Integrated Production, Quality and Maintenance Models under Various Preventive Maintenance Policies

by

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Dedicated to

my parents

May Allah have mercy on them

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After praise and thanks to the Lord of the World, the Almighty for having guided me at every stage of my life, and salutations to his noble prophet Mohammed (peace and blessings of Allah upon him) the last of the messengers, I express my profound gratitude and respect to my advisor Dr. Mohammed Ben Daya for his initiation, guidance, countless hours of dedication, extraordinary support and encouragement throughout all stages of this thesis report. Also, I express my appreciation for the cooperation and guidance extended by Dr. Salih O. Duffuaa and my committee members, Dr. Khalid Al-Sultan and Dr. Abdulrahim Al-Ghamdi.

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Abstract

Name: Mohammed Akram Abdulhai Makhdoum

Title: Integrated Production, Quality and Maintenance Models Under Various Preventive Maintenance Policies

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Production, quality and maintenance are three important aspects in any industrial process. Many attempts have been made in the literature to develop models which integrate them. However, these aspects have been generally treated separately. This thesis presents three integrated models for economic production quantity and economic design of control charts under various preventive maintenance (PM) policies. Taguchi's quadratic loss function is also incorporated into the development of the economic design of control charts for processes having general shift to failure distribution. A computer package to simulate all of the integrated models is developed. Computational results of various PM policies demonstrated that incorporating PM into production processes yields improvement in the product quality and consequently reduction in the total expected cost. Also, it demonstrated that incurring additional cost for performing PM might not be justifiable if the cost of PM is high to the point where it is not compensated for by the reductions in the quality control cost. Moreover, sensitivity analysis was conducted to study the effect of important parameters on the developed models.

Master of Science Degree

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ملخّص

الاسم: محمد أكرم عبد الحي مخدوم

العنوان: نماذج متكاملة للإنتاج والجودة النوعية والصيانة من خلال سياسات متعددة من الصيانة الوقائية

التخصص: هندسة النظم والأساليب

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إن الإنتاج والجودة النوعية والصيانة هي ثلاثة نواحي مهمة في أي عملية صناعية. وقد جرت في السابق عدة محاولات لإيجاد نماذج تكاملها. ولكن بصفة عامة تمت دراسة هذه النواحي كل على حدة. هذه الرسالة تقدم ثلاثة نماذج متكاملة لكمية الإنتاج الإقتصادية والتصميم الإقتصادي لرسومات المراقبة من خلال سياسات متعددة من الصيانة الوقائية. أيضا فإن معادلة تاجوتشي من الدرجة الثانية لحساب مقدار الخسارة قد دمجت في تطوير التصميم الإقتصادي لرسومات المراقبة لعمليات تحتوي على توزيع عام للإنحراف نحو العطل. كذلك قد تم تطوير عدة برامج للحاسب الآلي لمحاكاة جميع هذه النماذج. وقد وضحت النتائج الحسابية لسياسات متعددة من الصيانة الوقائية أن دمج الصيانة الوقائية في عمليات الإنتاج يؤدي الى تحسن في الجودة النوعية للمنتج وبالتالي إنخفاض في التكلفة المتوقعة الكلية. وقد وضحت أيضا أن التعرض لتكلفة إضافية للقيام بالصيانة الوقائية قد يكون غير ممكن تبريره إذا كانت تكلفة الصيانة الوقائية عالية لدرجة لايمكن تعويضها بالإنخفاض في تكلفة الجودة النوعية. إضافة إلى ذلك فإن عملية تحليل الحساسية قد أجريت لدراسة تأثير القيم المتغيرة على هذه النماذج.

درجة الماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران-المملكة العربية السعودية

Chapter 1

INTRODUCTION

1.1 General Background

Production, quality and maintenance are three important aspects in any industrial process. In today's manufacturing environment, new strategies that favor variety of products are replacing the old ones of mass production of a uniform product. Rapidly changing markets and the requirement to face the explosion of products variety have increased the need for automation and complex equipment which in turn increased the cost of the manufacturing processes. In order to get the best out of the expensive manufacturing processes and meet the quality challenge, production processes have to be maintained in good operating conditions. These facts have shifted the focus to maintenance and the need for effective maintenance policies.

The link between quality improvement and productivity is well established. Quality improvement can be achieved by elimination of waste such as scrap and rework. Production quantity, production quality and maintenance of the production process are interrelated problems.

In the past, production, quality and maintenance have been treated as three separate problems. Because of the interdependence between them, many attempts have been made to develop models that take into consideration more than one of the three aspects. The problem of improving the product quality in production systems and its integration with production quantity has received a significant amount of attention in the literature. Generally, this problem was approached by two ways. Many researches included product quality by considering the percentage of defective items in production systems. Rosenblatt and Lee [43] have studied the effect of imperfect processes on the optimal production quantity. They assumed that the production process deteriorates and produces some proportion of defective items. Other researches approached this problem by using control charts to monitor non-conforming items. Rahim [39] has considered the effect of the economic design of $\bar{x}$-control charts on the Economic Production Quantity (EPQ). He developed an integrated model for production quantity, inspection schedule and control chart design for a class of deteriorating processes where the in control period follows a general probability distribution with an increasing failure rate.

Although there were some attempts to develop integrated models for the economic design of control charts, EPQ and maintenance policies, these problems have been traditionally considered separately. Recently, Ben Daya [6] has extended Rahim's model [39] and developed an integrated model which introduces the imperfect preventive maintenance (PM) concept into the determination of EPQ and the economic design of $\bar{x}$-control charts. Ben Daya assumed that PM activities are carried out at each inspection interval. Also, he assumed that in the presence of PM activities the reduction in the shift rate to the out of control state is proportional to the PM level. Moreover, in Ben Daya's model it was assumed that the reduction in the age of the production process is always the same for a given PM level and a full PM will bring the process to the as good as new condition. In other words, the age of the process will drop to zero if a full PM is performed. However, a more realistic situation is to assume that the reduction in the age of the process will decrease, even for full PM, as the process ages. Also, performing PM at each inspection interval might not be cost effective. In this thesis, we will develop integrated models for maintenance, production quantity, inspection schedule and control chart design using various PM policies that manipulate the reduction in the age of the process and PM activities in more realistic situations.

Recently, Ben Daya and Duffuaa [7] have introduced Taguchi's Loss Function into the economic design of $\bar{x}$-control charts by extending Duncan's [17] model. They derived separate functions to compute both the in control and out of control quality costs using Taguchi's quadratic loss function. In their model, the probability that the process shifts to out of control state was assumed to follow an exponential distribution. In this thesis, we will also extend Ben Daya and Duffuaa's model to a process shift that follows a general distribution function. Finally, we incorporate the extended model into an integrated production, quality and maintenance model.

1.2 Problem Definition

The traditional approaches to the problem of determining the economic production quantity have always assumed implicitly that items produced are of perfect or acceptable quality. Product quality, however, is not always perfect, and it is usually a function of the state of the production process. When the production process is in a good condition, items produced may be of high or perfect quality. As time goes on, process ages and consequently the items produced may contain defectives or items that are of substandard quality. The relationships between maintenance and production and maintenance and quality are well established. In practice, maintenance of the production process is useful in improving the age of the production process and minimizing the tendency of the production process to shift to the out of control state. This in turn has a dual effect of improving the quality of produced items and increasing the potential of the production process. In this thesis, we consider a production process with a single machine producing a single kind of product. The $\bar{x}$-control chart is used to monitor the status of the production process. We also study the effect of maintenance on the quality and quantity of produced items.

Recently, Ben Daya [6] developed a model that integrates economic production quantity, economic design of control charts and imperfect preventive maintenance (PM). This was done for a class of deteriorating processes where the in control period follows a general probability distribution with an increasing failure rate. The status of the process is monitored by an $\bar{x}$-control chart and PM activities are coordinated with quality control inspections. The process is shut down at times h_1, $h_2 + h_1$,..., where h_j is the length of the j^{th} interval. At these times the process is inspected to determine the state of the process and PM tasks are carried out in parallel. If the process is found to be out of control production ceases until the accumulated on hand inventory is depleted. Otherwise. production continues. PM activities reduce the failure rate of the process proportional to PM cost. The length of each interval is chosen such that the integrated hazard over each interval is the same. The total expected cost model consists of the following:

1. the set up cost
2. the expected inventory holding cost
3. the expected quality cost; and
4. the expected maintenance cost.

The problem is to determine the optimal design parameters of the control chart, economic production quantity and PM level that minimize the total expected cost per unit time.

1.3 Thesis Objectives

The purpose of this thesis is to use the framework developed by Ben Daya to study the effect of various PM policies. The objectives of the thesis are:

I) To develop an "integrated model for the maintenance level, economic production quantity and economic design of control charts" for each of the following PM policies:

1. Perform PM at every l inspection intervals where l is a decision variable.
2. Perform PM if the process shift rate reaches a preset threshold which is considered as a decision variable.
3. Perform PM if two consecutive observations fall in a warning zone. The width of the warning zone is also a decision variable.

For each of the above three models, a FORTRAN code will be developed for simulation purpose.

II) To incorporate Taguchi's quadratic loss function into one of the above models for maintenance level, economic production quantity and economic design of control Charts. This will be accomplished by:

1. Extending the model developed by Ben Daya and Duffuaa to general probability distribution for the in control period; then
2. Introducing preventive maintenance concept into the extended model, and
3. Using one of the above integrated models to develop the complete production, quality and maintenance model.

Also, all of these models will be coded in FORTRAN for simulation purpose.

