Takagi, Sugeno and Kang in an effort to formalize a system approach to generate fuzzy rules from an input-output data set. A typical fuzzy rule in a Sugeno fuzzy model has the format

IF x is A and y is B THEN $z = f(x, y)$,

Where A, B are fuzzy sets in the antecedent; $Z = f(x, y)$ is a crisp function in the consequent. Usually $f(x, y)$ is a polynomial in the input variables x and y, but it can be any other functions that can appropriately describe the output of the system within the fuzzy region specified by the antecedent of the rule. When $f(x, y)$ is a first-order polynomial, we have the first-order Sugeno fuzzy model. When f is a constant, we then have the zero-order Sugeno fuzzy model, which can be viewed as a special case of the Mamdani FIS [31]

Advantages of the Mamdani Method

- The Mamdani FIS is more intuitive than Sugeno FIS.
- The Mamdani FIS has more widespread acceptance than Sugeno FIS.
- The Mamdani FIS is well suited to human input and understanding.

Advantages of the Sugeno Method

- The Sugeno method works well with linear techniques.
- The Sugeno method is more suitable with optimization and adaptive techniques.
- The Sugeno method is well suited to mathematical analysis.

Example for Fuzzy Inference system:

With the knowledge gained from the discussion about fuzzy logic and fuzzy inference system, we consider an example to explain the implementation of the fuzzy logic in real life.

Considering a tipping problem to determine the right amount to tip the waiter after dinning in a restaurant. We can state the problem as: Giving a rating between 0 and 10 for quality of service and quality of food (where 10 is excellent), what should be the tip?

The fuzzy approach to solve this problem begins with defining the rules. The criteria to solve this problem can be listed as:

1. If service is poor, then tip is cheap

2. If service is good, then tip is average

3. If service is excellent, then tip is generous

4. If food is rancid, then tip is cheap

5. If food is delicious, then tip is generous

Combining the above rules we can summarize the rule base for the tipping problem to three rules as :

1. If service is poor or the food is rancid, then tip is cheap.
2. If service is good, then tip is average.
3. If service is excellent or food is delicious, then tip is generous.

Since there are two factors affecting the amount of tip, we use a two input, one output Mamdani FIS as shown in figure 4.6 to determine the tip. There are five steps in the fuzzy inference system: fuzzification of input variables, application of the fuzzy operator (AND or OR) in the antecedent, implication from the antecedent to the consequent, aggregation of the consequents across the rules, and defuzzification.

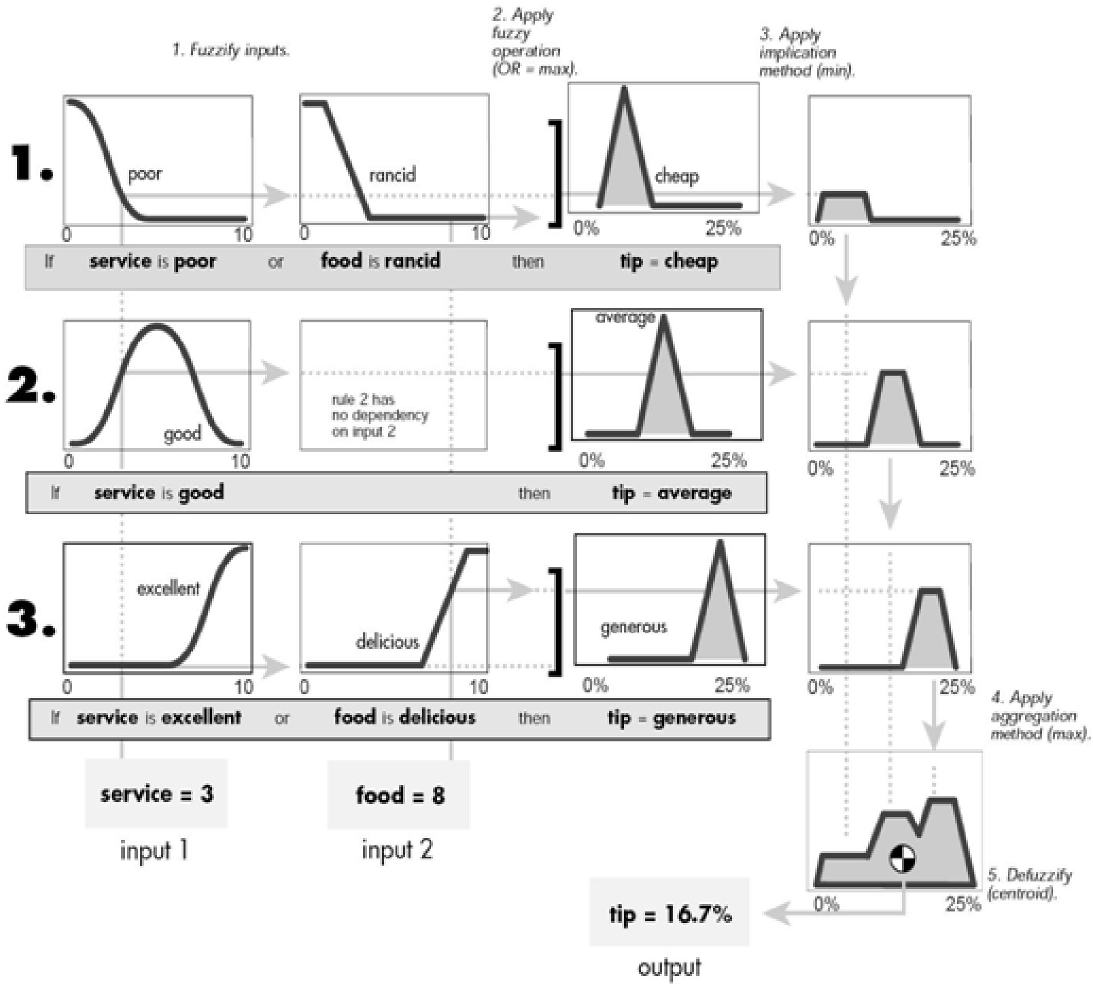

Figure 4-6 Tipping Example

Fuzzify inputs

The first step is to take the real inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions. We define the membership functions of the inputs as follows:

The first input to FIS, which is the quality of service, is divided into three membership functions defined by the linguistic variables poor, good and excellent. The other input quality of food is divided into two membership functions using two linguistic variables rancid and delicious. From the membership functions we can see how our rating on a scale of 10 qualifies to be member of a linguistic variable i.e. how our real world data is converted to a fuzzy input. The membership functions of the inputs are represented as illustrated in figure 4.6.

Apply the fuzzy operator

Once the inputs have been fuzzified, we know the degree to which each part of the antecedent has been satisfied for each rule. If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule. This number will then be applied to the output function. The input to the fuzzy operator is two or more membership values from fuzzified input variables. The output is a single truth value.

Apply Implication Method

Before applying the implication method, we must take care of the rule's weight. Every rule has a weight (a number between 0 and 1), which is applied to the number given by the antecedent. Generally this weight is 1 (as it is for this example) and so it has no effect at all on the implication process. Once proper weighting has been assigned to each rule, the implication method is implemented. A consequent is a fuzzy set represented by a membership function, which weights appropriately the linguistic characteristics that are attributed to it. The consequent is reshaped using a function associated with the antecedent (a single number). The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. Implication is implemented for each rule. Two built-in methods are supported, and they are the same functions that are used by the AND method: *min* (minimum), which truncates the output fuzzy set, and *prod* (product), which scales the output fuzzy set.

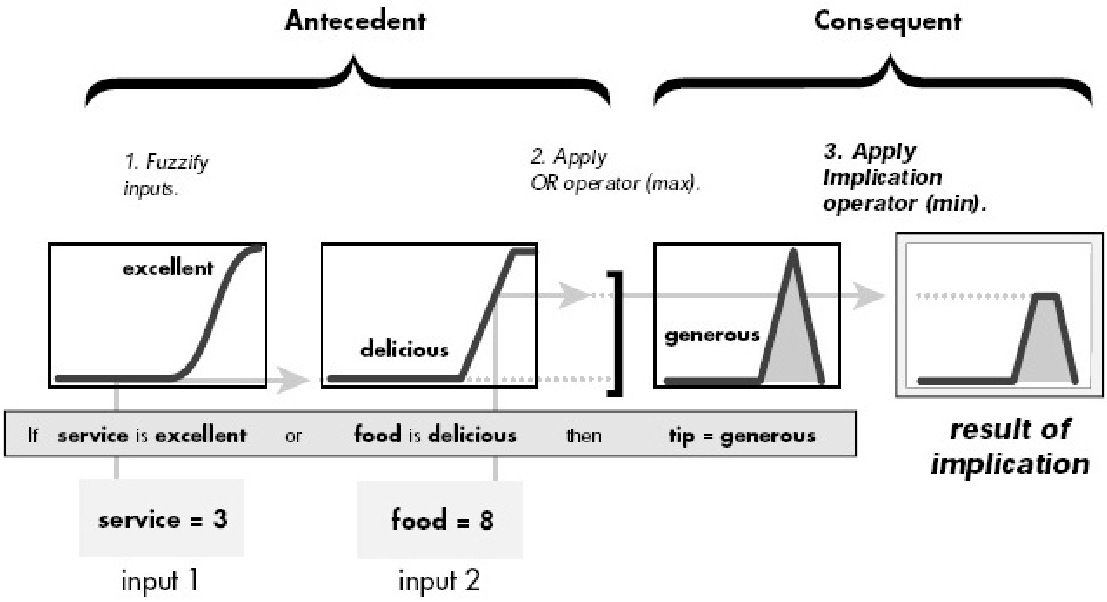

Figure 4-7 Implication Operation

The steps of applying the fuzzy operator and the Implication operation is illustrated in figure 4.7.

