Echo Cancellation using a Variable Step-Size NLMS Algorithm

by

Ahmed Iyanda Sulyman

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

ELECTRICAL ENGINEERING

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Date

Dedicated To My Parents.

Acknowledgements

All praises are for Allah (SWT), the Most Compassionate, Most Merciful. May peace and blessings be upon Prophet Muhammed, and His Household. I thank Almighty Allah (SWT) for all His blessings upon me. His divine help led to an incredibly speedy completion of this work. I ask Him (SWT) to forgive all my shortcomings before, and during the course of this work.

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Nomenclature

The following standard convention for writing vector and scalar equations were strictly adhered to throughout the report:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1N} \\ a_{21} & a_{22} & . & . & a_{2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{N1} & a_{N2} & . & . & a_{NN} \end{bmatrix}, \quad (\mathit{Matrix}), \tag{1}$$

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_N \end{bmatrix}, \quad (\mathit{Vector}), \tag{2}$$

$$\mathbf{a}^T = \begin{bmatrix} a_1 & a_2 & . & . & . & a_N \end{bmatrix}, \quad (\mathit{Vector}), \tag{3}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & 0 \\ 0 & 0 & 1 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & . & . & 1 \end{bmatrix}, \quad (DiagonalMatrix), \tag{4}$$

$$a = a\,, \quad (Scalar). \tag{5}$$

THESIS ABSTRACT

Name: AHMED IYANDA SULYMAN

Title: ECHO CANCELATION USING: A VARIABLE STEP-SIZE NLMS ALGORITHM

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In this work, an echo cancellation scheme using a variable step-size Normalized Least Mean-Square (VSS-NLMS) adaptive algorithm is proposed. Many algorithms proposed in the literature for this application were not given much attention in real-time applications due to their heavy complexities. The NLMS algorithm however, is given much attention because it has a good balance between performance and complexity. Conventional echo cancellers using a fixed step-size algorithm, usually results in a trade-off between residual error and speed of convergence. The variable step-size NLMS algorithm presented in this work gives an improved performance, eliminating such trade-off. The step-size variation makes it possible for the algorithm to converge faster and at the same time, yielding a lower steady-state error compared to the one obtained by the fixed step-size case. Derivations and analysis of the VSS-NLMS algorithm are presented and simulation results confirm the theoretical findings. The results indicate that the VSS-NLMS algorithm outperforms the traditional NLMS algorithm both in terms of convergence and steady-state error in case of low values of the step-size initializations. However, when high values of the step-size initializations are used, the VSS-NLMS algorithm still converge to a lower steady-state error than the traditional NLMS algorithm, but with same speed of convergence.

Keywords: Echo Cancelation, Variable Step-Size, NLMS algorithm.

King Fahd University of Petroleum and Minerals, Dhahran.
May 2000

الخلاصة

الإسم :أحمد إيندا سليمان

عنوان الرسالة :إلغاء الصدى بإستخدام خوارزمية NLMS متغيرة حجم الخطوة

الدرجة : ماجستير

التخصص : هندسة كهربائية

تاريخ التخرج :صفر ١٤٢١هـ

في هذا البحث تم عرض طريقة لإلغاء الصدى بإستخدام خوارزمية موفقـــة لمربعـــات المتوســـطات الصغرى المطبعة متغيرة حجم الخطوة (VSS-NLMS) .العديد من الخوارزميات المعروضة سابقا لم تنـــل كثيرا من الإهتمام في التطبيق الفعلي بسبب ما تحتويه من تعقيد. على أن خوارزميـــة NLMS نـــالت حظاً وافراً من الإهتمام بسبب ما تمنحه من إتزان بين الأداء ودرجة التعقيد. ملغيات الصدى العاديـــه ثابتة حجم الخطوة غالباً ما تنطوي على موازنة بين الخطأ والتقارب.خوارزمية NLMS متغيرة حجـــم الخطوة المعروضة في هذا البحث تعطي أداء محسنا بالتخلص من مثل هذه الموازنة .تغير حجم الخطـــوة يمكن الخوارزمية من التقارب بصورة أسرع وفي ذات الوقت ينتج عنها خطأ استقراري أقل مقارنـــةً بخوارزمية الخطوة ثابتة الحجم. تم عرض اشتقاق وتحليل خوارزمية (VSS – NLMS)، ونتائج المحاكـــاة التي تدعم النتائج النظرية. أوضحت النتائج أن خوارزمية (VSS-LMS) تفوق خوارزميـــة NLMS أداء في كل من التقارب والخطأ في حالة أخذ حجم الخطوة قيماً ابتدائية صغيرة. غير أنه حينما يـــأخذ حجم الخطوة قيما ابتدائية كبيرة فإن خوارزمية (VSS-NLMS) تظل تتقارب إلى خطأ استقراري أقـــل من خوارزمية NLMS ولكن بسر عة التقارب ذاتها.

جامعة الملك فهد للبترول والمعادن

الظهران ـــ المملكة العربية السعودية

Chapter 1

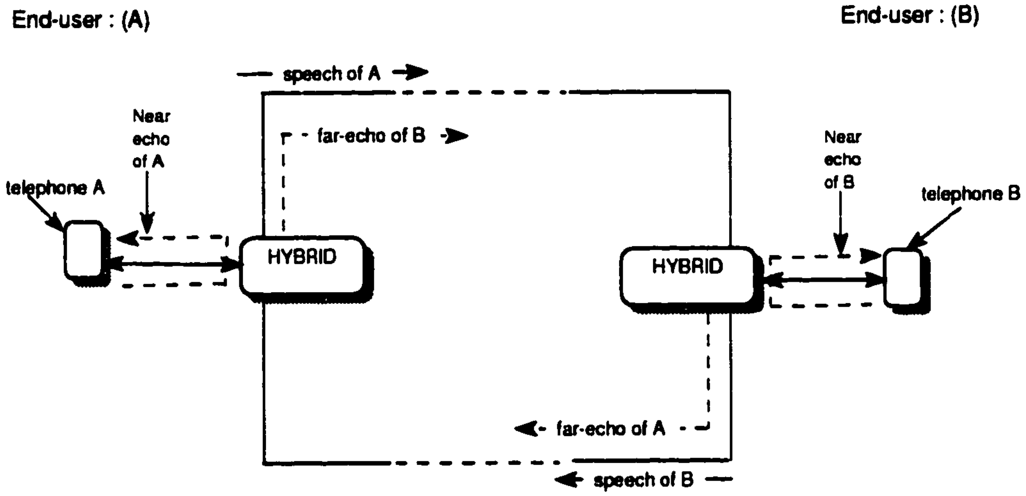
Introduction

Telecommunication systems, like any other elctrical system, often involve considerable delays. Therefore, if an echo generated at one end of a communication channel travels to its other end, the delay is often sufficient to make it noticeable and bothersome. Echoes encountered in communication systems belongs to two types:

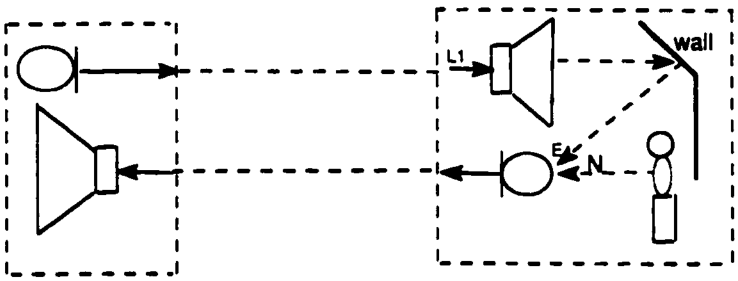
- **Electrical (Network) echo** which results from the reflection of unabsorbed electrical energy due to impedance mismatches at the interface between the two-wire links (local loop) and four-wire links (digital trunks) known as the hybrid, Figure 1.1 (a) details this situation, and

- **Acoustic echo** which is generated when a speech from a speaker is picked up by the microphone, and transmitted back to the source.
  Figure 1.1(b) depicts this clearly.

Acoustic echoes are especially severe in teleconferencing situations and hands-free (mobile) telephone systems. On the wired telephone system, though the loudspeaker and the microphone are combined in a hand-set designed to provide an attenuation of at least 45dB thereby eliminating most of the acoustic echo [2], the electrical echo in the form of reflections around the hybrid remains an odious problem.



(a) : Electrical Echo



(b) : Acoustic Echo

Figure 1.1: Echoes in Communication Systems:(a) Generation of Electrical (Network) echo at the hybrid, (b) Generation of Acoustic echo during hands-free telephony.

A modern practice cancels echo on the telephone network by adaptive filtering. The performance of the echo canceler depends mainly on the choice of the adaptive algorithm. The adaptive algorithm iteratively matches the impulse response of the echo path by minimizing the error (in the mean-square sense) between the synthesized echo and the true echo. Echo cancellation is achieved by subtraction of the synthesized echo from the true echo, and the resulting signal is referred to as the residual echo (residual error).

A commonly used algorithm for this application is the Normalized LMS (NLMS) algorithm [3, 4, 5]. The algorithm is much embraced for this application because of the good balance it has between cost and performance. Modern day communication requirements however demand for algorithms with better performance than the NLMS algorithm for this application. But the algorithms proposed so far are not given much attention in real-time implementations because the cost of implementing them far outweigh the gain in their performance. It has become clear that most often than not, a simple and robust algorithm will be given more attention in the industry than a highly sophisticated one promising some meager additional performance gain. Hence only an algorithm that still posses a good balance between cost and the promised performance improvement over the existing implementation can be well embraced as a replacement to the NLMS algorithm for this and related applications.

Our objective here therefore, is to concentrate our efforts on improving the performance of the NLMS algorithm without appreciably increasing its complexity. In the conventional NLMS algorithm, a fixed adaptation constant (step-size) is used for the adaptation of the weights of the adaptive filter. In this work, we use a variable step-size NLMS (VSS-NLMS) algorithm in which a particular value of the step-size is used at the onset, and then the step-size is adjusted (depending on how far the coefficients of the filter are from the optimal value) as the adaptation proceeds, such that at the steady-state condition, the step-size would have been reduced to a lower value and consequently a very small steady-state residual error will be attained without compromising the convergence speed of the algorithm (in comparison with the fixed step-size case).

The rest of the Chapter is organized thus: next we present the literature survey on this subject. In the following sections we discuss briefly the methods of echo control in communication systems, the echo cancellers and echo cancellation models, and the algorithms used. The last part of this chapter presents the objective of this thesis, its scope, and organization.

1.1 Literature Review

Echo cancellation using adaptive filtering has been an effective way of combating echo problem in communication systems that meets the modern day requirements [2, 6, 7, 8, 9, 10, 11, 12].

Broadly speaking, two main categories of adaptive filtering algorithms are available in the literature [13] that can be used for this and similar applications:

1. The first category is the method of least squares [14, 15, 16], which contains the whole class of the recursive least square (RLS) algorithms [17, 18, 19, 20].

2. The second category is known as the family of the stochastic gradient algorithms. The Least-Mean-Square (LMS) and the Normalized LMS (NLMS) algorithms belong to this category [20, 21, 22].

•

Of all the above mentioned adaptation algorithms, the LMS and the normalized LMS (NLMS) algorithms are generally known for their simplicity of implementations. Unfortunately however, the LMS algorithm behaves unstably with speech input signal and has therefore not been popular with speech applications [23]. More still, the tracking dynamics of the NLMS algorithm appears to be significantly less sensitive to a variety of input signal distribution aspects than holds for the LMS algorithm [21]. Therefore only the NLMS algorithm has been given much attention in the industry for echo cancellers [8]. This is because only this algorithm so far, is known to exhibit a good balance between cost and performance. There may be several other adaptive filtering algorithm with better performance than the NLMS algorithm but most often a simple and robust algorithm is given precedence, in the industry, over a highly sophisticated ones promising some meager additional performance gains. However, the adaptation performance of the NLMS algorithm, with speech excitation, may appears to be a little poor due to strong correlation of speech signals [9]. One way used to effectively overcome this problem is to prewhiten (decorrelate) the incoming excitation signal before passing it to the adaptive algorithm [24]. Decorelation filters have been widely used in the field of echo cancellation for this application [25, 26, 27, 28], and as such echo cancellers till date are mostly NLMS based [8].

Step-size variation on the other hand has been effectively used to remedy the trade-off that usually exist between the steady-state excess error and speed of convergence of the LMS algorithm [1, 29, 30, 31]. The approaches from a general point of view is to utilize a certain step-size value (that will ensure fast convergence) at the onset, and some how adapt the step-size such that a very small value (that would ensure low steady-state excess error) would have been assumed at the steady-state condition.

In this work, a variable step-size NLMS algorithm is investigated. The step-size variation method used here is originally due to Shin and Lee [32] and effectively incorporated with the LMS algorithm by Mathew and Xie [1]. Their resulting variable step-size LMS (VSS-LMS) algorithm recorded some performance improvements over the traditional LMS algorithm. In the same vein, here we incorporate the step-size variation method in [32] with the conventional NLMS algorithm and the resulting algorithm, which we call a Variable Step-Size NLMS (VSS-NLMS) algorithm is extensively analyzed for the mean and mean-square behaviors of the coefficient of the adaptive filter as well as those of the step-size sequence.

1.2 Echo Control in Communication Systems

1.2.1 Early Methods of Echo Control: Losses Insertion, Echo Suppressors

One of the earliest method used for echo control on relatively short circuit is to insert a loss, of say L dB, in each direction of transmission. This attenuates the signal by L dB while the (talker) echo is attenuated by $2L$ dB so that the resulting signal-to-echo ratio is enhanced by L dB [33]. However, this method is not suitable for long circuits (exceeding $5000km$) as it result in extremely low signal level at the receiver [34].

Another method suitable for longer-delay echoes is the use of an echo suppressor [35]. An echo suppressor is a device that detects human speech coming from one

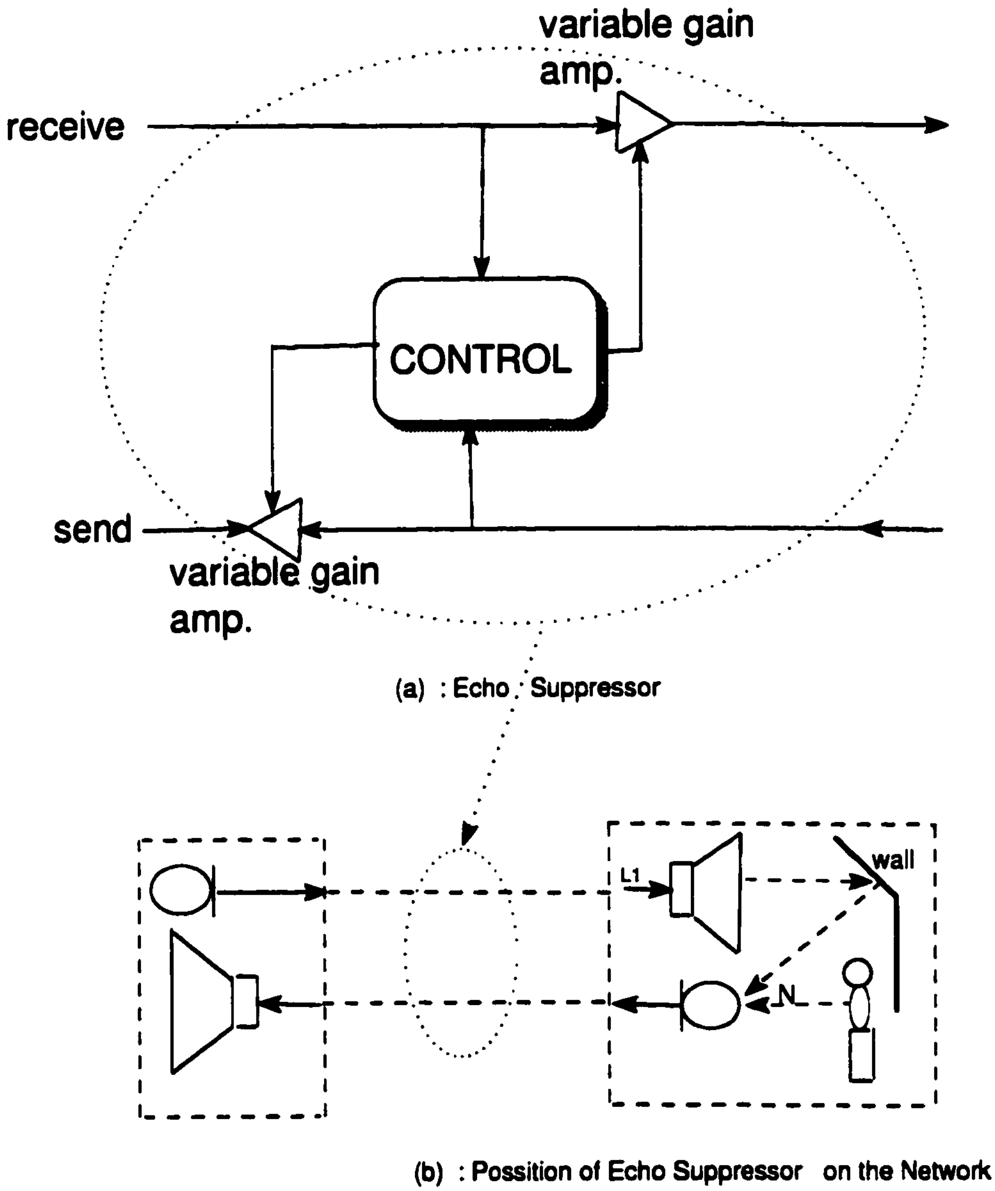


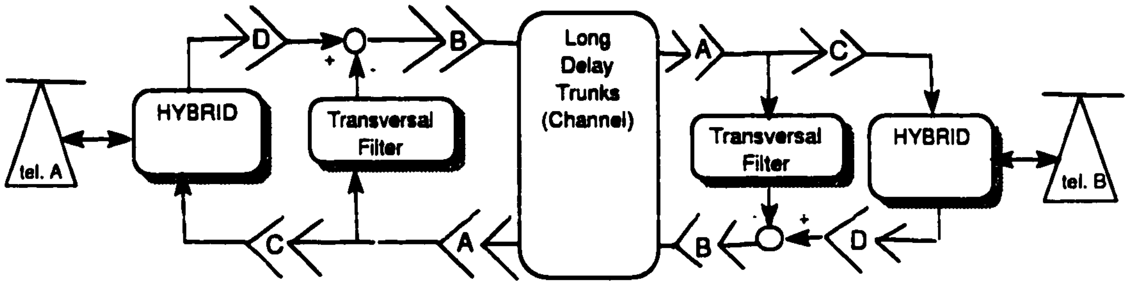
Figure 1.2: Echo Suppressor on the Telephone Network

end of the connection and suppresses all signals going the other way as shown in figure 1.2. The suppressor does this by dynamically altering the gains of the amplifier in the send and the receive path as shown. Usually the device increases the gain of the amplifier on the path with the higher signal (to a high positive value), while the gain of the other amplifier is reduced (to a high negative value). Thus echo suppressor inherently, allows only one-way Communication (half-duplex), usually favoring the louder speaker. Hence the use of echo suppressor requires highly disciplined speakers as it does not favor interruption, and more still, when used on circuits with round-trip delay more than $100ms$, the clipping and impairing interruption that characterizes echo suppressors become unacceptable, and echo cancellers are used instead [36].

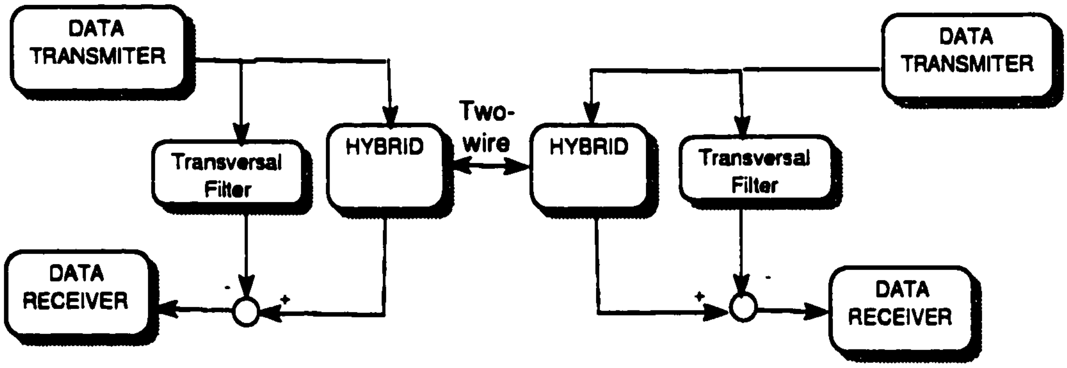
1.2.2 Modern Methods of Echo Control: Echo Cancellers

A modern method of echo control that has found wide application in the modern day communication is the use of echo cancellers. Echo Cancellers are adaptive filtering circuits that simulates (estimates) the impulse response of the echo path, generates a replica of the echo, and then subtract it from the signal delivered. With an echo Canceller on the line, full-duplex communication is not hampered. For this reason, echo cancellers have rapidly replaced echo suppressors in the modern day communication systems [35].

Figure 1.3(a) illustrates the use of an echo canceller on the telephone network for mitigating the echo on both directions of the connection. It is a four-terminal device which interfaces to both directions of the transmission. Ports C and D are coupled to the near-end talker through a hybrid, while ports A and B are coupled to the far-end talker via the channel. The echo canceller combats the echo representing the feed-through from port C through the hybrid to port D.



(a) : Split echo canceler configuration for two directional Voice transmission



(b) : Echo canceler configuration for Data transmission

Figure 1.3: Echo Canceller Configuration on the Telephone System

Echo cancellation is also used in a related application in data transmission as illustrated in fig. 1.3(b), where full-duplex data transmission is desired on a single two-wire facility. In a modem, containing (among other things) a transmitter and a receiver, for data signals, the near-end data signal $\mathbf{x}$ has to be transmitted while the far-end data signal, $\mathbf{s}$, has to be received. An hybrid directs signal $\mathbf{x}$ to, and signal $\mathbf{s}$ from a transmission medium. Applications of this include digital transmission on the subscriber loop and voice-band data transmission with interface to the network on a two-wire basis. In this application the bandwidth is not doubled relative to the bandwidth required for transmission in a single direction as in the other configuration, but implementation of this configuration is somewhat challenging [37]. The focus of this work is voice echo cancelling, and as such data echo cancellation will not be further elaborated upon. Information on data echo cancellation can be obtained from [38, 39].

1.3 Echo and Echo Cancellation Models

Figure 1.4 present the model that is used to represent the Echo Source (path) and Echo Canceller configuration as is used in communication systems. The impulse response of the echo path shown is denoted by $\mathbf{w}_k^*$.

In figure 1.4, only the far-end echo canceller is shown. An equivalent system will be present also at the near-end to cancel the far-end echo. It is assumed that the echo y_k^* is generated from the near-end speech (reference input) $\mathbf{x}_k$ by a linear time-varying system, and corrupted by an observation noise ζ_k which is assumed to be white. $\mathbf{x}_k$ and y_k^* are correlated because the latter is obtained by passing the former through the echo channel. It is further assumed that the near-end speech (reference input)

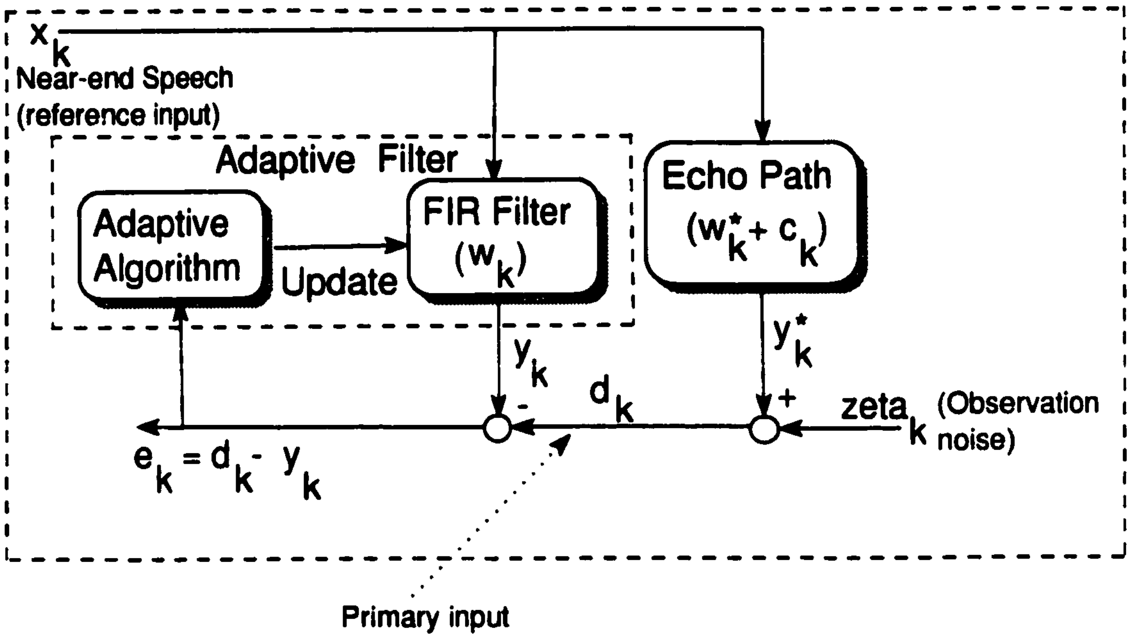


Figure 1.4: Echo and Echo Cancellation Model

that generates the echo, $\mathbf{x}_k$, is statistically independent of the observation noise, ζ_k. Thus the primary input to the echo canceller, d_k, is a sum of two statistically independent components: the echo y_k^* and the observation noise, ζ_k. Based on these assumptions, which are quite accurate in most practical cases, we model the echo generation system as a linear filter with an impulse response of $\mathbf{w}_k^*$, and the echo as the convolution of the reference input signal with this impulse response. The adaptive echo canceller dynamically estimates $\mathbf{w}_k^*$ by a $\mathbf{w}_k$, and generates an estimate of the echo, y_k^*, at each time instant k. The adaptation (updating) algorithm 'seeks' $\mathbf{w}_k^*$ by $\mathbf{w}_k$ following the iterative procedure of the algorithm.

1.4 Adaptive Algorithms for Echo Cancellers

1.4.1 The Algorithmic Proceedure

Figure 1.4 presents a general model of the echo source and the scheme of echo cancellation where

- $\mathbf{w}_k^*$ is the time-varying impulse response of the echo path,
- $\mathbf{c}_k$ is the disturbance vector for the impulse response modeling the time-varying echo path.
- $\mathbf{x}_k$ is the reference input signal (near-end speech),
- ζ_k is the observation noise,
- d_k is the desired response (true echo, y_k^*, currupted with observation noise ζ_k)
- y_k is the synthesized echo,
- e_k is the residual error at each time instant given by: $e_k = d_k - y_k$, and
- k is the time index.

  This model has been used for both Network (Electrical) and Acoustic echoes [6, 9, 8]. From the model we can write the following equation:

$$d_k = \sum_{j=0}^{N-1} w^*(j)x(k-j) + \zeta_k \tag{1.1}$$

From (1.1), it can be observed that echo cancellation requires cancelling of the echo component y_k^* embedded in the primary signal d_k, being delivered back to the near-end site. To do so:

1. An estimate of the echo is synthesized, y_k.

2. An estimation error e_k is generated by comparing the echo estimate with the actual echo (desired response), and therefore we can write:

$$\begin{aligned} e_k &= d_k - y_k \\ &= y_k^* - y_k + \zeta_k \\ &= \sum_{j=0}^{N-1} [w^*(j) - w(j)]x(k-j) + \zeta_k \end{aligned} \tag{1.2}$$

   This error is then used in the update of the coefficients of the adaptive filter.

From (1.2), it is clear that once the synthesized echo y_k is equal to the true echo y_k^*, then the echo is completely cancelled, and the signal transmitted to the near-end is the far-end speech only. This is the goal of an echo cancller. The job of matching the weights of the adaptive filter with the impulse response of the echo path is done by the adaptive filtering algorithm following the procedure described above.

1.4.2 The Algorithms

Broadly speaking, two main categories of adaptation techniques are available in the literature optimizing different variations of the cost function [13]:

1. The first category known as the method of least squares [14, 15, 16], which contains the whole class of the recursive least square (RLS) algorithms.

2. The second category, referred to as the family of the stochastic gradient algorithms, minimizes the cost function $J_n = E[e^{2n}]$, where $n \geq 1$, and $E[.]$ denotes statistical expectation. The Least-Mean-Square (LMS) algorithm belongs to this category [20, 21, 22]. The normalized LMS (NLMS) algorithm also belongs to the family of stochastic gradient algorithms.