1.4 Thesis Organization

This thesis is presented in five chapters. Chapter 2 presents a comprehensive survey of relevant literature. Chapter 3 presents the necessary assumptions and notation needed throughout the context of this thesis. Also, it presents the development of three integrated production, quality and maintenance models under various preventive maintenance policies. Computational results illustrating some of the main aspects of the three developed models will be also presented. Chapter 4 presents the development of an integrated production, quality and maintenance model using Taguchi's quadratic loss function and some illustrative examples. Chapter 5 concludes the thesis.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to survey research dealing with models integrating production, quality and preventive maintenance. Several attempts to develop integrated models dealing with these issues have been conducted in the literature. The following sections discuss some of these attempts:

2.2 Integrated Models for Production and Quality

Generally, two different approaches to integrate quality cost with the determination of EPQ have been followed. Some authors modelled the quality cost by computing the expected percentage of defective items in production systems and others by using control charts to monitor non-conforming product quality characteristics. Under the first approach, Rosenblatt and Lee [43] have developed a model which addresses the problem of determining EPQ for imperfect production processes. In their model, the time of the process shift to the out of control state was assumed to follow an exponential distribution. Three different dynamic process deterioration schemes were investigated, namely, linear, exponential and generalized multiple state deterioration. For both linear and exponential deterioration, they found that the production run lengths are shorter than that of the classical EPQ model. For the multiple state deterioration, they also showed that the production run length is even smaller than or equal to the linear and exponential deterioration. Porteus [37] and [38], has also developed models which study the relationships between lot size and quality. He showed that investment in quality improvement can be done by three options: (i) reducing the probability that the process moves to out of control state. (ii) reducing setup costs and (iii) simultaneously using the two previous options. He also proposed that the lot sizes should be reduced to compensate for poor quality if no effective inspection is possible. Illustrative examples of his models using these three investment options have demonstrated that lower costs will be yielded for all of the three options than that of the classical EPQ model. Liou *et al.* [30] have addressed the problem of determining the optimal inspection number and optimal EPQ of a single item. They have assumed that Type I and Type II inspection errors exist when inspecting the imperfect production system. They derived an approximate formula for total cost difference between the general case, $\alpha \neq 0$, and $\beta \neq 0$, and the degenerate case, $\alpha = 0$, and $\beta = 0$. Computational results of their model demonstrated that under some conditions this cost difference is an increasing function of the demand-supply ratio and the cost difference of the post-sale and pre-sale defective costs. Cheng [13] proposed a general equation to model the relationship between production set-up and process reliability and flexibility.

Cheng [12] also proposed an EPQ model with demand-dependent unit production cost and imperfect production processes. In both [13] and [12] he formulated the EPQ problem as a geometric programming problem and derived closed loop solutions. Khouja and Mehrez [25] have proposed an extension to the economic production lot size model to cases where the production rate is a decision variable. They assumed that the quality of the production process deteriorates with increased production rate. They showed that for cases where increases in the production rate lead to a significant deterioration in quality, the optimal production rate may be smaller than the rate that minimizes unit production cost. For cases where quality is largely independent of production rate, the optimal production rate may be larger than the rate that minimizes unit production cost.

Under the second approach, Peters *et al.* [36] have developed a model for the joint determination of optimal inventory and quality control policy. They presented a cost model that combines a fixed order quantity inventory control system with Baysian quality control system for a lot-by-lot attribute acceptance sampling plan. Different simulation examples showed that their model is superior to both inventory and quality control systems when they are considered separately. This superiority becomes even more significant when the difference between cost of defective units exiting the quality control system and cost of repairing defective units increases. Cheng [14] has considered the determination of EPQ for imperfect production processes. He developed a simple model which relates unit production cost with process capability and quality assurance and he showed that unit production cost increases with increase in process capability and quality assurance expenses. Chen and Chung [11] have also considered the problem of jointly determining the optimal production run and the quality selection. They have developed a model for the joint determination of the optimal process mean and the production run for a process having an exponential shift to failure distribution. Separate expressions to compute the expected profit per item during the in control and out of control periods, optimal production run and optimal process mean setting were developed. Tagaras [45] has developed an economic model which integrates both statistical process control and maintenance procedures and simultaneously optimizes their parameters. He assumed that preventive maintenance activities are performed at equal intervals and that the effectiveness of the preventive maintenance remains unchanged.

A closely related area where production and quality have been considered jointly is the area of determining optimal production runs for processes with random drift that can occur when a tool wears out or a major component deteriorates. The relationship between tools wear, production run and quality has been considered by many authors. Rahim and Banerjee [40] have developed an optimal decision rule for tools resetting for processes with random linear drift to minimize the cost of rejected items and the loss of production time due to repetitive resetting. Schneider and Tang [44] have also developed a model for optimal control of a production process subject to random deterioration caused by tools wear. They developed a decision criterion of selecting the starting level of the process mean and the level at which the process mean should be adjusted back to the starting level to minimize production cost, tools adjustment and loss of defective items. Simple approximation solutions were developed to evaluate the optimal policy. A survey in this area is provided by Al-Sultan and Rahim [3] and Al-Fawzan and Al-Sultan [2].

Goyal [19] has reviewed the literature available on integrating the various aspects of production, lot sizing, inspection and rework under suitable classification scheme. He also presented a framework for developing an efficient production design system to motivate new models for modern manufacturing.

Recently, Rahim [39] has considered the determination of EPQ and design of the $\bar{x}$-control chart jointly. He developed an integrated model for both the inventory and quality control problems for a class of deteriorating processes where the in control period follows a general probability distribution with an increasing failure rate. The inspections are carried out at an increasing sampling frequency to determine the state of the process and the quality of the product using $\bar{x}$-control charts.

2.3 Integrated Models for Production and Maintenance

The problem of integrating production and maintenance has been generally approached in the literature in two different ways. Some authors, approached this problem by determining the optimal preventive maintenance schedule in the production system and others by taking maintenance as a constraint to the production system. Under the first approach, Lee and Rosenblatt [29] have developed a model to determine the EPQ of a single product and the schedule of inspection for process maintenance. An exponential probability distribution function and uniform sampling inspection intervals were assumed to compute the elapsed time for the process to be in the in control state. A decision criterion to either perform a maintenance or not at a given inspection interval was derived based on a comparison between the restoration cost and the defective items cost that may occur during that inspection interval. Computation schemes for other decision variables such as the production time, EPQ and number of inspection intervals were also derived based on the same comparison criterion. Groenevelt *et al.* [20] have also developed a model which studies the problem of selecting the economic lot size for an unreliable manufacturing facility with a constant failure rate and general randomly distributed repair times. The quantity of the safety stock to be used over those time intervals when the machine is being repaired was derived based on the target service level (desired fraction of total demand satisfied) constraint. The optimal lot size which minimizes the average running cycle cost was also derived as a function of setup cost, demand rate and preventive maintenance level which is carried out at the beginning of the production cycle. Three different functions of the mean time between failures ($MTBF$) were implemented to illustrate the relationships between the maintenance budget level and the expected total cost. Groenevelt *et al.* [21] have also developed another model to study the effect of machine breakdowns and corrective maintenance on the economic lot sizing decisions. Two production control policies were proposed to accommodate stochastic interruptions caused by unexpected machine breakdowns. The first policy assumes that production of the interrupted lots is not resumed after a breakdown and the next cycle is initiated when the on hand inventory is depleted before . On the other hand, the second policy assumes the production is immediately resumed. Lee and Rosenblatt [27] have also developed a joint model for production planning and maintenance schedule. They assumed that the cost of process restoration is a function of the detection delay. They have addressed the problem of simultaneous determination of the economic production quantity and the schedule of inspection of the process for maintenance. The process deterioration was assumed to follow an exponential distribution. also, they have assumed equal- interval inspections to construct the complete inspection schedule during the whole production cycle. Makis and Fung [32] have studied the problem of the joint determination of the lot size, inspection interval and preventive replacement time. They assumed that the shift to failure time of the process follows a general distribution. The also have studied the relationship between the preventive replacement time and the mean time to failure of the process. Lee and Park [26] have also considered production- maintenance policy for a deteriorating production system. The assumed that inspections are carried out at equally spaced intervals. They derived an cost model to determine the production cycle and inspection intervals jointly. This was done by considering the difference between reworking cost before sale and warranty cost after sale.

Under the second approach, Rishel and Christy [41] have studied the impact of incorporating alternative scheduled maintenance policies into the material requirements planing (MRP) system. They used four performance measures to evaluate the impact of incorporating maintenance into the production schedule, namely, the number of on-time orders, number of scheduled maintenance actions, number of equipment failures and total maintenance cost. Also, they have studied the feasibility of incorporating predicted emergency maintenance into the production schedule. Six different material requirements plans were used in their study to (i) determine if integrating scheduled maintenance with production schedule improves the measures of performance, (ii) determine whether it is more appropriate to schedule maintenance as independent and/or dependent demand item. (iii) determine if it is beneficial to incorporate a predicted failure into the production schedule and (iv) determine whether it is more appropriate to predict emergency maintenance on independent or dependent demand item. Results of their simulation showed that scheduled maintenance does make a significant impact on the number of failures and allow an increase in the production potential. Also, they showed that by including emergency maintenance in the MRP system and treating it as a dependent demand item will provide the ability to make more informed planning, scheduling and control decisions in both the production and maintenance functions. Brandolese *et al.* [10] have considered the problem of planning and management of a multi-product and one-stage production system made up of flexible machines operating in parallel. They developed a model to find the optimal schedule of both production and maintenance intervention which was considered as capacity-consuming jobs to be scheduled on the production resources. Their model was designed to meet release and due dates, minimize the total cost (sum of the expected maintenance cost, the set up cost and the production cost) and to minimize the total plant utilization time (sum of the total job processing time, the total machine idle time and the total maintenance time). A Weibull reliability function was used to compute the expected preventive maintenance cost and the system age was assumed to become as good as new after each preventive maintenance activity.

2.4 Integrated Models for Maintenance and Quality

The economic design of $\bar{x}$-control charts was first introduced by Duncan [17] in 1956. Duncan's model has stimulated most of the work on this area and has been extended by many authors. Under the assumption of exponential distributed in control period, Lorenzen and Vance [31] proposed a unified model which applies to situations where production continues or ceases during search or repair time. Moskowitz *et al.* [34] have investigated the effects of the choice of process failure mechanism on the design parameters of control charts for both exponential and general distributed in control periods. Other extensions by Baker [4], Montgomery and Heikes [33], Heikes *et al.* [22] and Banerjee and Rahim [5] have been also conducted to generalize the in control period to non-exponential probability distribution functions. Chiu and Huang [16] have developed a model which introduces preventive maintenance into the economic design of $\bar{x}$-control charts. They assumed a non-uniform distribution for the in control period with an increasing hazard rate and fixed sampling intervals. Also, they assumed that the system will become as good as new i.e. the age of the system will drop to zero after each preventive maintenance schedule. Results of their simulation showed that a small-scale restoration will significantly reduce the total expected cost and relatively large-scale restoration will still yield lower costs than that for non-maintained systems. Effect of maintenance on the economic design of $\bar{x}$-control charts for general time to shift distribution with increasing hazard rate was also studied by Ben Daya and Rahim [8].