Aggregate All Outputs

Since decisions are based on the testing of all of the rules in an FIS, the rules must be combined in some manner in order to make a decision. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Aggregation only occurs once for each output variable, just prior to the fifth and final step, defuzzification. The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule. The output of the aggregation process is one fuzzy set for each output variable. In figure 4.6, all three rules have been placed together to show how the output of each rule is combined, or aggregated, into a single fuzzy set whose membership function assigns a weighting for every output (tip) value.

Defuzzify

The input for the defuzzification process is a fuzzy set (the aggregate output fuzzy set) and the output is a single number. As much as fuzziness helps the rule evaluation during the intermediate steps, the final desired output for each variable is generally a single number. However, the aggregate of a fuzzy set encompasses a range of output values, and so must be defuzzified in order to resolve a single output value from the set. There are five built-in methods supported: centroid, bisector, middle of maximum (the average of the maximum value of the output set), largest of maximum, and smallest of maximum. Perhaps the most popular defuzzification method is the centroid calculation, which returns the center of area under the curve as given in figure 4.6. In our example, we can see that the tip is 16.7% of final bill amount if the quality of service is 3 and quality of food is 8.

CHAPTER 5

FUZZY BASED WEIGHTED PIC

In this chapter, we propose a fuzzy inference system for determining the weighs in Fuzzy Weighted PIC scheme. In section 5.1, proposed detector is introduced and the fuzzy inference system for determining weights is developed. In Section 5.2, performance of proposed technique is investigated and compared with decorrelating detector.

5.1 Fuzzy based Parallel Interference Cancellation

As seen in the previous chapters, the effectiveness of Interference Cancellation is related to the reliability of the tentative decision involved in interference estimates. Since at low SNR's the probability of inaccurate estimation is greater than at higher SNR, we can conclude that reliability of the estimates depends on the Signal to Interference plus noise ratio (SINR). Thus, the optimal cancellation weights could be related to signal to noise ratios and the amplitude of the interferers. An adaptive weighted scheme for PIC is presented here which uses these factors to determine the weight of each interference cancellation path. According to the estimated interference reliability, the weight of the interfering users can be estimated by fuzzy logic system for each user based on the effective number of interferers and Signal to noise ratio of that user. The soft decision making ability of fuzzy inference system is briefed in Appendix B.

A multistage PPIC scheme based on FIS for multiuser detection is presented and the first two stages are shown in figure 5.1.

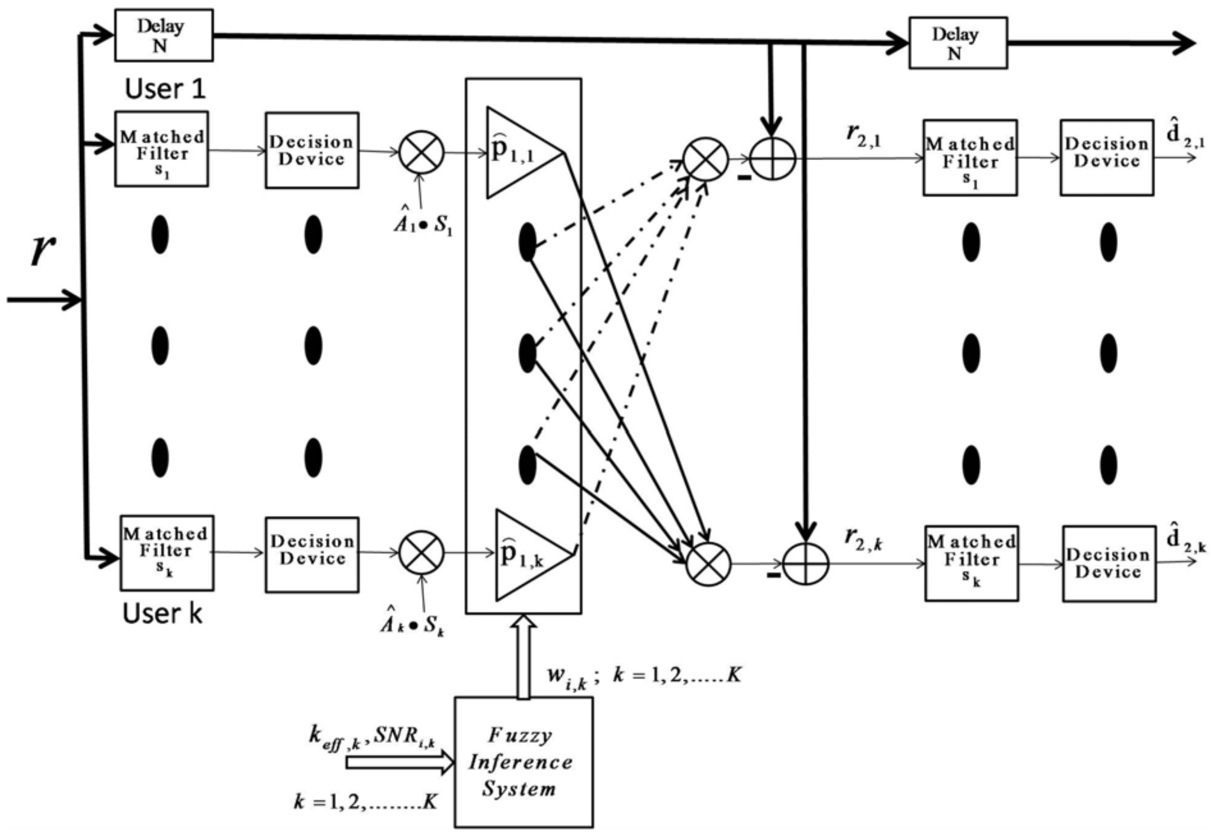

Figure 5-1 Proposed Fuzzy based Weighted PIC detector

The first stage in a proposed PPIC scheme is the matched filter and for second stage, the weight vector is evaluated by FIS. The detection and interference elimination procedure is similar to that of weighted PIC. We present the FIS system for the determining the weights as follows:

The effective numbers of users for user k are defined as:

$$k_{eff,k} = \frac{\sum_{i=1}^{K} \hat{a}_i}{\hat{a}_k} \tag{5.1}$$

where a_k is the amplitude of the k^{th} user.

The fuzzy inference system is given effective number of users and the SNR of each user as inputs to obtain the optimal weight. The fuzzy relationship between the output optimal weighting factors and inputs SNR and effective number of users is established by experiment. It requires investigation of BER with different weights for WPIC scheme by computer simulation and obtaining the optimal weights that results in least bit error rate. In our work, these relationships are obtained from [46].

In general, the relationship between the optimal weights and the inputs of FIS can be stated as- If K_{eff} increases or SNR decreases, the BER is increased and therefore the reliability of the desired user becomes poorer. In this case, a smaller cancellation weight should be selected for the next stage.

We use a Mamdani type Fuzzy inference system with SNR and effective number of users as two inputs and one output, the partial cancellation weight. Based on the relationships from experimentation in [46], the inputs to the FIS are fuzzified by using five Gaussian distribution membership functions to cover the entire universe of discourse of two inputs, K_{eff} and SNR_k and six for output p_k respectively, as shown in the figure. In figure 5.1, there are five linguistic terms: negative low (NL), zero (ZE), positive low (PL), positive medium (PM), and positive high (PH), chosen to cover the entire universe of discourse for SNR_k. In figure 5.2 there are five linguistic terms, very few (VF), few (F), medium (MED), many (M), and great many (GM), chosen to cover the entire universe of discourse for $K_{eff,k}$. In Fig.5.3, there are six linguistic terms, almost zero (AZ), small (S), medium (MED), large (L), very large (VL), and almost one (AO), chosen to cover its universe of discourse for $p_{k,i}$. The fuzzy set in each interval of $\lfloor C_i^-, C_i^+ \rfloor$ of Gaussian membership functions (MBF) is declared as F_i^l and the universe of discourse U can be expressed by:

$$\mu_{F_i^l}(x_i) = exp\left[-\frac{1}{2}\left(\frac{x_i - {x_i}^{-l}}{\sigma_i^l}\right)\right]$$

where $\forall\, l \in \{1,2,\dots,5\}$, $\forall\, i \in \{1,2,\dots,5\}$, $x_i \in \lfloor C_i^-, C_i^+ \rfloor$ and ${x_i}^{-l}$ is the mean and σ_i^l is the variance of Gaussian MBF, respectively.