. The LMS and the normalized LMS (NLMS) algorithms are generally known for their simplicity of implementations. But only the NLMS algorithm has been given much attention in the industry for echo cancellers [6]. This is because this algorithm is well known to exhibit a good balance between cost and performance [8]. While LMS algorithm performs badly with speech excitation, the performance of NLMS algorithm is quite acceptable for most applications [23]. The performance superiority (with just trivial complexity increase) of the NLMS algorithm over the LMS algorithm can be appreciated from their recursive equations. The recursive equation for dynamically updating the weights (or coefficients), $\mathbf{w}_k$, of an adaptive filter according to the LMS [40] and NLMS [5, 41] algorithm are given respectively as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu e_k \mathbf{x}_k \tag{1.3}$$

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \frac{e_k \mathbf{x}_k}{\|\mathbf{x}_k\|^2} \tag{1.4}$$

where

- $\mathbf{w}_{k+1}$ is the next tap weight value of the adaptive filter.
- $\mathbf{w}_k$ is the present tap weight value of the adaptive filter.

- μ is a fixed adaptation constant (step-size). For the LMS algorithm μ is constrained by [40]:$0 < \mu < \frac{2}{\lambda_{max}}$, while for the NLMS algorithm, it is constrained by [21, 42, 43, 44]: $0 < \mu < 2$
- e_k is the error (mismatch) between the desired response of the filter and the actual response.
- $\mathbf{x}_k$ is the reference input, and
- $||\mathbf{x}_k||^2$ is the norm of the input vector.

The normalization in the NLMS algorithm makes it to behave more stably, less sensitive to colored input signal (as the effect of the eigen value spread of the input vector is reduced) [45], and converges faster (but to a higher steady-state error) [21] than the LMS algorithm.

1.5 The Traditional Normalized LMS (NLMS) Adaptive Filtering Algorithm (fixed step-size)

We now discuss the use of the Normalized LMS (NLMS) algorithm for the adaptation of the coefficient of the adaptive filter (echo canceller). Generally speaking, the NLMS adaptive filtering algorithm, as well as other adaptive filtering algorithms, adapts the adaptive filter weights $\mathbf{w}_k$ by minimizing the error between the true echo and the synthesized echo. Once the error is minimized, $\mathbf{w}_k$ is the impulse response of the echo path. As a result, the synthesized echo is the true echo. The speed with which this convergence is reached is referred to as the speed of convergence of the algorithm, while the residual error still left after convergence is known as the steady-state excess error.

Let the impulse response vector at time k be defined by

$$\mathbf{w}_k^* = [w_k^*(0), w_k^*(1), \ldots, w_k^*(N-1)]^T \tag{1.5}$$

and the vector simulated by the adaptive filter at time k by

$$\mathbf{w}_k = [w_k(0), w_k(1), \ldots, w_k(N-1)]^T \tag{1.6}$$

where T stands for transpose, and N is the length of the filter. We also define the input vector by

$$\mathbf{x}_k = [x_k, x_{k-1}, x_{k-2}, \ldots, x_{k-N+1}]^T \tag{1.7}$$

Then we see from (1.1) that

$$d_k = \mathbf{w}_k^{*^T} \mathbf{x}_k + \zeta_k \tag{1.8}$$

and

$$y_k = \mathbf{w}_k^T \mathbf{x}_k \tag{1.9}$$

The error then becomes:

$$\begin{aligned} e_k &= d_k - y_k \\ &= (\mathbf{w}_k^{*^T} - \mathbf{w}_k^T)\mathbf{x}_k + \zeta_k \\ &= \mathbf{x}_k^T(\mathbf{w}_k^* - \mathbf{w}_k) + \zeta_k \end{aligned} \tag{1.10}$$

In the conventional Normalized LMS (NLMS) Algorithm, the weights of the adaptive filter are updated using the NLMS algorithm according to Equation 1.4.

1.6 NLMS Algorithm in Comparison with the LMS Algorithm: Motivation For the VSS-NLMS Algorithm

Comparing the recursive (1.4) for the NLMS with the corresponding recursion for the conventional LMS in (1.3), we observe that setting

$$\mu_k = \frac{\mu}{\|\mathbf{x}_k\|^2} \tag{1.11}$$

for the NLMS algorithm, we may view the NLMS algorithm as an LMS algorithm with a sort of "time-varying step-size" parameter. Several works have investigated the performance of the NLMS algorithm in comparison with the LMS algorithm [5, 21, 42]. We hold a view that the observed performance improvement of the NLMS algorithm over the LMS algorithm may be due to the above stated fact, and this gave us the **motivation** to investigate the performance of the NLMS algorithm when the step-size is even purposefully varied in order to enhance more, the good properties observed to be exhibited by the conventional NLMS algorithm. The resulting algorithm which we call a variable step-size normalized LMS (vss-nlms) algorithm, promises an impressive improvement over the conventional NLMS algorithm.

1.7 Thesis Objective, Scope, and Organization

1.7.1 Thesis Objective

Several algorithms have been proposed in the literature which promised one form of improvement or the other, over the NLMS algorithm. But however, these algorithm were not given much attention in the industry mainly due to their heavy complexities which in most cases make their implementations not viable from an economic point of view. Till date, echo cancellers implemented in the industry are mostly based on the Normalized LMS (NLMS) algorithm with fixed step-size [8]. The Objective of this work is to evolve an improved version of this algorithm that will not essentially increase the complexity of the conventional NLMS algorithm, by the use of step-size variation.

1.7.2 Scope Of Work

Our approach in this thesis work is to consider echo cancellation as an example of a general problem of adaptive system identification. This work will evolve a new improved version of the NLMS algorithm to deal more effectively with this problem, the algorithm's performance will be examined using the general model for adaptive system identification, and then tested with echo cancellation Insha Allah. Therefore this work will focus essentially on developing the proposed algorithm.

1.7.3 Thesis Organization

In Chapter 2, we derive the proposed Variable Step-Size Normalized LMS (VSS-NLMS) algorithm. The derivations are based on some simplifying assumptions which are often justifiable in real-time situations.

In Chapter 3, the derived performance behaviors of the algorithm are further analyzed for steady-state conditions in stationary as well as in non-stationary environments[1], both of which are encountered in real-time situations. These steady-state performances of the algorithm so obtained (from the mathematical analysis) are then compared with those of other algorithms in the literature towards the end of this chapter.

Since an accurate mathematical analysis (i.e. without assumptions) is generally not popular in this types of adaptive system identification analysis [1, 21, 42, 46, 47, 48, 49, 50, 51, 52, 53], the analytical results presented in chapters 2 and 3 will usually not be enough to guarantee the algorithm's performances shown by the derivations. In chapter 4 therefore, we present a detailed simulation result that investigates the performances of the proposed algorithm both in stationary and non-stationary environments. Simulation results for the influences of the various parameters of the algorithm on its performances are also presented. The analytical and empirical (simulation) results are then compared. Towards the end of this chapter, simulation results when the algorithm is used for echo canceling are presented.

[1]The terms: Stationary and Non-Stationary environments, have been widely used in this content in the literature to refer to Static and Dynamic (Time-varying) environments, respectively [45].

Chapter 5 then enumerate the contributions of this work, draw its conclusions, and present some suggestions for prospective future works.

Chapter 2

The Proposed Variable Step-Size Normalized LMS (VSS-NLMS) Algorithm

2.1 introduction

The most simple and perhaps the most popular adaptive filtering algorithm is the Least Mean Square (LMS) algorithm given by the equation [17, 47, 48]:

$$\mathbf{w_{k+1}} = \mathbf{w}_k + \mu \mathbf{x}_k e_k. \tag{2.1}$$

The algorithm was so widely embraced because of its simplicity of implementation both in software and hardware. However, this algorithm suffers from relatively slow convergence, and it is data-dependent. More still, an attempt to choose a larger step-size parameter that will give higher convergence speed usually result in higher misadjustment error. Hence the users of the algorithm have realized that the choice of the step-size parameter is a trade-off between misadjustment and the speed of adaptation. Several works have thus presented the idea of variable step-size LMS algorithms [1, 29, 30, 31] to remedy this problem. The aim is to improve on the performance of the gradient adaptive filter without this trade-off between the convergence speed and the steady-state excess error or, misadjustment. The idea, from a general point of view, is to somehow sense how far away the adaptive filter coefficients are from the optimal filter coefficients and use step-sizes that are small when adaptive filter coefficients are close to the optimal values and use large step-sizes otherwise.

The resulting update is of the form [1, 29, 30, 31]:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_k \mathbf{x}_k e_k \tag{2.2}$$

where μ_k is the time-varying step-size. The approach is heuristically sound and has resulted in several *ad hoc* techniques, where the selection of the step-size parameter is based on the magnitude of the estimation error [54], polarity of the successive samples of the estimation error [29], measurement of the cross correlation of the estimation error with input data [32, 55],the kurtosis of error estimation [31] and so on.

Following these approaches, this work presents a variable step-size Normalized LMS (VSS-NLMS) algorithm. The time-varying step-size selection method utilized here was originally introduced by Shin and Lee [45], and successfully incorporated with the conventional LMS algorithm by

Mathews and Xie [1]. The idea is to change the time-varying convergence parameters in such a way that the change is proportional to the negative of the gradient of the squared estimation error with respect to the convergence parameter.

2.2 VSS-NLMS algorithm

Following the approaches used for variable step-size LMS algorithms [1, 29, 30, 31] as defined in Equation (2.1), we shall study here a variable step-size NLMS algorithm. The algorithm, as shown later, improves on the performance of the conventional NLMS algorithm and eliminate the well known trade-off between the convergence speed and the steady-state excess error. Our approach here is to adapt the step-size sequence using a gradient descent algorithm so as to reduce the squared-estimation error at each time iteration.This approach will result in the following algorithm:

$$\begin{aligned} \mathbf{w}_{k+1} &= \mathbf{w}_k + \frac{\mu_k e_k \mathbf{x}_k}{\|\mathbf{x}_k\|^2} &\quad (2.3)\\ &= \mathbf{w}_k + \frac{\mu_k \mathbf{x}_k}{\|\mathbf{x}_k\|^2}.[d_k - \mathbf{x}_k^{\mathbf{T}}\mathbf{w}_k] \\ &= (1 - \frac{\mu_k \mathbf{x}_k \mathbf{x}_k^{\mathbf{T}}}{\|\mathbf{x}_k\|^2})\mathbf{w}_k + \frac{\mu_k d_k \mathbf{x}_k}{\|\mathbf{x}_k\|^2} &\quad (2.4)\end{aligned}$$

where the error signal e_k is defined as:

$$e_k = d_k - \mathbf{x}_k^{\mathbf{T}}\mathbf{w}_k, \quad (2.5)$$

and μ_k is the "time-varying" step-size.

Note that it is possible to adapt the step-size corresponding to each coefficient of the adaptive filter individually. Such an approach was considered in [56], but here we follow the approach used in [1, 32], where the step-sizes are all simultaneously adapted using a single recursive equation. Therefore, the step-size sequence $\mu_{\mathbf{k}}$ here is a diagonal matrix of equal elements, i.e.,

$$\mu_{\mathbf{k}} = \begin{bmatrix} \mu_1 & 0 & . & . & 0 \\ 0 & \mu_2 & 0 & . & 0 \\ 0 & 0 & \mu_3 & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & \mu_N \end{bmatrix} \tag{2.6}$$

and $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_N$.

Each diagonal element of $\mu_{\mathbf{k}}$ will be adapted according to [32]:

$$\begin{aligned} \acute{\mu}_k &= \mu_{k-1} - \frac{\rho}{2}\frac{\partial e_k^2}{\partial \mu_{k-1}} \\ &= \mu_{k-1} - \frac{\rho}{2}\frac{\partial^T e_k^2}{\partial \mathbf{w}_k}\cdot\frac{\partial \mathbf{w}_k}{\partial \mu_{k-1}} \\ &= \mu_{k-1} + \rho e_k \mathbf{x}_k^{\mathbf{T}}\cdot\frac{\partial \mathbf{w}_k}{\partial \mu_{k-1}}\,, \end{aligned} \tag{2.7}$$

where ρ is a small positive constant that controls the adaptive behavior of the step-size sequence μ_k.

Notice that we have written $\acute{\mu}_k$ and not μ_k in (2.7). This is because the convergence requirements put limits on the the values that μ_k can assume during the adaption. Therefore, the computed step-size value in (2.7) will not be used as the adaptation step-size value μ_k at any particular time iteration if it falls outside the convergence region. The adaptation rule for the step-size sequence μ_k will be discussed in the following section. Now, we proceed by using (2.3) to write:

$$\frac{\partial \mathbf{w}_k}{\partial \mu_{k-1}} = \frac{e_{k-1}\mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}, \tag{2.8}$$

and substituting (2.8) into (2.7) will yield:

$$\acute{\mu}_k = \mu_{k-1} + \frac{\rho e_k e_{k-1}\mathbf{x}_k^{\mathbf{T}}\mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}. \tag{2.9}$$

2.3 Convergence Requirements

The convergence of the mean weight vector of an adaptive algorithm is not sufficient to guarantee a finite mean-square error [49]. It is therefore important that we choose the algorithm parameters that will guarantee also a finite mean square error as well as a finite variance for the weight vector.

These two criteria are strongly related [21]. This means that it suffices us to confirm that one of them will be finite at steady state. Here we choose to concentrate on the analysis of the weight error vector and it suffices us therefore, to ensure that at the steady state condition, the mean value as well as the mean square value of the weight error vector are both finite. To guarantee (mean-squared) convergence of the weight of the adaptive filter we will restrict μ_k to assume only values within the range that would ensure convergence. A sufficient and necessary condition on μ (for the stationary input signal, and fixed step-size case) to ensure mean-squared convergence of the NLMS adaptive filter is [21, 45]:

$$0 < \mu < 2 \, .$$

Hence we will restrict the step-size sequence μ_k here within the limit $0 < \mu_k < 2$. To do this we adopt the following rule:

$$\mu_k = \begin{cases} \mu_{max} & \text{if } \acute{\mu}_k > \mu_{max} \\ \mu_{min} & \text{if } \acute{\mu}_k < \mu_{min} \\ \acute{\mu}_k & \text{otherwise.} \end{cases} \tag{2.10}$$

where $0 < \mu_{min} < \mu_{max} < 2$.

2.4 Environment Model Assumed for the Derivations

Let $\mathbf{w}_k^*$ denote the optimal coefficient vector for estimating the desired response signal d_k using $\mathbf{x}_k$. Assume $\mathbf{w}_k^*$ is time varying, therefore:

$$\mathbf{w}_{k+1}^* = \mathbf{w}_k^* + \mathbf{c}_k \tag{2.11}$$

where $\mathbf{c}_k$ is a zero-mean and white vector disturbance process with covariance matrix $\sigma_c^2\mathbf{I}$. Our derivations will be made for the general case in which the optimal coefficient vector is time-varying as written in Equation (2.11) above. For analysis of the algorithm's behavior in stationary environment, we will only need to set $\sigma_c^2\mathbf{I} = 0$.

2.5 Simplifying Assumptions used in the Derivations

Following the traditions of adaptive signal processing, we shall make use of the following extensively used simplifying assumptions [1, 42, 21]:

1. $\mathbf{x}_k$andd_k are jointly Gaussian and zero-mean random processes. $\mathbf{x}_k$ is a stationary process. More over, $\{\mathbf{x}_k, d_k\}$ is uncorrelated with $\{\mathbf{x}_n, d_n\}$ if $n \neq k$.

2. The auto-correlation matrix $\mathbf{R}$ of the input vector $\mathbf{x}_k$ is a diagonal matrix and is given by:

$$\mathbf{R} = \sigma_x^2\mathbf{I}$$

   This is a fairly restrictive assumption but has been used [1] because of analytical difficulty.
   We will also assume that the triplet $\{\mathbf{x}_k, \mathbf{c}_k, \zeta_k\}$ are statistically independent random processes.

3. We will also assume μ_k to be statistically independent of $\mathbf{x}_k$, $\mathbf{w}_k$ and e_k. While this assumption is always true for the fixed step-size case, it will only be approximately true here if ρ is chosen very small.

4. We will assume that [1]:

$$E[e_k^2\mathbf{x}_k\mathbf{x}_k^T|\mathbf{w}_k] \approx E[e_k^2\mathbf{x}_k\mathbf{x}_k^T]\,. \tag{2.12}$$

2.6 Observations Following the Assumptions

The following observations, some of which directly follow from the various assumptions above, will also be used in the derivations.

1. Let ζ_k corresponds to the optimal estimation error process, and defined as follows:

$$\zeta_k = d_k - \mathbf{x}_k^T \mathbf{w}_k^* . \tag{2.13}$$

Then pre-multiplying the above expression by $\mathbf{x_k}$ and taking the expectation yields:

$$\begin{aligned} E[\mathbf{x}_k \zeta_k] &= E[\mathbf{x}_k d_k] - E[\mathbf{x}_k \mathbf{x}_k^T]\mathbf{w}_k^* \\ &= \mathbf{p} - \mathbf{R}\mathbf{w}_k^* , \end{aligned} \tag{2.14}$$

where $p = E[\mathbf{x}_k d_k]$ and $R = E[\mathbf{x}_k \mathbf{x}_k^T d_k]$.
Since $\mathbf{w}_k^* = \mathbf{R}^{-1}\mathbf{p}$ [13, 50], therefore

$$E[\mathbf{x}_k \zeta_k] = \mathbf{p} - \mathbf{R}\mathbf{R}^{-1}\mathbf{p} = \mathbf{0} . \tag{2.15}$$

The uncorrelatedness of ζ_k with $\mathbf{x}_k$ in equation (2.15) means that since both ζ_k and $\mathbf{x}_k$ are jointly Gaussian, then they are independent as well. This goes in line with our earlier model where the reference signal is modeled as independent of the observation noise.

2. It is straight forward to show that

$$\sigma_{e_k}^2 = E[e^2] = \xi_{min} + \sigma_x^2 tr[\mathbf{G}_k] \tag{2.16}$$

   where $\xi_{min} = E[\zeta_k \zeta_k]$ = minimum MSE and $\mathbf{G}_k$ as defined in the following sections, is given by $\mathbf{G}_k = E[\mathbf{v}_k \mathbf{v}_k^T]$.

3. Also, since $\mathbf{x}_k$ and $\mathbf{x}_n$ are uncorrelated for $j \neq n$ and Gaussian, they are independent as well. Hence, from (2.23) (see below), $\mathbf{v}_k$ and $\mathbf{x}_k$ are independent. This is a well known consequence of the independent assumption [45].

4. Based on observation 3, $\mathbf{B}$, $\mathbf{H}$, $\mathbf{D}_k$, and hence $\mathbf{G}_k$ in (2.26) (see below) are diagonal matrices.

2.7 Derivations of the Mean Behavior of the Weight Vector for the VSS-NLMS Algorithm.

Let the coefficient misalignment vector at time k be:

$$\mathbf{v}_k = \mathbf{w}_k - \mathbf{w}_k^* . \tag{2.17}$$

Then the error e_k can be expressed as:

$$\begin{aligned} e_k &= d_k - y_k && (2.18)\\ &= \zeta_k - \mathbf{x}_k^T \mathbf{v}_k . && (2.19) \end{aligned}$$

Using (2.3) we can also express the error e_k in (2.18) in another form as:

$$\begin{aligned} e_k &= d_k - \mathbf{x}_k^T \mathbf{w}_k \\ &= d_k - \mathbf{x}_k^T \mathbf{w}_{k-1} - \frac{\mathbf{x}_k^T e_{k-1}\mu_{k-1}\mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2} . \end{aligned} \tag{2.20}$$

It is pertinent also to point out that from (2.11) and (2.13), d_k can be expressed as:

$$d_k = \mathbf{x}_k^T \mathbf{w}_{k-1}^* + \mathbf{x}_k^T \mathbf{c}_{k-1} + \zeta_k \tag{2.21}$$

We now proceed by substituting (2.19) into (2.3) to obtain:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\mu_k \mathbf{x}_k \zeta_k}{\|\mathbf{x}_k\|^2} - \frac{\mu_k \mathbf{x}_k \mathbf{v}_k^T \mathbf{x}_k}{\|\mathbf{x}_k\|^2} . \tag{2.22}$$

Subtracting $\mathbf{w}^*_{k+1}$ from both sides of (2.22) and using (2.11) results in the following expression for the weight error vector:

$$\mathbf{v}_{k+1} = (\mathbf{I} - \frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2})\mathbf{v}_k + \frac{\mu_k \mathbf{x}_k \zeta_k}{||\mathbf{x}_k||^2} - \mathbf{c}_k \, . \tag{2.23}$$

Taking the expectations of both sides of the expression in (2.23), and taking into consideration the previously mentioned assumptions and observations results in the following recursion for the mean behavior of the weight vector:

$$E[\mathbf{v}_{k+1}] = \{\mathbf{I} - E[\mu_k]E[\frac{\mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2}]\}E[\mathbf{v}_k] \, . \tag{2.24}$$

The expression derived in (2.24) above is the evolution of the weight error vector as the adaptation proceeds. Notice that the convergence of the weight vector of the adaptive algorithm to the optimal values implies that this recursion will gradually reduce to a value close to null vector at the steady-state condition. It is easy to see, from the above equation, that this convergence is dependent on the behavior of the mean behavior of the step-size sequence $E[\mu_k]$ which will be treated in the in the following sections.

2.8 Derivations of the Mean-Square Behavior of the Weight Vector for the VSS-NLMS Algorithm.

Post-multiplying both sides of (2.23) by $\mathbf{v}_k^T$ we get:

$$\begin{aligned}
&\mathbf{v}_{k+1}\mathbf{v}_{k+1}^T \\
&= (\mathbf{I} - \frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2})\mathbf{v}_k\mathbf{v}_k^T(\mathbf{I} - \frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2}) + (\mathbf{I} - \frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2})\mathbf{v}_k[\frac{\mu_k \zeta_k \mathbf{x}_k}{||\mathbf{x}_k||^2}]^T \\
&\quad -(\mathbf{I} - \frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2})\mathbf{v}_k(\mathbf{c}_k^T) + (\frac{\mu_k \zeta_k \mathbf{x}_k}{||\mathbf{x}_k||^2})\mathbf{v}_k^T - (\frac{\mu_k \zeta_k \mathbf{x}_k}{||\mathbf{x}_k||^2})(\frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2})\mathbf{v}_k^T \\
&\quad + \frac{\mu_k^2 \zeta_k^2 \mathbf{x}_k \mathbf{x}_k^T}{(||\mathbf{x}_k||^2)^2} - \frac{\mu_k \zeta_k \mathbf{x}_k}{||\mathbf{x}_k||^2}\mathbf{c}_k^T - \mathbf{c}_k\mathbf{v}_k^T + \mathbf{c}_k(\frac{\mu_k \mathbf{x}_k \mathbf{x}_k^T}{||\mathbf{x}_k||^2})\mathbf{v}_k^T \\
&\quad -\mathbf{c}_k\frac{\mu_k \zeta_k \mathbf{x}_k}{||\mathbf{x}_k||^2} + \mathbf{c}_k\mathbf{c}_k^T
\end{aligned}$$

which can be written in a shorter form as:

$$\begin{aligned}
&= \mathbf{v}_k\mathbf{v}_k^T - \mu_k[\frac{\mathbf{x}_k\mathbf{x}_k^T\mathbf{v}_k\mathbf{v}_k^T}{||\mathbf{x}_k||^2} + \frac{\mathbf{v}_k\mathbf{v}_k^T\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}] + \mu_k^2[\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}\mathbf{v}_k\mathbf{v}_k^T\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}] \\
&\quad +\mu_k^2\zeta_k^2\frac{\mathbf{x}_k\mathbf{x}_k^T}{(||\mathbf{x}_k||^2)^2} + \mathbf{c}_k\mathbf{c}_k^T + \mathbf{f}(\mu_k, \mathbf{x}_k, \mathbf{v}_k, \zeta_k, \mathbf{c}_k) \qquad (2.25)
\end{aligned}$$

where $\mathbf{f}(\mu_k, \mathbf{x}_k, \mathbf{v}_k, \zeta_k, \mathbf{c}_k)$ represents the terms of the above expression that are not explicitly written in Equation (2.25). Next we take the expected values of both sides of (2.25). Under our previously mentioned assumptions, $E[\mathbf{f}] = \mathbf{0}$.

Let $\mathbf{G}_k = E[\mathbf{v}_k\mathbf{v}_k^T]$ denote the second moment matrix of the mis-alignment vector. Taking this expectation with the aid of the independence assumptions and the uncorrelatedness of μ_k with other data, we get:

$$\mathbf{G}_{k+1} = \mathbf{G}_k - E[\mu_k][\mathbf{B}\mathbf{G}_k + \mathbf{G}_k\mathbf{B}] + E[\mu_k^2]\mathbf{D}_k + E[\mu_k^2]\xi_{min}\mathbf{H} + \sigma_c^2\mathbf{I} \quad (2.26)$$

where $\mathbf{B}$, $\mathbf{D}_k$, and $\mathbf{H}$ are defined, respectively, as:

$$\mathbf{B} = E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}] \quad , \quad (2.27)$$

$$\mathbf{D}_k = E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}\mathbf{G}_k\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}] \, , \quad (2.28)$$

$$\mathbf{H} = E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{(||\mathbf{x}_k||^2)^2}] \, . \quad (2.29)$$

To make the analysis easier to handle, we next proceed to make use of some approximations. We will make use of the approximation [45]:

$$E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}] \approx \frac{E[\mathbf{x}_k\mathbf{x}_k^T]}{E[||\mathbf{x}_k||^2]} \, . \quad (2.30)$$

The approximation in (2.30) was suggested in [45] to be a valid approximation that can be employed to simplify this and similar analysis.

We therefore evaluate $\mathbf{B}$, $\mathbf{D}_k$, and $\mathbf{H}$ in (2.27),(2.28), and (2.29) as follows:

$$\begin{aligned}\mathbf{B} &= E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{||\mathbf{x}_k||^2}] \approx \frac{E[\mathbf{x}_k\mathbf{x}_k^T]}{E[||\mathbf{x}_k||^2]} \\ &= \frac{\mathbf{I}}{N} \, . \end{aligned} \quad (2.31)$$

Similarly,

$$\mathbf{D}_k = E[\frac{\mathbf{x}_k\mathbf{x}_k^T\mathbf{G}_k\mathbf{x}_k\mathbf{x}_k^T}{(||\mathbf{x}_k||^2)^2}] \approx \frac{E[\mathbf{x}_k\mathbf{x}_k^T\mathbf{G}_k\mathbf{x}_k\mathbf{x}_k^T]}{E[(||\mathbf{x}_k||^2)^2]} \, . \quad (2.32)$$

Since the entries of $\mathbf{x}_k$ are zero-mean and white Gaussian random variables and realizing that by the Gaussian moment factoring theorem, fourth-order expectations can be expressed as a sum of products of second-order expectation [45, 57], we evaluate the expectation on the numerator of expression in the right hand-side of (2.32) as:

$$E[\mathbf{x}_k\mathbf{x}_k^T\mathbf{G}_k\mathbf{x}_k\mathbf{x}_k^T] = 2\sigma_x^4\mathbf{G}_k + \sigma_x^4 tr[\mathbf{G}_k]\mathbf{I} \tag{2.33}$$

Also,

$$\begin{aligned} E[(\|\mathbf{x}_k\|^2)^2] &= E[\mathbf{x}_k^T\mathbf{x}_k\mathbf{x}_k^T\mathbf{x}_k] \\ &= N\sigma_x^4\,. \end{aligned} \tag{2.34}$$

Therefore,

$$\mathbf{D}_k = \frac{1}{N}\{2\mathbf{G}_k + tr[\mathbf{G}_k]\mathbf{I}\}\,. \tag{2.35}$$

Finally,

$$\begin{aligned} \mathbf{H} &= E[\frac{\mathbf{x}_k\mathbf{x}_k^T}{(\|\mathbf{x}_k\|^2)^2}] \approx \frac{E[\mathbf{x}_k\mathbf{x}_k^T]}{E[(\|\mathbf{x}_k\|^2)^2]} \\ &= \frac{\mathbf{I}}{N\sigma_x^2}\,. \end{aligned} \tag{2.36}$$

Substituting $\mathbf{B}$, $\mathbf{D}_k$, and $\mathbf{H}$ into (2.26) and using (2.16) to re-write the resulting expression we obtain the following evolution equation for the second moment of the coefficient error vector:

$$\mathbf{G}_{k+1} = \{\mathbf{I} - \frac{2\mathbf{I}}{N}E[\mu_k]\}\mathbf{G}_k + \frac{1}{N}E[\mu_k^2]\{2\mathbf{G}_k + \frac{\sigma_{e_k}^2\mathbf{I}}{\sigma_x^2}\} + \sigma_c^2\mathbf{I}\,. \qquad (2.37)$$

Notice that the mean-square behavior of the weight vector is dictated by the behavior of the evolution equation for the coefficient error vector in (2.37) above. The weight vector of the algorithm converges, in the mean-square sense, if and only if (2.37) is finite at the steady-state. The necessary condition for this is that the mean behavior and the mean-square behavior of the step-size sequence must converge to finite steady-state values. We later show in the steady-state analysis that this requirements are fulfilled at the steady-state condition of the algorithm. From (2.30)-(2.36), it becomes relatively easy to agree with observation 4 that $\mathbf{G}_k$ is a diagonal matrix. Also we carefully observe the system of evolution equation in (2.37) and note that since at the onset, all the weights of the filter are initialized to zero values, then given already that the step-size sequence corresponding to each coefficient are equal and are also adapted using a single recursive equation as stated earlier, it becomes easy to note that the diagonal elements of the diagonal matrix $\mathbf{G}_k$ in (2.37) are equal.