Another model by Chiu and Huang [15] has been also developed to design both $\bar{x}$ and S^2 control charts with preventive maintenance. Two different preventive maintenance policies with both uniform and non-uniform sampling schemes were compared to determine the advantages of maintained systems over non-maintained systems.

Lee and Rosenblatt [28] considered the costs of different policies for providing early detection of process shifts and restoration capabilities. They have considered four different combinations of inspection and restoration policies: i) continuous inspection and restoration based on the process time to failure, ii) periodic inspection with restoration if needed, iii) continuous inspection and periodic restoration and iv) periodic inspection and restoration. The optimal cost models for all policies were derived for processes with in control periods following exponential distributions.

Tapiero [46] formulated a problem of continuous quality production and maintenance of a machine. He assumed that quality is a known function of the machine's degradation states. He used different applications of a specific quality function to obtain analytical solutions to an open-loop and feedback stochastic control maintenance problems.

Ben Daya and Duffuaa [9] discussed the relationship between maintenance and quality and proposed a broad framework for modelling the maintenance-quality relationship. They have proposed two approaches for linking and modelling this relationship. The first approach is based on the idea that maintenance affects the failure pattern of the equipment and that it should be modelled using the concept of imperfect maintenance. That is the equipment failure rate is decreased by a certain amount after each preventive maintenance which in turn amounts to a reduction in the age of the equipment. The second approach is based on the Taguchi's approach of quality i.e. using maintenance to reduce the deviations of products quality characteristics from their target value.

2.5 Joint Model for Maintenance, Production and Quality

Huang and Chiu [24] have attempted to integrate production planning, inspection and preventive maintenance. The have developed two cost models for monitoring policies with and without preventive maintenance. They have assumed that preventive maintenance are performed at equal intervals and the shift time of the production process follows a general distribution. Also, they have assumed that the cost of process restoration and the percentage of defective items depend on detection delay. Moreover, they have used linear forms to compute both preventive maintenance and restoration costs and determine when would be necessary to perform preventive maintenance.

Recently, Ben Daya [6] has extended Rahim's [39] model by introducing imperfect preventive maintenance concept into the determination of EPQ and the economic design of $\bar{x}$-control chart for a class of deteriorating processes where the in control period follows a general probability distribution with an increasing failure rate. Preventive maintenance activities are carried out at each inspection interval with an associated cost. The effect of the preventive maintenance was incorporated into Rahim's model by adapting the actual age of the machine at each inspection interval according to the amount of the preventive maintenance level. A linear relationship between the reduction in the actual machine age and the maintenance level was assumed. Also, it was assumed that the age of the process completely drops to zero if full PM activity was performed.

2.6 Taguchi's Loss Function and Economic Design of Control Charts

Recently, there were few attempts to introduce Taguchi's concept into the economic design of $\bar{x}$-control charts. Alexander, S. *et al.* [1] attempted to embellish Duncan's cost model with Taguchi's quadratic loss function (QLF) to incorporate losses that result from inherent variability due to assignable causes. The expected total cost of Duncan's model was modified by simply computing the out of control quality cost using Taguchi's QLF and also adding the in control quality cost which is also computed using Taguchi's QLF. Elsayed and Chen [18] have also attempted to develop an economic design of $\bar{x}$-control chart using Taguchi's QLF for processes with continuous operations. They considered and developed cost models for all four types of quality characteristics, namely, (i) symmetric nominal, (ii) asymmetric nominal, (iii) the-smaller-the-better and (iv) the-larger-the-better.

Most recently, Ben Daya and Duffuaa [7] have attempted to embed Taguchi's loss function into the economic design of control charts for processes with exponentially distributed shift to failure. They have imposed the parameters of the control charts on the loss function to obtain the expected costs for in control and out of control states. Also, they have used the basic relationships in Duncan's model to develop the complete model for the economic design of control charts.

2.7 Summary

Although there were several attempts in the literature to integrate production, quality and maintenance there were very few models dealing with the integration of all these aspects. Also, there is a lack of adequate models dealing with studying the effect of maintenance on the quality and production. Moreover, the integration of Taguchi's quadratic loss function and the economic design of control charts has been done only for processes having exponential time to shift distribution and there is a need to extend this integration for processes having general time to shift distribution.

Chapter 3

INTEGRATED PRODUCTION, QUALITY AND MAINTENANCE MODELS UNDER VARIOUS PREVENTIVE MAINTENANCE POLICIES

3.1 Introduction

In the literature, different attempts have been made to model the effect of PM on the age of the process. However, many models have assumed that the process becomes as good as new after any PM. In practice this assumption is often not true. The process after PM usually becomes younger or in fact might be worse than before PM because of faulty procedure [35]. Recently, Ben Daya [6] developed a model that integrates economic production quantity, economic design of control charts and imperfect preventive maintenance (PM). This was done for processes with in control periods following general distribution with increasing failure rates. Ben Daya assumed that the process is monitored by an

$\bar{x}$-control chart to determine its status and it is shutdown at each inspection interval for both the PM and quality control inspection activities. He also assumed that the frequency of quality control inspection increases as the process ages.

In Ben Daya's model, it was assumed that the reduction in the shift failure rate to the out of control state due to PM activities is proportional to the PM level and that a full PM will always yield an as good as new process. In the following sections, we will use the framework developed by Ben Daya to develop three integrated production, quality and maintenance models under various preventive maintenance policies. However, we will also assume that the age of the production process will always be somewhere between as good as new and as bad as old even if full PM was performed and the reduction in the age of the production process for a given PM will decrease due to process aging.

3.2 Notation and Assumptions:

In this section, we state all necessary notation and assumptions needed to develop the three integrated models for the joint determination of EPQ, preventive maintenance level and the economic design parameters of the control chart.

3.2.1 Notation

Decision variables

m:	number of inspection intervals
n:	sample size
h_i:	length of the i^{th} sampling (inspection) interval
k:	control limit coefficient of the $\bar{x}$-control chart
C_{pm}:	cost of actual PM activities
l:	frequency at which PM activities should be performed
r_{max}:	failure rate threshold beyond which PM activities should be performed
w_l:	warning limit coefficient of the $\bar{x}$-control chart

other notation:

Z_1^{max}:	the expected time to perform the highest level of a PM
Z_1:	the expected time to perform a PM
Z_2:	the expected time to repair the process if a failure is detected
a:	fixed sampling cost
b:	cost per unit sampled
C_f:	cost per false alarm
C_a:	cost to locate and repair the assignable cause
C_{in}:	quality cost per unit time while producing in control
C_{out}:	quality cost per unit time while producing out of control
α:	Pr(exceeding control limits | process in control)

β: Pr(not exceeding control limits | process out of control)

t_j: time at the end of j^{th} interval

y_j: actual age of the process right before the j^{th} PM

w_j: actual age of the process right after the j^{th} PM

$F(t)$: cumulative time to shift distribution function

$\bar{F}(t) =$ $1 - F(t)$

p_j: conditional probability that the process shifts to the out of control state during the time interval (t_{j-1}, t_j) given that it was in the in-control state at time t_{j-1}

$L(t)$: salvage value for working equipment of age t

$E(T)$: expected total length of production cycle including PM time

$E(QC)$: expected quality control cost per production cycle including the repair cost (PM cost not included)

for inventory holding cost:

D: demand rate

P: production rate

C_h: inventory holding cost per unit per unit time

S_0: set-up cost for each production cycle

$E(HC)$: expected inventory holding cost per inventory cycle *for preventive maintenance:*

C^o_{pm}:	cost of the maximum PM level
C_{pm}:	cost of actual PM activities
γ_j:	fraction used to compute the reduction in the process age at time t_j
η:	imperfectness factor
n_{pm}:	expected number of PMs during the production cycle
$E(PM)$:	expected preventive maintenance cost per production cycle.

3.2.2 Assumptions

The joint production, maintenance and quality model is based on the following assumptions:

1. The duration of the in control period is assumed to follow an arbitrary probability distribution, $f(t)$, having an increasing hazard rate $r(t)$, and cumulative distribution function $F(t)$.

2. The process is inspected at times $h_1, h_1 + h_2, \ldots$, to determine its state and the output quality of the product is monitored by an $\bar{x}$-control chart.

3. Quality inspection activities are carried out at the end of each interval. If a PM criterion is met at the end of the interval, production ceases for an amount of time Z_1 to carry out preventive maintenance activities.

4. For mathematical convenience, we assume that if any inspection shows that the state of the process is out of control, production ceases until the accumulated on hand

inventory is depleted to zero. If the process is found to be in control, production continues until the next sampling is due or the predetermined level of inventory is accumulated.

5. The production cycle begins when a new component is installed and ends either with a true alarm or after a specified number m of inspection intervals, whichever occurs first. In other words, if no true alarm is observed by time t_{m-1} then the cycle is allowed to continue for an additional time h_m. At time t_m necessary maintenance work is carried out. Therefore, there is no cost of sampling and charting during the m^{th} sampling interval. The process is brought back to the in control state by repair and/or replacement. Thus, a renewal occurs at the end of each cycle. This type of renewal process (Ross [42]) has the property that the expected cost per unit time can be expressed as the ratio of the expected cost per cycle to the expected length of the cycle.

6. The preventive maintenance action reduces the age of the equipment proportional to the cost of maintenance.

7. For the same level of PM, the reduction in the age of the equipment decreases as the process ages.

8. If a PM is not performed, the time to search for assignable cause is assumed to be negligible.

9. A salvage value is incorporated in the model since the residual life beyond a certain age for processes involving increasing hazard rate will be rather short.

3.3 Policy 1: PM at Every l Sampling Interval

In this section we develop an integrated production, quality and maintenance model under the following preventive maintenance policy:

Policy 1 *The process is inspected at times $h_1, h_1 + h_2, \ldots$, to determine the state of the process. At times $t_l, t_{2l}, t_{3l}, \ldots$, where l is a decision variable, the process is shut down and both quality control inspections and PM tasks are carried out in parallel.*

The total expected cost consists of the setup cost S_0, the expected quality control cost $E(QC)$, the expected inventory holding cost $E(HC)$ and the expected preventive maintenance cost $E(PM)$.

3.3.1 Quality Control Cost

In this section we derive expressions for both the expected quality control cost per cycle $E(QC)$ and the expected cycle length $E(T)$. The expected cycle length includes:

1. the expected time for inspection intervals when the process is in control
2. the expected time for detecting the presence of an assignable cause
3. the preventive maintenance time and
4. the repair time.