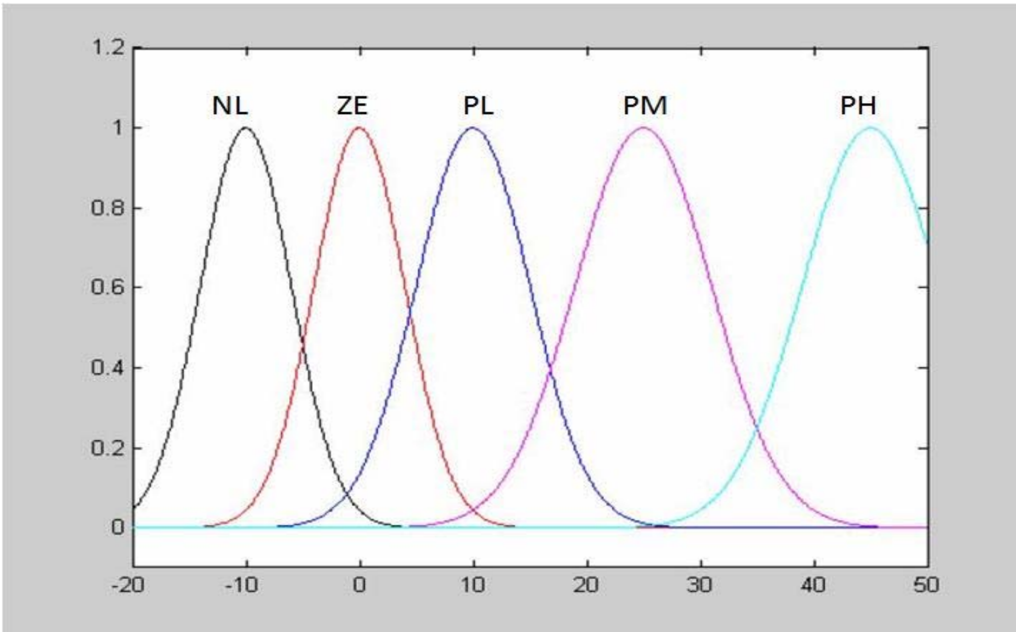

Figure 5-2 Membership function for SNR

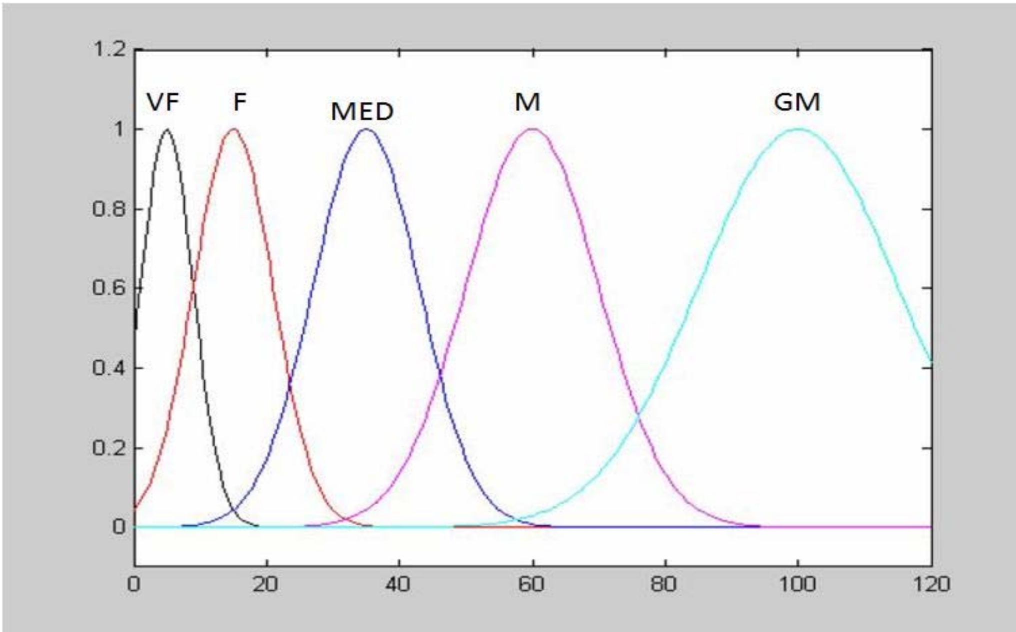

Figure 5-3 Membership function for effective number of users

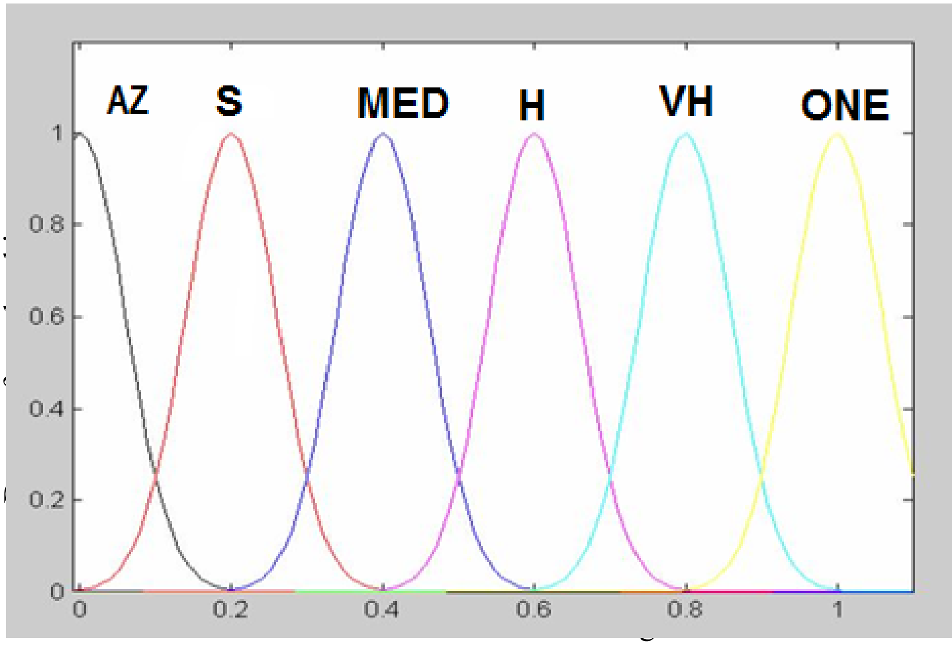

Figure 5-4 Membership function for partial weight

We can establish the fuzzy rules by matching input output pairs through an adaptive procedure. Hence the fuzzy control rules for a two input fuzzy system can be determined as:

R^j : IF SNR_K is $F_1^{l_1}$ and $K_{eff,k}$ is $F_2^{l_2}$, THEN $p_k = F_3^{l_3}$

Where j is the index of rule , $\forall j \in \{1, 2, \dots, 5\}$ and $F_1^{l_1}$, $F_2^{l_2}$, and $F_3^{l_3}$ are the linguistic terms of the two input variable SNR_K, $K_{eff,k}$ and one output variable p_k, respectively , $\forall l_1, l_2 \in \{1, 2, \dots, 5\}$ and $l_3 \in \{1, 2, \dots, 6\}$.

For example, if input membership function corresponding to the SNR is positive low (PL) and input membership function corresponding to effective number of users (K_{eff}) is few (F), then output membership function for weight estimation is high (H) and the crisp value for the weight is determined using Centroid method as described in chapter 4. The complete rule base with two inputs and one output is given as in table 5.1.