2.9 Derivations of the Mean Behavior of the Step-Size $E[\mu_k]$

For the purpose of analysis, we shall presume that the computed value of the step-size by (2.9) at the k^{th} iteration falls within the convergence region so that only the case $\mu_k = \acute{\mu_k}$ need be analyzed. We start with taking the statistical expectation of (2.9):

$$E[\mu_k] = E[\mu_{k-1}] + \rho E[e_k e_{k-1} \frac{\mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}] \, . \tag{2.38}$$

Expanding e_k as in (2.20), substituting for d_k and $\mathbf{v}_k$ from (2.21) and (2.17) in the expansion, and then using (2.19) and (2.15) together with our various assumptions while evaluating the second expectation on the right-hand side of (2.38) will lead to:

$$E[e_k e_{k-1} \frac{\mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}] = a - E[\mu_{k-1}]b \tag{2.39}$$

where

$$a = E[\frac{\mathbf{x}_k^T \mathbf{v}_{k-1} \mathbf{x}_{k-1}^T \mathbf{v}_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}] \, ,$$

and

$$b = E[\frac{e_{k-1} e_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{(\|\mathbf{x}_{k-1}\|^2)^2}] \, .$$

With the benefit of our various assumptions and approximations, a can be evaluated as:

$$a = \frac{\sigma_x^4 tr[\mathbf{G}_{k-1}]}{N\sigma_x^2} = \frac{\sigma_x^2}{N} tr[\mathbf{G}_{k-1}] . \tag{2.40}$$

Expanding e_{k-1} as in (2.19) and using observation 1, we also evaluate b as:

$$\begin{aligned} b &= E[\frac{\mathbf{x}_k^T \mathbf{x}_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{(\|\mathbf{x}_{k-1}\|^2)^2}(\zeta_{k-1} - \mathbf{x}_{k-1}^T \mathbf{v}_{k-1})(\zeta_{k-1} - \mathbf{x}_{k-1}^T \mathbf{v}_{k-1})] \\ &= E[\frac{\mathbf{x}_k^T \mathbf{x}_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{(\|\mathbf{x}_{k-1}\|^2)^2}(\xi_{min} - 2\zeta_{k-1}\mathbf{x}_{k-1}^T \mathbf{v}_{k-1} + \mathbf{x}_{k-1}^T \mathbf{v}_{k-1} \mathbf{v}_{k-1}^T \mathbf{x}_{k-1})] \end{aligned}$$

Furthermore, we use our independence assumption, the uncorrelatedness of ζ_k and $\mathbf{x}_k$, (2.34), and the approximation in (2.30), to write:

$$\begin{aligned} b \approx E[\mathbf{x}_k^T \mathbf{x}_k \frac{E[\mathbf{x}_{k-1}^T \mathbf{x}_{k-1}]}{E[(\|\mathbf{x}_{k-1}\|^2)^2]}]\xi_{min} \\ + tr[\ E[\mathbf{x}_k^T \mathbf{x}_k \mathbf{v}_{k-1} \mathbf{v}_{k-1}^T \frac{E[\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T \mathbf{x}_{k-1}\mathbf{x}_{k-1}^T]}{E[(\|\mathbf{x}_{k-1}\|^2)^2]}]\ ] \end{aligned}$$

$$= \xi_{min} + tr[\ E[\mathbf{x}_k^T \mathbf{x}_k \mathbf{v}_{k-1} \mathbf{v}_{k-1}^T \frac{E[\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T \mathbf{x}_{k-1}\mathbf{x}_{k-1}^T]}{N\sigma_x^4}]\ ] . \tag{2.41}$$

Employing the Gaussian moment factoring theorem, we evaluate the innermost expectation on the numerator of the second expression on the right-hand side of (2.41) as:

$$E[\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T \mathbf{x}_{k-1}\mathbf{x}_{k-1}^T] = (2+N)\sigma_x^4 \mathbf{I} . \tag{2.42}$$

Using (2.42), the second expression on the right-hand side of (2.41) can be evaluated as:

$$tr\left[\frac{E[\mathbf{x}_k^T\mathbf{x}_k\mathbf{v}_{k-1}\mathbf{v}_{k-1}^T\ E[\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T]\ ]}{N\sigma_x^4}\right] = \frac{(2+N)\sigma_x^2 tr[\mathbf{G}_{k-1}]}{N}, \quad (2.43)$$

and on substituting (2.43) into (2.41) we have:

$$b = \xi_{min} + \frac{(2+N)\sigma_x^2 tr[\mathbf{G}_{k-1}]}{N}. \quad (2.44)$$

Finally, on substituting for (2.40) and (2.44) in (2.39) we have:

$$E[e_k e_{k-1}\frac{\mathbf{x}_k^T\mathbf{x}_{k-1}}{||\mathbf{x}_{k-1}||^2}] \approx \frac{\sigma_x^2 tr[\mathbf{G}_{k-1}]}{N} - E[\mu_{k-1}]\{\xi_{min} + \frac{(2+N)\sigma_x^2 tr[\mathbf{G}_{k-1}]}{N}\}. \quad (2.45)$$

Using (2.16) we re-write (2.45) as :

$$E[e_k e_{k-1}\frac{\mathbf{x}_k^T\mathbf{x}_{k-1}}{||\mathbf{x}_{k-1}||^2}] \approx \frac{\sigma_x^2}{N}tr[\mathbf{G}_{k-1}] - \frac{E[\mu_{k-1}]}{N}\{N\sigma_{e_{k-1}}^2 - N\sigma_x^2 tr[\mathbf{G}_{k-1}] + 2\sigma_x^2 tr[\mathbf{G}_{k-1}] + N\sigma_x^2 tr[\mathbf{G}_{k-1}]\}$$

$$= \frac{\sigma_x^2 tr[\mathbf{G}_{k-1}]}{N} - \frac{E[\mu_{k-1}]\{N\sigma_{e_{k-1}}^2 + 2\sigma_x^2 tr[\mathbf{G}_{k-1}]\}}{N}. \quad (2.46)$$

Finally putting (2.46) into (2.38) and re-arranging terms, we derive the following recursion for the mean-behavior of μ_k:

$$E[\mu_k] = E[\mu_{k-1}]\{1 - \rho(\sigma_{e_{k-1}}^2 + \frac{2\sigma_x^2}{N}tr[\mathbf{G}_{k-1}])\} + \rho\frac{\sigma_x^2}{N}tr[\mathbf{G}_{k-1}]. \quad (2.47)$$

The evolution of the step-size sequence follows the derived recursion in (2.47) above. From this equation, it is clear that the adaptive behavior of the step-size sequence is controlled by a positive, fixed, constant ρ as was earlier mentioned. It can be observed also from this equation that the value chosen for ρ has to be conservatively small in order to ensure smooth evolution of the step-size sequence. In all our simulation experiments, the results of which are displayed later, we used the value, $\rho = 8.0 \times 10^{-4}$. Finally it should be noted that mean-squared convergence of the algorithm coefficient can be guaranteed only when the step-size sequence μ_k is restricted within the range that would ensure convergence following the adaptation rule for step-size sequence stated in (2.10).

2.10 Derivations of the Mean-Square Behavior of the Step-Size, $E[\mu_k^2]$ for the VSS-NLMS Algorithm.

Squaring both sides of (2.9), we have:

$$\mu_k^2 = (\mu_{k-1} + \rho e_k e_{k-1} \frac{\mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2})(\mu_{k-1} + \rho e_k e_{k-1} \frac{\mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2})$$

and on expanding the brackets and taking the expectations of both sides, with the help of the independence assumptions, we get:

$$E[\mu_k^2] = E[\mu_{k-1}^2] + 2\rho E[\mu_{k-1}].E[\frac{e_k e_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{\|\mathbf{x}_{k-1}\|^2}] + \rho^2 E[\frac{e_k^2 e_{k-1}^2 \mathbf{x}_k^T \mathbf{x}_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{(\|\mathbf{x}_{k-1}\|^2)^2}] . \quad (2.48)$$

The third expectation on the right hand side of (2.48) has been solved in (2.46). The last expectation can be evaluated using (2.30) and the fact that e_k and e_{k-1} are uncorrelated, as:

$$E[\frac{e_k^2 e_{k-1}^2 \mathbf{x}_k^T \mathbf{x}_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{(\|\mathbf{x}_{k-1}\|^2)^2}] \approx E[e_k^2 \mathbf{x}_k^T \frac{E[e_{k-1}^2 \mathbf{x}_{k-1}^T \mathbf{x}_{k-1}]}{E[(\|\mathbf{x}_{k-1}\|^2)^2]} \mathbf{x}_k] \quad (2.49)$$

Notting that e_k is a zero-mean and Gaussian signal when conditioned on the coefficient vector $\mathbf{w}_k$, the inner expectation in the numerator of (2.49) can be expanded using Gaussian moment factoring theorem. Then with the help of the approximation in (2.12), equation (2.19), and the uncorrelatedness of $\mathbf{x}_k$ and ζ_k in (2.15), the resulting expression simplifies to:

$$E[e_{k-1}e_{k-1}\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T] = \xi_{min}\sigma_x^2\mathbf{I} + \sigma_x^4 tr[\mathbf{G}_{k-1}]\mathbf{I} + 2\sigma_x^4\mathbf{G}_{k-1} \;. \quad (2.50)$$

Using (2.16), (2.50) becomes:

$$E[e_{k-1}e_{k-1}\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T] = \mathbf{\Gamma}_{k-1} \,,$$

where

$$\mathbf{\Gamma}_{k-1} = \sigma_{e_{k-1}}^2\sigma_x^2\mathbf{I} + 2\sigma_x^4\mathbf{G}_{k-1} \,.$$

Using (2.34), we have:

$$\begin{aligned} E[e_k^2\mathbf{x}_k^T\frac{E[e_{k-1}^2\mathbf{x}_{k-1}\mathbf{x}_{k-1}^T]}{E[(\|\mathbf{x}_{k-1}\|^2)^2]}\mathbf{x}_k] &\approx tr[\,E[e_k^2\mathbf{x}_k\mathbf{x}_k^T\frac{\mathbf{\Gamma}_{k-1}}{N\sigma_x^4}]\,] \\ &= tr[\frac{\mathbf{\Gamma}_k\mathbf{\Gamma}_{k-1}}{N\sigma_x^4}] \end{aligned} \quad (2.51)$$

Substituting (2.46) and (2.51) into (2.48) and re-arranging we obtain the mean-square behavior of the step-size as:

$$\begin{aligned} E[\mu_k^2] = E[\mu_{k-1}^2]\{1 - \tfrac{2\rho}{N}(N\sigma_{e_{k-1}}^2 + 2\sigma_x^2 tr[\mathbf{G}_{k-1}])\} + \tfrac{2\rho}{N}E[\mu_{k-1}]\sigma_x^2 tr[\mathbf{G}_{k-1}] \\ +\rho^2 tr[(\sigma_{e_k}^2\mathbf{I} + 2\sigma_x^2\mathbf{G}_k)(\sigma_{e_{k-1}}^2\mathbf{I} + 2\sigma_x^2\mathbf{G}_{k-1})\tfrac{1}{N}] \end{aligned} \quad (2.52)$$

From the recursion derived for the mean-square behavior of the step-size sequence in (2.52) above, it can be observed that the convergence of μ_k^2 depends on both the step-size adaptation constant ρ as well as the mean behavior of the step-size sequence, $E[\mu_k]$. Since the convergence of

$E[\mu_k]$ in turn depends on ρ, it becomes clear that the convergence of μ_k^2 depends primarily on ρ. Deriving conditions on ρ so that convergence of the algorithm parameters can be guaranteed appears to be a difficult task. However, choosing ρ to be conservatively small, and restricting μ_k to be such that it always stay within the range that would ensure convergence, we can guarantee the convergence of the algorithm's parameters.

The mean-square error behavior of the proposed VSS-NLMS algorithm, which depends both on $E[\mu_k]$ and $E[\mu_k^2]$, is completely described by the derivations in (2.37), (2.47), and (2.52). In the next chapter, we shall proceed to analyze these system of evolution equations for the steady-state mean-square error behavior of the VSS-NLMS algorithm.

Chapter 3

Steady-State Analysis

3.1 Steady State Properties of the VSS-NLMS Algorithm

The system of evolution equations derived in chapter 2, namely expressions (2.37), (2.47), and (2.52) will converge provided that the step-size sequence is adapted according to the rule stated in (2.10) above. Under this condition, we study in this chapter, the steady-state behavior of the VSS-NLMS adaptive filtering algorithm, and then compare its performance with those of other algorithms.

As the steady-state is approached, let the algorithm parameters approach their respective steady-state values as:

$$E[\mu_k] \longrightarrow \overline{\mu}_\infty \,, \tag{3.1}$$

$$E[\mu_k^2] \longrightarrow \overline{\mu_\infty^2} \,, \tag{3.2}$$

$$\sigma^2_{e_k} \longrightarrow \sigma^2_{e_\infty} , \tag{3.3}$$

and

$$\mathbf{G}_k \longrightarrow \mathbf{G}_\infty . \tag{3.4}$$

On substituting (3.1)-(3.4) into (2.16), (2.37), (2.47), and (2.52) we get:

$$\sigma^2_{e_\infty} = \xi_{min} + \sigma_x^2 tr[\mathbf{G}_\infty] , \tag{3.5}$$

$$\mathbf{G}_\infty = \mathbf{G}_\infty - \frac{2\mathbf{I}\overline{\mu}_\infty}{N}\mathbf{G}_\infty + \frac{\overline{\mu^2_\infty}}{N}\{2\mathbf{G}_\infty + \frac{\sigma^2_{e_\infty}}{\sigma_x^2}\mathbf{I}\} + \sigma_c^2\mathbf{I} , \tag{3.6}$$

$$\overline{\mu}_\infty = \overline{\mu}_\infty - \overline{\mu}_\infty\rho\{\sigma^2_{e_\infty} + \frac{2\sigma_x^2}{N}tr[\mathbf{G}_\infty]\} + \frac{\rho\sigma_x^2}{N}tr[\mathbf{G}_\infty] , \tag{3.7}$$

and

$$\begin{aligned} \overline{\mu^2_\infty} \;&=\; \overline{\mu^2_\infty} - \overline{\mu^2_\infty}.\frac{2\rho}{N}\{N\sigma^2_{e_\infty} + 2\sigma_x^2 tr[\mathbf{G}_\infty]\} + \frac{2\rho}{N}\overline{\mu}_\infty\sigma_x^2 tr[\mathbf{G}_\infty] \\ &\quad + \rho^2 tr[(\sigma^2_{e_\infty}\mathbf{I} + 2\sigma_x^2\mathbf{G}_\infty)(\sigma^2_{e_\infty}\mathbf{I} + 2\sigma_x^2\mathbf{G}_\infty)\frac{1}{N}] . \end{aligned} \tag{3.8}$$

Since $\mathbf{G}_\infty$ is a diagonal matrix of equal diagonal elements, we can let $\mathbf{G}_\infty = k_\infty\mathbf{I}$ in (3.6), (3.7) and (3.8), and proceed with the analysis using (3.5) to obtain closed form expressions for the steady-state values of each of the algorithm's parameters in terms of other parameters as follows: For k_∞ we have:

$$k_\infty \;=\; \frac{\overline{\mu^2_\infty}\xi_{min} + N\sigma_x^2\sigma_c^2}{2\sigma_x^2\overline{\mu}_\infty - \overline{\mu^2_\infty}\sigma_x^2(2+N)} , \tag{3.9}$$

similarly for $\overline{\mu}_\infty$ we have:

$$\overline{\mu}_\infty = \frac{\sigma_x^2 k_\infty}{\xi_{min} + \sigma_x^2 k_\infty (N+2)}, \tag{3.10}$$

and finally, for $\overline{\mu_\infty^2}$ we obtained:

$$\overline{\mu_\infty^2} = (\overline{\mu}_\infty)^2 + \frac{\rho}{2}[\xi_{min} + \sigma_x^2 k_\infty (N+2)]. \tag{3.11}$$

We now proceed to make few simplifications in (3.9)-(3.11) which will enable us to get a much better insight into the behavior of the algorithm.

1. In most practical applications involving stationary environments, the excess mean-squared error is much smaller than the minimum mean-squared estimation error [1] so that:

$$\xi_{min} \gg (N+2)\sigma_x^2 k_\infty . \tag{3.12}$$

2. Under the same circumstances of (3.12), we observe that since $\overline{\mu}_\infty$ itself is very small for stationary environments and $\overline{\mu_\infty^2}$ is of the order of $(\overline{\mu}_\infty)^2$, therefore:

$$\overline{\mu}_\infty \gg \overline{\mu_\infty^2}(N+2)\sigma_x^2 . \tag{3.13}$$

3.1.1 Approximate Expressions for the steady-state Performances of the VSS-NLMS Algorithm in Stationary Environments

Using (3.12) we can approximate $\overline{\mu}_\infty$ and $\overline{\mu^2_\infty}$, respectively, as:

$$\overline{\mu}_\infty \approx \frac{\sigma_x^2 k_\infty}{\xi_{min}}, \tag{3.14}$$

and

$$\overline{\mu^2_\infty} \approx (\overline{\mu}_\infty)^2 + \frac{\rho}{2}\xi_{min} . \tag{3.15}$$

Similarly, using (3.13), (3.15) and setting $\sigma_c = 0$ for stationary case, we approximate k_∞ as:

$$k_\infty \approx \frac{[(\overline{\mu}_\infty)^2 + \frac{\rho}{2}\xi_{min}]\xi_{min}}{2\sigma_x^2\overline{\mu}_\infty} . \tag{3.16}$$

Substituting for $\overline{\mu}_\infty$ as approximated in (3.14) into (3.16) we have:

$$k_\infty \approx \frac{\frac{\sigma_x^4 k_\infty^2}{\xi_{min}} + \frac{\rho}{2}\xi_{min}^2}{2\sigma_x^2 . \frac{\sigma_x^2 k_\infty}{\xi_{min}}},$$

that is,

$$k_\infty^2 [\frac{2\sigma_x^4}{\xi_{min}} - \frac{\sigma_x^4}{\xi_{min}}] \approx \frac{\rho}{2}\xi_{min}^2$$

We therefore obtain an approximate expression for k_∞ as:

$$k_\infty \approx \sqrt{\frac{\rho\xi_{min}^3}{2\sigma_x^4}} \tag{3.17}$$

This value given by equation (3.17) above represent the approximate mean-squared value of the coefficient fluctuations for the VSS-NLMS in stationary environment. It should be noted that since ρ, ξ_{min}, and σ_x^4 are all finite, the steady-state mean-squared value of the coefficient fluctuation k_∞ and consequently the steady-state mean-squared norm of the coefficient error vector $\mathbf{G}_\infty$ are finite as well. Hence we conclude that the system of evolution equations derived in Chapter 2 will all converge at the steady-state conditions.

3.1.2 Comparison between the Steady-State Performances of the VSS-NLMS and the NLMS algorithm in Stationary Environments.

The measure of performance used is the mis-adjustment defined [47] as:

$$M_k = \frac{e_{ex_k}}{\xi_{min}}, \tag{3.18}$$

where e_{ex_k} is the excess error at each time iteration k. The steady-state mis-adjustment can be expressed from (3.18) as

$$M_\infty = \frac{e_{ex_\infty}}{\xi_{min}}, \tag{3.19}$$

where e_{ex_∞} is the steady-state value of the excess mean-squared estimation error. The steady-state mean-squared estimation error can be calculated from (2.16) as:

$$\sigma^2_{e_\infty} = \xi_{min} + \sigma_x^2 tr[\mathbf{G}_\infty] \ .$$

This show that the steady-state excess mean-squared estimation error (MSE) is given by:

$$\begin{aligned} e_{ex_\infty} &= \sigma_x^2 tr[\mathbf{G}_\infty] \ . \\ &= N\sigma_x^2 k_\infty \ . \end{aligned} \tag{3.20}$$

Substituting (3.17) into (3.20) we have:

$$e_{ex_\infty, VSS-NLMS} \approx N\sigma_x^2 \sqrt{\frac{\rho\xi^3_{min}}{2\sigma_x^4}} \ . \tag{3.21}$$

Hence, the steady-state mis-adjustment for VSS-NLMS algorithm is given by:

$$\begin{aligned} M_{\infty, VSS-NLMS} &= \frac{e_{ex_\infty, VSS-NLMS}}{\xi_{min}} \\ &\approx N\sigma_x^2 \sqrt{\frac{\rho\xi_{min}}{2\sigma_x^4}} \ .(3.22) \end{aligned}$$

The authors in [1] have shown for their variable step-size LMS algorithm (in stationary environment), that:

$$e_{ex_\infty, VSS-LMS} \approx N\sigma_x^2 \sqrt{\frac{\rho\xi^3_{min}}{2}} \ , \tag{3.23}$$

and therefore the steady-state mis-adjustment for the VSS-LMS algorithm in [1] can be calculated from (3.23) as:

$$M_{\infty,VSS-LMS} = N\sigma_x^2\sqrt{\frac{\rho\xi_{min}}{2}} \,. \tag{3.24}$$

Conclusions for Stationary Environment

Comparing equations (3.21) and (3.22) with (3.23) and (3.24), respectively, we observe that since generally $\sigma_x^4 < 1$ in most practical situations, then we can conclude that in a stationary environment, our VSS-NLMS algorithm will converge to a higher steady-state excess MSE and consequently higher mis-adjustment error than the counterpart VSS-LMS algorithm when both utilize similar step-size variation method. This result is not unexpected since the authors in [21] have shown that NLMS algorithm, fixed step-size, will converge faster[1] than the LMS algorithm (fixed step-size), but to a higher mis-adjustment error. Therefore, when similar step-size variation method is incorporated with these two algorithms, their relative performances, obtainable without step-size variations, is expected to be observed. More importantly however, the result of this analysis allows us to have an insight into the performance of our VSS-NLMS algorithm with respect to the traditional NLMS algorithm. The authors in [1] showed in their simulation results that after about 50000 iterations, the squared norm of the mis-adjustment vector for their variable step-size LMS algorithm is more than $20dB$ smaller than that of the

[1] the performance superiority of the NLMS algorithm over the LMS algorithm in terms of convergence speed is well depicted only at high step-size values [21] as can be observed from our simulation result in figures 4.1 and 4.3

NLMS algorithm. From the expressions in (3.21)-(3.24), we observe here that the difference between the steady-state coefficient fluctuation for the VSS-NLMS and the VSS-LMS algorithm is $\frac{1}{\sqrt{\sigma_x^4}}$. This implies that such wide difference (20 dB) cannot exist between the mean-squared error performance of our variable step-size NLMS algorithm and its counterpart variable step-size LMS algorithm at the steady-state condition of the two algorithms. Hence we can reasonably conclude from this argument that the proposed VSS-NLMS algorithm has improved on the steady-state error performance of the traditional NLMS algorithm. We later confirm this conclusion by computer simulation experiments.

3.1.3 Approximate Expressions For the steady-state Performances of the VSS-NLMS Algorithm in Non-Stationary Environments

If we assume that the degree of non-stationarity of the environment is small so that σ_c is very small, then the inequalities in (3.12) and (3.13) still hold. Under such circumstances, from (3.9) we have:

$$k_\infty \approx \frac{\overline{\mu_\infty^2}\xi_{min} + N\sigma_x^2\sigma_c^2}{2\sigma_x^2\overline{\mu}_\infty}. \tag{3.25}$$

Substitution of (3.14) and (3.15) into (3.25) and re-arranging terms will yield the following approximate expression for the mean-square value of the coefficient fluctuation for the VSS-NLMS algorithm in non-stationary environment:

$$k_\infty \approx \sqrt{\frac{\rho}{2\sigma_x^4}\xi_{min}^3 + \frac{\sigma_c^2}{\sigma_x^2}\xi_{min}} \,. \tag{3.26}$$

3.1.4 Comparison between the Steady-State Performances of VSS-NLMS and the NLMS algorithm in Non-Stationary Environments

From (3.26),the steady-state excess estimation error and the Mis-adjustment for the VSS-NLMS algorithm in non-stationary environments are given respectively by:

$$\begin{aligned} e_{ex_\infty,VSS-NLMS} &= N\sigma_x^2 k_\infty \\ &\approx N\sigma_x^2\sqrt{\frac{\rho}{2\sigma_x^4}\xi_{min}^3 + \frac{\sigma_c^2}{\sigma_x^2}\xi_{min}} \,, \end{aligned} \tag{3.27}$$

and

$$\begin{aligned} M_{\infty,VSS-NLMS} &= \frac{e_{ex_\infty,VSS-NLMS}}{\xi_{min}} \\ &\approx N\sigma_x^2\sqrt{\frac{\rho}{2\sigma_x^4}\xi_{min} + \frac{\sigma_c^2}{\sigma_x^2\xi_{min}}} \,. \end{aligned} \tag{3.28}$$

Similarly, the steady-state excess error for the VSS-LMS algorithm, $e_{ex_\infty,VSS-LMS}$, in non-stationary environment was found to be [1]:

$$e_{ex_\infty,VSS-LMS} = N\sigma_x^2\sqrt{\frac{\rho}{2}\xi_{min}^3 + \frac{\sigma_c^2}{\sigma_x^2}\xi_{min}} \,. \tag{3.29}$$

From which we compute it's mis-adjustment as:

$$\begin{aligned} M_{\infty,VSS-LMS} &= \frac{e_{ex_{\infty},VSS-LMS}}{\xi_{min}} \\ &\approx N\sigma_x^2\sqrt{\frac{\rho}{2}\xi_{min} + \frac{\sigma_c^2}{\sigma_x^2\xi_{min}}} \end{aligned} \quad . \tag{3.30}$$

Conclusions for Non-Stationary Environment

Comparing (3.27) and (3.28) with (3.29) and (3.30), respectively, we again note that the steady-state excess error (and hence the mis-adjustment) that a VSS-NLMS algorithm converges to, is also greater than that given by it's counterpart VSS-LMS algorithm when both of them still operate in non-stationary environments. Since the same factor σ_x^4 is still the difference between the expressions in (3.27), (3.28) and (3.29), (3.30) therefore the conclusion we drew for the stationary case, that the steady-state MSE performance of our VSS-NLMS algorithm will be better than that of the traditional NLMS algorithm, is also drawn here in the case of non-stationary environment. Simulation results shown later confirm these theoretical findings.

3.2 Conclusions of the Analysis

The Conclusion we arrive here is that whether the operating environment is stationary or non-stationary, our VSS-NLMS algorithm will converge to a smaller steady-state excess error, and consequently lower Mis-adjustment, than the traditional NLMS algorithm. In other words, the statistics of the environment, stationary or non-stationary, has no influence on the performance superiority of the VSS-NLMS over the conventional NLMS algorithm.

Chapter 4

Simulation Results

This Chapter presents the results of the computer simulations we made to investigate the performance behaviors of the VSS-NLMS algorithm presented in this work. The results of our simulations demonstrate several good properties of our algorithm (VSS-NLMS) and its performance superiority over the traditional NLMS algorithm (with fixed step-size). It is well known that $\mu = 1.0$ for NLMS algorithm, gives the fastest convergence speed of the algorithm. However, if low steady-state MSE is the performance quantity more desired, then choosing a very low value of the step-size for the traditional NLMS (fixed step-size) must be the the compromise to be made. This means that the optimum convergence speed compounded with low steady-state MSE cannot be achieved with the conventional NLMS algorithm. Here, we show that the proposed VSS-NLMS algorithm can utilize the optimum step-size at initialization (for fastest convergence speed), and still converge to a very low MSE at the steady-state condition. All performances of our algorithm examined in this work are adequately compared with the corresponding performances of the traditional NLMS algorithm, and the results in all cases considered showed that our algorithm records better performance gains over the conventional NLMS algorithm.