We assume that the expected time to perform a PM is proportional to the PM level and is given by:

$$Z_1 = Z_1^{max}(C_{pm}/C_{pm}^0).$$

The expected quality control cost includes:

1. the expected cost of operating while in control with no alarm
2. the expected cost of false alarm
3. the expected cost of operating while out of control with no alarm
4. the repair cost and
5. the cost of sampling
6. minus the salvage value for working equipment of age t

The expected cycle length and expected quality control cost per cycle are given by the following theorem:

Theorem 3.1 *Under the assumptions described in Section 3.2.2 and the requirements of policy 1, $E(T)$ is given by:*

$$\begin{aligned} E(T) &= Z_2\left[\sum_{j=1}^{m-1}(F(y_j)-F(w_{j-1}))+\bar{F}(w_{m-1})\right]+\sum_{j=1}^{m}h_j\bar{F}(w_{j-1})+Z_{1_j}\sum_{j=1}^{m-1}\bar{F}(w_{j-1}) \\ &+ \beta\sum_{j=1}^{m-1}(F(y_j)-F(w_{j-1}))\left[\sum_{i=j+1}^{m}(h_i+Z_{1_j})\beta^{i-j-1}+h_m\beta^{m-j-1}\right], \end{aligned} \tag{3.1}$$

where

$$Z_{1_j} = \begin{cases} Z_1 & \text{if } j = l, 2l, 3l, ... \\ 0 & \text{otherwise} \end{cases}$$

The quality control cost $E(QC)$ is given by:

$$\begin{aligned} E(QC) &= (a+bn)\left\{1 + \sum_{j=1}^{m-2} \bar{F}(w_j) + \beta \sum_{j=1}^{m-2} (F(y_j) - F(w_{j-1}))\right. \\ &\quad \left. \cdot \left[(1-\beta)\sum_{i=1}^{m-j-1} i\beta^{i-1} + (m-j-1)\beta^{m-j-1}\right]\right\} \\ &+ (C_{in} - C_{out})\sum_{j=1}^{m}\int_{w_{j-1}}^{y_j} tf(t)dt + (C_{out} - C_{in})\sum_{j=1}^{m} y_j(F(y_j) - F(w_{j-1})) \\ &+ C_{in}\sum_{j=1}^{m} h_j\bar{F}(w_{j-1}) \\ &+ C_a[\sum_{j=1}^{m}(F(y_j) - F(w_{j-1})) + \bar{F}(y_m)] \\ &+ C_{out}\beta\left[\sum_{j=1}^{m-1}(F(y_j) - F(w_{j-1}))\sum_{i=j+1}^{m} h_i\beta^{i-j-1}\right] \\ &+ \alpha C_f \sum_{j=1}^{m-1} \bar{F}(w_j) - \bar{F}(y_m)L(t_m). \end{aligned} \tag{3.2}$$

where y_j and w_j are given by: (3.6) and (3.7), respectively.

The proof of this theorem is given in Appendix A.1.

3.3.2 Inventory Holding Cost

The total expected inventory holding cost, $E(HC)$, is defined by:

$$E(HC) = C_h \int_0^{T_I} I(t)dt, \tag{3.3}$$

where T_I is the inventory cycle length and $I(t)$ is the function of the inventory level.

The integral in (3.3) is determined by computing the expected area $E(A)$ under the function $I(t)$. Hence

$$E(HC) = C_h \int_0^{T_I} I(t)dt = C_h E(A).$$

In order to compute $E(A)$, the expression of the inventory levels at times $t_j + Z_{1_j}$ is required and is given by the following lemma:

Lemma 3.1 *Let I_j be the inventory level at time $t_j + Z_{1_j}$, $j = 1, 2, \ldots, m-1$, I_m be the inventory level at time t_m, then*

$$I_j = \begin{cases} I_{j-1} + (P-D)h_j - DZ_1 & \text{if } j = l, 2l, 3l, \ldots \\ I_{j-1} + (P-D)h_j & \text{otherwise and for } j = m \end{cases}$$

where $I_0 = 0$ and if $I_j < 0$ it is set equal to zero.

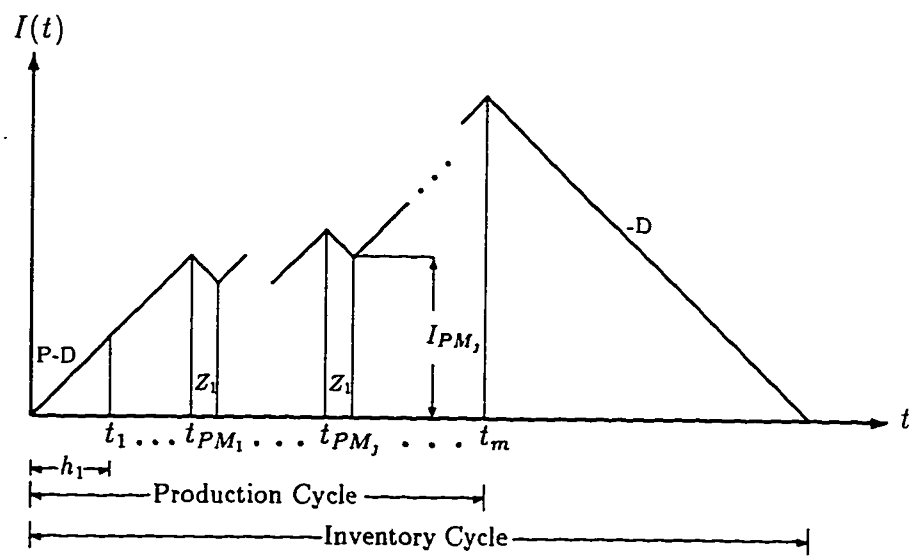


Figure 3.1: Inventory levels; where t_{PM_j} is the time at which the j^{th} PM is performed

The proof of this lemma is clear from Figure 3.1.

The expected area $E(A)$ under the function $I(t)$ is given by the following theorem:

Theorem 3.2 *Let*

$$U_j = \begin{cases} \left[\begin{array}{l} [2I_{j-1} + (P-D)h_j]\dfrac{h_j}{2} + \dfrac{[I_{j-1} + (P-D)h_j]^2}{2D} \\ \text{for } I_j = 0 \\ [2I_{j-1} + (P-D)h_j]\dfrac{h_j}{2} + [I_{j-1} + (P-D)h_j + I_j]\dfrac{Z_1}{2} \\ \text{for } I_j > 0 \end{array} \right] & \text{if } j = l, 2l, 3l, \ldots \\ \left[[2I_{j-1} + (P-D)h_j]\dfrac{h_j}{2} \right] & \text{otherwise and for } j = m \end{cases} \tag{3.4}$$

and let $B_j = I_j^2/2D, \ \ j = 1, 2, \ldots, m$. *Then* $E(A)$ *is given by:*

$$\begin{aligned} E(A) &= \sum_{j=1}^{m} U_j \bar{F}(w_{j-1}) + (1-\beta) \sum_{j=1}^{m-1} B_j(F(y_j) - F(w_{j-1})) \\ &+ \ \beta \sum_{j=1}^{m-1} (F(y_j) - F(w_{j-1})) \left[\beta^{i-j-1} U_i + \beta^{m-j-1} B_m\right] + B_m \bar{F}(w_{m-1}). \end{aligned} \tag{3.5}$$

The proof of this theorem is given in Appendix A.2.

3.3.3 Preventive Maintenance Cost

In this section, we model the effect of maintenance on the time to shift distribution. We show how this model can be used in the joint determination of EPQ, economic design of control chart, and optimal preventive maintenance schedule.

The objective of preventive maintenance is to prevent, mitigate, or detect the onset of failure using diagnostic techniques.

It can be assumed that the failure rate of the equipment is decreased after each PM. This amounts to a reduction in the age of the equipment. In this thesis, we assume as in Ben Daya's model [6] that the reduction in the age of the equipment is proportional to the cost of preventive maintenance C_{pm}. Moreover, we also assume that the effect of PM on the age of the equipment degrades as the process ages. Hence the age of the equipment after a PM, t_a, is related to its age before PM, t_b, by the following equation:

$$t_a = (1 - C_{pm}/C^0_{pm})t_b.$$

So, if PM activities are carried out at times $t_1, t_2, \cdots, t_j, \cdots$, then the fractions $\gamma_j s$ needed to compute the reductions in the process age are given by:

$$\gamma_j = \eta^{j-1}(C_{pm}/C^0_{pm}),\ j = 1, ..., m-1$$

where $0.0 < \eta < 1.0$ is the imperfectness factor in the PM effect. Therefore, the process age at time t_j, right before and after PM is given by:

$$\left.\begin{array}{l} y_1 = h_1 \\ y_j = w_{j-1} + h_j,\ j = 2, ..., m \end{array}\right\} \tag{3.6}$$

and

$$w_j = \begin{cases} (1-\gamma_j)y_j & \text{if } j = l, 2l, 3l, ... \\ y_j & \text{otherwise} \end{cases} \tag{3.7}$$

No PM is carried out at the end of the m^{th} interval and the times Z_{1_j} needed for PM activities are not counted in the age of the process.

This change in the age of the equipment will affect the time to shift distribution and consequently the design of the control chart. It will also affect the length of the production run and consequently EPQ. Thus the imperfect maintenance concept allows the integration of EPQ, economic design of control chart, and the optimization of the preventive maintenance effort. The value of C_{pm} that produces the least total expected cost corresponds to the optimal PM level.

The expected PM cost is given by the following theorem:

Theorem 3.3 *The expected maintenance cost during a complete cycle is given by:*

$$\begin{aligned} E(PM) &= \sum_{j=1}^{m-1} C_{pm_j}\bar{F}(w_j) + \beta \sum_{j=1}^{m-1} C_{pm_j}(F(y_j) - F(w_{j-1})) \\ &+ C_{pm_j}\beta \sum_{j=1}^{m-1}(F(y_j) - F(w_{j-1})) \\ &\cdot \left[\sum_{j=1}^{m-j-1} i(1-\beta)\beta^{i-1} + (m-j-1)\beta^{m-j-1}\right], \end{aligned} \tag{3.8}$$

where

$$C_{pm_j} = \begin{cases} C_{pm} & if\ j = l, 2l, 3l, \ldots \\ 0 & otherwise \end{cases}$$

The proof of this theorem is given in Appendix A.3.