Keff ***SNR***	VF	F	MED	M	GM
NL	MED	S	AZ	AZ	AZ
ZE	H	MED	S	S	AZ
PL	ONE	H	MED	S	AZ
PM	ONE	VH	H	MED	AZ
PH	ONE	VH	H	MED	S

Table 5-1 Fuzzy Rule Base

There are twenty-five fuzzy IF-THEN rules for interference cancellation. As discussed in the previous chapter, the next step after rule base is to defuzzified to produce a useful output. Here the Centroid calculation for the defuzzification method is adopted as following:

$$p_k = \frac{\sum_{i=1}^{q} z_i U_{F_3^l}(z_i)}{\sum_{i=1}^{q} U_{F_3^l}(z_i)}$$

where q is the number of output-quantized levels under the aggregated MBF's, z_i is the amount of inference output at the i^{th} quantization level, and $U_{F_3^l}$ is the membership value of the output fuzzy set $F_3^{l_3}$. This defuzzified output is the weighting factor for the Weighted PIC. Thus, we can find the vector for the weighting factors about which the estimated interference is scaled before it is subtracted. The results are presented here to show that the performance of the Parallel interference cancellation scheme improves with number of stages and converges to that of decorrelating detector even when the eigenvalues are of correlation matrix are greater than 2.

5.2 Improved Fuzzy based Weighted PIC by gradually increasing the membership functions with stages. (VarMFFPIC)

Using the fact that the interference variation decreases with number of stages in PIC, an improved fuzzy PIC is presented in this section. Improved performance can be achieved by gradually increasing membership functions with number of stages. At first, three membership functions are used to cover the universe of discourse of the weighting factor and with number of stages, membership functions are also increased The membership functions along with the rule base at each stage is given below:

First stage- Three MF:

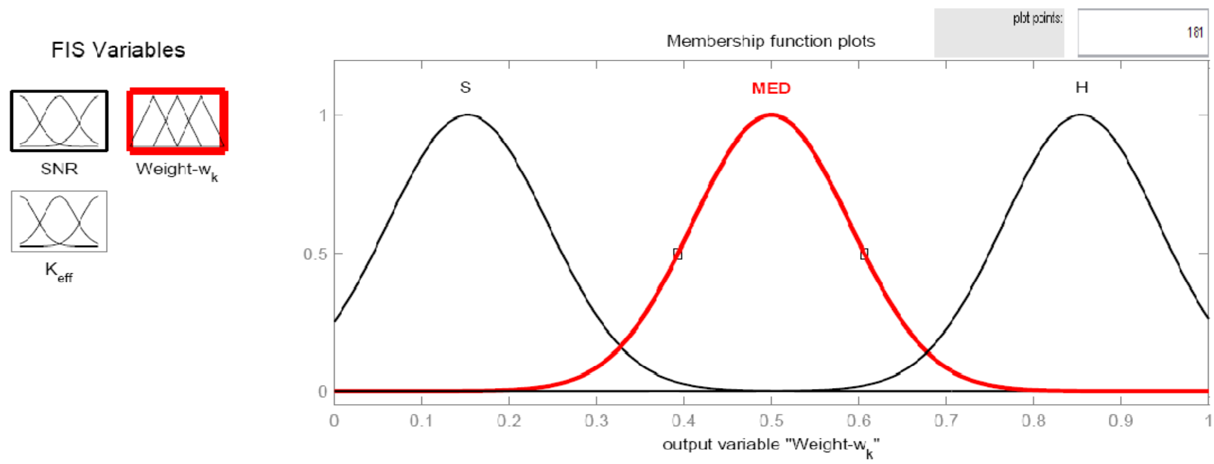

Table 5-2 First Stage Rule Base

Keff / ***SNR***	VF	F	MED	M	GM
NL	MED	S	S	S	S
ZE	MED	MED	MED	S	S
PL	H	MED	MED	S	S
PM	H	MED	MED	MED	S
PH	H	H	MED	MED	S

Second Stage – Four MF:

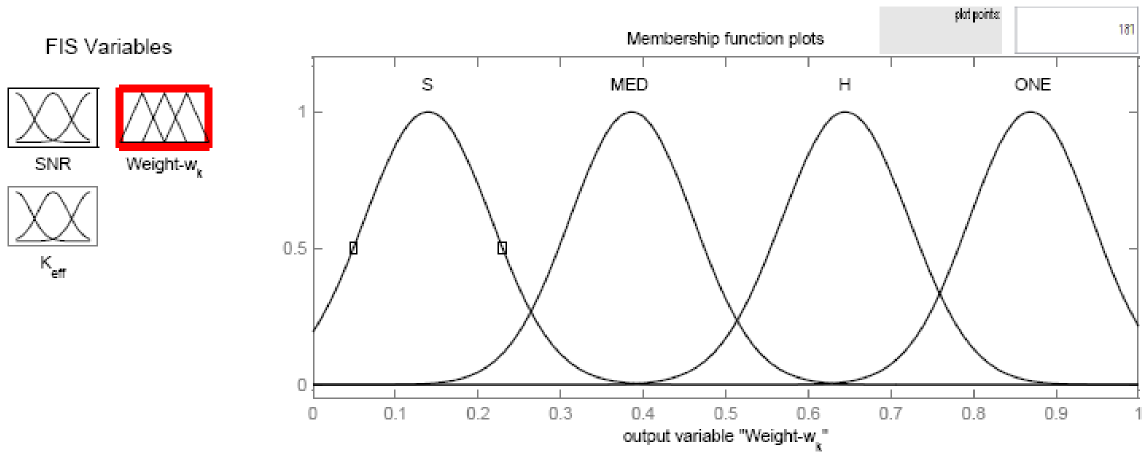

Table 5-3 Second Stage Rule Base

Keff / ***SNR***	VF	F	MED	M	GM
NL	MED	S	S	S	S
ZE	MED	MED	MED	S	S
PL	H	H	MED	S	S
PM	1	H	H	H	MED
PH	1	H	H	MED	S

Third Stage - Five MF:

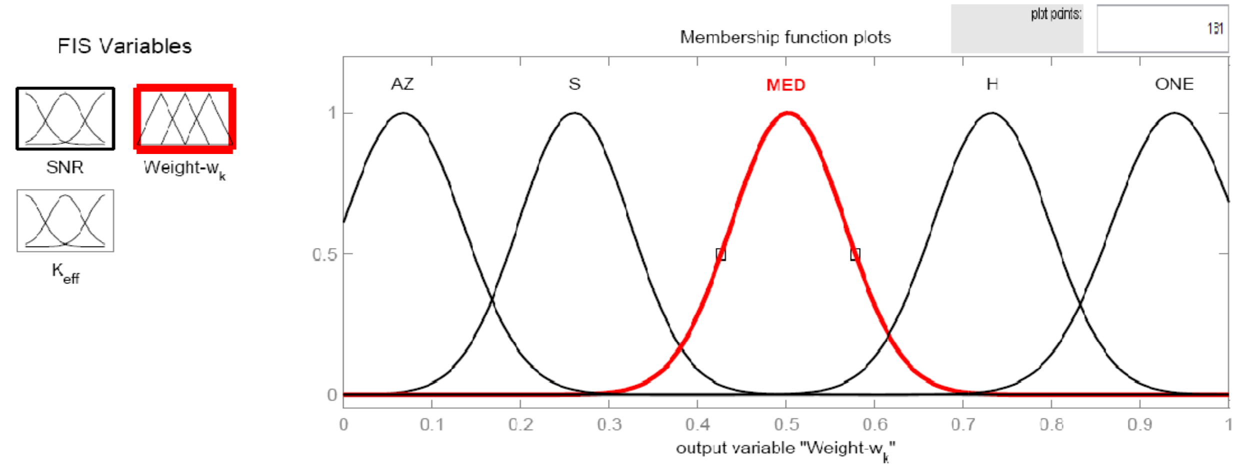

Table 5-4 Third Stage Rule Base

Keff / ***SNR***	VF	F	MED	M	GM
NL	MED	S	S	AZ	AZ
ZE	H	MED	MED	S	AZ
PL	ONE	H	MED	S	S
PM	ONE	H	H	MED	S
PH	ONE	ONE	H	MED	S

Fourth Stage - Six MF:

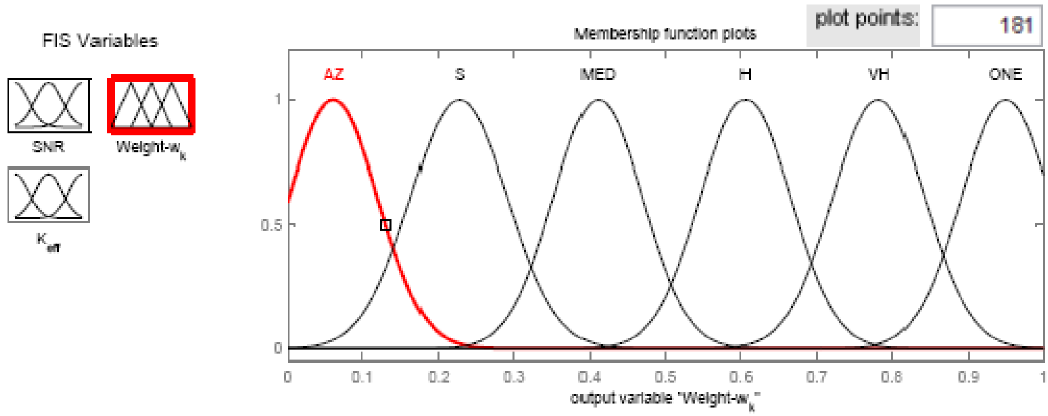

Table 5-5 Fourth Stage Rule Base

Keff / ***SNR***	VF	F	MED	M	GM
NL	MED	S	S	S	AZ
ZE	H	MED	MED	S	S
PL	1	VH	MED	S	S
PM	1	VH	H	MED	S
PH	1	VH	VH	MED	S

<u>Fifth Stage - Seven MF:</u>

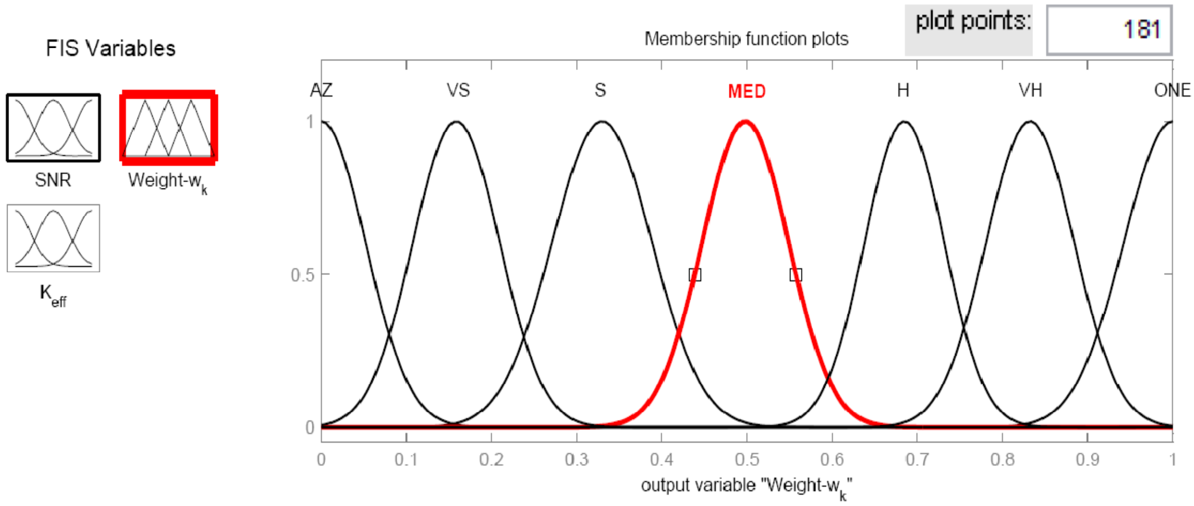

Table 5-6 Fifth Stage Rule Base

Keff ***SNR***	VF	F	MED	M	GM
NL	MED	S	S	VS	AZ
ZE	H	MED	MED	S	AZ
PL	ONE	VH	MED	S	VS
PM	ONE	VH	H	MED	S
PH	ONE	ONE	VH	MED	S

5.3 Performance of the Proposed Fuzzy based Weighted PIC

The Performance of the proposed Fuzzy based PIC detector is studied in AWGN and Rayleigh fading channel. The results obtained are as follows:

The BER performance of the of Fuzzy Weighted PIC using PN sequence having maximum eigenvalue of code correlation matrix greater than 2 for a 10 user system is shown in figure 5.5. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^4$ bits, using PN codes of length 31 and having equal received power. From this plot, we can see that the performance enhances with increasing number of stages and approaches the BER of the decorrelating detector.

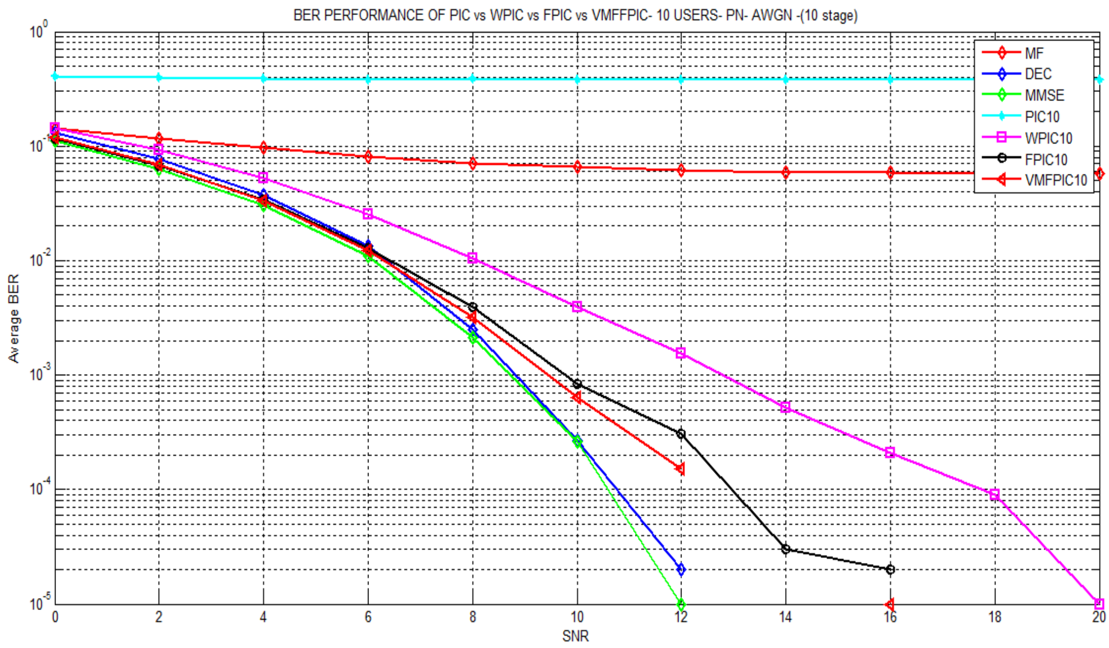

Figure 5-5 BER Performance of Fuzzy PIC using PN Codes- 10 equal power users

Convergence of above Fuzzy PIC using PN sequence having maximum Eigenvalue of code correlation matrix greater than 2 for a 10-user system in AWGN channel is shown in figure 5.6. The result indicates that FPIC is not divergent even when the Eigenvalues are greater than 2 and it converges towards decorrelating detector. The VarMF FPIC is performing better as it converges faster and offers lower BER than FPIC.

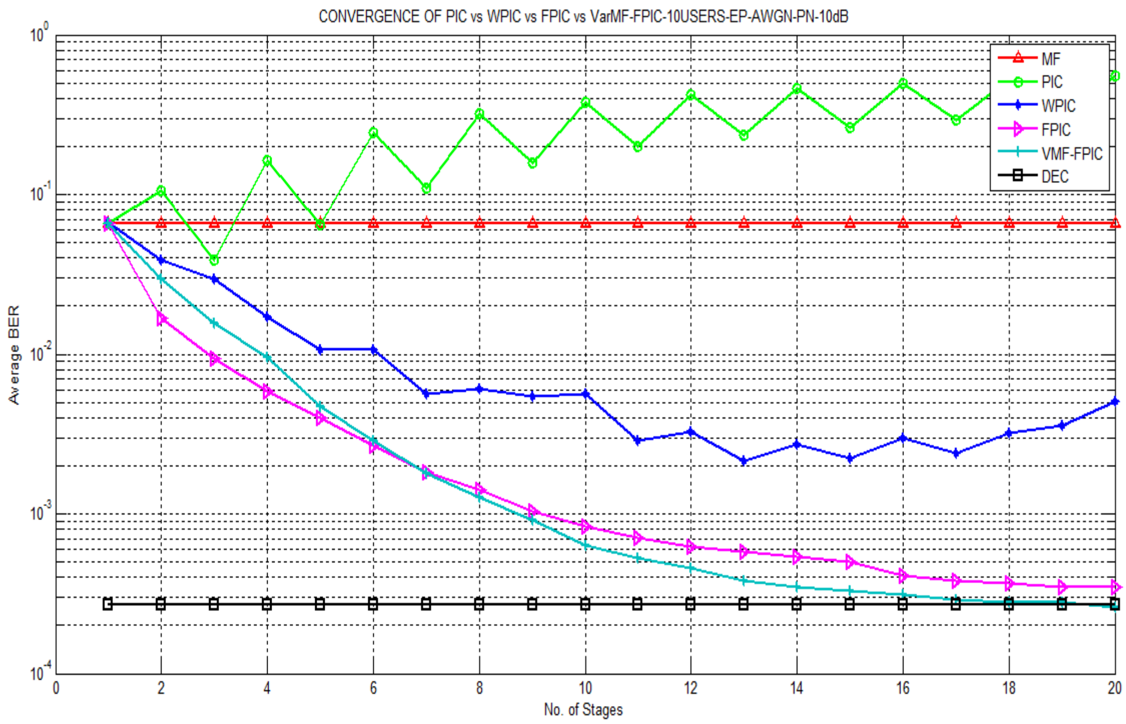

Figure 5-6 Convergence of Fuzzy PIC using PN Codes- 10 equal power users

On comparison of figure 3.6, 3.8 and 5.6, we can clearly see that the performance of our fuzzy PIC scheme is better and convergence is achieved in lesser number of stages.