In our simulations, the following performance quantities were investigated for our algorithm:

* steady-state mean-square error (MSE),
* initial convergence time,
* recovery time after changes in the environment (echo path).

All simulation results presented are averages over 50 independent runs. For the case of the non-stationary environment, ensemble averaging was used.

The simulation part consists of two cenario. In the first one, the performance of the VSS-NLMS algorithm is studied for a system identification problem, whereas in the second one the algorithm performance is evaluated for an echo canceller configuration.

4.1 The Adaptive System Identification Problem Considered

The following identification problem considered in [1] is also used, in our simulation, to investigate the performance of our algorithm:

The coefficient of the unknown system (to be identified by the adaptive filter) is $\{\mathbf{w}_i \;\; i = 0, 1, 2, 3, 4\} = \{0.1, 0.3, 0.5, 0.3, 0.1\}$.

The input signal $\mathbf{x}_k$ is a zero-mean, and Gaussian process obtained as the output of the all-pole filter with the transfer function

$$H(z) = \frac{0.44}{1 - 1.5z^{-1} + z^{-2} - 0.25z^{-3}} \tag{4.1}$$

when the input to the filter was zero-mean Gaussian noise with unit variance. The desired response signal d_k was obtained by corrupting the output of the system in (4.1) with zero-mean Gaussian additive observation noise. The observation noise was uncorrelated with the input signal and its variance was 0.01. The adaptive filter was run using five coefficient (longer coefficient will be used later while testing the algorithm for the particular case of echo cancellation, but notice that the eigen value spread of the resulting input auto-correlation matrix is more than 140 [1], and as such colored (speech) input is closely modelled), and all the coefficients of the system identifier were initialized to zero values at the onset.

4.2 Results on the comparison between the variable step-size NLMS (VSS-NLMS) and a counterpart variable step-size LMS (VSS-LMS) algorithms

In Figure 4.1, we have plotted the sum of the mean-squared deviations of each coefficient from its mean value (mean-squared norm of the coefficient error vector) as a function of time for the first 10000 iterations for both the VSS-NLMS algorithm and the counterpart VSS-LMS algorithm in [1]. The parameters used for both algorithms are: $\rho = 0.0008$, and $\mu_0 = 0.5$. The maximum and minimum possible value of μ_k dur-

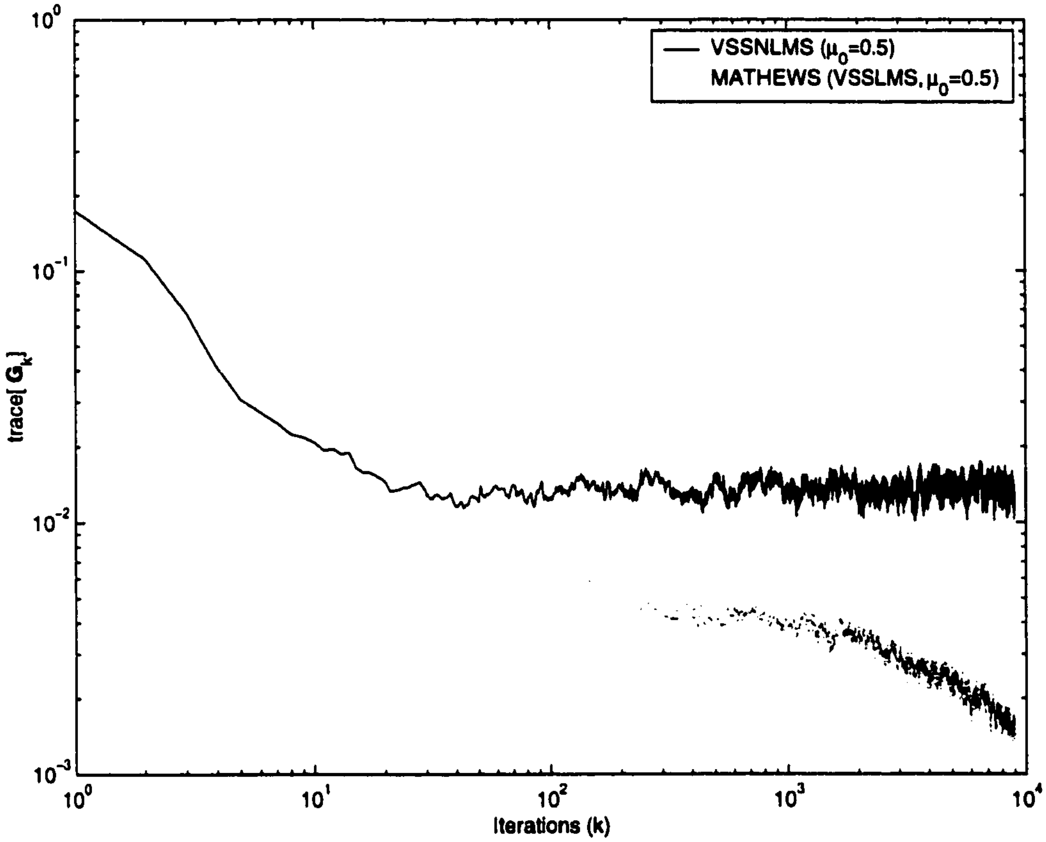


Figure 4.1: Comparison of the performance of the variable step-size NLMS algorithm and the counterpart variable step-size LMS algorithm in [1].

ing adaptation was set in accordance with the rule stated in (2.10) (i.e. $0 < \mu_{min} < \mu_{max} < 2$) so as to meet the requirement for convergence of the algorithm. We used the values: $\mu_{min} = 10^{-8}$ and $\mu_{max} = 1.9999999$. The step-size was limited to the same maximum and minimum values in all other simulation experiments made in this work.We can observe from the figure that the VSS-NLMS algorithm presented here converges faster

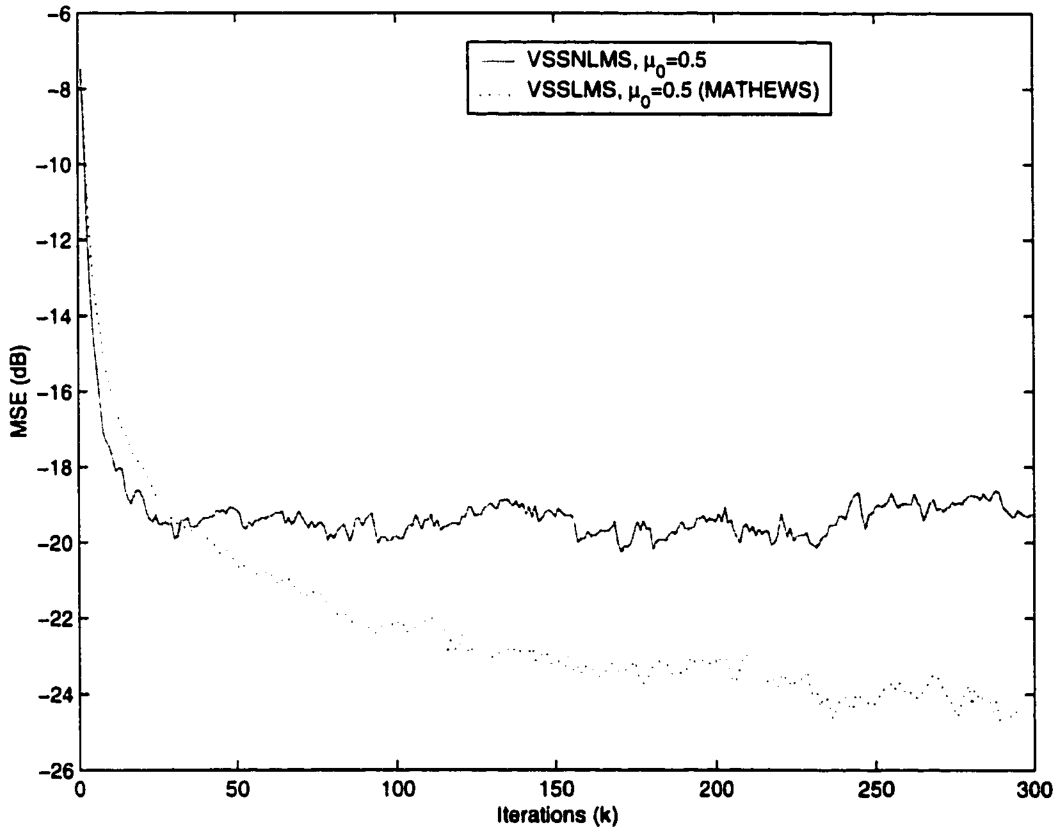


Figure 4.2: Comparison of the convergence speed and the initial MSE at convegence of the variable step-size NLMS (VSS-NLMS) algorithm and its counterpart VSS-LMS (Mathews') algorithm.

than Mathews VSS-LMS algorithm, but to a higher steady-state misadjustment error than Mathews' algorithm. This result supports what the analytical result in Chapter 3 suggested. These two results (analytical and simulation) presented here are in line with what the authors in [21] have found to be the relative performances of the conventional NLMS and LMS algorithms. In Figure 4.2 we have displayed the curves of the mean-square error (MSE) of the two algorithms for the same experiment. This figure shows more clearly, the relative convergence speed, and the initial MSE at convergence time for the two algorithms. The result displayed in this figure is equivalent to what the two traditional algorithms (NLMS and LMS algorithms) will perform when step-size variation are not incorporated with them. However, because of the step-size variation here, the steady-state MSE of the two variable step-size algorithms will drop considerably to very small values later at higher time iterations (as suggested by the mean behavior of the step-size sequence shown in figures 4.4-4.6.

4.3 Results on the Properties of the VSS-NLMS Algorithm

In Figure 4.3 (a), the mean-square norm of the coefficient error vector for the VSS-NLMS is shown for several values of μ_0, and fixed $\rho = 0.0008$. As would be expected, the convergence speed of the algorithm is better for higher values of μ_0. However, we note that even for the case μ_0 very close to zero (10^{-8}), (when the traditional NLMS as shown later in Figure 4.16 is non-convergent at all), the proposed algorithm still display good convergence speed. Figure 4.3 (b) duplicates Mathews' corresponding results for the VSS-LMS algorithm.

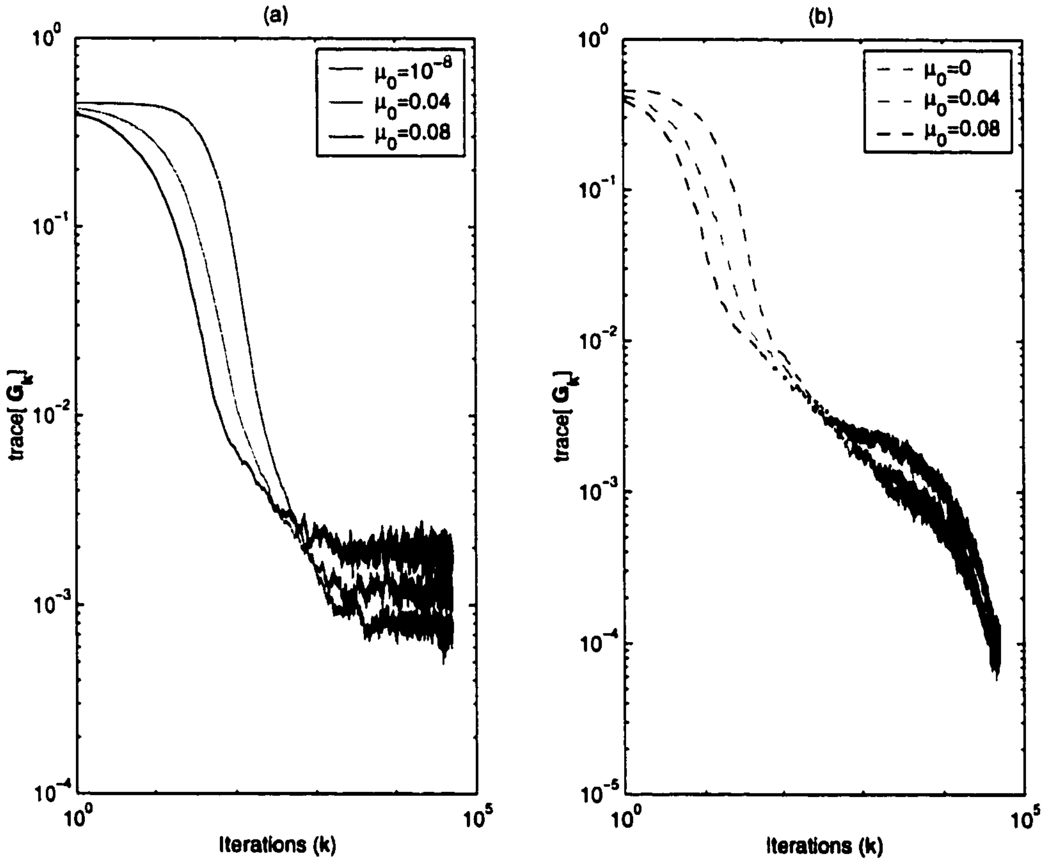


Figure 4.3: Performance of the VSS-NLMS adaptive algorithm for different values of μ_0 and fixed ρ = 0.0008: (a) VSS-NLMS algorithm, (b) Mathews' VSS-LMS algorithm.

The mean behavior of the step-size, $E[\mu_k]$, is documented in Figure 4.4-4.6 for different initializations μ_0, and fixed ρ in stationary environment. Note that for large values of μ_0 (that already insures fast convergence speed, e.g. $\mu_0 = 1.0$, $\mu_0 = 0.5$ as shown in Figure 4.4), the step-size remains constant for awhile and then decreases swiftly as the adaptation proceeds (so as to achieve low MSE at steady-state). Note that the step-size do not need to rise up at the onset in this case as the value of the step-size started with is good enough for fast convergence of the algorithm. But for low values of μ_0 (that are not good enough to insure fast convergence, e.g. $\mu_0 = 10^{-8}$ and $\mu_0 = 0.04$ as shown in Figure 4.6), the step-size quickly rises up to a value that will be good enough for fast convergence speed at the onset, it then remains constant on the value so attained for awhile and later also decreases swiftly as the adaptation proceeds (so as to achieve low MSE at steady-state).

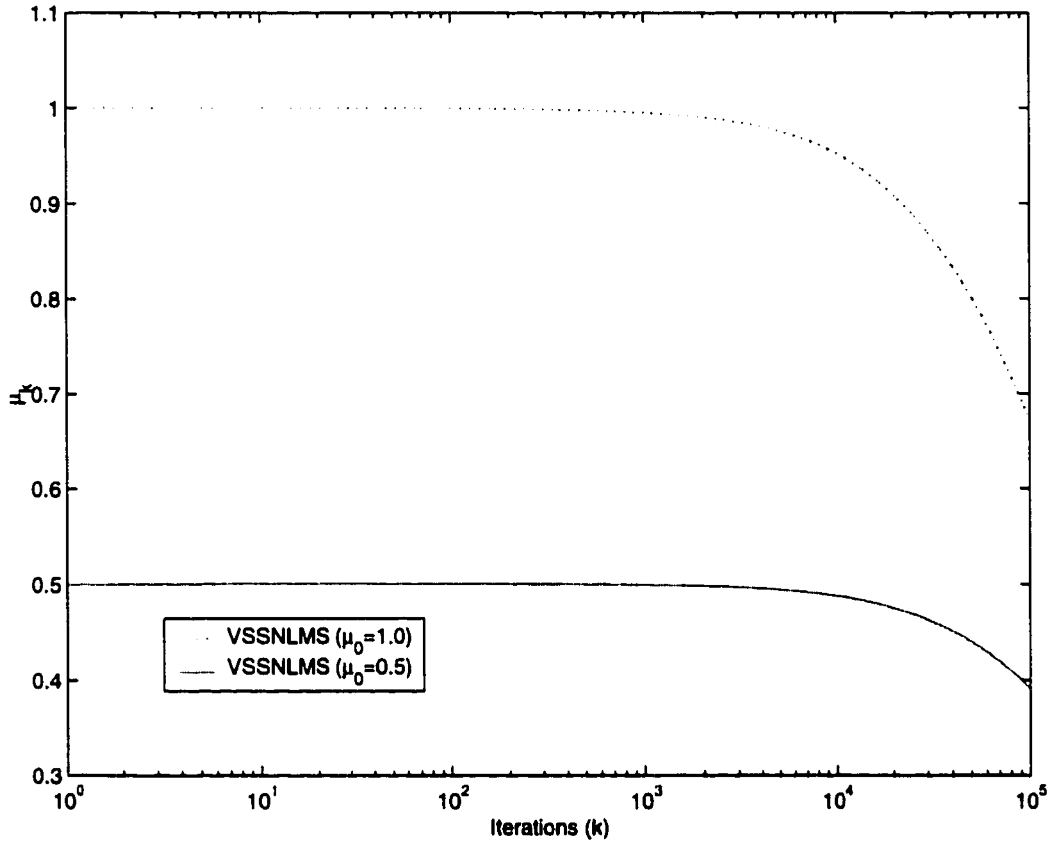


Figure 4.4: Mean Behavior of the step-size, μ_k, for the VSS-NLMS algorithm in Stationary Environment for the case: $\rho = 0.0008$, $\mu_0 = 1.0$, and $\mu_0 = 0.5$.

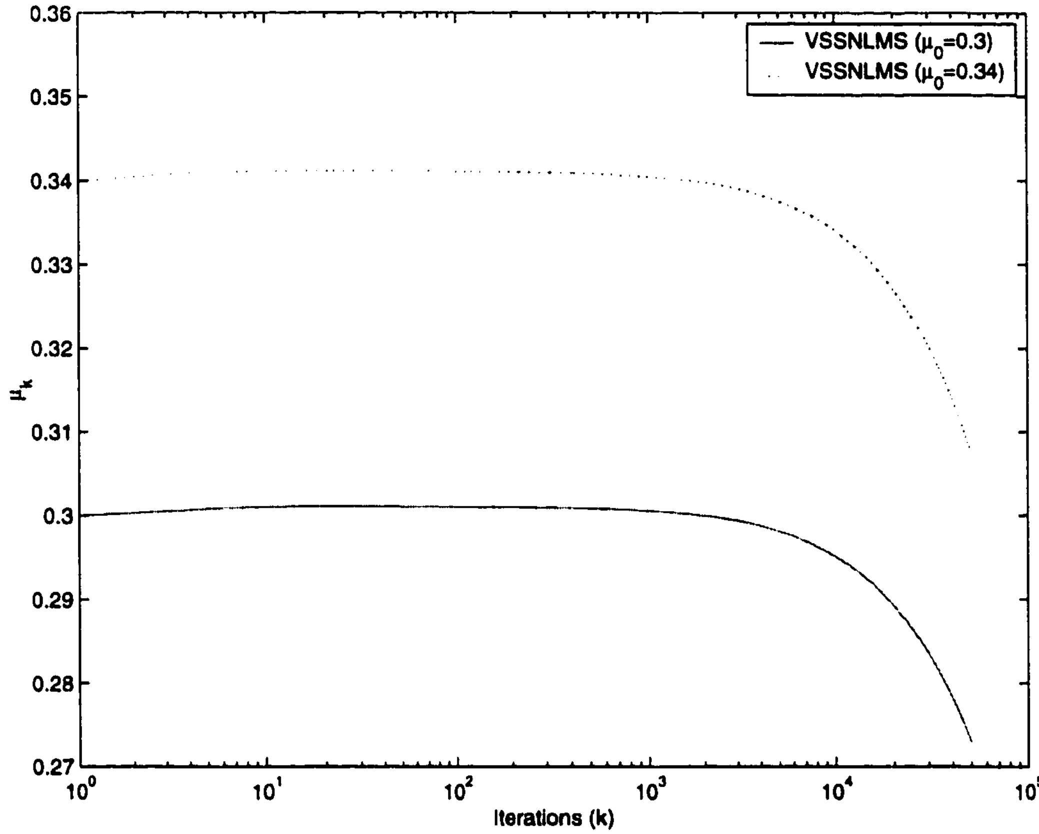


Figure 4.5: Mean Behavior of the step-size, μ_k, for the VSS-NLMS algorithm in Stationary Environment for the case: $\rho = 0.0008$, $\mu_0 = 0.3$, and $\mu_0 = 0.34$.

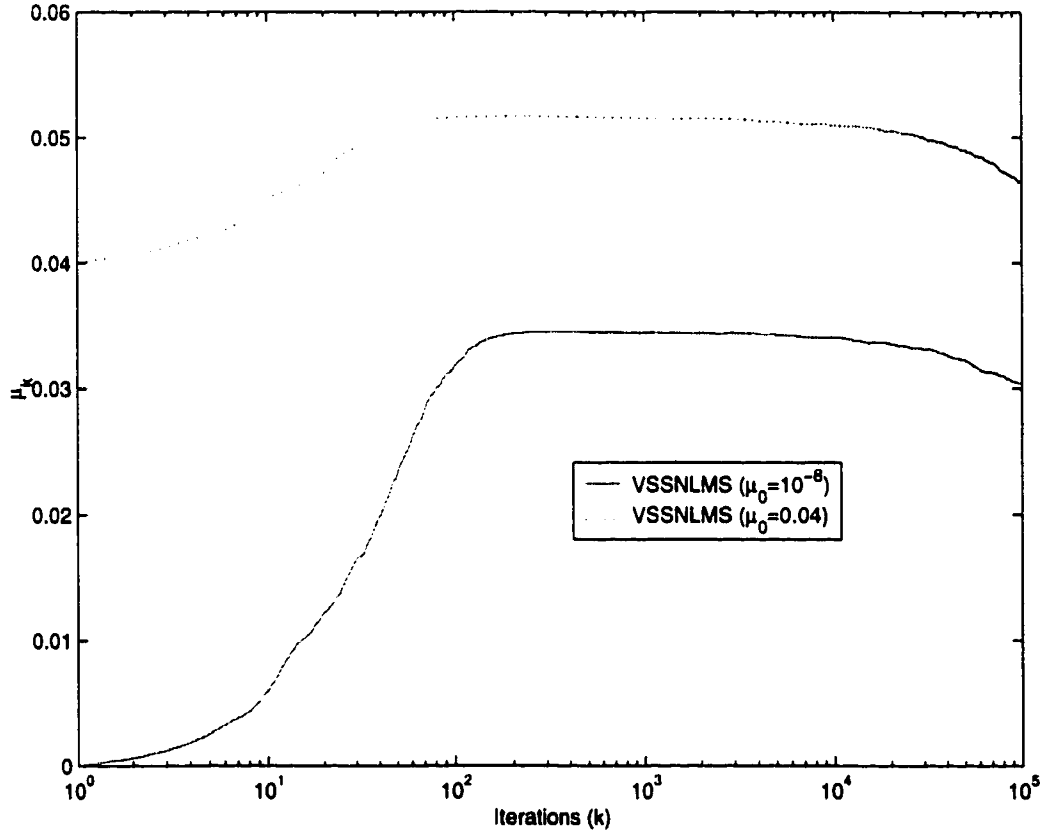


Figure 4.6: Mean Behavior of the step-size, μ_k, for the VSS-NLMS algorithm in Stationary Environment for the case: $\rho = 0.0008$, $\mu_0 = 10^{-8}$, and $\mu_0 = 0.04$.

For medium values of μ_0 (e.g. $\mu_0 = 0.3$, $\mu_0 = 0.34$), the step-size did rise up at the onset (this can be gathered by carefully observing the curve for this case shown in Figure 4.5. It can be seen that the curve is not a perfect straight line at the beginning. Results shown later in Figure 4.19 clarify this point better). As usual after the curve rose to a level, it remains constant on this value for awhile, and then decreases swiftly as the adaptation proceeds. Because of these good behaviors of the step-size sequence, the algorithm is able to combine the two much coveted performance quality, fast speed of convergence and low steady-state MSE, without having to compromise one for the other -a performance which cannot be achieved by the traditional NLMS algorithm. This is a classical performance improvement that the VSS-NLMS algorithm has contributed to the performance of the traditional NLMS algorithm.

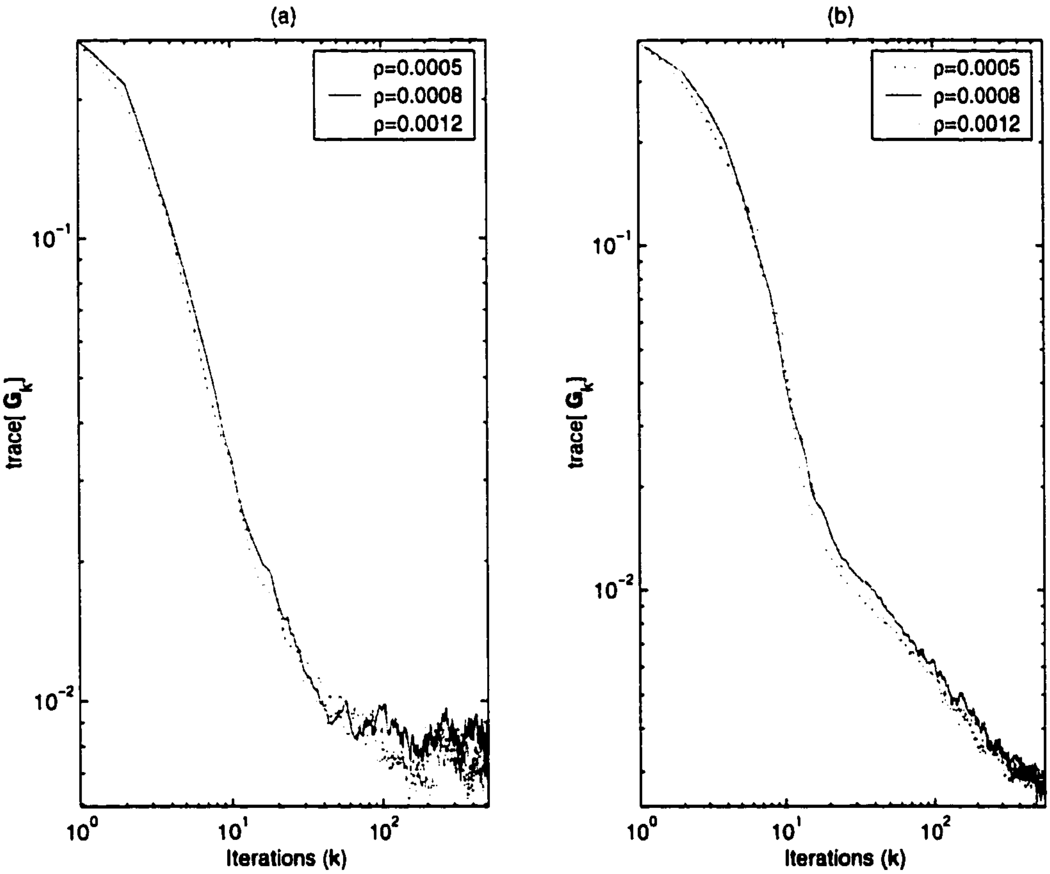


Figure 4.7: Performances of: (a)VSS-NLMS adaptive algorithm for $\mu_0 = 0.3$,$\rho = 0.0012$, $\rho = 0.0008$, $\rho = 0.0005$ (b)VSS-LMS adaptive algorithm for $\mu_0 = 0.08$,$\rho = 0.0012$, $\rho = 0.0008$, $\rho = 0.0005$.

Figure 4.7(a) displays curves for the second moment of the coefficient error vector (for an experiment similar to those of Figures 4.4-4.6) for several values of ρ and fixed $\mu_0 = 0.3$. Since the initial value of μ_0 was not low here, the initial convergence speed is more or less insensitive to the choice of ρ. As shown in Chapter 3, the steady-state behavior does depend on ρ, but it will take a very large number of iterations before the differences can show up in a significant fashion. But it is fairly displayed in this figure. Figure 4.7(b) displays a similar curve for the counterpart VSS-LMS algorithm. On the basis of Figures 4.4-4.7, we can reasonably infer that for an optimum utilization of the good properties of this algorithm, it is advisable to select the (highest) step-size that gives the fastest convergence speed of the traditional NLMS algorithm (which has been shown [21, 42] to be $\mu = 1.0$) as the initial step-size, μ_0, for the VSS-NLMS algorithm. This will guarantee the optimum convergence speed. Then ρ can be used to influence the steady-state MSE as desired.