Note that the expected number of PMs n_{pm} during the production cycle is given by:

$$n_{pm} = \begin{cases} E(PM)/C_{pm} & \text{if } C_{pm} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.9}$$

3.4 Policy 2: PM if Process Shift Rate Reaches a Preset Threshold

In this section we develop an integrated production, quality and maintenance model under the following preventive maintenance policy:

Policy 2 *The process is inspected at times $h_1, h_1 + h_2, \ldots,$ to determine the state of the process. PM activities are performed only at those intervals at which the failure rate of the process reaches a preset threshold. At those intervals, the process is shut down and both quality control inspections and PM tasks are carried out in parallel.*

3.4.1 Quality Control Cost

Under the assumptions described in Section 3.2.2 and the requirements of policy 2, the expected cycle length $E(T)$ is the same as equation (3.1) except that Z_{1_j} is redefined as follows:

$$Z_{1_j} = \begin{cases} Z_1 & \text{if } r(t_j) \geq r_{max} \\ 0 & \text{otherwise} \end{cases}$$

where $r(t_j)$ is the failure rate at time t_j and r_{max} is the preset threshold.

The quality control cost $E(QC)$ per cycle is exactly the same as equation (3.2).

The derivations of the above two equations are the same as of theorem 3.1.

3.4.2 Inventory Holding Cost

The total expected inventory holding cost, $E(HC)$, is defined by (3.3) and the inventory levels are given by the following lemma:

Lemma 3.2 *Let I_j be the inventory level at time $t_j + Z_{1_j}$, $j = 1, 2, ..., m-1$, I_m be the inventory level at time t_m, then*

$$I_j = \begin{cases} I_{j-1} + (P-D)h_j - DZ_1 & \textit{if } r(t_j) \geq r_{max} \\ I_{j-1} + (P-D)h_j & \textit{otherwise and for } j = m \end{cases}$$

where $I_0 = 0$ and if $I_j < 0$ it is set equal to zero.

The proof of this lemma is clear from Figure 3.1.

The expected area $E(A)$ under the function $I(t)$ is the same as equation (3.5) except that U_j is redefined as follows:

$$U_j = \begin{cases} \left[\begin{array}{l} [2I_{j-1} + (P-D)h_j] \dfrac{h_j}{2} + \dfrac{[I_{j-1} + (P-D)h_j]^2}{2D} \\ \textit{for } I_j = 0 \\ [2I_{j-1} + (P-D)h_j] \dfrac{h_j}{2} + [I_{j-1} + (P-D)h_j + I_j] \dfrac{Z_1}{2} \\ \textit{for } I_j > 0 \end{array} \right] & \text{if } r(t_j) \geq r_{max} \\ \left[[2I_{j-1} + (P-D)h_j] \dfrac{h_j}{2} \right] & \text{otherwise and for } j = m \end{cases}$$

3.4.3 Preventive Maintenance Cost

The expected maintenance cost during a complete cycle is the same as equation (3.8) except that C_{pm_j} is redefined as follows:

$$C_{pm_j} = \begin{cases} C_{pm} & \text{if } r(t_j) \geq r_{max} \\ 0 & \text{otherwise} \end{cases}$$

The expected number of PMs n_{pm} during the production cycle is given by (3.9).

Note that the age of the equipment at time t_j right before PM is given by (3.6) and right after the PM is given by:

$$w_j = \begin{cases} (1-\gamma_j)y_j & \text{if } r(t_j) \geq r_{max} \\ y_j & \text{otherwise} \end{cases}$$

3.5 Policy 3: PM if Two Consecutive Observations fall in the Warning Zone

In this section we develop an integrated production, quality and maintenance model under the following preventive maintenance policy:

Policy 3 *The process is inspected at times $h_1, h_1 + h_2, \ldots$, to determine the state of the process. PM activities are performed only at those intervals at which two consecutive values of sample means fall in the warning zone. At those intervals, the process is shut down and both quality control inspections and PM tasks are carried out in parallel.*

The shaded areas in Figure 3.2 bounded the upper/lower control limits and upper/lower warning limits define the warning zone of interest.

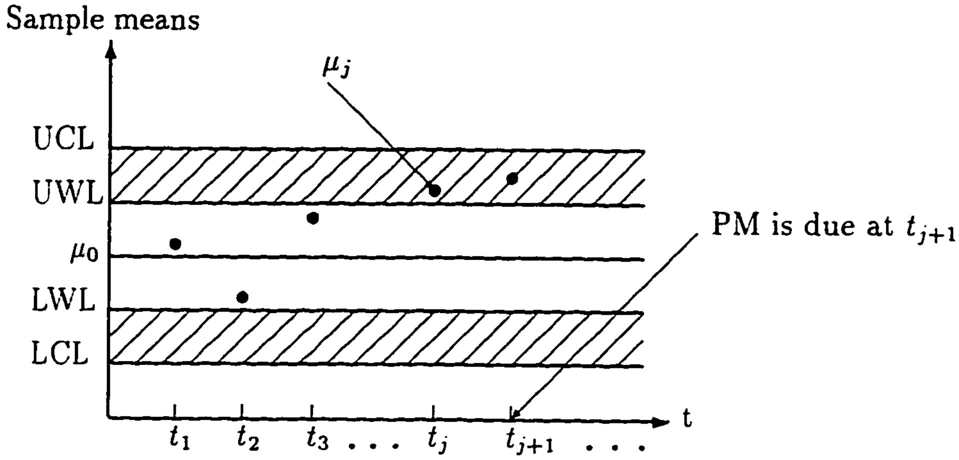


Figure 3.2: Two consecutive sample means at t_j and t_{j+1} falling in the warning zone

3.5.1 Quality Control Cost

Under the assumptions described in Section 3.2.2 and the requirements of policy 3, the expected cycle length $E(T)$ is the same as equation (3.1) except that Z_1 is replaced by $p_w^2 Z_1$, where p_w is the probability that a sample mean falls in the warning zone.

The quality control cost $E(QC)$ per cycle is exactly the same as equation (3.2).

The derivations of the above two equations are the same as of theorem 3.1.

3.5.2 Inventory Holding Cost

The total expected inventory holding cost, $E(HC)$, is defined by (3.3) and the inventory levels are given by the following lemma:

Lemma 3.3 *Let I_j be the inventory level at time $t_j + p_w^2 Z_1$ $j = 1, 2, ..., m-1$, I_m be the inventory level at time t_m, then*

$$I_j = (1 - p_w^2)[I_{j-1} + (P - D)h_j] + p_w^2[I_{j-1} + (P - D)h_j - DZ_1], \ j = 1, 2, ..., m-1$$
$$I_m = I_{m-1} + (P - D)h_m$$

where $I_0 = 0$ and if $I_j < 0$ it is set equal to zero.

The proof of this lemma is clear from Figure 3.1.

The expected area at a given inspection interval can be computed as: (the probability of performing PM)×(inventory area if PM is performed) + (the probability of not performing PM)×(inventory area if PM is not performed). The expected area $E(A)$ under the function $I(t)$ is the same as equation (3.5) except that U_j is redefined as follows:

$$U_j = \begin{cases} p_w^2 \left([2I_{j-1} + (P - D)h_j] \dfrac{h_j}{2} + \dfrac{[I_{j-1} + (P - D)h_j]^2}{2D} \right) \\ +(1 - p_w^2) \left([2I_{j-1} + (P - D)h_j] \dfrac{h_j}{2} \right) \ \mathit{for}\ I_j = 0, \ j = 1, 2, ..., m-1 \\ p_w^2 \left([2I_{j-1} + (P - D)h_j] \dfrac{h_j}{2} + [I_{j-1} + (P - D)h_j + I_j] \dfrac{Z_1}{2} \right) \\ +(1 - p_w^2) \left([2I_{j-1} + (P - D)h_j] \dfrac{h_j}{2} \right) \ \mathit{for}\ I_j > 0, \ j = 1, 2, ..., m-1 \\ [2I_{j-1} + (P - D)h_j] \dfrac{h_j}{2} \ \mathit{for}\ j = m \end{cases}$$

3.5.3 Preventive Maintenance Cost

The expected maintenance cost during a complete cycle is given by:

$$E(PM) = C_{pm}n_{pm}, \tag{3.10}$$

where n_{pm} is given by:

$$n_{pm} = (m-1)p_w^2. \tag{3.11}$$

Again the expected age of the equipment at a given inspection interval can be computed as: (the probability of performing PM)×(age of the equipment if PM is performed) + (the probability of not performing PM)×(age of the equipment if no PM is performed). So. the age of the equipment at time t_j right before PM is given by (3.6) and right after the PM is given by:

$$w_j = p_w^2(1-\gamma)y_j + (1-p_w^2)y_j.$$

3.6 Complete Integrated Production, Maintenance and Quality Models

This section provides the complete production, quality and maintenance integrated models for all of the above developed policies.

Recall that the expected total cost of the integrated model consists of the following:

1. the set up cost S_o

2. the expected inventory holding cost $E(HC)$

3. the expected quality cost $E(QC)$; and

4. the expected maintenance cost $E(PM)$

Therefore the total expected cost per unit time is given by:

$$ETC = \frac{S_0 + E(HC) + E(QC) + E(PM)}{E(T_I)}.$$

The expression of $E(T_I)$ is given by:

$$E(T_I) = \frac{P}{D}(E(T) - n_{pm}Z_1)).$$

3.7 Solution Procedure

In this section we discuss the problem of solving all of the above developed integrated production, quality and maintenance models to obtain the optimal values for the decision variables. Also, we will discuss the way by which the frequency of sampling should be regulated and the optimization procedure used to determine the optimal design parameter values and the optimal preventive maintenance effort.

3.7.1 Decision Variables

The problem is to determine simultaneously the optimal production run time and hence the optimal EPQ, the optimal preventive maintenance level, and the optimal design parameters of the $\bar{x}$-control chart, namely $h_1, h_2, ..., h_m$, the sample size n, and the control limit coefficient k. In addition, the optimal values of l, r_{max} and warning limit coefficient for policies 2,3 and 4, respectively, will be also determined simultaneously with the above mentioned decision variables.