Convergence of Fuzzy PIC using PN sequence having maximum Eigen value of code correlation matrix greater than 2 for a 25-user system in AWGN channel is shown in figure 5.7.

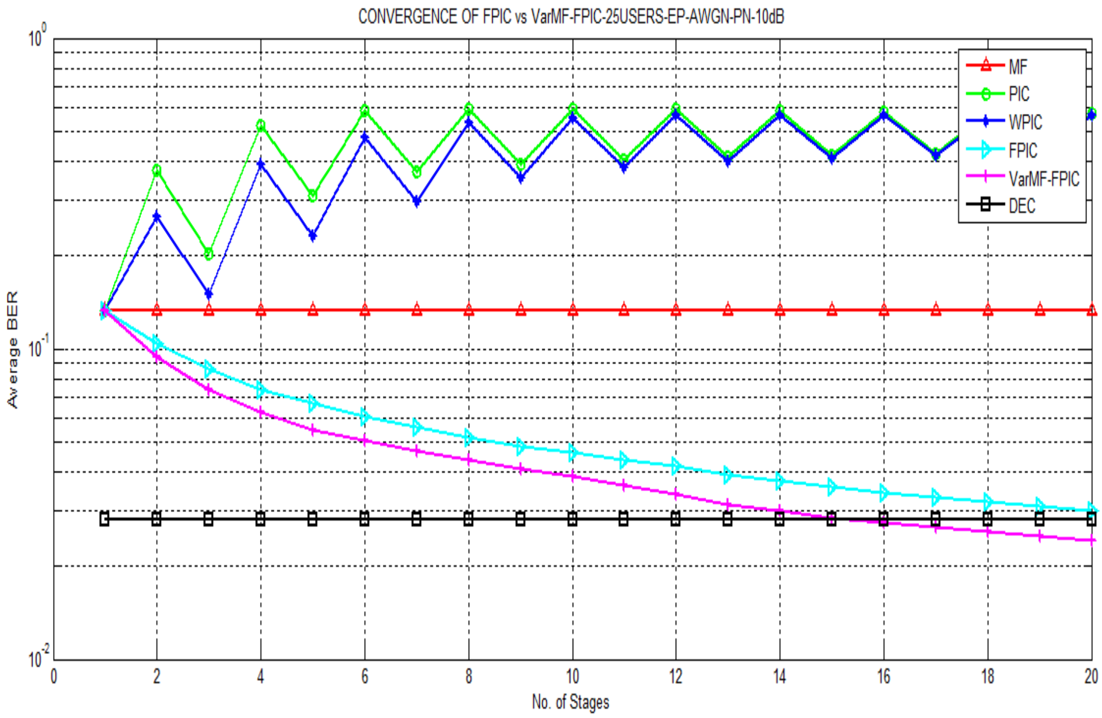

Figure 5-7Convergence of Fuzzy PIC using PN Codes- 25 equal power users

The result in figure 5.7 indicates that PIC and Weighted PIC are divergent as the eigenvalues of the cross correlation matrix are greater than 2 and as the system load becomes high. However, the Fuzzy PIC and the VMF Fuzzy PIC converges towards the decorrelating detector even for highly loaded system.

The BER performance of the of Fuzzy PIC using signature sequence having maximum eigenvalue of code correlation matrix less than 2 for a 10 user system is shown in figure 5.8. The simulations are performed for synchronous CDMA system using BPSK modulation in AWGN channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^5$ bits, using PN codes of length 31 and having equal received power.

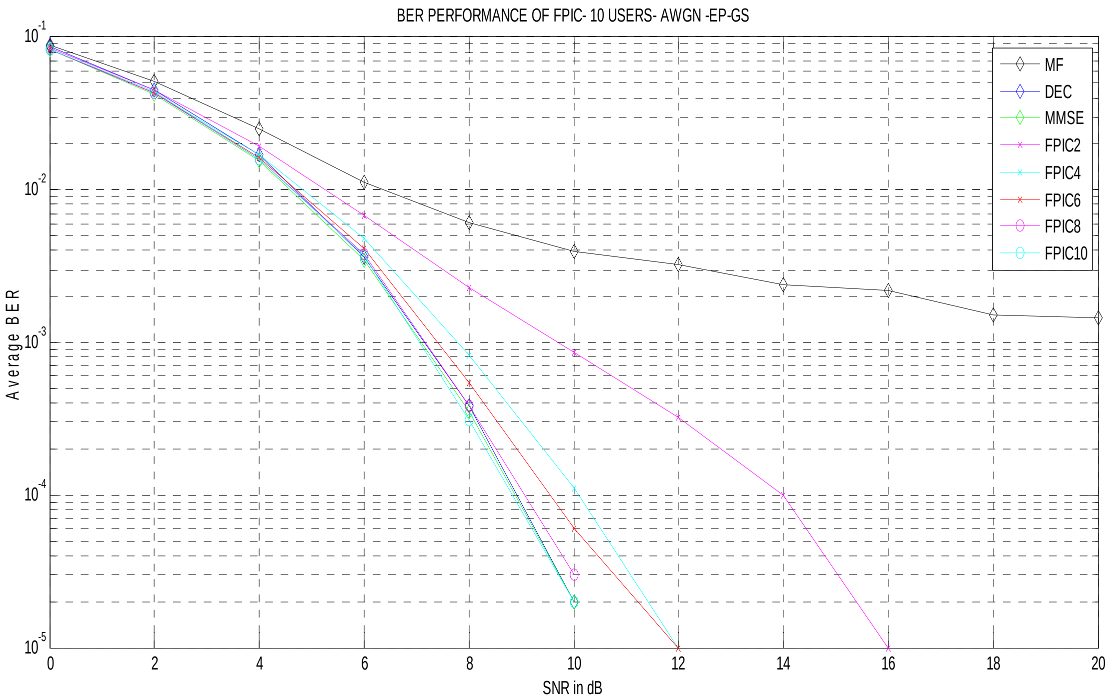

Figure 5-8 BER Performance of Fuzzy PIC using Gold Codes- 10 equal power users

Convergence of Fuzzy PIC using signature sequence having maximum eigenvalue of code correlation matrix less than 2 for a 10-user system with equal received power in AWGN channel is shown in figure 5.9. We can see that the FPIC converges in 12 stages.

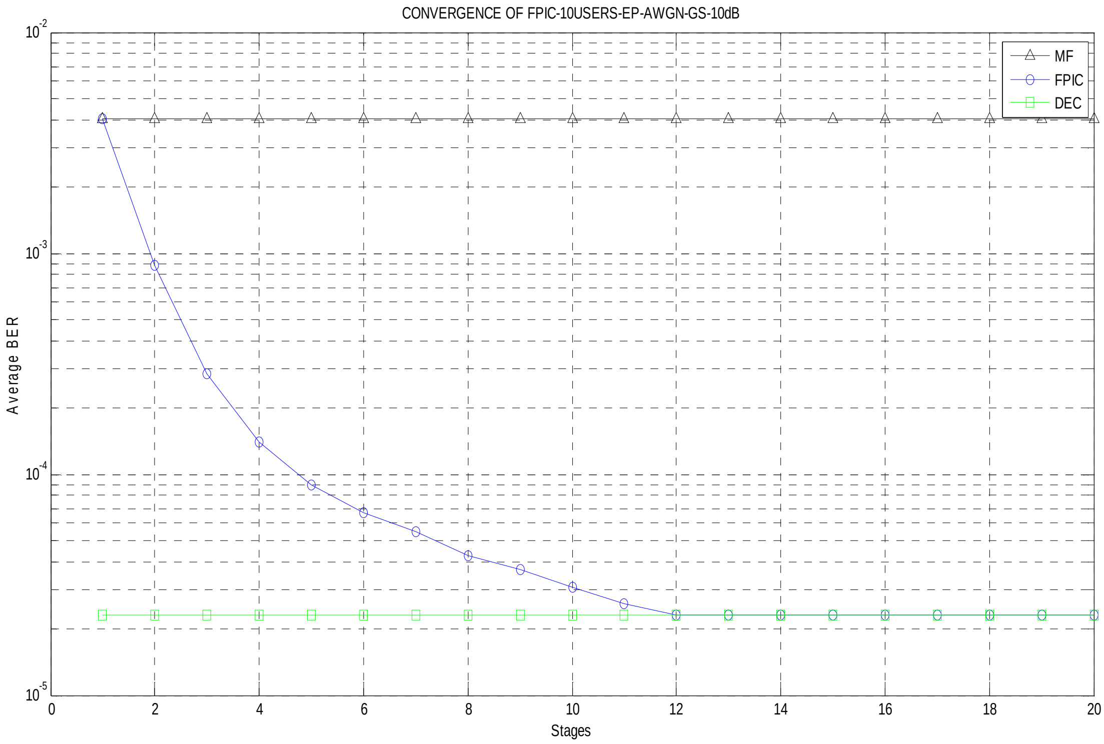

Figure 5-9 Convergence of Fuzzy PIC using Gold Codes- 10 equal power users

Comparison of figures 3.4 and 5.9 indicates that the fuzzy PIC scheme converges in 12 stages as compared to 14 stages for PIC when the eigenvalues of the code correlation matrix are less than two.