4.4 Results on the Comparison between the VSS-NLMS filter and NLMS filter in Stationary Environment

4.4.1 Results for medium values of step-size initializations for the VSS-NLMS filter

In Figure 4.8, we compare the second moment of the coefficient error vector for the VSS-NLMS filter and the NLMS filter. This result stems from the observed behaviors of the step-size when moderately small values of the step-size initializations are used for the VSS-NLMS algorithm. As shown earlier, with relatively low values of μ_0, the step-size sequence μ_k for the VSS-NLMS algorithm will rise up at the onset to a higher value that gives faster convergence speed. For this reason, the convergence speed of the VSS-NLMS filter for $\mu_0 = 0.3$ is observed to be equivalent to that of the NLMS filter for $\mu = 0.5$. Hence VSS-NLMS filter converges faster, and it goes without saying that this is compounded with lower steady-state MSE since the values of the step-size sequence will have been reduced to lower values at the steady-state condition. In Figure 4.9, the MSE curve for the same consideration of Figure 4.8 is shown where the convergence speed of the two filters can be clearly seen to coincide.

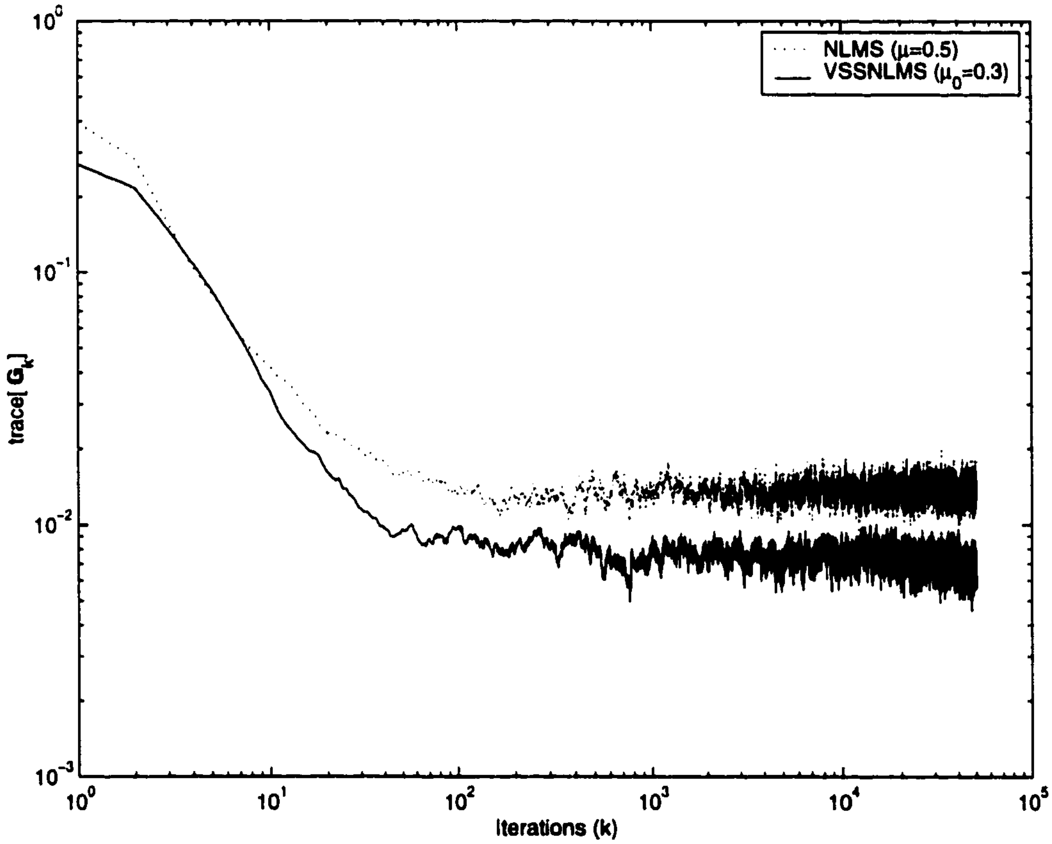


Figure 4.8: Comparison of the convergence speed and steady-state performances of the VSS-NLMS adaptive algorithm (for $\mu_0 = 0.3$) and the traditional NLMS algorithm (fixed step-size, $\mu = 0.5$).

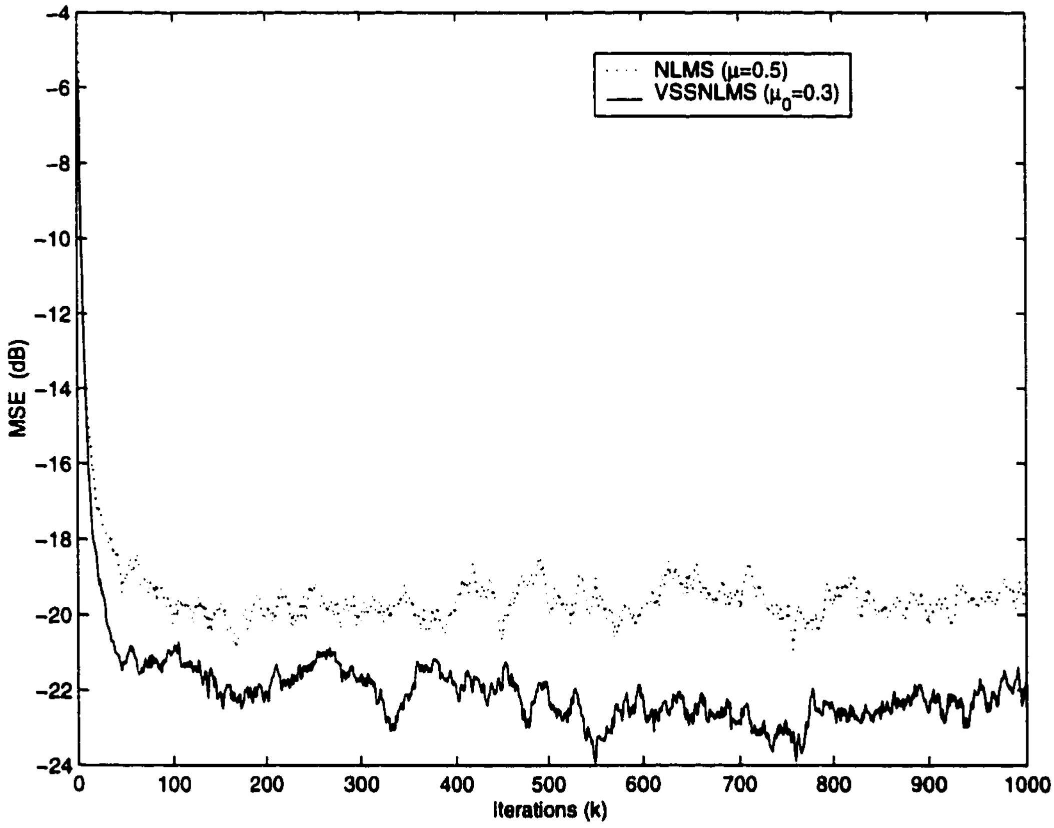


Figure 4.9: Comparison of the MSE curve for the VSS-NLMS algorithm and the NLMS algorithm of figure 4.8.

4.4.2 Results for High values of step-size initializations for the VSS-NLMS filter

In Figures 4.10 and 4.11, the mean-square norm of the coefficient error vector as a function of time is plotted for both the VSS-NLMS ($\rho = 0.0008$, $\mu_0 = 1.0$) and the conventional NLMS ($\mu = 1.0$) filter for the case when the step-size initialization of the VSS-NLMS filter is very high. From Figure 4.10, it can be observed that while the NLMS filter finally converges at steady-state to a constant MSE dictated by the value of the step-size, $\mu = 1.0$ used, the mean-square norm for the VSS-NLMS filter started decreasing as from about 10000 iteration. In Figure 4.11(a), we re-plot the curves of Figure 4.10 for the last 50000 iterations, while Figure 4.11(b) shows the same curves for the last 50 iterations (99950 – 100000). From this figure, the performance improvement recorded by the VSS-NLMS filter over the traditional NLMS filter can be observed.

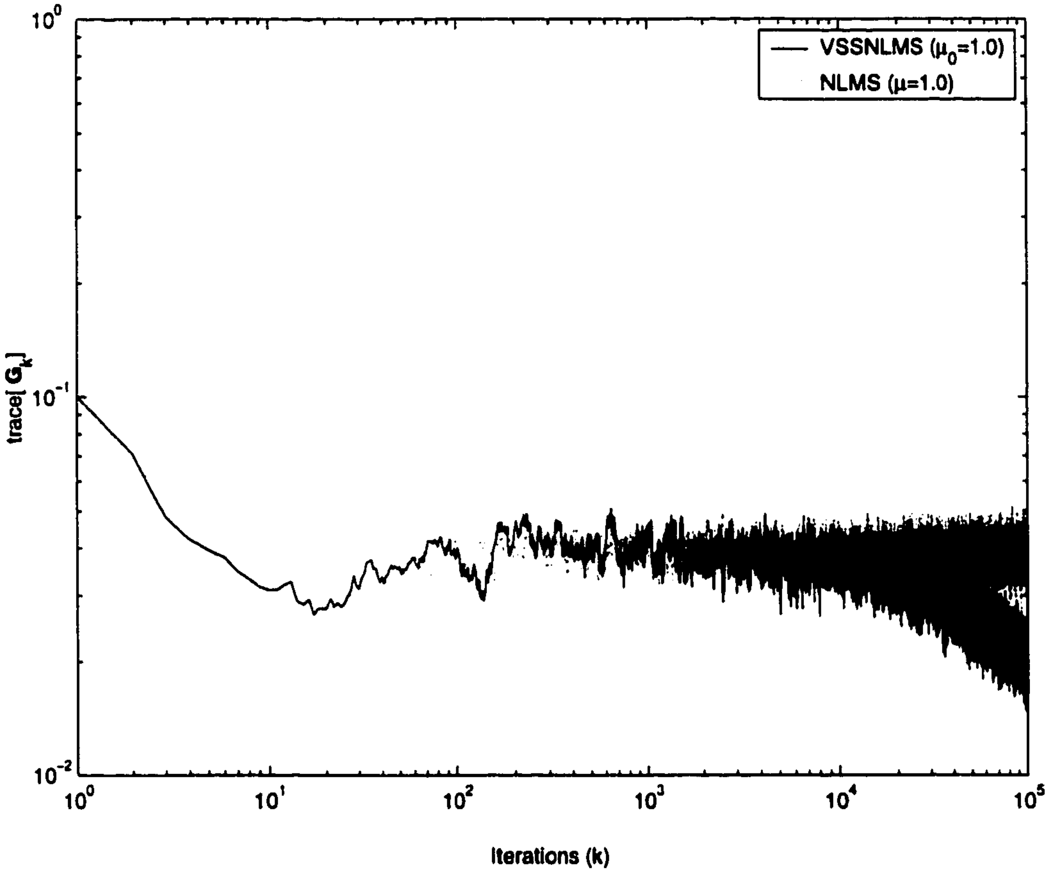


Figure 4.10: Comparison of the mean-square norm of the coefficient error vector for the VSS-NLMS algorithm ($\mu_0 = 1.0$) and the NLMS algorithm (fixed step-size, $\mu = 1.0$).

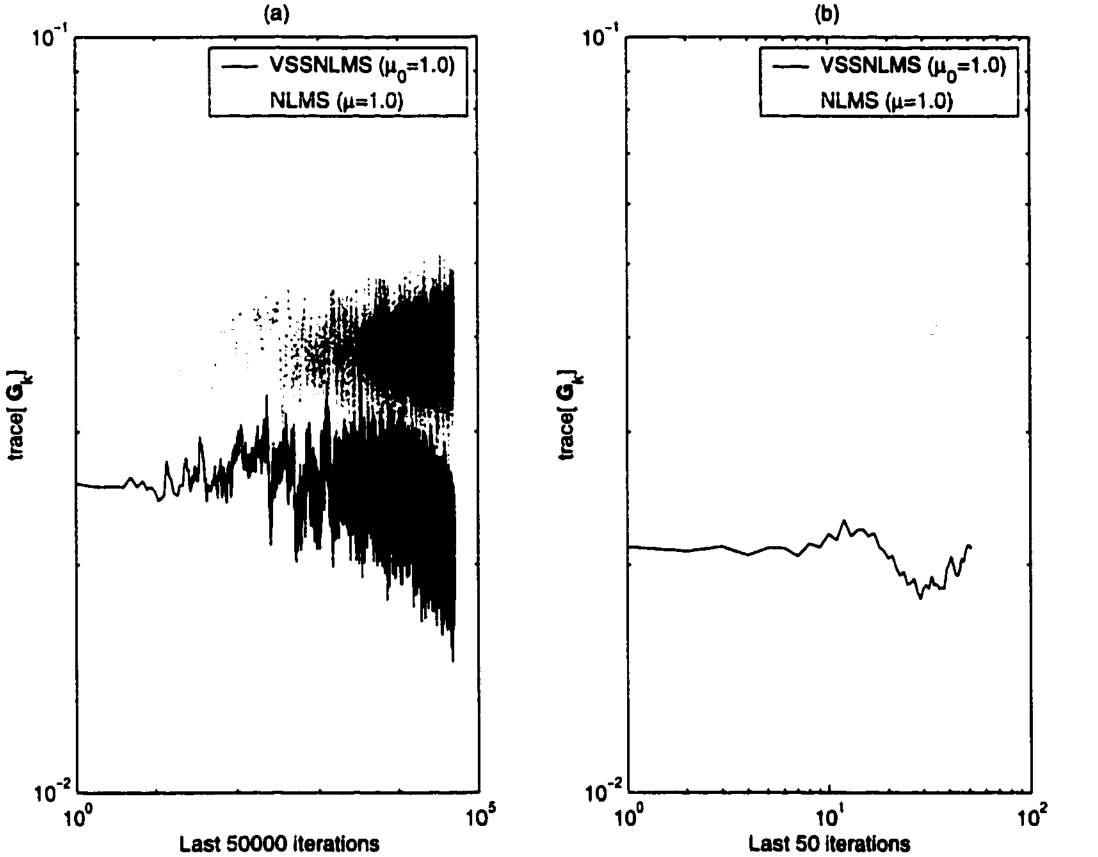


Figure 4.11: Comparison of the mean-square norm of the coefficient error vector for the VSS-NLMS algorithm ($\mu_0 = 1.0$) and the NLMS algorithm (fixed step-size, $\mu = 1.0$): (a)Curves of Figure 4.10 shown for the last 50000 iterations, (b)Curves of Figure 4.10 shown for the last 50 iterations.

In Figure 4.12 and 4.13, the MSE performance of the two filters for the same consideration are shown. From Figure 4.13(b), it is also clear to see that VSS-NLMS algorithm converges to an excess MSE that is about 4 dB less than that of the NLMS algorithm at about 100000 iteration. More gains is however, still expected as the filters gets closer more and more, to the steady-state condition.

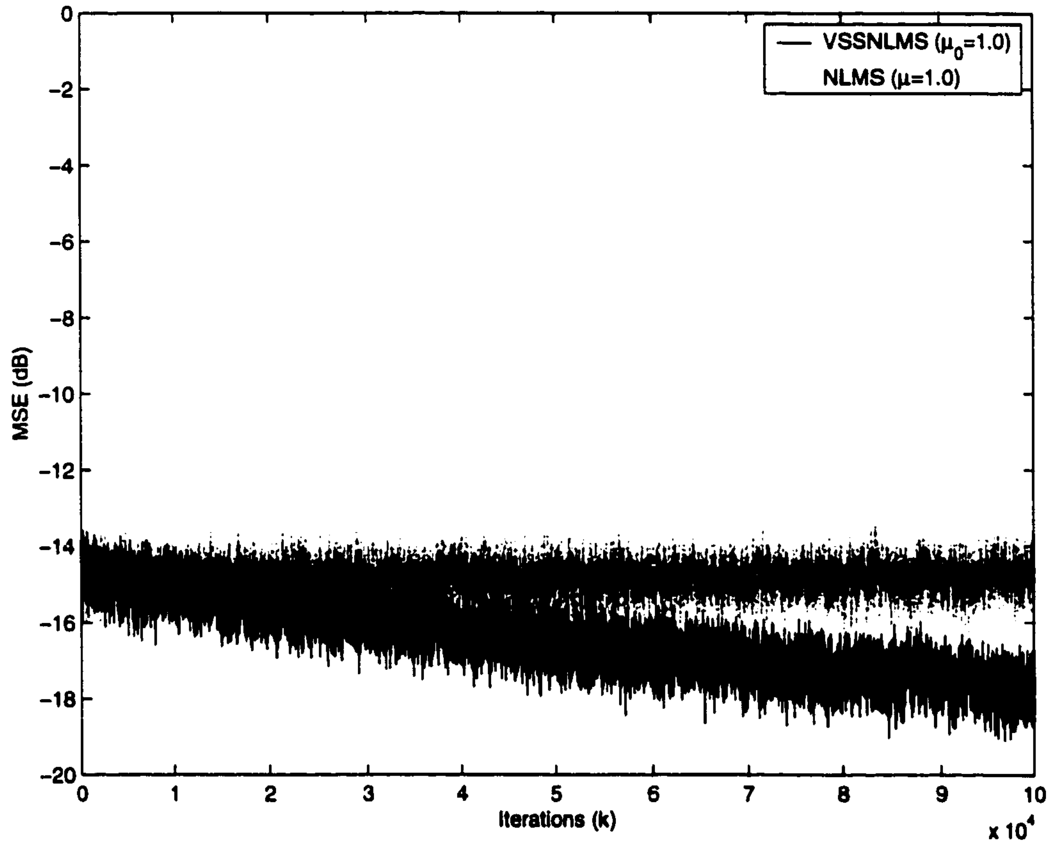


Figure 4.12: Comparison of the MSE performances of the adaptive algorithms for the experiment of Figure 4.10.

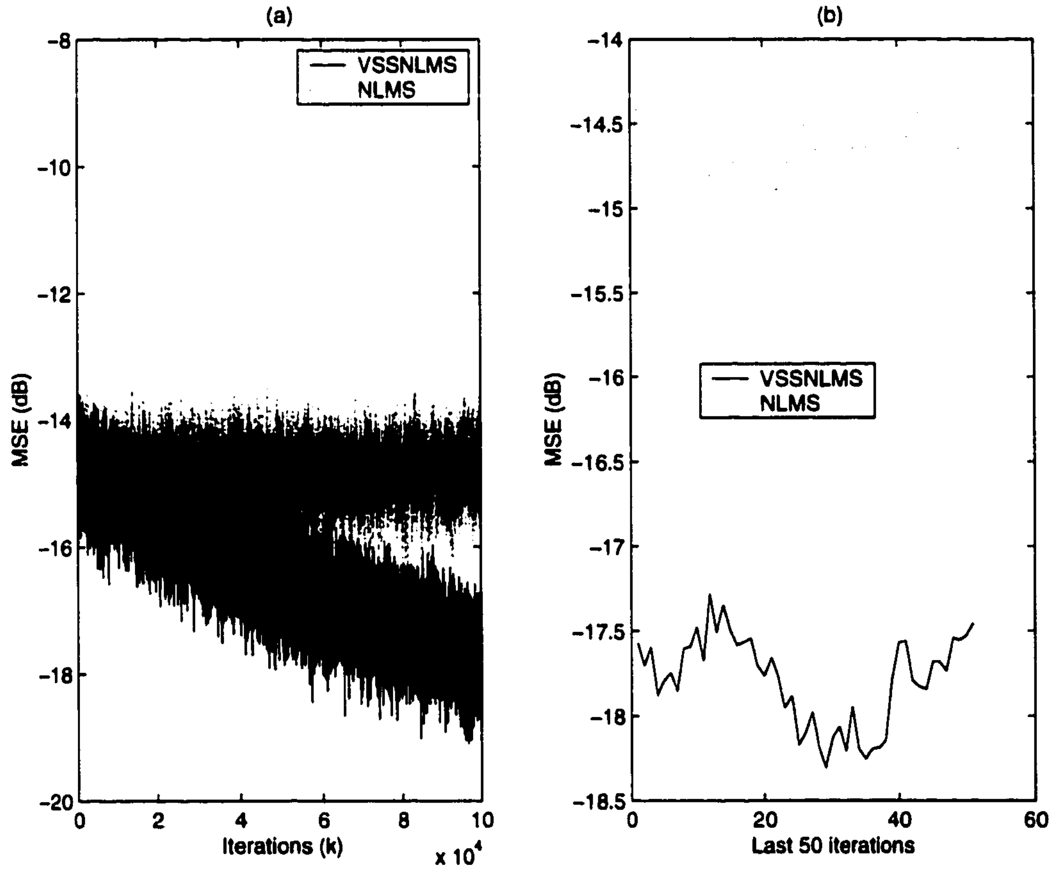


Figure 4.13: Comparison of the MSE performances of the adaptive algorithms for the experiment of Figure 4.10: (a)re-plot of the MSE curves of Figure 4.12, (b)MSE curves of Figure 4.12 shown for the last 50 iterations.

4.4.3 Results for Low values of step-size initializations for the VSS-NLMS filter

In Figures 4.14-4.17, we present the results of our comparison between the VSS-NLMS algorithm and the conventional NLMS algorithm for the case when a very low value of step-size initialization, μ_0, is used for the VSS-NLMS algorithm. Figures 4.14 and 4.15 show the result for the case when $\mu_0 = 0.04$. From Figure 4.14(a) we can see that the VSS-NLMS algorithm converges faster than the NLMS algorithm, and at the convergence time the MSE of the VSS-NLMS filter is 2 dB lower than that of the LMS filter as shown in Figure 4.14(b). This however, is not yet all of the performance improvement that the VSS-NLMS filter will give at the steady-state. As can be observed from the mean behavior of the step-size for the VSS-NLMS algorithm shown in Figure 4.15, the performance improvement (in terms of steady-state error) will well be shown at a time beyond 100000 iteration when the mean value of the step-size would have reduced considerably to a very low value, thereby converging to a much lower steady-state MSE than is shown in Figure 4.14.

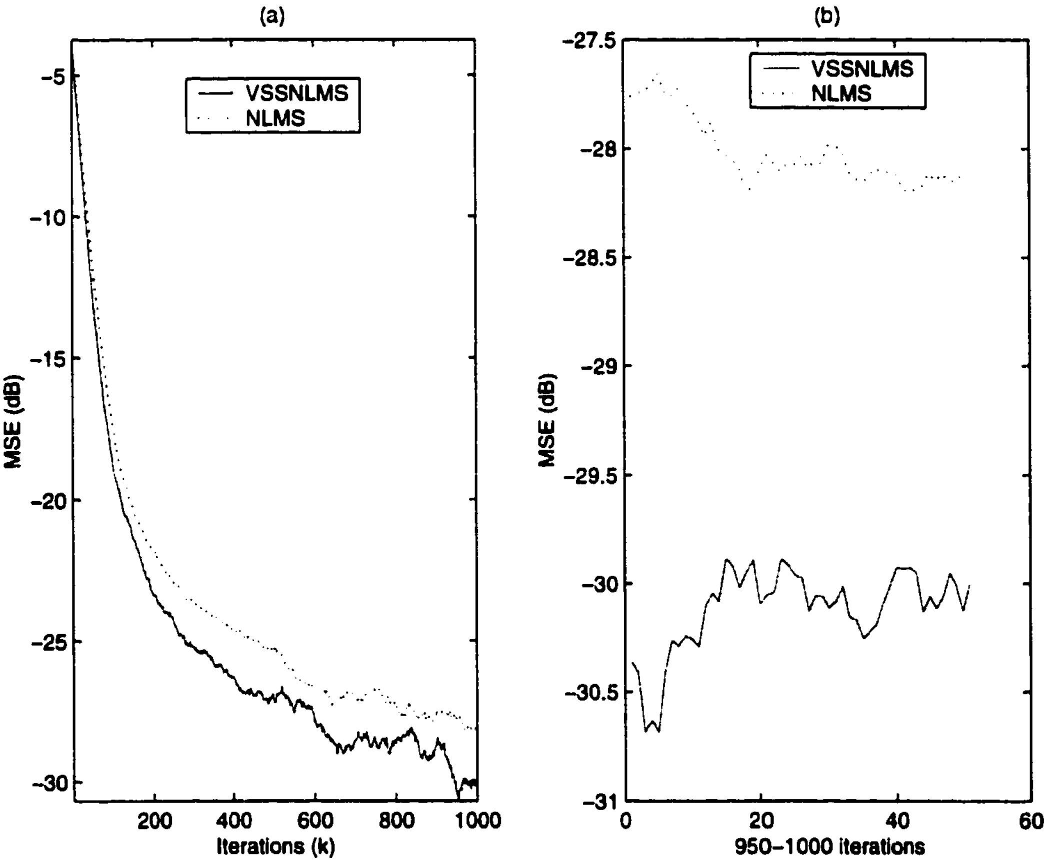


Figure 4.14: Comparison between the performances of the VSS-NLMS algorithm (for the case $\mu_0 = 0.04$) and the NLMS algorithm (for the case $\mu = 0.04$): (a) MSE curve for the two algorithms, (b) MSE at convergence for the two algorithms.

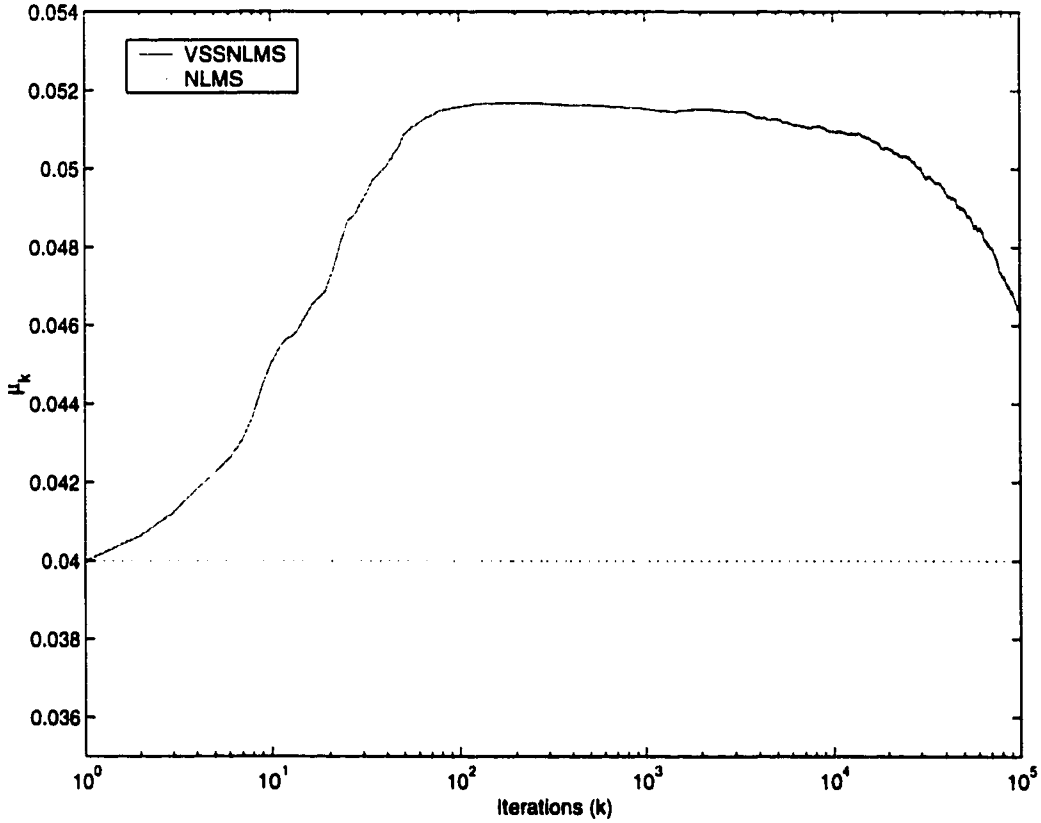


Figure 4.15: Mean behavior of the step-size for the two algorithms explaining the observed performance behaviors in Figure 4.14.

In Figures 4.16 and 4.17, the same result for the case when $\mu_0 = 10^{-8}$ is presented. This experiment was made deliberately to drive home our points on the performance improvements of the proposed algorithm over the conventional NLMS algorithm. The convergence requirements for the traditional NLMS algorithm specify that the step-size should be chosen within the limits $0 < \mu < 2$. From this Figure we see clearly that when μ is chosen too close to zero (such as $\mu = 10^{-8}$ shown here), NLMS filter will not be able to converge. Even for this case, the proposed VSS-NLMS algorithm still display good convergence time (approximately 1000 iterations) that would be adequate for most applications. More still, we can see from Figure 4.16(b) that VSS-NLMS algorithm converges to an MSE that is 25 dB lower than that of the NLMS algorithm. The performance behavior of the VSS-NLMS algorithm shown in Figure 4.16 stem from from the behavior of the step-size sequence of the algorithm as shown in Figure 4.17. The high rise in the mean value of the step-size, μ_k, drives the coefficients of the filter away from this 'non-convergent zone', where the NLMS filter's coefficients are inescapably 'trapped' because the algorithm utilizes the same step-size all through. We say this is another 'safe' property of the VSS-NLMS algorithm presented here.