3.7.2 Frequency of Sampling

For a Markovian shock model, a uniform sampling scheme provides a constant integrated hazard over each interval. Rahim and Banerjee [5] extended this fact to non-Markovian shock models by choosing the length of sampling intervals such that the integrated hazard over each interval is the same for all intervals, that is

$$\int_{t_j}^{t_{j+1}} r(t)dt = \int_{0}^{t_1} r(t)dt \tag{3.12}$$

where $r(t)$ is the hazard function defined by:

$$r(t) = \frac{f(t)}{\bar{F}(t)}. \tag{3.13}$$

Since the failure rate is reduced at the end of each interval because of PM activities, condition (3.12) becomes;

$$\int_{w_{j-1}}^{y_j} r(t)dt = \int_{0}^{h_1} r(t)dt, \quad j = 2, ..., m \tag{3.14}$$

If the time that the process remains in the in control state follows a Weibull distribution. that is, its probability density function is given by:

$$f(t) = \lambda\nu t^{\nu-1}e^{-\lambda t^{\nu}}, \quad t > 0, \ \ \nu \geq 1 \ \ \lambda > 0,$$

then using (3.14), the length of the sampling intervals h_j, $j = 2, ..., m$ can be determined recursively as follows:

$$h_j = [(w_{j-1})^{\nu} + h_1^{\nu}]^{1/\nu} - w_{j-1}, j - 2, ..., m. \tag{3.15}$$

3.7.3 Optimization Procedure

The problem is to determine the values of the decision variables m, n, k, h_1 and γ, which defines the PM level, that minimize the expected total cost ETC. Recall that the age of the equipment after a PM is reduced proportional to the PM cost C_{pm}. The cost function is minimized using the pattern search technique of Hooke and Jeeves [23]. The search starts with a local exploration in small steps around some starting point. If the exploration is a success, i.e., the cost reduces during local exploration, the step size grows; if the exploration is a failure, the step size is reduced. If a change of direction is necessary, the method starts all over again with a new pattern. The search is terminated when the step size is reduced to a predetermined value or when the number of iterations equals a predetermined value, whichever occurs first.

3.8 Computational Results

In this section, we present numerical examples to illustrate important aspects of the developed integrated models. In all examples, the process shift mechanism is assumed to follow a Weibull distribution. The Weibull scale and shape parameters are $\lambda = 0.05$ and $\nu = 2$, respectively, unless specified otherwise. The following data will be used as the basis for all examples: $Z_1^{max} = 0.1$, $Z_2 = 1.0$, $a = \$2.0$, $b = \$0.5$, $C_f = \$500$, $C_a = \$1100$, $C_{in} = \$50$, $C_{out} = \$950$, $\delta = 0.5$, $L(t_m) = 1100e^{-t_m}$, $D = 1400$ units, $P = 1500$ units, $C_h = \$0.1$, $S_o = \$20$ and $\eta = 0.99$ (the reduction in the age factor due to process aging). Note that in Ben Daya's [6] model, it was assumed that $\eta = 1.0$ or, in other words, the age of the equipment is reduced by the same amount after each PM task.

3.8.1 Effect of PM level

In this subsection, two different cases of relationship between the cost of PM (C_{pm}) and the improvement in the age of the equipment will be used for the sake of demonstrating the effect of PM level on both quality control and total expected costs. The two cases are (1) linear and (2) non-linear improvement relationship. For the first case, the age of the equipment after PM is defined as $t_a = (1-\gamma)t_b$ and for the second case as $t_a = (1-\sqrt{\gamma})t_b$. Figure 3.3 shows the relationship between the PM level and the process age improvement for both cases.

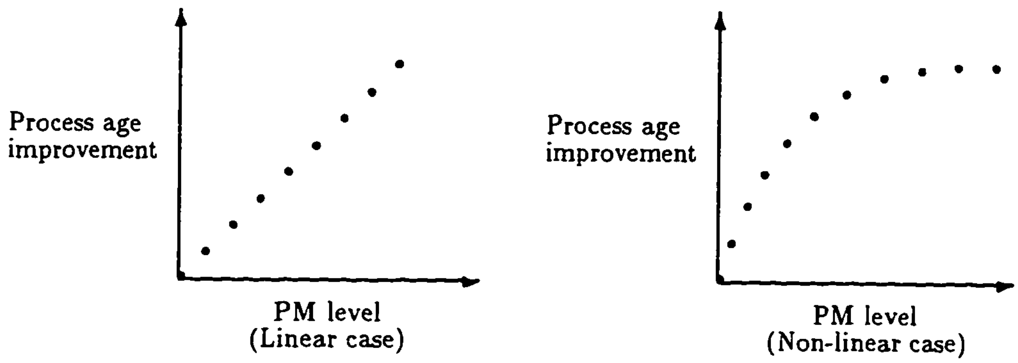


Figure 3.3: Linear and non-linear relationships between the PM level and the process age improvement

The results of policy 1 with l set equal to1 obtained for different PM levels corresponding to $C^o_{pm} = 300$ for both cases are summarized in Tables 3.1 and 3.2.

C_{pm}	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
0	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0	6502	429.7
100	20	52	2.57	1.41	0.0102	0.8499	14.3	309.5	21.9	121.5	20584	454.3
200	20	62	2.54	1.46	0.0111	0.9187	22.7	244.6	13.3	164.6	32226	423.5
300	17	69	2.52	1.49	0.0116	0.9484	27.2	205.7	9.2	177.1	38355	392.8

Table 3.1: Case 1: effect of PM level for linear improvement

C_{pm}	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
0	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0	6502	429.7
100	18	50	2.58	1.18	0.0098	0.8300	14.9	246.2	27.1	110.9	21571	385.5
200	20	67	2.53	1.46	0.0115	0.9412	25.9	207.2	7.2	123.4	43624	332.4
300	17	69	2.52	1.49	0.0116	0.9484	27.2	205.7	9.2	177.1	38355	392.8

Table 3.2: Case 2: effect of non-linear PM level for non-linear improvement

These results illustrate clearly the trade-offs between PM levels and quality control costs. The increase in PM level yields reductions in quality control costs. With no PM, the quality control cost is \$400.1. With a PM level of \$100, the quality control cost is reduced to \$309.5 and \$246.2 for both case 1 and case 2, respectively. The optimum PM level when $C^o_{pm} = 300$ is obtained when $C_{pm} = \$300$, leading to a quality cost of \$205.7 and an overall cost of \$392.8 much less than without PM (\$429.75). One might also notice that at low values of C_{pm}, more reductions in both quality and overall costs are obtained for case 2 than that for case 1. This is because that for the same cost of PM, case 2 yields much more reduction in the age of the equipment than case 1. At the full PM level ($C_{pm} = \$300$) both cases yield the same amount of improvement in the age of the equipment and hence the same reduction in both quality and overall costs.

Another interesting point is to see also if PM activities have any effects of the economic production quantity EPQ. As a matter of fact, we can notice from Tables 3.1 and 3.2 that PM does affect the production cycle (t_m) and for higher PM levels we have longer production cycles. This is because when PMs are performed during the production cycle quality control costs are reduced and hence longer production cycle will be still feasible. This in turn affect the value of EPQ (note that $EPQ = [E(T) - n_{pm} Z_1]\, P$), where $E(T)$ is the expected production cycle including PM time, n_{pm} is the expected number of PMs during the production cycle, Z_1 is the time needed to perform a PM and P is the production rate. So we can see that PM tasks have indirect influence on the determination of EPQ value.

3.8.2 Frequency of PM tasks

In this subsection, we study the frequency by which PM tasks should be performed. Tables 3.3 and 3.4 demonstrate the results of policies 1 and 2, respectively, obtained by performing PM at different levels where $C_{pm} = C^{o}_{pm}$. Both l and r_{max} are included in the tables as decision variables, where l is the frequency at which PM should be performed and r_{max} is the threshold beyond which PM should be performed.

C_{pm}	l	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
250	1	19	68	2.52	1.48	0.0116	0.9451	29.9	203.1	8.2	151.0	42185	362.9
300	1	17	69	2.52	1.49	0.0116	0.9484	27.2	205.7	9.2	177.1	38355	392.8
350	2	12	64	2.52	1.62	0.0116	0.9301	14.6	264.3	29.9	117.3	21199	413.0

Table 3.3: Effect of the parametr l

C_{pm}	r_{max}	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
250	0.1	19	68	2.52	1.48	0.0116	0.9451	29.9	203.1	8.2	151.0	42185	362.9
300	0.1	17	69	2.52	1.49	0.0116	0.9484	27.2	205.7	9.2	177.1	38355	392.8
350	0.2	12	64	2.52	1.62	0.0116	0.9302	14.6	264.4	30.0	117.2	21209	413.0

Table 3.4: Effect of the parametr r_{max}

The above tables show clearly that when the cost of PM is relatively low, the optimum scheme would be to perform PM at every inspection interval. This is shown in table 3.3 where the optimum value of l is 1 for both $C_{pm} = 250$ and 300. Table 3.4 also re-emphasizes the same argument where the optimum value of r_{max} 0.1 for the same C_{pm} values. However, when the cost of PM becomes large to the point where incurring C_{pm} cost at every inspection interval is not compensated for by a reduction in the quality control cost, then the optimum scheme is not to perform PM at every inspection interval.

So, at $C_{pm} = C^{o}_{pm} = 350$, Table 3.3 shows that the optimum value of l is 2 and Table 3.4 shows that the optimal threshold setting is 0.2.

3.8.3 Effect of Mean Time to Failure

In this subsection, three different values of Weibull scale λ and shape γ parameters will be used to generate different mean times to failure: 2.46, 3.96 and 8.86 hours to demonstrate their effect on PM policies. For each mean time to failure value, the three PM policies are presented with their corresponding optimal values, namely, C_{pm}, l, r_{max} and the warning limit coefficient w_l. Tables 3.5, 3.6 and 3.7 show all results of all policies obtained for the three mean times to failure, respectively.

When the mean time to failure is small (high failure rate frequency), one should expect that the quality cost would be high and performing PM tasks would be justifiable even for relatively high costs of PMs. Table 3.5 shows that performing PM would always yields reductions in both quality and overall costs for all policies even when C_{pm}=\$450. However, when the mean times to failure are larger (lower failure rates frequency), high costs of PMs might not be justifiable. This is because for higher values of mean times to failure the quality control costs will be rather smaller and high PM costs will not be compensated for by the reductions in the quality control costs. Table 3.6 shows that performing PM at C_{pm}=\$450 is not justifiable for all policies and Table 3.7 shows that performing PM even at C_{pm}=\$350 is not justifiable for all policies and the optimal policy is not to perform PM ($C_{pm} = 0$).