The BER performance of the of Fuzzy Weighted PIC using signature sequence having maximum eigenvalue of code correlation matrix less than 2 for a 10 user system is shown in figure 5.10. The simulations are performed for synchronous CDMA system using BPSK modulation in flat Rayleigh fading channel. BER is evaluated in SNR range 0-20 dB where each user is transmitting $1*10^5$ bits, using PN codes of length 31 and having equal received power.

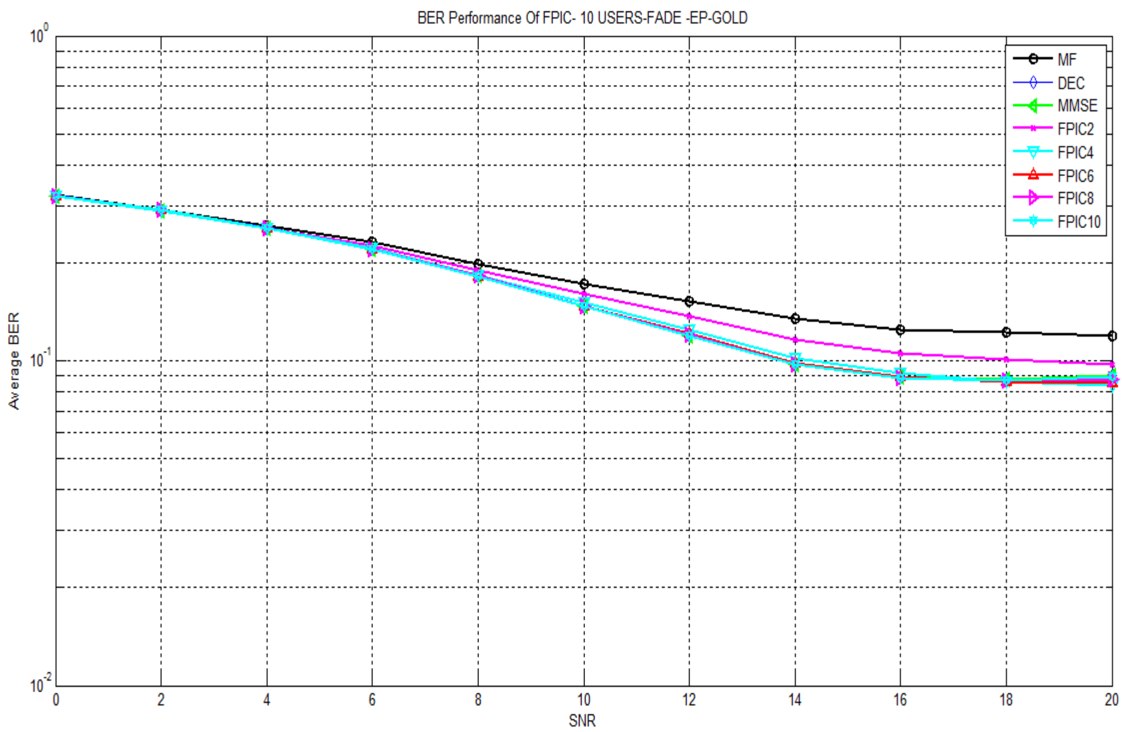

Figure 5-10 BER Performance of Fuzzy PIC using Gold Codes- 10 equal power users under Rayleigh flat fading Channel

Convergence of Fuzzy PIC using random GS sequence for a 10-user system with equal received power in Rayleigh fading channel is shown in figure 5.11.

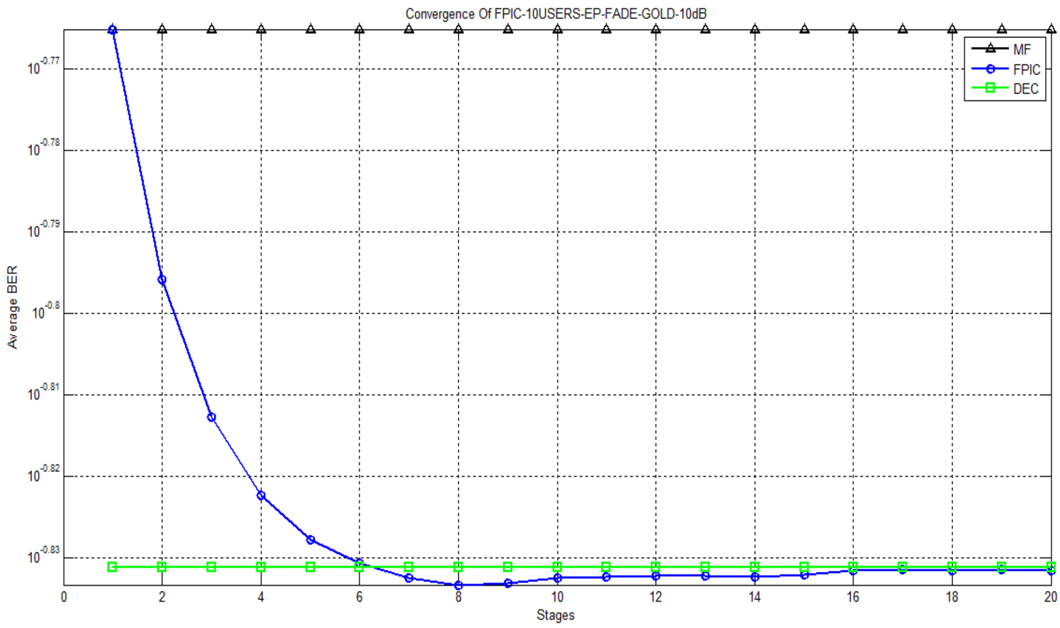

Figure 5-11 Convergence of Fuzzy PIC using Gold Codes- 10 equal power users under Rayleigh flat fading Channel

We can see that the Fuzzy PIC under fading channel condition performs better than weighted PIC scheme given in figure 3.12 which converges in 19 stages whereas FPIC converges is 16 stages.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusion

In this thesis, we studied the matrix algebraic analysis of parallel interference cancellation. A fuzzy logic system is introduced in the multistage PIC scheme to estimate the interference cancellation weights. The proposed technique is equipped with a set of adaptive weights that are selected through a fuzzy inference system to reduce the poor mutual user interferences estimates in the initial stages that result in reduced performance of the conventional multistage schemes. The need for adaptive weighting factors for each user at every stage is described. It is shown that the fuzzy PIC estimation of the weighting factors gives improved performance. Condition for the convergence of PIC is studied and the convergence of the proposed technique is compared in AWGN and Rayleigh fading channel. It is conferred from the simulation results that the performance of Fuzzy PIC improves with the number of stages and approaches to that of the decorrelating detector.

6.2 Future Work

Previous work shows that non-linear decision functions in the intermediate stages such as hyperbolic tangent, clip function or even hard decision may perform much better. This could be studied to achieve improved performance.

In this work, we studied the synchronous model of CDMA system using the matrix algebra. As asynchronous model is more realistic for the uplink channel of a cellular mobile system, the performance of the proposed scheme could be studied in asynchronous system.

We have considered the shape of membership functions as Gaussian. Other membership functions may be used to study the effect of the shapes on membership functions on the performance of fuzzy PIC.

The optimal number of membership functions for a given stage and a formula to determine the appropriate width of the membership function will give further significant improvement in the performance and is further topic of interest.