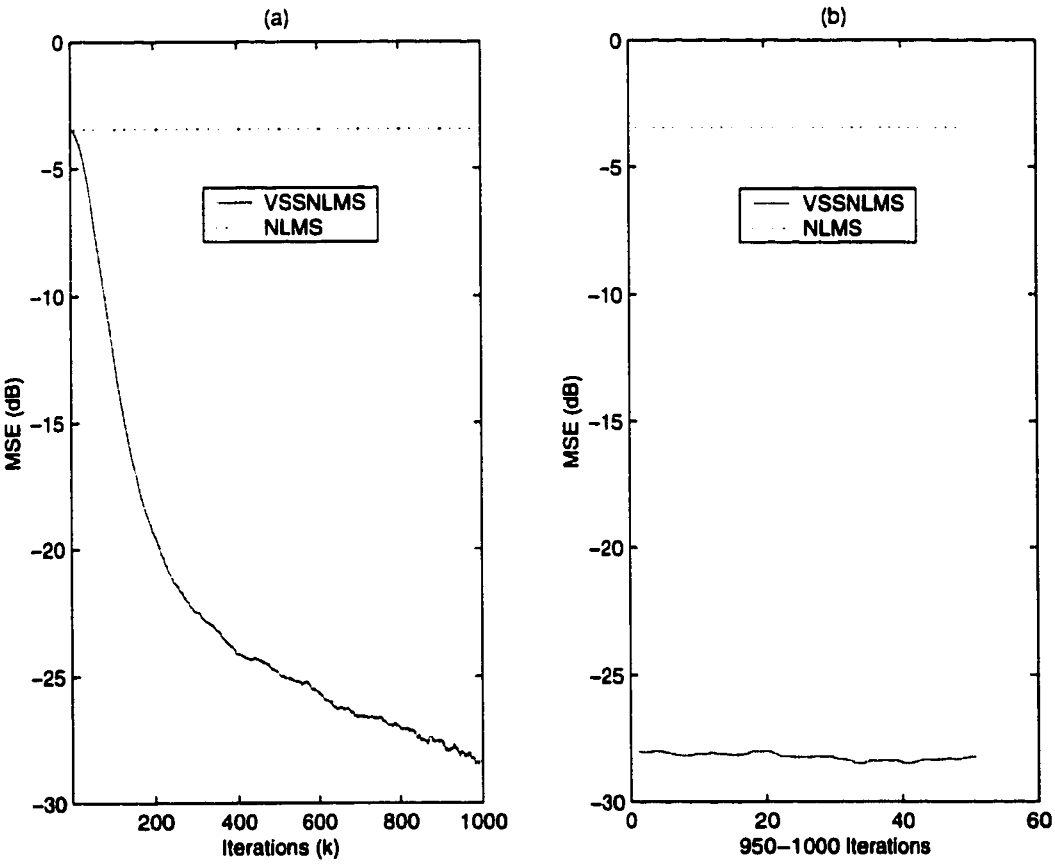


Figure 4.16: Comparison between the performances of the VSS-NLMS algorithm (for the case $\mu_0 = 10^{-8}$) and the NLMS algorithm (for the case $\mu = 10^{-8}$): (a) MSE curve for the two algorithms, (b) MSE at convergence for the two algorithm.

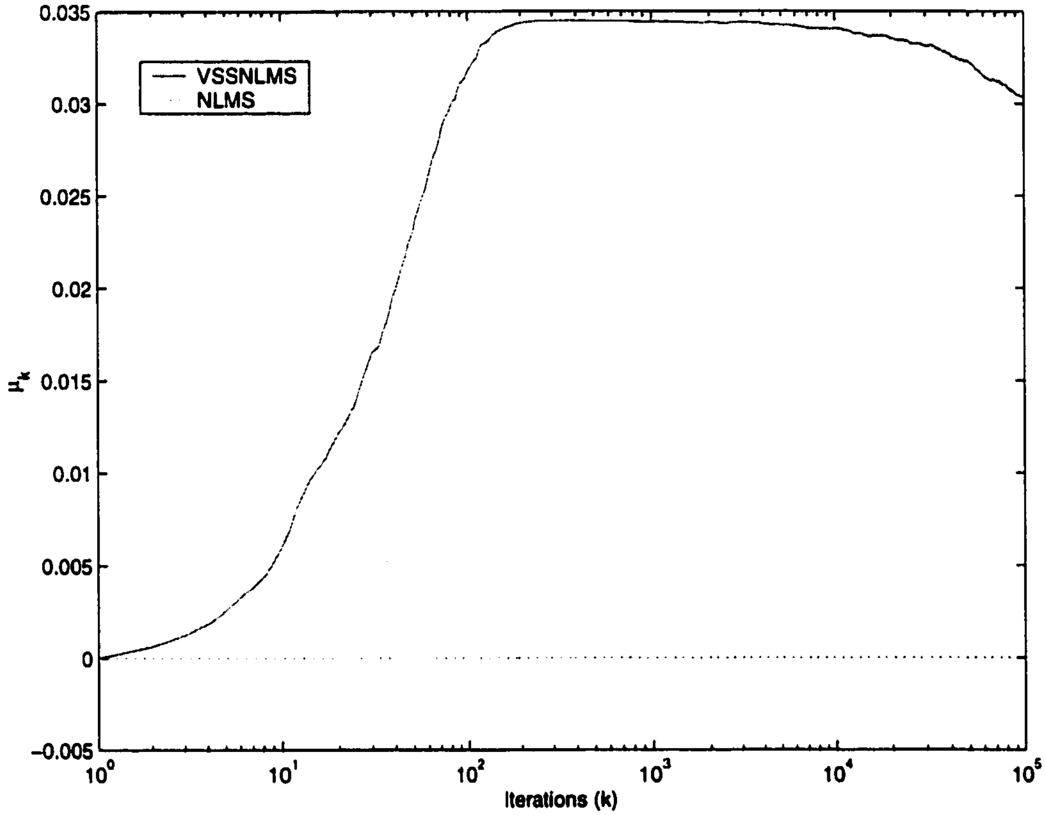


Figure 4.17: Mean behavior of the step-size for the two algorithms explaining the observed performance behaviors in Figure 4.16.

4.5 Results on the performance of the VSS-NLMS filter when the environment being tracked changes abruptly

In this section, we present the results of our studies on the performances of VSS-NLMS adaptive filter when the unknown environment that is being tracked changes abruptly. The non-stationarity model considered in our analysis in Chapter 3 will be investigated later in the next section. The input signals and the optimal coefficient set were the same as in the previous experiments from time $k = 0$ to $k = 10000$. At $k = 10001$, the coefficients of the unknown system were all changed to their corresponding negative values. The mean-squared coefficient behavior and the mean step-size behavior are shown in Figures 4.18 and 4.19, respectively.

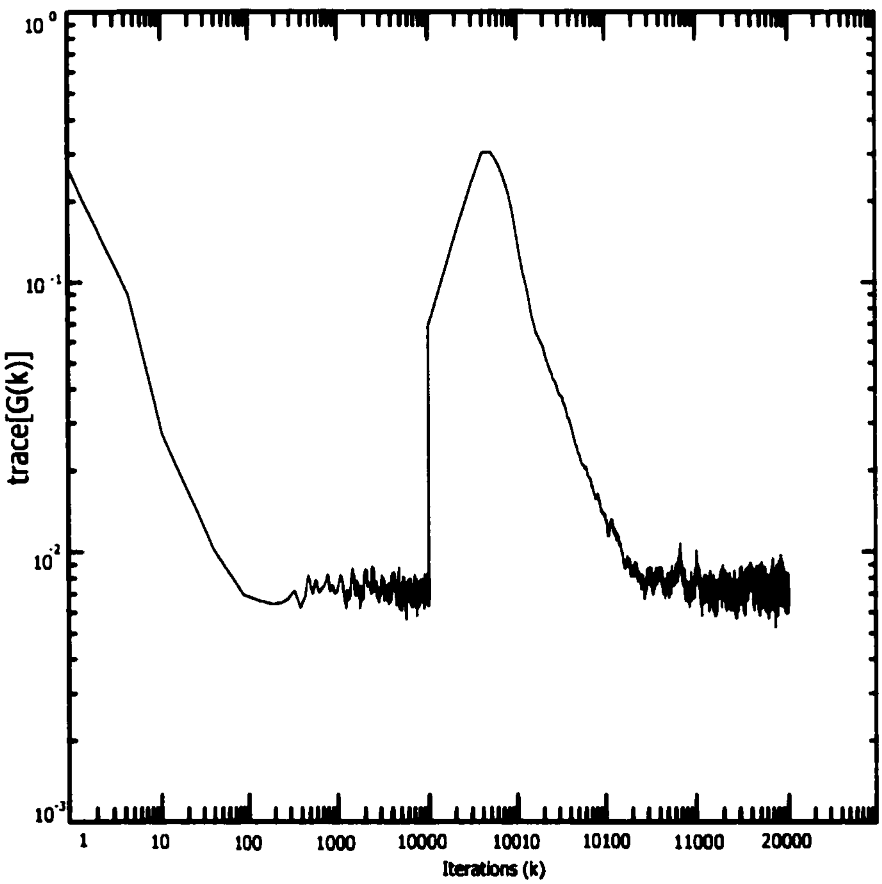


Figure 4.18: Response of the VSS-NLMS adaptive filter to an abrupt change in the environment.

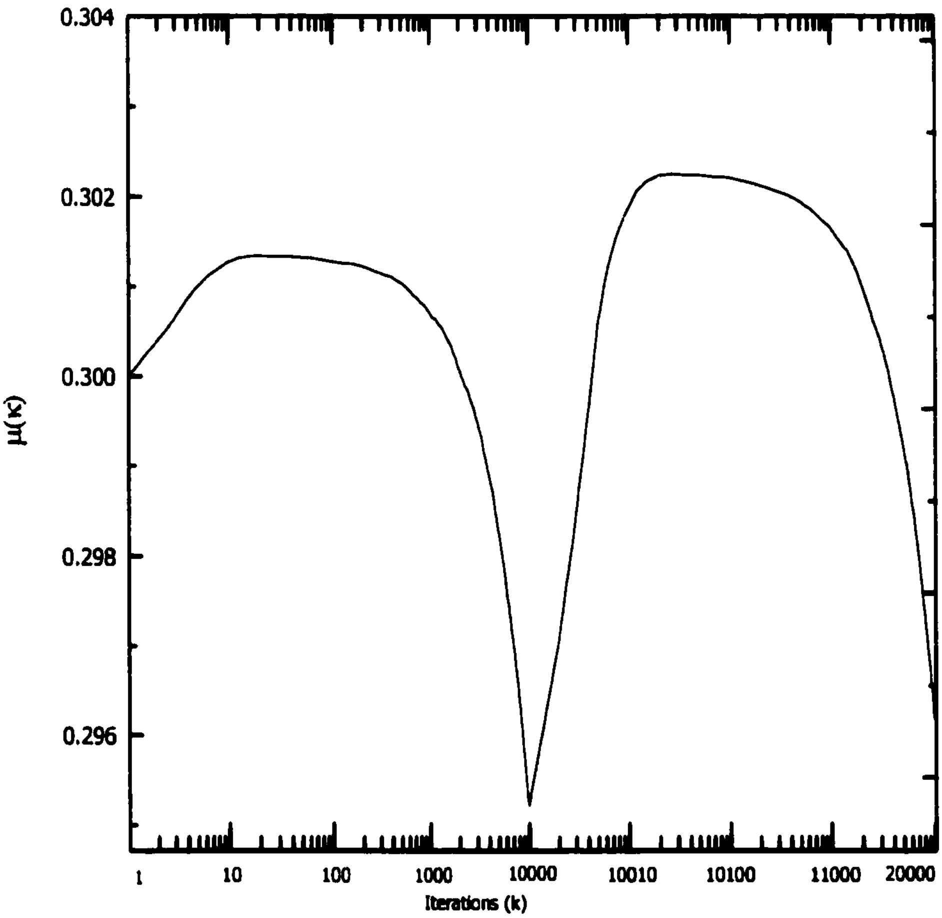


Figure 4.19: Mean behavior of μ_k for the VSS-NLMS adaptive filter when there is an abrupt change in the environment.

We observe that the step-size increases very quickly immediately after the environment changes and therefore the algorithm is able to track the abrupt changes in the operating environment very well. Note the change in scales after 10000 time iteration in the two plots. This was introduced in order to prevent the details after this time point from being compressed together, since for a normal log scale the next time point on the plot after 10000 time point would have been 100000.

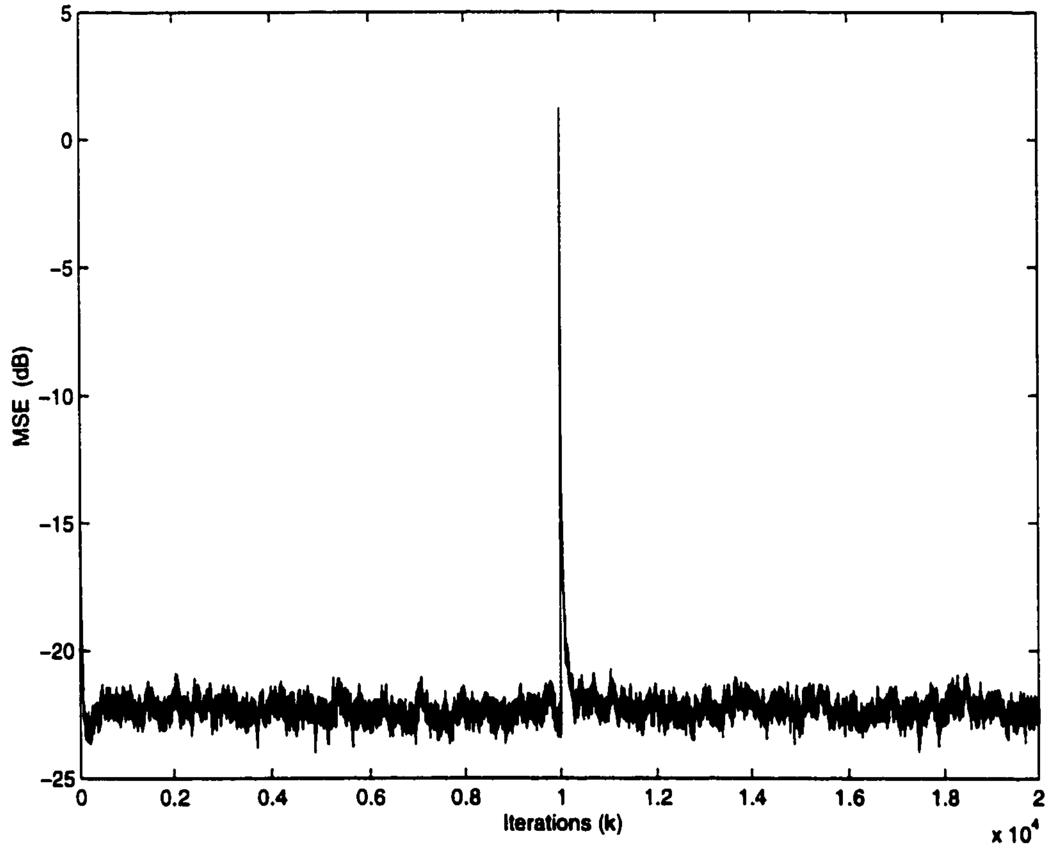


Figure 4.20: Response of the VSS-NLMS adaptive filter (MSE) to an abrupt change in the environment.

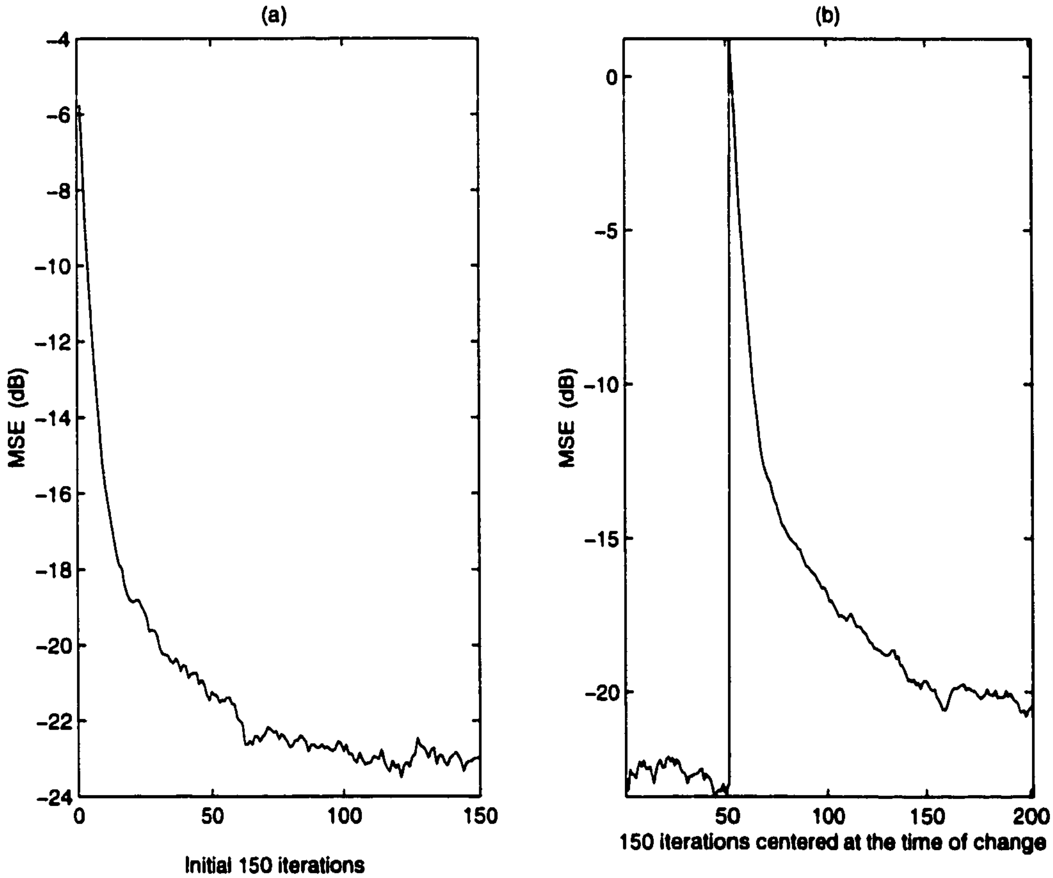


Figure 4.21: Response of the VSS-NLMS adaptive filter to an abrupt change in the environment: (a) Initial convergence time, (b) Recovery time after the abrupt change

The MSE plot for the same consideration on hand is presented in Figure 4.20. In Figure 4.21(a), the MSE curve for the first 150 time iterations after initialization is shown, while in Figure 4.21(b) the MSE curve for 150 time iteration centered at the time of change is shown. From these figures, it can be noted that the recovery time (convergence after the change) takes a little longer than the initial convergence time. The parameters of the adaptive filter for this investigation (experiment) are $\rho = 0.0008$ and $\mu_0 = 0.3$.

4.6 Comparison between the Analytical and Empirical (Simulation) Results for the Performances of the VSS-NLMS filter

The results presented in this section are intended to demonstrate the validity of the analysis in Chapter 2.

4.6.1 Stationary Environment

Here we again consider indentifying the same system identified in the experiment of Figure 4.1. The observation noise variance is still 0.01 as in that experiment, and the parameters of the adaptive filter for each experiment are as indicated on the plots.

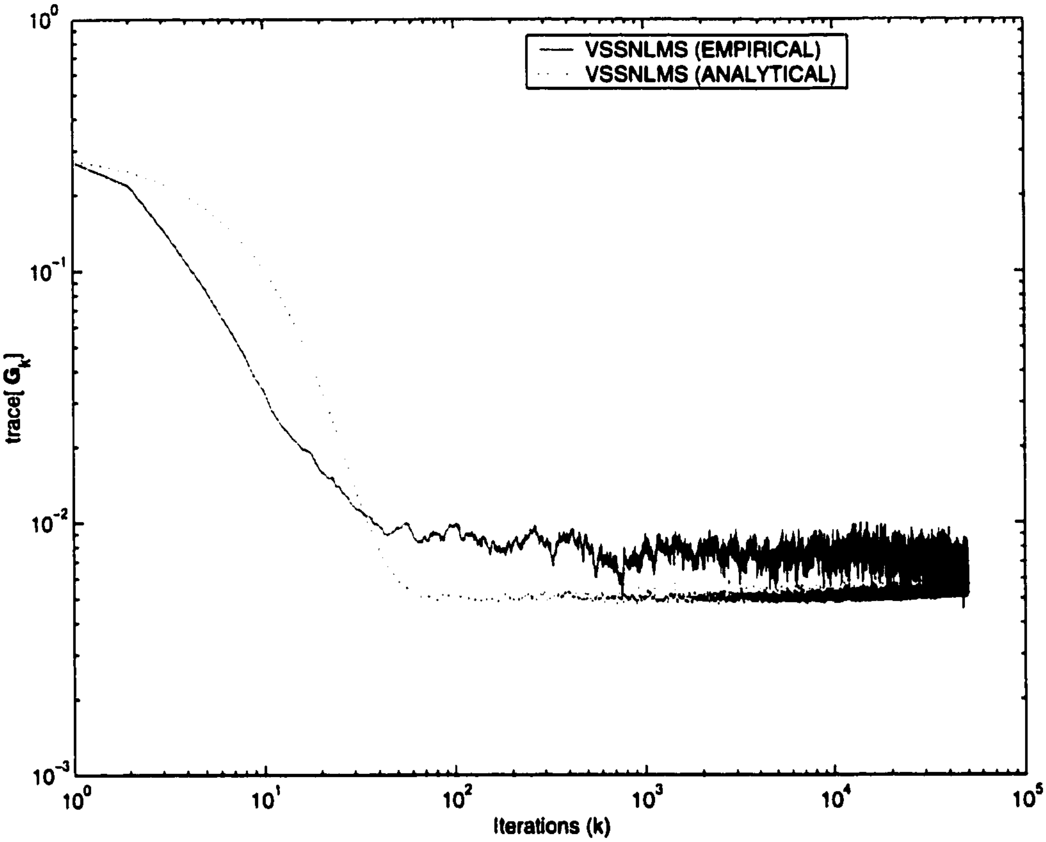


Figure 4.22: Comparison of empirical (simulation) and analytical (from equation 2.37) results for the mean-square behavior of the coefficients of the VSS-NLMS adaptive algorithm. The parameters of the adaptive filter for these curves are: $\rho = 0.0008$, and $\mu_0 = 0.3$

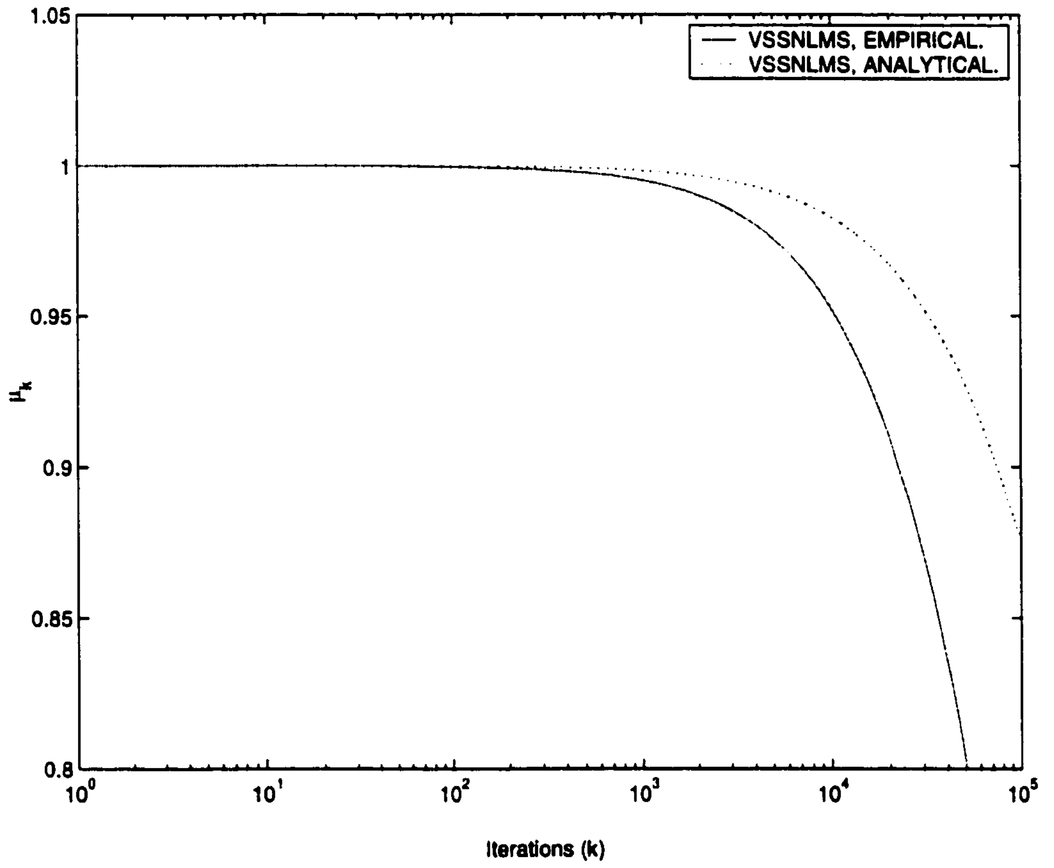


Figure 4.23: Comparison of empirical (simulation) and analytical (from equation 2.47) results for the mean behavior of the step-size sequence of the VSS-NLMS adaptive algorithm. The parameters of the adaptive filter for these curves are: $\rho = 0.0008$, and $\mu_0 = 1.0$

In Figure 4.22 we have plotted the trace of the second moment matrix of the coefficient misalignment vector obtained from the theoretical analysis presented in Chapter 2, and also from our simulation experiments. We observed that the experimental and analytical results match closely. This close matchness so observed between the two results do not only confirm the correctness of our derivations, but also proof the usefulness of the various simplifying assumptions and approximations we used in deriving those results. Similarly, Figure 4.23 shows the empirical (simulation) and analytical curves for the mean behavior of the step-size for the initialization, $\mu_0 = 1.0$. Again note that the analytical and empirical results do match well.

4.6.2 Non-Stationary Environments

In this experiment, we consider the identification of a time-varying system. The time varying coefficients, $\mathbf{w}_k^*$, of the system are modeled using a random disturbance process:

$$\mathbf{w}_k^* = \mathbf{w}_{k-1}^* + \mathbf{c}_k, \tag{4.2}$$

where $\mathbf{c}_k$ is zero-mean white vector process with co-variance matrix

$$\sigma_c^2 \mathbf{I} = 10^{-4} \mathbf{I} \tag{4.3}$$

The initial values of the optimal coefficients were as in the first experiment (Figure 4.1). The mean-squared norm of the coefficient error vector obtained using analysis and simulation experiment are plotted in Figure 4.24. Other parameters of the system, except the coefficients that are time-varying, are the same as before. The parameters of the adaptive filter are: $\rho = 0.0008$ and $\mu_0 = 1.0$.

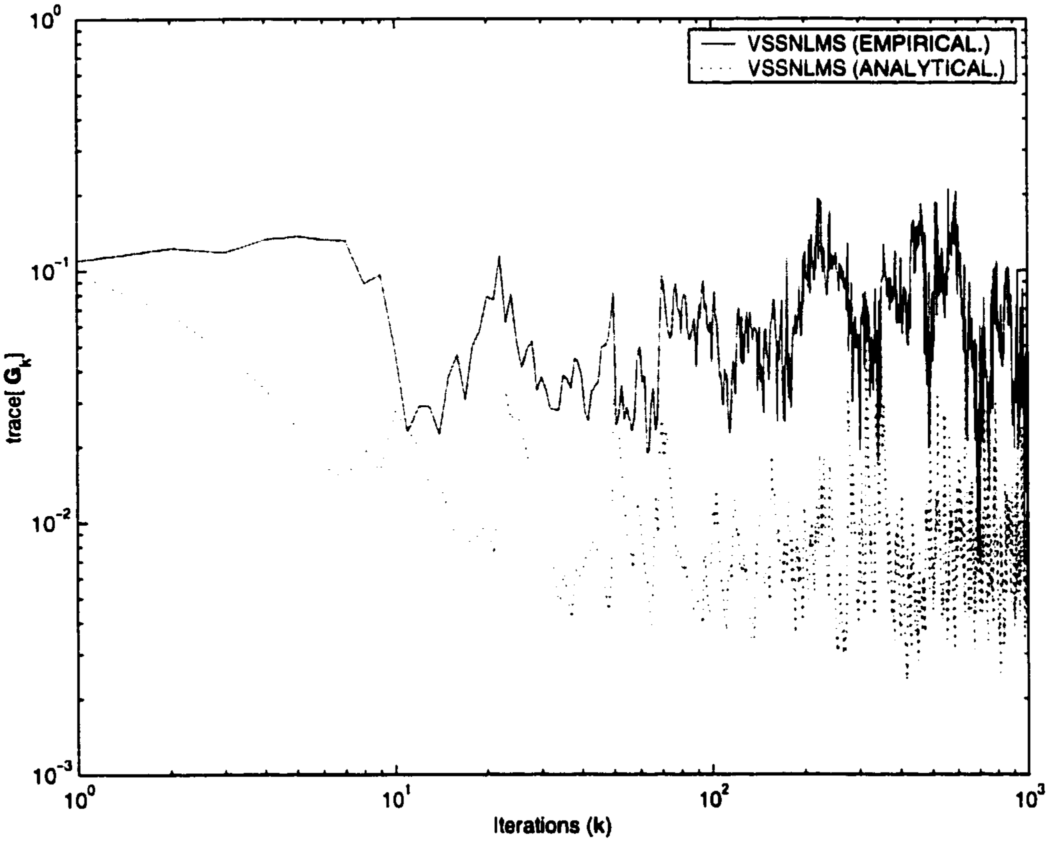


Figure 4.24: Comparison of empirical and analytical results for the mean-square coefficient behavior of the VSS-NLMS algorithm in non-stationary environments.