Examples for all three developed PM policies under Weibull Shock Model																	
Policy	C^0_{pm}	C_{pm}	l	r_{max}	w_l	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
1	0	0	-	-	-	23	46	2.46	1.60	0.0138	0.8237	2.8	557.9	15.6	0.0	4227	580.2
	250	250	1	-	-	16	74	2.50	1.37	0.0125	0.9643	25.0	281.0	5.7	150.6	35309	438.1
	350	350	1	-	-	14	78	2.50	1.45	0.0124	0.9723	23.3	292.9	6.7	195.2	32999	495.6
	450	450	2	-	-	12	68	2.51	1.41	0.0119	0.9461	12.9	350.9	18.5	172.4	18587	543.3
2	0	0	-	-	-	23	46	2.46	1.60	0.0138	0.8237	2.8	557.9	15.6	0.0	4227	580.2
	250	250	-	0.2	-	16	74	2.50	1.37	0.0125	0.9644	25.0	280.9	5.7	150.7	35259	438.0
	350	350	-	0.3	-	14	78	2.50	1.45	0.0124	0.9723	23.3	292.9	6.7	195.2	32979	495.5
	450	450	-	0.5	-	12	68	2.51	1.41	0.0119	0.9461	12.9	350.9	18.5	172.4	18587	543.3
3	0	0	-	-	-	23	46	2.46	1.60	0.0138	0.8237	2.8	557.9	15.6	0.0	4227	580.2
	250	250	-	-	0.24	18	53	2.42	1.26	0.0153	0.8880	21.5	303.3	4.3	155.7	30308	464.2
	350	350	-	-	0.24	15	54	2.41	1.38	0.0160	0.8972	19.9	323.0	5.5	192.7	28255	522.2
	450	450	-	-	0.24	13	52	2.40	1.45	0.0165	0.8863	18.4	337.1	8.4	229.1	26195	575.7

Table 3.5: Effect of Weibull scale and shape parameters; $\lambda = 0.1, \nu = 2.25$;
Mean time to failure=2.46

Examples for all three developed PM policies under Weibull Shock Model

Policy	C_{pm}^0	C_{pm}	l	r_{max}	w_l	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
1	0	0	-	-	-	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0.0	6502	429.7
	250	250	1	-	-	19	68	2.52	1.48	0.0116	0.9451	29.9	203.1	8.2	151.0	42185	362.9
	350	350	2	-	-	12	64	2.52	1.62	0.0116	0.9301	14.6	264.3	29.9	117.3	21199	413.0
	450	0	-	-	-	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0.0	6502	429.7
2	0	0	-	-	-	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0.0	6502	429.7
	250	250	-	0.1	-	19	68	2.52	1.48	0.0116	0.9451	29.9	203.1	8.2	151.0	42185	362.9
	350	350	-	0.2	-	12	64	2.52	1.62	0.0116	0.9302	14.6	264.4	30.0	117.2	21209	413.0
	450	0	-	-	-	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0.0	6502	429.7
3	0	0	-	-	-	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0.0	6502	429.7
	250	250	-	-	0.25	22	52	2.46	1.43	0.0139	0.8743	27.6	223.3	7.4	149.4	38946	380.9
	350	350	-	-	0.24	13	56	2.43	1.65	0.0151	0.9054	19.6	249.6	19.4	166.8	27946	436.8
	450	0	-	-	-	26	49	2.51	1.88	0.0120	0.8387	4.3	400.1	25.4	0.0	6502	429.7

Table 3.6: Effect of Weibull scale and shape parameters; $\lambda = 0.05, \nu = 2.0$;
Mean time to failure=3.96

Examples for all three developed PM policies under Weibull Shock Model

Policy	C_{pm}^0	C_{pm}	l	r_{max}	w_l	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
1	0	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
	250	250	2	-	-	8	60	2.55	2.49	0.0109	0.9079	14.3	182.1	47.4	50.4	21066	281.2
	350	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
	450	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
2	0	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
	250	250	-	0.1	-	6	62	2.52	3.06	0.0118	0.9218	10.4	219.3	43.9	22.9	15410	288.0
	350	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
	450	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
3	0	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
	250	250	-	-	0.25	10	52	2.49	2.19	0.0129	0.8684	18.8	159.4	39.4	91.5	27210	291.4
	350	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7
	450	0	-	-	-	14	50	2.54	3.01	0.0110	0.8399	8.3	240.6	45.8	0.0	12417	288.7

Table 3.7: Effect of Weibull scale and shape parameters; $\lambda = 0.01, \nu = 2.0$;
Mean time to failure=8.86

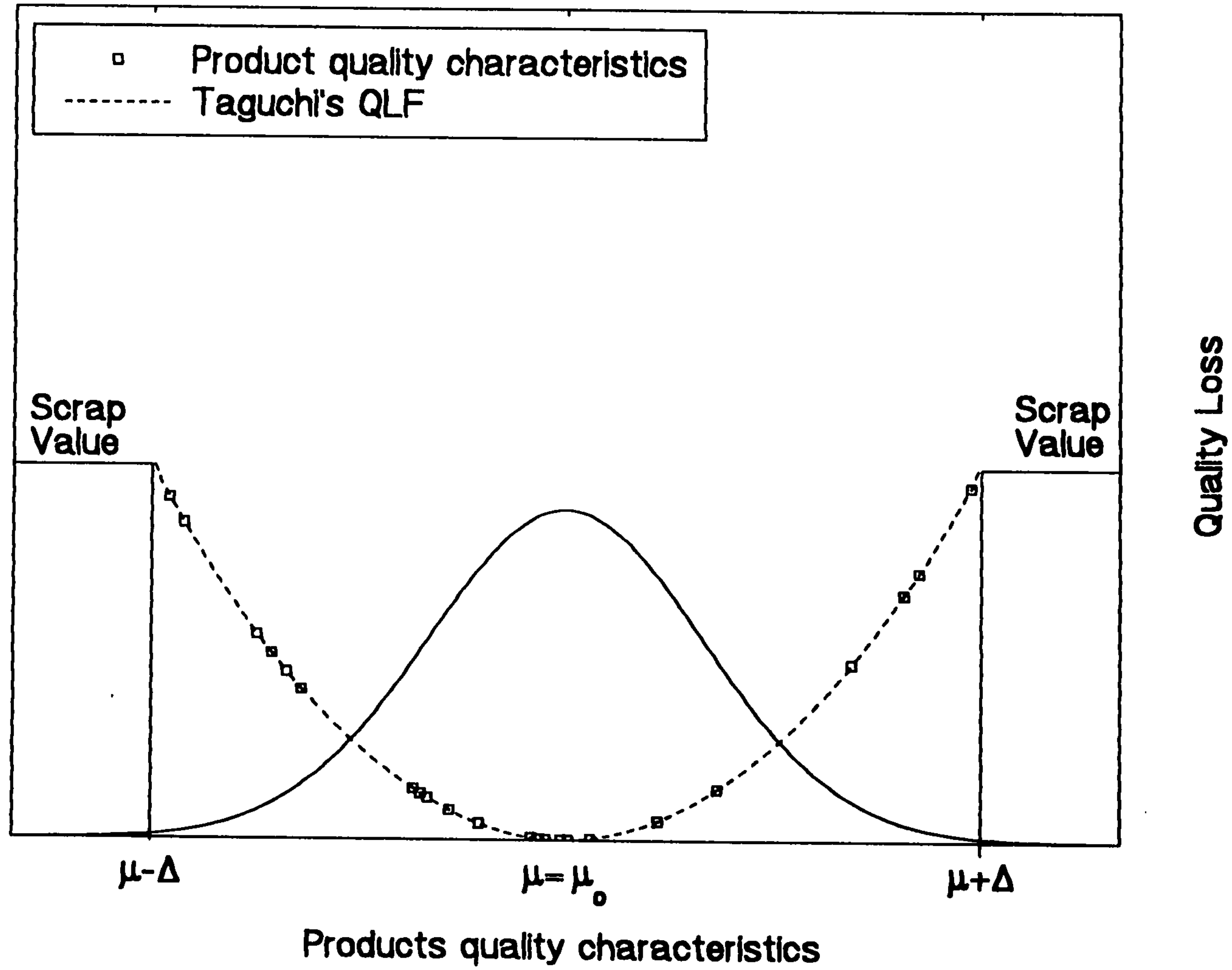
Chapter 4

INTEGRATED PRODUCTION, QUALITY AND MAINTENANCE MODEL USING TAGUCHI'S QUADRATIC LOSS FUNCTION

4.1 Introduction

Traditionally, quality loss is considered as the cost incurred by products having quality characteristics falling outside the specification limits. All products falling within the specification limits are considered to be having the same quality irrespective of their quality characteristics deviations from the target value. However, products with quality characteristics closed to the target value have less quality cost than those closed to the upper or lower specification limits. Taguchi defines quality loss as the "loss to society caused by the product after it is shipped out". He proposed a quadratic loss function (QLF) to estimate the quality loss of a product when its quality characteristics deviates from the target value.

Figure 4.1: Taguchi's QLF and quality losses



When the deviation of a product's quality characteristics from the target value is larger or equal to the tolerance value Δ, the quality loss would be equal to the scrap or rework cost. Figure 4.1 shows the Taguchi's QLF, specification limits and the quality loss associated with each product's quality characteristics deviation from the target value.

Until recently, the Taguchi's QLF has not been used in the economic design of control charts. Although there were few attempts to do so, Taguchi's QLF was not directly imposed on the economic design of control chart parameters, especially n, the sample size and k, the control limit coefficient. Most recently, Ben Daya and Duffuaa [7] have attempted to embed Taguchi's QLF into the economic design of control charts for processes with exponentially distributed shift to failure. They have directly imposed Taguchi's QLF on the parameters of the control chart to obtain the expected costs for in control and out of control states. Also, they have used the basic relationships in Duncan's model to develop the complete model for the economic design of control charts.

In this chapter, we will develop an integrated production, quality and maintenance model using Taguchi's QLF. This will be done by using Taguchi's QLF to evaluate the quality losses for processes with in control periods following general distribution with increasing failure rates. The complete integrated model using Taguchi's QLF will be developed as follows: (1) by extending Ben Daya and Duffuaa's model to processes having general distributed shifts to failures, (2) introducing preventive maintenance concept into the economic design of control charts and finally (3) using (1) and (2) to develop the complete integrated production, quality and maintenance model.