Appendix

6.3 A. Relationship between Steepest Descent Method and Weighted PIC [24]

A linear detector $\boldsymbol{G}$ is an N x K linear matrix filter $\boldsymbol{G} = (\boldsymbol{g_1}, \boldsymbol{g_2}, \boldsymbol{g_3}, \dots, \boldsymbol{g_K})^{\boldsymbol{T}}$ where $\boldsymbol{g_1}, \boldsymbol{g_2}, \boldsymbol{g_3}, \dots, \boldsymbol{g_K}$ are column vectors of length N. The filter output is then the following estimate of the transmitted data symbols:

$$\boldsymbol{y} = \boldsymbol{G^H r} = \boldsymbol{G^H}(\boldsymbol{SAd} + \boldsymbol{n}) \tag{3.18}$$

The corresponding Mean Square Error (MSE) is given by

$$J = E\,\{\|\boldsymbol{y} - \boldsymbol{d}\|^2\} = E\,\{\|\boldsymbol{G^H r} - \boldsymbol{d}\|^2\} = \sum_{k=1}^{K} E\,\left\{\left|\boldsymbol{g_k^H r} - \boldsymbol{d_k}\right|^2\right\} \tag{A.1}$$

Differentiating with respect to $\boldsymbol{g_k}$ yields:

$$\begin{aligned} \frac{\partial J}{\partial \boldsymbol{g_k}} &= E\left\{\frac{\partial\left|\boldsymbol{g_k^H r} - \boldsymbol{d_k}\right|^2}{\partial \boldsymbol{g_k}}\right\} \\ &= E\left\{\boldsymbol{r}^*(\boldsymbol{g_k^H r} - \boldsymbol{d_k})^2\right\} \\ &= E\,\{(\boldsymbol{rr^H})^*\}\,\boldsymbol{g_k^*} - E\,\{\boldsymbol{d_k}\,\boldsymbol{r}^*\} \\ &= [\boldsymbol{SS^H} + \sigma^2\,\boldsymbol{I}]^*\boldsymbol{g_k^*} - \boldsymbol{a_k^*} \end{aligned} \tag{A.2}$$

The gradient with respect to $\boldsymbol{g_k}$ is then

$$\nabla_k J = 2\,\frac{\partial J}{\partial g_k} = 2([\boldsymbol{AA^H} + \sigma^2\,\boldsymbol{I}]^*\boldsymbol{g_k^*} - \boldsymbol{a_k^*}) \tag{A.3}$$

The steepest descent method gives the following recursion for finding the minimum MSE

$$\boldsymbol{g}_{\boldsymbol{k}}^{*}(i) = \boldsymbol{g}_{\boldsymbol{k}}^{*}(i-1) - \frac{1}{2}\, p_i \nabla_k J$$

$$= [\boldsymbol{I} - p_i(\boldsymbol{S}\boldsymbol{S}^H + \sigma^2 \boldsymbol{I})^*]\boldsymbol{g}_{\boldsymbol{k}}^{*}(i-1) + p_i \boldsymbol{a}_{\boldsymbol{k}}^{*} \tag{A.4}$$

where p_i is a variable step size of the current stage.

Treating the *K* filters as a filter bank, we have:

$$\boldsymbol{G}_{\boldsymbol{i}}^{\boldsymbol{H}} = \boldsymbol{G}_{\boldsymbol{i-1}}^{\boldsymbol{H}} - p_i\,[(\boldsymbol{S}\boldsymbol{S}^{\boldsymbol{H}} + \sigma^2 \boldsymbol{I}) - \boldsymbol{S}^{\boldsymbol{H}}\,] \tag{A.5}$$

where $\boldsymbol{G}_{\boldsymbol{0}} = 0$.

The equivalent one-shot filter for an i-stage PIC detector in non recursive form is then

$$\boldsymbol{G}_{\boldsymbol{i}}^{\boldsymbol{H}} = \sum_{l=1}^{i} p_i\, \boldsymbol{S}^{\boldsymbol{H}} \prod_{j=l+1}^{i}(I - p_j\,(\boldsymbol{S}\boldsymbol{S}^{\boldsymbol{H}} + \sigma^2 \boldsymbol{I})) \tag{A.6}$$

Note that $\boldsymbol{G}_{\boldsymbol{i}}^{\boldsymbol{H}}\boldsymbol{S}\boldsymbol{S}^{\boldsymbol{H}} = \boldsymbol{R}\,\boldsymbol{G}_{\boldsymbol{i}}^{\boldsymbol{H}}$. Therefore (3.24)

$$\boldsymbol{G}_{\boldsymbol{i}}^{\boldsymbol{H}} = \boldsymbol{G}_{\boldsymbol{i-1}}^{\boldsymbol{H}} - p_i\left[(\boldsymbol{R} + \sigma^2 \boldsymbol{I})\, \boldsymbol{G}_{\boldsymbol{i-1}}^{\boldsymbol{H}} - \boldsymbol{S}^{\boldsymbol{H}}\,\right] \tag{A.7}$$

Post multiplying with $\mathbf{r}$ gives

$$\boldsymbol{y}_{\boldsymbol{i}} = [\boldsymbol{I} - p_i\,(\boldsymbol{R} + \sigma^2 \boldsymbol{I})]\, \boldsymbol{y}_{\boldsymbol{i-1}} + p_i\, \boldsymbol{S}^{\boldsymbol{H}}\boldsymbol{r} \tag{A.8}$$

The above equation matches with (3.17) for Weighted PIC.

6.4 B. Decision making and Fuzzy Logic System

To explore the estimation of weight in weighted Fuzzy PIC, an outline on how soft decisions are made using fuzzy logic systems is given. This could be illustrated using a typical pattern classification problem [47] where the input to the FLS structure is represented by a vector in a feature space. The space is represented by c possible classes I_1, I_2,.... I_c. A possible way to represent pattern classifier is in terms of a set of discriminant functions $\{g_i(x), i = 1,2, \dots, c\}$, where x is a feature vector. The classifier assigns x to class i if $g_i(x) > g_j(x), \forall j \neq i$. The feature space is therefore partitioned into c disjoint regions $\Gamma_1, \Gamma_2, \dots, \Gamma_c$. These regions can be represented by c characteristic functions defined on the feature space as follows:

$$\mu_{\Gamma_1}(x) = \begin{cases} 1, if\ x \epsilon \Gamma_1 \\ 0, \text{otherwise} \end{cases} \qquad i = 1,2, \dots, c. \qquad \text{(B-1)}$$

Using the above expression, the classification result for x can be expressed as a fuzzy singleton B_x whose membership function is a function of x, i.e.,

$$\mu_{B_x}(I_i) = \frac{\mu_{\Gamma_i}(x)}{\sum_{l=1}^{c} \mu_{\Gamma_i}(x)} \qquad i = 1,2, \dots, c. \text{ (12)} \qquad \text{(B-2)}$$

Note that for each x there is only one value of i for which $\mu_{B_x}(I_i)$ is nonzero; therefore, this classification is a hard decision.

In fuzzy classifications scheme, we consider the set of classes $V = \{I_1, I_2, \dots . I_c\}$ as a universe of discourse on which fuzzy sets are defined to represent the concept of vague classes.

A fuzzy class is a fuzzy set $G \subset V$ with fuzzy membership function $\mu_G(I)$, where $I \epsilon V$. For example, $G = \{0.8/I_1,\ 0.4/I_2, 0.1/I_3\}$ is a fuzzy set representation of similar to class I_1.

Now if we generalize the $\mu_{\Gamma_1}(x)$'s in equation (B.1) into fuzzy membership functions i.e., $\mu_{\Gamma_i}(x)$ assumes a value between zero and one and $\mu_{\Gamma_i}(x)$ can be nonzero for multiple values of i for the same x. This makes the classification output in equation (B.2) a non-singleton fuzzy set and B_x now becomes a soft decision.

Since the classifier is now defined by the functions in equation (B.2), the classification problem has been translated into the problem of approximating these functions. FLS's can be used as approximators for these functions.

Nomenclature

MUD:	Multiuser Detection
CDMA:	Code Division Multiple Access
MAI:	Multiple Access Interference
PIC:	Parallel Interference Cancellation
SIC:	Successive/Serial Interference Cancellation
MF:	Matched Filter
MLS:	Maximum likelihood sequence
MMSE:	Minimum mean square error
IC:	Interference cancellation
LMS:	Least mean square
RLS:	Recursive least square
BER:	Bit error rate
SNR:	Signal to noise ratio
AWGN:	Additive white Gaussian noise

R:	Received signal vector
S:	Matrix of signature codes
A:	Matrix of received amplitudes of users
D:	Vector of data bits
n:	Vector of AWGN samples
N:	Processing gain
K:	Number of users
s_k:	Signature sequence of k^{th} user
a_k:	Received amplitude of k^{th} user
d_k:	Data bit of k^{th} user
y_k:	Decision statistics of k^{th} user
σ^2:	Variance of noise
m:	Stage index

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