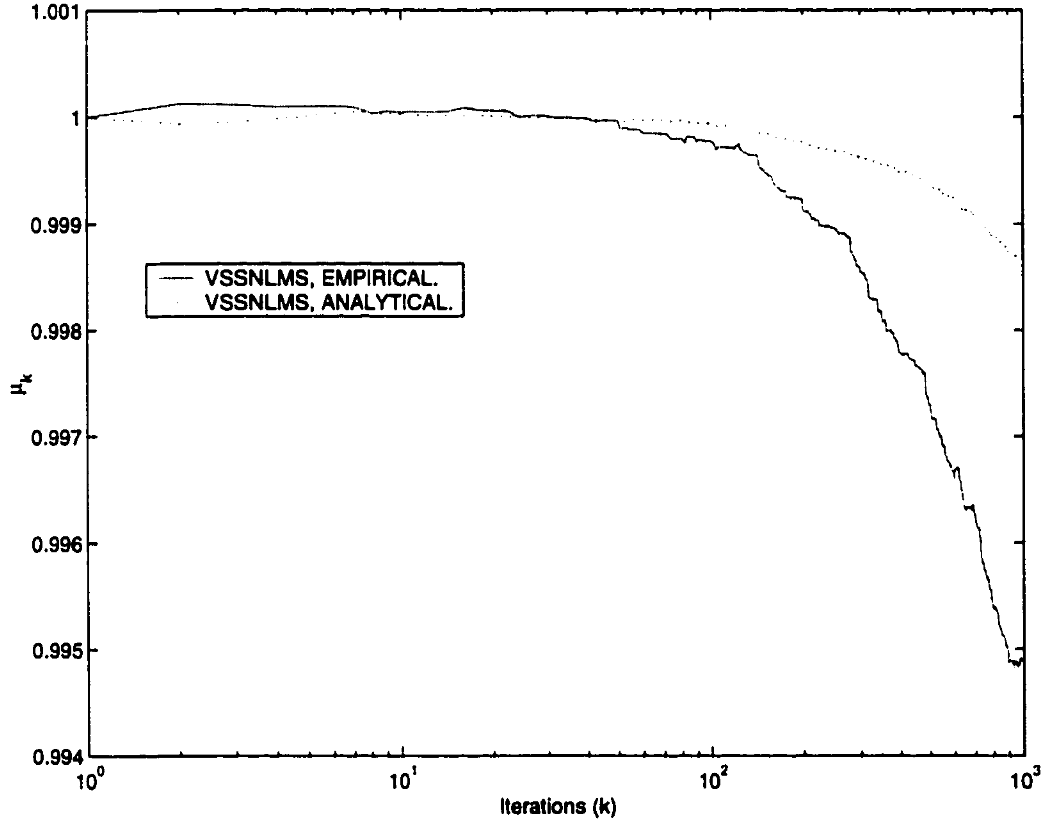


Figure 4.25: Comparison of the empirical and analytical curves for the mean behavior of the step-size sequence of the VSS-NLMS algorithm, μ_k, in non-stationary environments for the case: $\mu_0 = 1.0$

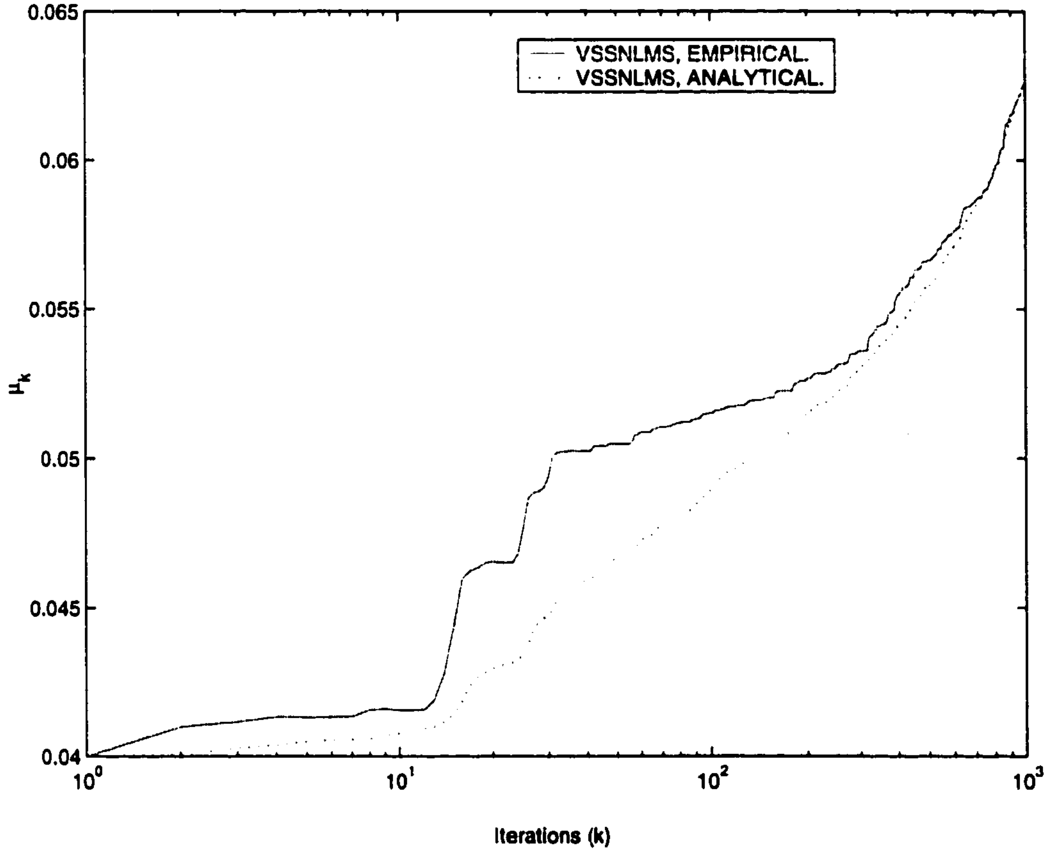


Figure 4.26: Comparison of the empirical and analytical curves for the mean behavior of the step-size sequence of the VSS-NLMS algorithm, μ_k, in non-stationary environments for the case: $\mu_0 = 0.04$

We notice here that the mean-square norm of the coefficient error vector predicted by the analysis is a little lower than the empirical value obtained from simulation. However, the two results still closely match within some reasonable degree of accuracy. Also presented in Figure 4.25 is the mean behaviors of the step-size of the adaptive filter for this experiment ($\mu_0 = 1.0$). In figure 4.26 we present curves, similar to those of Figure 4.25, for the case of low values of step-size initialization ($\mu_0 = 0.04$). Again note from these two figures that the same behaviors of the step-size sequence observed in the stationary case are still observed here. This confirms therefore the conclusions of our mathematical analysis of this algorithm, that the VSS-NLMS algorithm presented in this work will always outperform the traditional NLMS algorithm whatever the nature of the environment (stationary or non-stationary) in which it operates. Hence the statistics of the environment being tracked has no influence on the performance improvement that our algorithm records over the NLMS algorithm.

4.7 Results on the Comparison between the VSS-NLMS algorithm and the NLMS algorithm in non-stationary environment

The results presented in this section complete our comparison between the VSS-NLMS algorithm presented in this work, and the NLMS algorithm. We make comparison between the the performances of the VSS-NLMS algorithm and the NLMS algorithm in non-stationary environment. The comparison made here are similar to those made in the stationary environment. The non-stationarity model just defined in the last section was used in these experiments, and the parameters of the VSS-NLMS adaptive filter are: $\rho = 0.0008$, and (a) $\mu_0 = 0.3$, (b) $\mu_0 = 1.0$. In case (a), the step-size of the NLMS filter was chosen to be $\mu = 0.5$, for investigation purpose, while in case (b), the step-size of the NLMS filter was chosen as $\mu = \mu_0 = 1.0$.

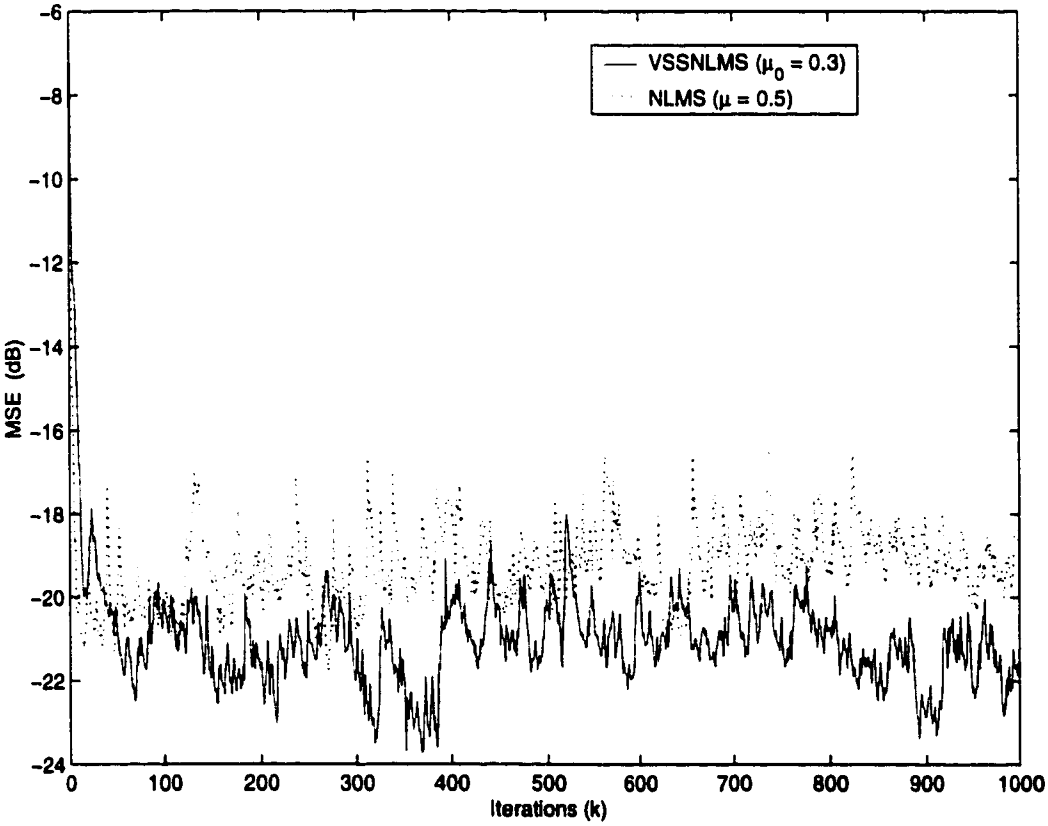


Figure 4.27: Comparison of the convergence speed and mean-square error (MSE) after convergence of the VSS-NLMS algorithm ($\rho = 0.0008$, $\mu_0 = 0.3$), and the NLMS algorithm ($\mu = 0.5$) in non-stationary environments.

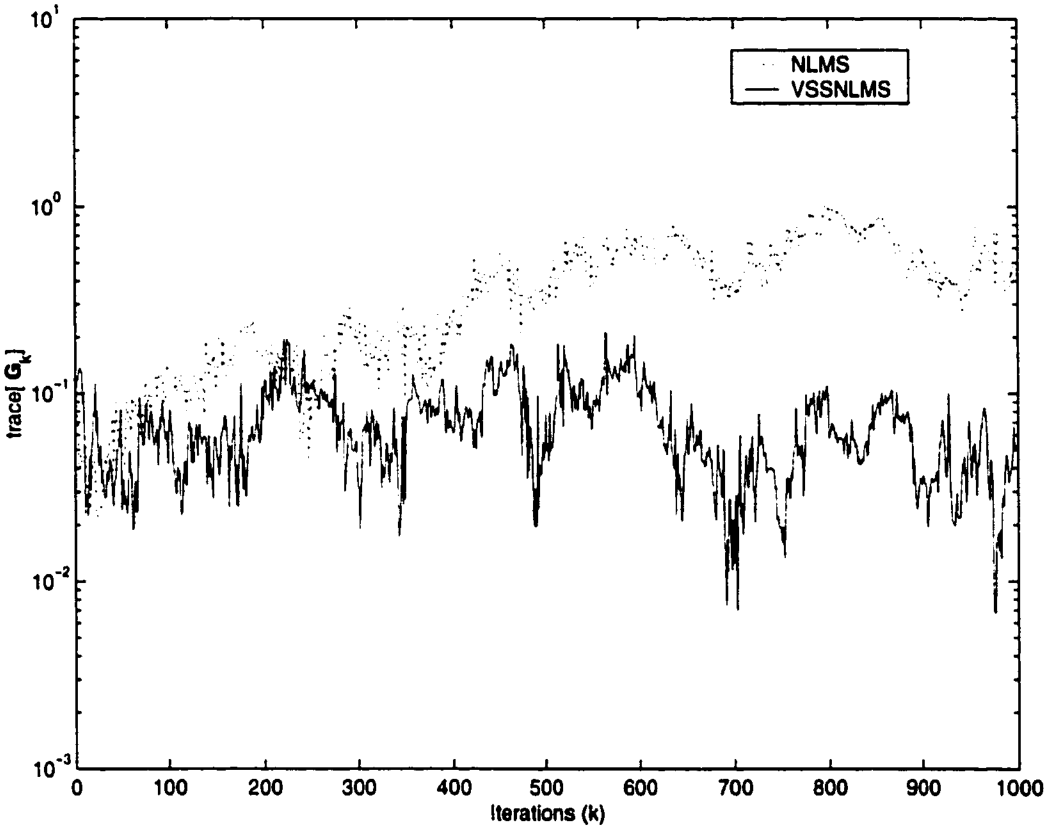


Figure 4.28: Comparison of the mean-square behavior of the coefficient of the VSS-NLMS algorithm (ρ = 0.0008, μ_0 = 1.0), and the conventional NLMS algorithm (μ = 1.0) in non-stationary environments.

The results of the experiments are presented in Figures 4.27 and 4.28. Figure 4.27 shows the performances of the two algorithms for the case when the step-size initialization of the VSS-NLMS filter is $\mu_0 = 0.3$. As can be seen from the figure, the two algorithm have the same convergence speed, while the VSS-NLMS filter converge to a lower MSE than the NLMS filter. Again what we observe here, similar to what has been observed in the case of stationary environment, is that: with relatively low value of step-size initialization (μ_0) for the VSS-NLMS algorithm, the filter converges faster than the NLMS algorithm. The reason for this (same as for the case of the stationary environment) is that the step-size sequence quickly rises up after initialization, as shown in Figure 4.26, to a high value. For this reason, the convergence speed of the VSS-NLMS algorithm for $\mu_0 = 0.3$ becomes the same as that of the NLMS algorithm for $\mu = 0.5$, and morestill, the VSS-NLMS filter should naturally converge to a lower MSE than the NLMS filter as shown.

Figure 4.28 presents the result obtained for the mean-squared norm of the coefficient error vector of the two algorithm for the experiment of case (b) above. Notice that because of the continued variation in the environment, it becomes more and more difficult for the NLMS algorithm to track the changes in the environment as time goes on, and hence we can see from the figure that the curve of the second moment of the coefficient error vector of the filter rises as the adaptation proceeds. But the VSS-NLMS algorithm on the other hand is able to adequately track this time variation in the unknown environment time to time. This represent another major performance superiority of the VSS-NLMS algorithm over the NLMS algorithm observed also here in the non-stationary environment.

4.8 Results on Echo Cancellation using the proposed VSS-NLMS algorithm

In this section, we present the results of our simulation experiment that we made in order to investigate echo cancellation using the proposed variable step-size NLMS (VSS-NLMS) algorithm.

4.8.1 Echo path Model

The echo path is modeled, for simplicity, using a single pole single zero digital filter [58]. The transfer function of the echo path is:

$$\begin{aligned} H(z) &= \frac{z}{z-a} \\ &= (1-az^{-1})^{-1} \\ &= 1 + az^{-1} + a^2 z^{-2} + \cdots \end{aligned} \tag{4.4}$$

where $0 < a < 1$. Two models of the echo path were used in our simulations. In the first model, we assumed that the impulse response of the echo path is significant only in the first 100 series, and therefore the series afterwards can be neglected without any significant loss of accuracy. The feedback coefficient a of the echo path filter for this model is chosen arbitrarily in such a way that the power level of the impulse response will be attenuated by 60 dB at 100 samples. Thereafter, the series is truncated. The feedback coefficient is calculated as:

$$20log_{10}a^{99} = -60 \ \ dB \quad \text{or } a = 0.932603346$$

The impulse response of the echo path so obtained for this model is as shown in Figure 4.29.

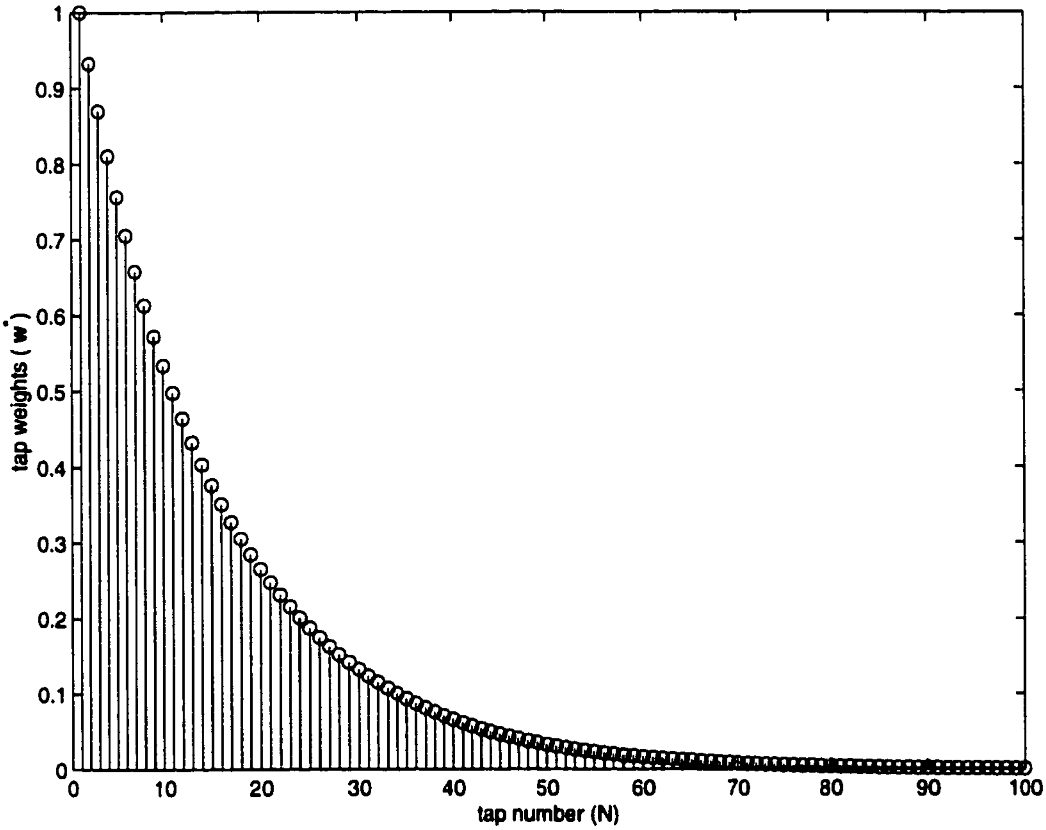


Figure 4.29: Impulse response sequence of the echo path with length $N = 100$.

This model represents a relatively short echo path, and can be assumed for electrical (network) echo that occur during telephone speech transmissions. This is because in this situation, the echo duration is usually several tens of milliseconds; thus the impulse response of the echo path as well as the required number of taps of the adaptive filter, can be on the order of a hundred or few hundreds. In the simulation experiments involving this model, the length of the adaptive filters were also chosen as $N = 100$.

In the second model considered in our simulation, we assumed that the impulse response of the echo path is significant up to the first 500 series, and therefore the series afterwards can similarly be neglected without any significant loss of accuracy. The feedback coefficient a of the echo path filter for this model was also chosen arbitrarily in a way similar to the first model such that the power level of the impulse response will be attenuated by 60 dB at 500 samples. Thereafter, the series is truncated. The feedback coefficient is calculated as:

$$20 log_{10} a^{499} = -60 \ dB \quad \text{or } a = 0.9862522$$

The impulse response of the echo path so obtained for this model is as shown in Figure 4.30.

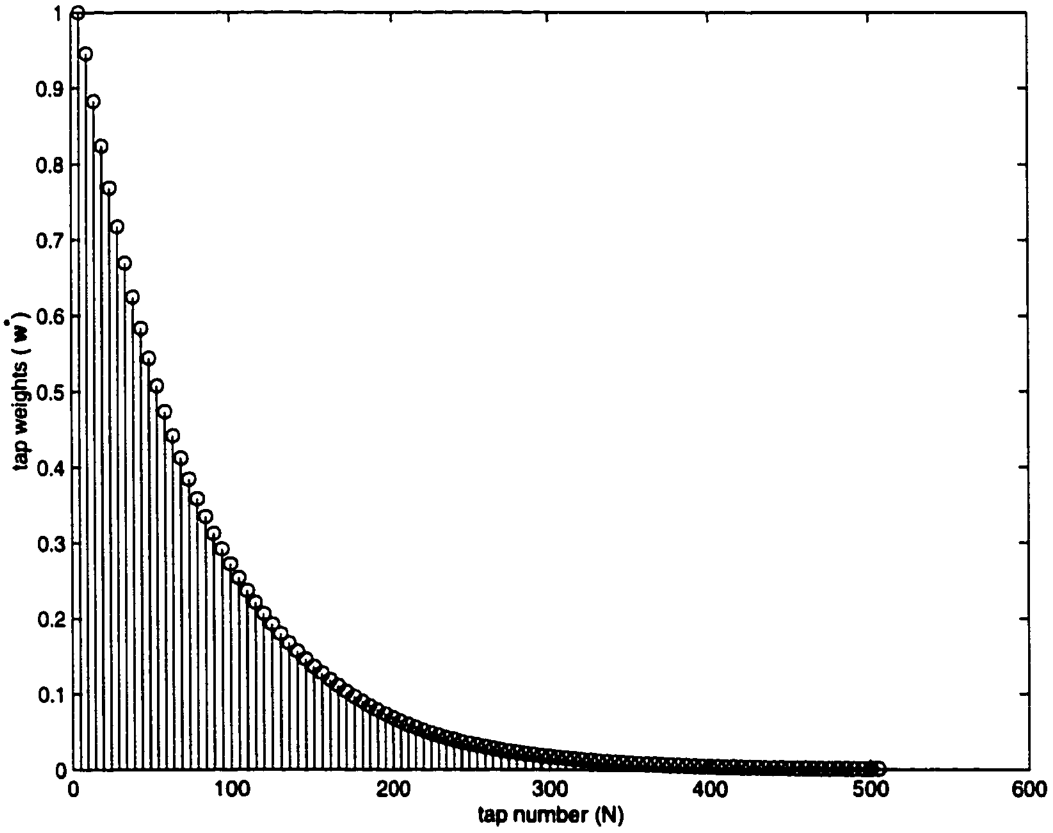


Figure 4.30: Impulse response sequence of the echo path with length $N = 500$.

The model in Figure 4.30 represent a relatively longer echo path. For the acoustic echo in teleconferencing systems, where the echoes are generated by sound waves with a much longer propagation delay, the impulse response of the echo path as well as the order of the adaptive filter required will be much greater than 500. Typically, straight forward FIR adaptive filter implementation of echo canceler for use in such situation will require about $4,000$ taps [59]. But such a straight forward implementation results in a prohibitively complex hardware that are not viable from an economic/performance efficiency point of view. One approach that has been effectively used to solve this problem is to split the signal into several frequency bands and provide a separate echo canceler for each band [59]. This way, the long echo path is decimated into smaller units or bands that are independently identified by separate adaptive filters. Since the tap adjustments are done in each band independently, consequently the convergence speed is enhanced. The effect of residual echo at the band gap, however prohibits the use of more than $4-8$ bands [60].More information on implementations of sub-Band acoustic echo cancelers can be obtained from [59], while information on the use of sub-Band NLMS can be obtained from [23]. Going by this provision, an echo path impulse response of length $N = 4,000$ can be decimated and identified in 8 sub-Bands of length $N = 500$ each. Therefore, our second model for echo path shown in Figure 4.30 fits in well for acoustic echo.

4.8.2 Simulation Results

Simulation experiments were made in turn with each of the two echo path models and the results are as discussed below.

Our simulation set up is according to the block diagram earlier shown in Figure 1.4. For stationary echo path, we set $\sigma_c^2\mathbf{I} = 0$, while non-stationary echo path was modeled by randomly disturbing the impulse response of the echo path by a disturbance vector, $\sigma_c^2\mathbf{I} = 10^{-4}$ at each time iteration. As usual, time averaging of 50 independent runs were used for the simulations involving stationary echo path, while enssemble averaging of large numbers of adaptive filters were used for simulations involving non-stationary echo path.

As stated above, the echo path was truncated at 100 and 500 samples respectively for the first and second consideration, and similarly the order of our adaptive filter (echo canceler) was 100 and 500 respectively. The input signal is the same colored signal used in all the previous experiments and was set such that the signal-to-noise ratio (SNR) was about 40 dB for some experiments, and 15 dB for others throughout the simulations. We have used 40 dB to model applications utilizing very high power, while we used 15 dB to model systems with limited processing powers.

The measure of performance used is the echo return loss enhancement (ERLE) defined as [9, 61]:

$$ERLE = 10log_{10}\frac{E[y_k^{*2}]}{E[\{y_k^* - y_k\}^2]} \; dB$$

where y_k^* and y_k are as defined in Chapter 1. The result of the simulations are as shown in Figure 4.31- 4.34, where the proposed VSS-NLMS algorithm ($\mu_0 = 0.04$) is compared with the traditional NLMS algorithm ($\mu = 0.04$).

In Figure 4.31, we compare ERLE for the VSS-NLMS algorithm and the traditional NLMS algorithm for the case when the two algorithms are tracking a stationary echo path. The signal-to-noise ratio (SNR) used for the simulation in this figure is $SNR = 40\ dB$, and the length of the adaptive filter as well as the echo path impulse response is $N = 100$. From this figure, it can be observed that the proposed VSS-NLMS algorithm converges faster, and at the same time achieves a higher echo return loss enhancement than that obtained by the traditional NLMS algorithm. A gain of about 15 dB of ERLE improvement over the NLMS algorithm is obtained in this case, at about 5000 iterations.

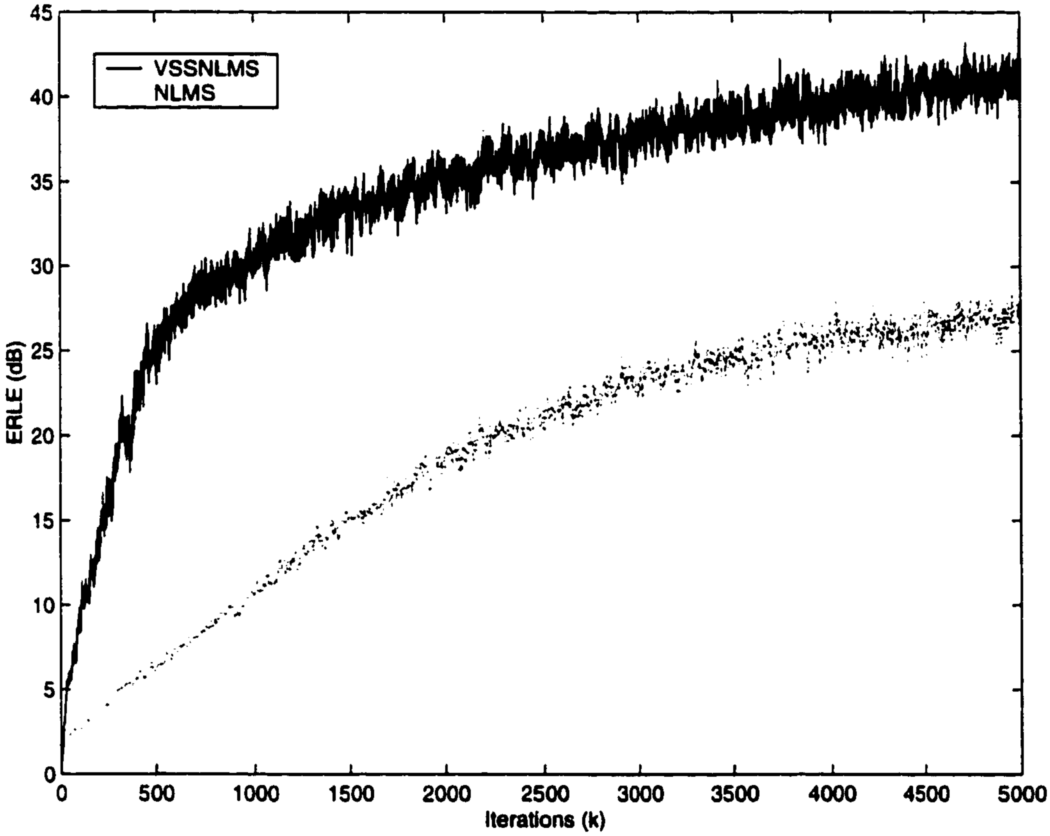


Figure 4.31: Comparison of the convergence characteristics and echo return loss enhancement for the VSS-NLMS algorithm and the traditional NLMS algorithm with colored (speech) input signals. The parameters used for this experiment are: $N = 100$ and $SNR = 40\ dB$.

In Figure 4.32, we compare ERLE for the VSS-NLMS algorithm and the traditional NLMS algorithm for similar consideration of Figure 4.31, but with lower signal-to-noise ratio. Here, $SNR = 15\ dB$ was used. Our result shown in Figure 4.32 shows that the VSS-NLMS algorithm converges faster than the NLMS algorithm. From this figure also, it can be observed that ERLE achieved by the two filters are a little lower than their respective performances in shown in the experiments of Figure 4.31 as would be expected. However, the ERLE achieved by the VSS-NLMS algorithm is still considerably higher than that of the traditional NLMS algorithm. A gain of about 6 dB of ERLE was observed with the proposed algorithm over the performance of the traditional NLMS algorithm.

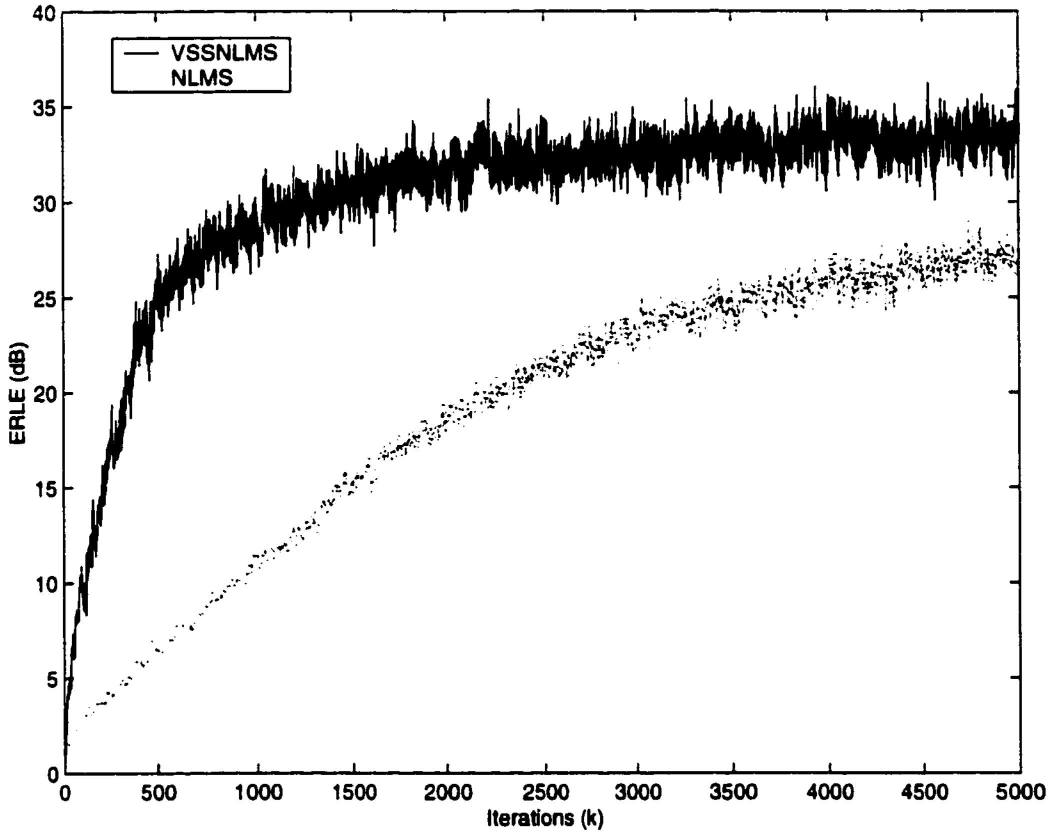


Figure 4.32: Comparison of the convergence characteristics and echo return loss enhancement for the VSS-NLMS algorithm and the traditional NLMS algorithm with colored (speech) input signals. The parameters used for this experiment are: $N = 100$ and $SNR = 15\,dB$.

In Figure 4.33, we have displayed the curves of the ERLE for the VSS-NLMS algorithm and the traditional NLMS algorithm for the case when the echo path impulse response is stationary, and of length $N = 500$. The signal-to-noise ratio used for this study is $SNR = 15\ dB$. This experiment models acoustic echo cancellation using the proposed algorithm. From this figure, it can be observed that the two algorithm both display higher tracking errors which leads to greater fluctuations in their ERLE curves than those of the previous figures. However, the ERLE achieved by the VSS-NLMS algorithm is still far much higher than that of the traditinal NLMS algorithm. At the same time, VSS-NLMS algorithm converges much faster than the traditional NLMS algorithm.

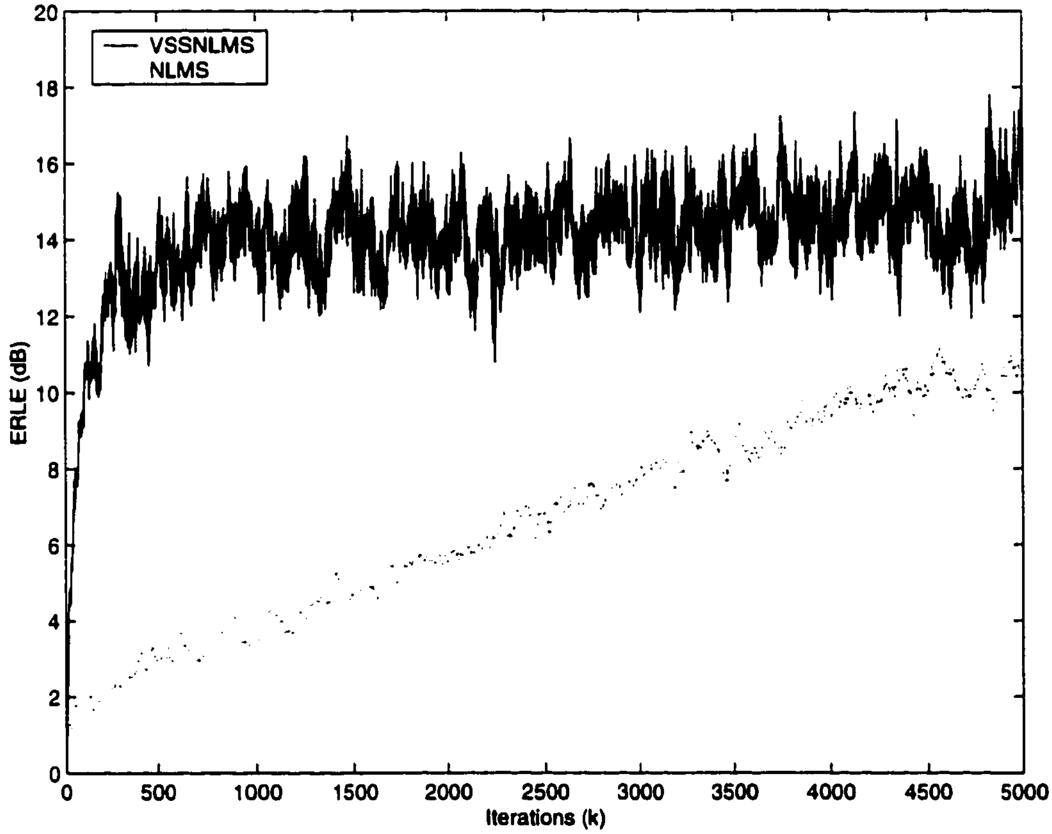


Figure 4.33: Comparison of the convergence characteristics and echo return loss enhancement for the VSS-NLMS algorithm and the traditional NLMS algorithm with colored (speech) input signals. The parameters used for this experiment are: $N = 500$ and $SNR = 15\,dB$.

In Figure 4.34, we have displayed the curves of the ERLE for the VSS-NLMS algorithm and the traditional NLMS algorithm for the case when both algorithm track a time-varying echo path. The variation in the echo path was set very high so as to ensure that real-time situations are taken care of. The length of the impulse response of the echo path as well as that of the adaptive filter is $N = 100$, and the signal-to-noise ratio used is $SNR = 40\ dB$. From this figure, we note that even though the echo path is highly varying, which results in high fluctuations in the coefficients of the adaptive filters, we can still clearly observe that VSS-NLMS algorithm converges faster and at the same time achieves higher ERLE than the traditional NLMS algorithm.

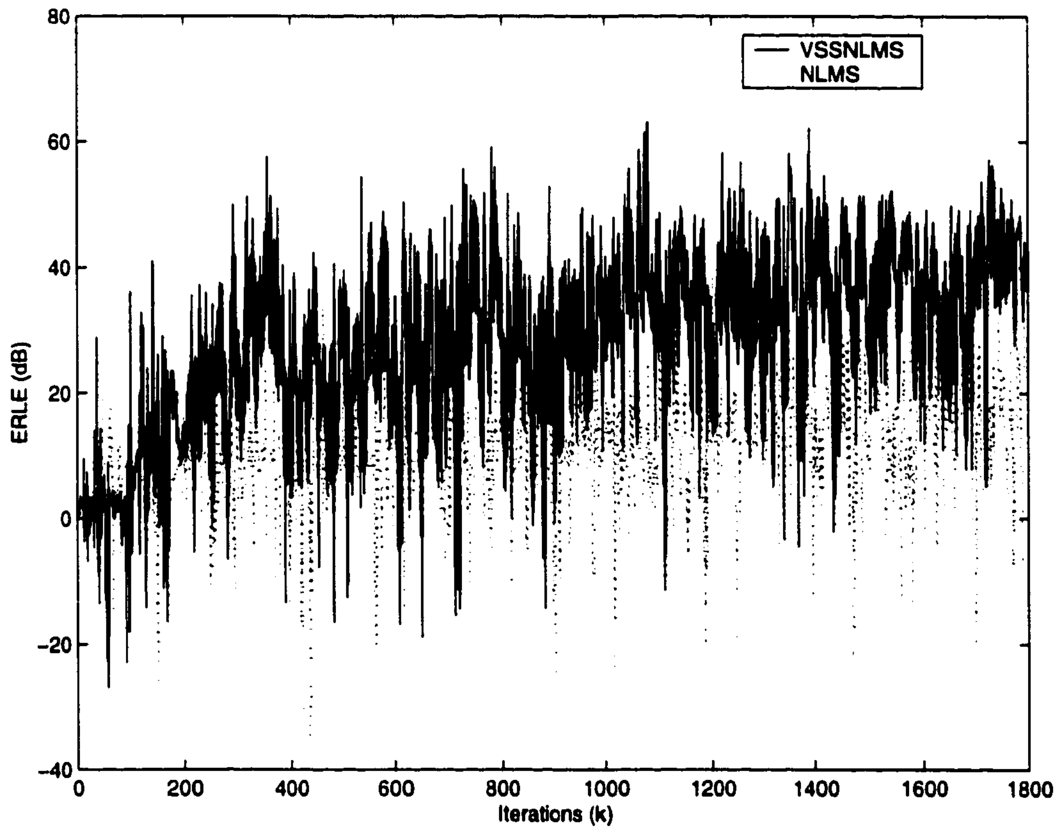


Figure 4.34: Comparison of the convergence characteristics and echo return loss enhancement for the VSS-NLMS algorithm and the traditional NLMS algorithm with colored (speech) input signals. The parameters used for this experiment are: time-varying echo path of $N = 100$, and $SNR = 40\ dB$.

Finally, in Figure 4.35-4.37, we have displayed the results of our experiments on the response of the proposed VSS-NLMS algorithm to abrupt changes in the echo path. Abrupt changes in echo path is a phenomenon that occur very frequently in the modern day telephone network, due to the use of the advanced electronics switching systems. Notice that the echo path model considered in this experiment is quite different from the time-varying echo path considered in the experiment of Figure 4.34 above.

The input signal and the impulse response of the echo path in this experiment were the same as in the experiment of 4.31 form time $k = 0$ to $k = 5,000$. At $k = 5,001$, the impulse response of the echo path was then changed abruptly to their corresponding negative values, while the input signal remains the same. The echo return loss enhancement achieved by the VSS-NLMS algorithm for this consideration is as illustrated in 4.35 and 4.36, while the mean behavior of the step-size that explains the observed behavior of the algorithm is shown in 4.37.

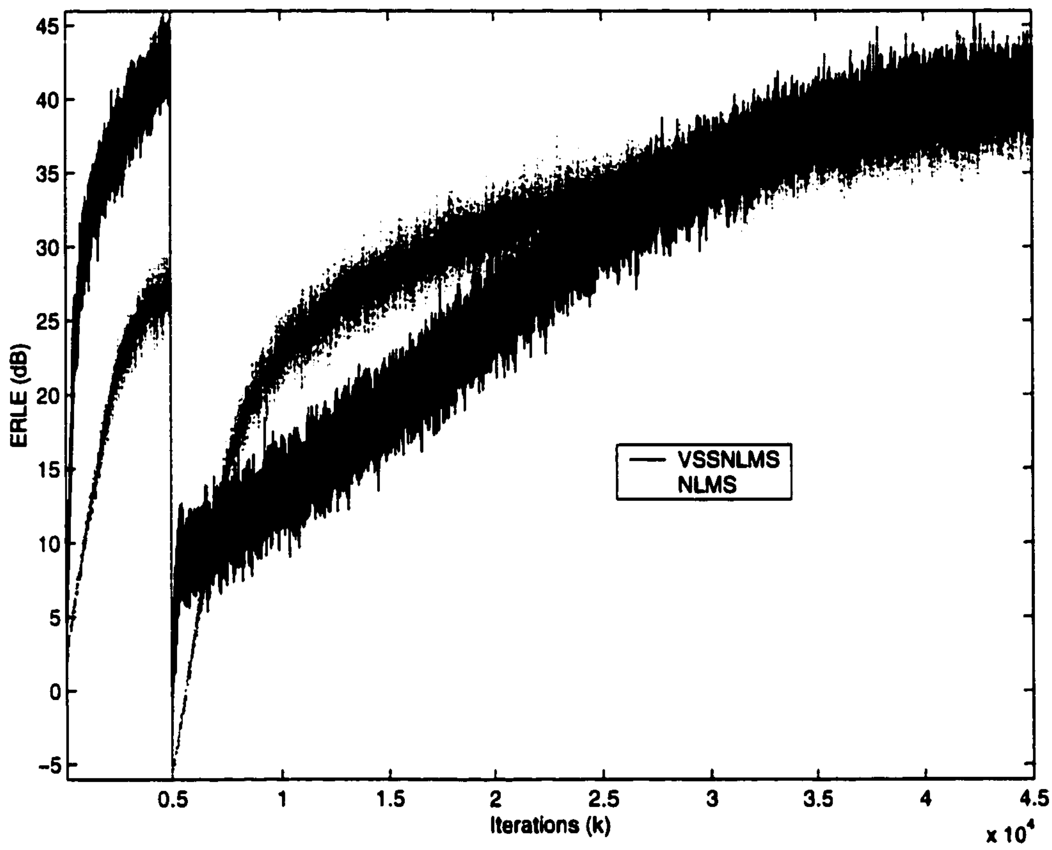


Figure 4.35: Comparison of the convergence characteristics and echo return loss enhancement for the VSS-NLMS algorithm and the traditional NLMS algorithm with colored (speech) input signals, for the case when the echo path changes abruptly. The parameters used for this experiment are: echo path of length $N = 100$, and $SNR = 40\,dB$.

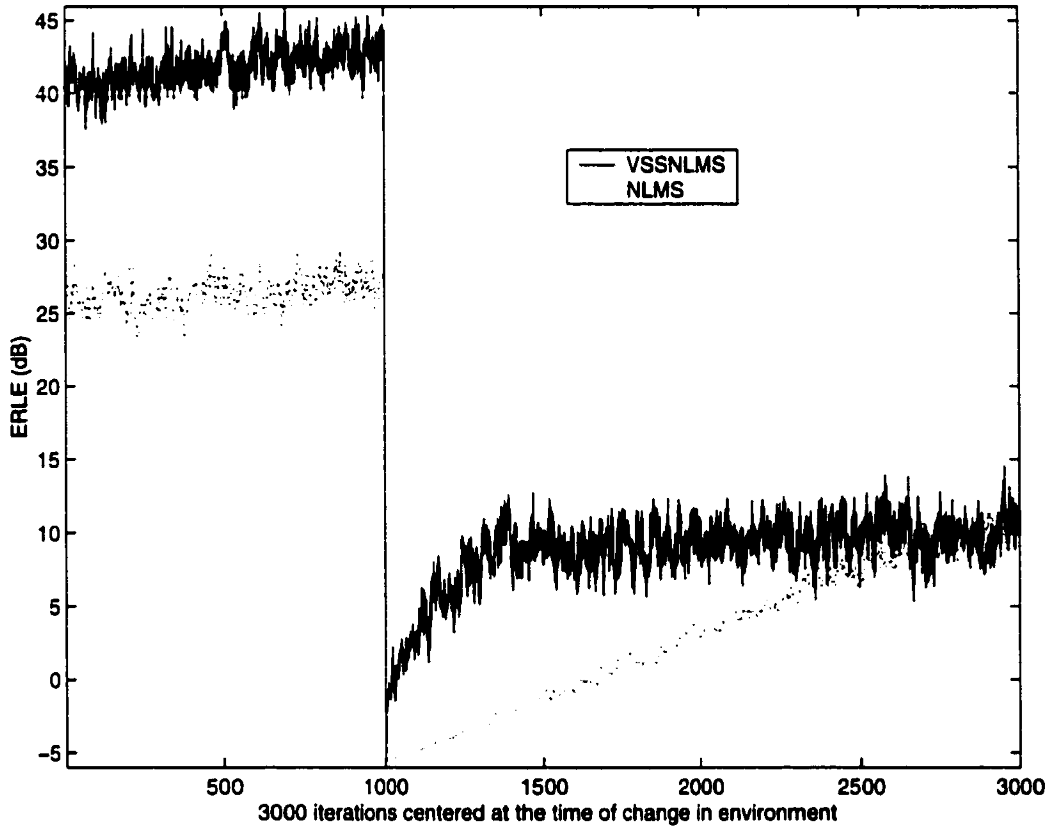


Figure 4.36: Comparison of the tracking capability and echo return loss enhancement after an abrupt change in the echo path, for the VSS-NLMS algorithm and the traditional NLMS algorithm. This plot is a re-plot of Figure 4.35 centered at the time of change.

From 4.37, we observe that the step-size increases very quickly immediately after the environment changes and therefore the algorithm is able to track the abrupt changes in the echo path faster than the NLMS algorithm as shown on the expanded plot, centered around the time of the change, in 4.36. It can also be observed from 4.37 that at the steady-state, the step-size value has dropped back to lower values. Consequently, the ERLE achieved by the proposed VSS-NLMS algorithm becomes higher than that achieved by the NLMS algorithm. Hence, the proposed algorithm combines the dual feature of faster tracking of changes in the echo path, as well as yielding, at steady-state, higher ERLE than the traditional NLMS algorithm.

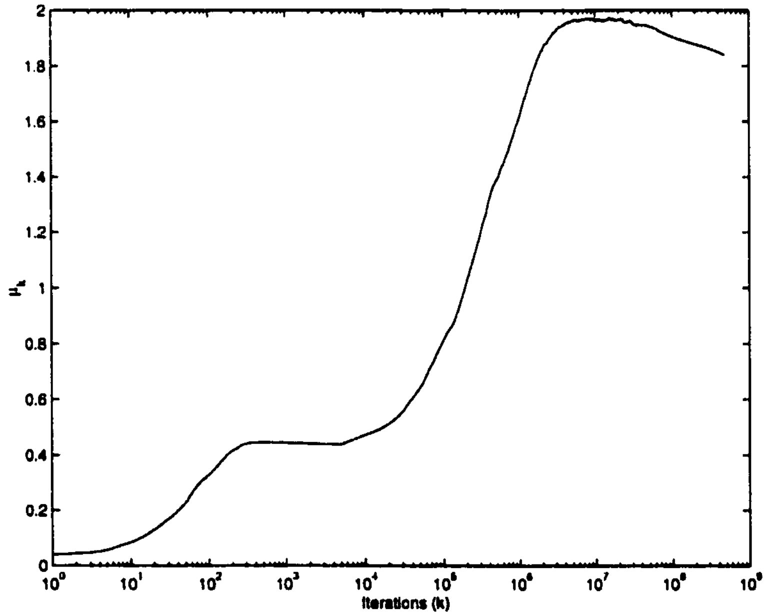


Figure 4.37: Mean behavior of μ_k for the VSS-NLMS algorithm explaining the performance behavior of the algorithm observed in the experiment of Figure 4.35.

4.8.3 Conclusion of Echo Cancellation Using VSS-NLMS Algorithm

The conclusion reached from our studies on echo cancellation using the proposed VSS-NLMS algorithm is that VSS-NLMS algorithm will always outperform the traditional NLMS algorithm both in convergence speed as well as ERLE. This performance gains observed with the proposed algorithm was found to be consistent and independent of the statistics of the echo path being tracked (i.e stationary or time-varying). This results obtained here tallies well with the earlier results we obtained in the previous sections where the general problem of adaptive system identification was considered. As echo cancellation is an example of such problems, such agreement between the results of these two considerations is well expected.

Finally it is also shown that the proposed VSS-NLMS algorithm combines the dual good properties of faster tracking of changes in echo path, and at the same time achieving higher ERLE, -a performance that is not possible with the traditional NLMS algorithm.

4.9 Notes on the Complexity of the proposed VSS-NLMS algorithm

The VSS-NLMS algorithm presented in this work (see chapter 2) is restated:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\mu_k e_k \mathbf{x}_k}{\|\mathbf{x}_k\|^2}, \tag{4.5}$$

and

$$\mu_k = \mu_{k-1} + \frac{\rho e_k e_{k-1} \mathbf{x}_k^T \mathbf{x}_{k-1}}{||\mathbf{x}_{k-1}||^2} . \tag{4.6}$$

While the conventional NLMS algorithm is given by:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\mu e_k \mathbf{x}_k}{||\mathbf{x}_k||^2} , \tag{4.7}$$

where the error signal e_k in each case is given by: $e_k = d_k - \mathbf{x}_k^T \mathbf{w}_k$. From equations (4.5)-(4.7), it can be observed that the complexities of the proposed VSS-NLMS algorithm and the NLMS algorithm are as shown in table 4.1 below:

Operations	NLMS	VSS-NLMS	VSS-NLMS' complexity increase.
$*/\div$	$3N+2$	$5N+6$	$2N+4$
$+/-$	$3N+1$	$5N+2$	$2N+1$
RamSpaces	-	$N+4$	$N+4$

Table 4.1: Comparison between the Complexities of the VSS-NLMS algorithm and the conventional NLMS algorithm.

However, the capability of the modern day Digital Signal Processors (DSP) has reduced most of the above stated complexity increases to very trivial ones. Therefore we can say that the complexity of the proposed VSS-NLMS is comparable to that of the conventional NLMS algorithm. Hence in concluding we reiterate that our algorithm remains essentially as simple as the conventional NLMS algorithm, whereas it is more robust, and promises several performance improvements over the NLMS algorithm.

Chapter 5

Thesis Contributions, Conclusions and Recommendations for Future Works

5.1 Thesis Contributions

This work successfully incorporates a simple step-size variation algorithm with the traditional NLMS algorithm. The resulting algorithm which we call a variable step-size NLMS (VSS-NLMS) algorithm is derived and analyzed for its steady-state performances. The major contribution of the results of this work to the traditional NLMS algorithm comes in two forms:

1) The problem of trade-off between convergence speed and steady-state error that confronts the traditional NLMS algorithm is effectively sur-

mounted in the new algorithm presented here. Choosing a high step-size that ensures fast convergence will no longer leads to a high steady-state error (as is obtained with the NLMS algorithm) when the VSS-NLMS algorithm is used;

2)The second contribution in terms of the convergence speed of the algorithm comes also in two forms:

a) for the case of low values of step-size initialization for the VSS-NLMS, the algorithm records an improvement over the convergence speed of the NLMS algorithm besides the improvement of the steady-state error stated in (1) above, b) but when high values of step-size initialization are used for the VSS-NLMS algorithm, the algorithm still record the improvement in the steady-state error over the NLMS algorithm as stated in (1) above, but the convergence speed of the two algorithms are essentially the same. The implication of this particular performance behavior of our algorithm is that it is possible to obtain the fastest speed of convergence (using optimum step-size) coupled with low steady-state error. Such a performance is not possible with the traditional NLMS algorithm.

5.2 Thesis Conclusions

In this thesis work, we derived a simple and robust variable step-size NLMS (VSS-NLMS) algorithm for echo cancellers. The step-size adaptation here was controlled by a gradient algorithm designed to minimize the squared estimation error. We present a theoretical performance analysis of the algorithm, and as well made simulation experiment to verify the derived results and investigate other properties of the algorithm. Our results showed the followings:

1. That when high values of step-size initializations are used for the VSS-NLMS algorithm, e.g. $\mu_0 = 1.0$, the convergence speed of the VSS-NLMS algorithm remains the same as that of the conventional NLMS algorithm, but VSS-NLMS algorithm will converge to a much lower steady-state MSE than the NLMS algorithm.
2. That with medium or low values of the step-size initializations, VSS-NLMS algorithm clearly converges faster than the NLMS algorithm, and at the same time converges to a much lower steady-state MSE than the NLMS algorithm.
3. That the performance improvements of the VSS-NLMS algorithm over the NLMS algorithm can be observed both in stationary and non-stationary environments, and as such the algorithm presented here is very robust.
4. That even with all the performance superiorities of the VSS-NLMS algorithm over the NLMS algorithm in above mentioned items, the VSS-NLMS algorithm remains simple and did not essentially increase the complexity of the NLMS algorithm.

These properties makes the VSS-NLMS algorithm to be more suitable for implementing modern day echo cancellers than the existing implementations using the NLMS algorithm. We therefore believe that the algorithm will be embraced for commercial production of echo canellers in the near future.

5.3 Recommendations for Future Works

In the light of the foregoing conclusion, we will like to suggest that a future work in this direction can be to investigate the real-time implementation of echo cancellers on DSP chips based on the VSS-NLMS algorithm presented here. Similar work was done for the case of the NLMS algorithm in [12]. The result of such work will provide the final information needed to actualize commercial production of echo cancellers based on this algorithm.

Also, we will like to point out that so far we have left the choice of the step-size adaptation constant ρ, in the proposed algorithm, to an arbitrary selection. A future work in this direction can consider deriving an exact condition on ρ that will guarantee convergence of the algorithm such that the arbitrariness in the selection of this parameter will be eliminated.

Lastly, we should mention that the proposed algorithm can be applied to an equalization problem for the case when the channel introduces large eigenvalue spread to the transmitted data. Hence, a future work can investigate the performance of the proposed VSS-NLMS algorithm in such situation.

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