4.2 Notation and Assumptions:

The notation and assumptions that will be used throughout this chapter will be the same as those stated in section 3.2 in addition to the following which are needed for the development of the in control and out of control cost models using Taguchi's QLF:

4.2.1 Notation

μ:	process mean
σ:	process standart deviation
δ:	magnitude of the shift in the mean
Δ:	acceptable deviation of the quality characteristic from its target value
$\phi()$:	Standard normal probability density function
$\Phi()$:	Standard normal cumulative function
A:	cost of rework or scrap of a unit with quality characteristic deviating by Δ or more from the target

4.2.2 Assumptions

1. The production process starts in the in control state with mean μ and standard deviation σ.

2. During the in control period, the process is centered at $\mu = \mu_0$, where μ_0 is the target value.

3. The out of control state is encountered when an assignable cause of magnitude δ results in a shift in the process mean from μ to $\mu \pm \delta\sigma$. The shifts of the process mean to $\mu + \delta\sigma$ or $\mu - \delta\sigma$ are assumed to occur with equal probability of 0.5 each.

4. The process standard deviation σ is assumed to remain the same when the process encounters an out of control state.

5. The process is monitored by an $\bar{x}$-control chart with center line μ and control limits $\mu \pm k\sigma/\sqrt{n}$.

6. The occurrence of the assignable cause follows a general probability distribution.

4.3 Economic Design of Control Charts Using Taguchi's QLF

In this section, we first present Ben Daya and Duffuaa's [7] model which was done for processes with exponential distributed shifts to failures. All of their complete model derivations will be also presented. At the end of this section, we will develop our model which extends Ben Daya and Duffuaa's model to processes having general shifts to failures distribution. The generalized model will be used to develop the rest of the models presented in this chapter.

4.3.1 Exponentially Distributed Shifts to Failures Model

Ben Daya and Duffuaa have derived separate expressions for both the in control and out of control costs. In this section, we re-derive these two cost expressions so that this thesis is self contained:

Using Taguchi's quadratic loss function, it is clear from Figure 4.2 that the in control cost is given by:

$$L_{in}(y_1, n, k) = \frac{A}{\Delta^2}\left\{\int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} (y_1-\mu)^2 f(y_1)dy_1\right\} \tag{4.1}$$

where Y_1 is a random variable denoting sample means of the quality characteristic and $f(y_1)$ is its normal density function with mean μ and standard deviation $\sigma/\sqrt{n}$. It is obvious from equation 4.1 that any deviation (regardless of its value) from the mean will incur a loss. Similarly, it is also clear from Figure 4.2 that the out of control cost is given by:

$$\begin{aligned} L_{out}(y_2, n, k) &= \frac{A}{\Delta^2}\left\{\int_{\mu-\Delta}^{\mu-k\sigma/\sqrt{n}} (y_2-\mu)^2 f(y_2)dy_2 + \int_{\mu+k\sigma/\sqrt{n}}^{\mu+\Delta} (y_2-\mu)^2 f(y_2)dy_2\right\} \\ &+ A\left\{\int_{-\infty}^{\mu-\Delta} f(y_2)dy_2 + \int_{\mu+\Delta}^{\infty} f(y_2)dy_2\right\} \end{aligned} \tag{4.2}$$

where Y_2 is a random variable denoting sample means of the quality characteristic and $f(y_2)$ is its normal density function with mean $\mu \pm \delta\sigma$ and standard deviation $\sigma/\sqrt{n}$.

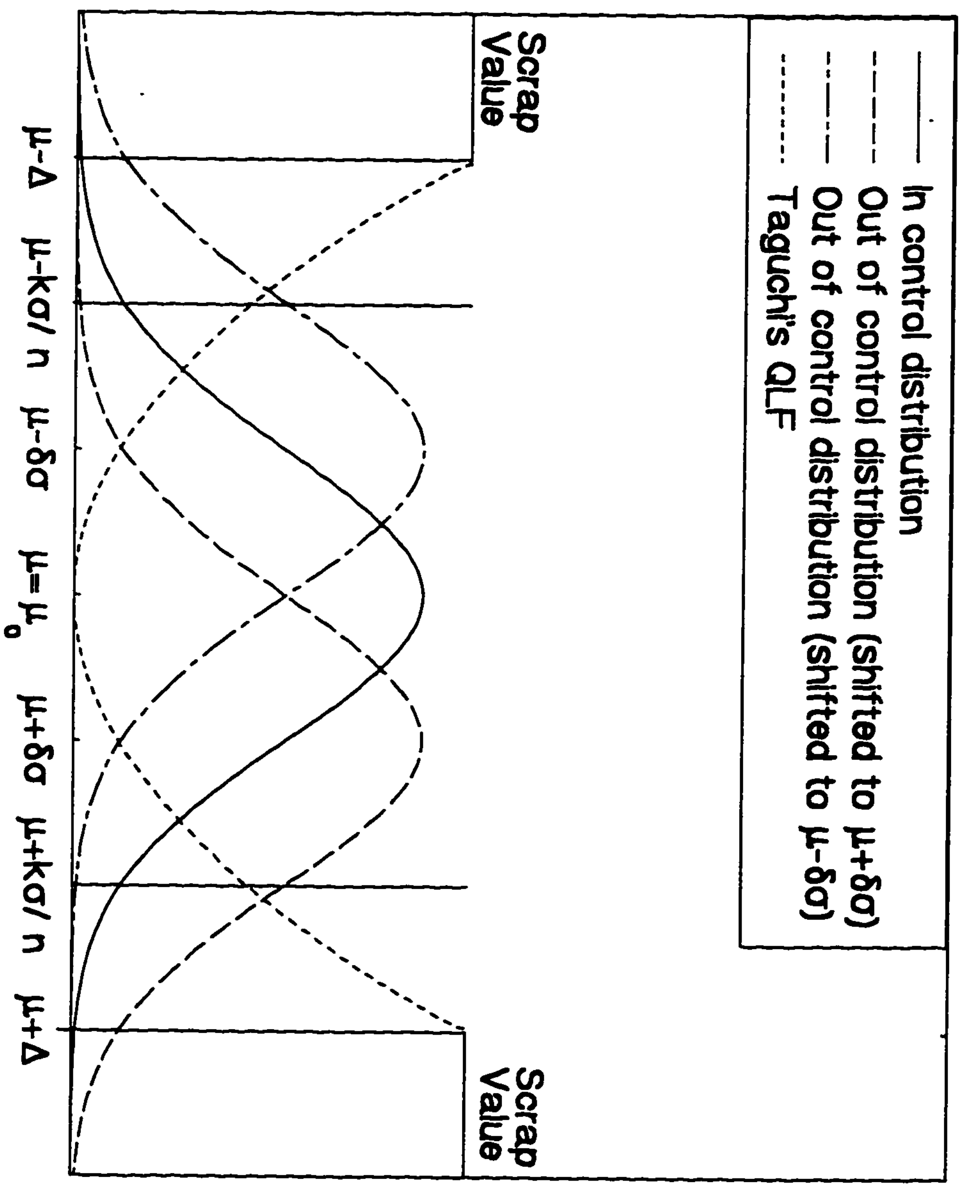


Figure 4.2: Taguchi's QLF, in control and out of control distributions

The values of $L_{in}(y_1, n, k)$ and $L_{out}(y_2, n, k)$ are given by the following two lemmas:

Lemma 4.1 *Assume that the process mean is on the target value, that is $\mu = \mu_0$. Then the in control cost is given by:*

$$L_{in}(k) = \frac{A\sigma^2}{n\Delta^2}\left\{1 - \frac{2k}{\sqrt{2\pi}}e^{-k^2/2} - 2\Phi(-k)\right\} \quad (4.3)$$

Proof: From equation (4.1), we have

$$L_{in} = \frac{A}{\Delta^2}\left\{\int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} (y_1 - \mu)^2 f(y_1) dy_1\right\}$$

transforming the above equation to standard normal, we get

$$L_{in} = \frac{A\sigma^2}{n\Delta^2}\left\{\int_{-k}^{k} z^2\phi(z)dz\right\}$$

integrating by parts,

$$L_{in} = \frac{A\sigma^2}{n\Delta^2}\left\{\left[-\frac{z}{\sqrt{2\pi}}e^{-z^2/2}\right]_{-k}^{k} + \int_{-k}^{k}\phi(z)dz\right\}$$

or simply,

$$L_{in} = \frac{A\sigma^2}{n\Delta^2}\left\{-\frac{k}{\sqrt{2\pi}}e^{-k^2/2} - \frac{k}{\sqrt{2\pi}}e^{-k^2/2} + \Phi(k) - \Phi(-k)\right\}.$$

Further simplification of the above expression yields equation (4.1).

Lemma 4.2 *Assume that during the out of control state the process mean shifts to $\mu \pm \delta\sigma$ while the standard deviation remains at σ. Then the out of control cost is given by:*

$$\begin{aligned}
L_{out}(y_2, n, k) &= \frac{A\sigma^2}{n\Delta^2}\left\{\frac{1}{\sqrt{2\pi}}\left(z_1 e^{-z_1^2/2} - z_2 e^{-z_2^2/2} + z_3 e^{-z_3^2/2} - z_4 e^{-z_4^2/2}\right)\right. \\
&+ (1 + n\delta^2)(\Phi(z_2) - \Phi(z_1) + \Phi(z_4) - \Phi(z_3)) \\
&+ \left.\frac{2\delta\sqrt{n}}{\sqrt{2\pi}}\left(e^{-z_1^2/2} - e^{-z_2^2/2} + e^{-z_3^2/2} - e^{-z_4^2/2}\right)\right\} \\
&+ A\left\{1 + \Phi(z_1) - \Phi(z_3)\right\} \qquad (4.4)
\end{aligned}$$

where $z_1 = -\Delta\sqrt{n}/\sigma - \delta\sqrt{n}$, $z_2 = -k - \delta\sqrt{n}$, $z_3 = \Delta\sqrt{n}/\sigma - \delta\sqrt{n}$ and $z_4 = k - \delta\sqrt{n}$.

Proof: Form equation (4.2), we have:

$$\begin{aligned}
L_{out}(y_2, n, k) &= \frac{A}{\Delta^2}\left\{\int_{\mu-\Delta}^{\mu-k\sigma/\sqrt{n}} (y_2 - \mu)^2 f(y_2)dy_2 + \int_{\mu+k\sigma/\sqrt{n}}^{\mu+\Delta} (y_2 - \mu)^2 f(y_2)dy_2\right\} \\
&+ A\left\{\int_{-\infty}^{\mu-\Delta} f(y_2)dy_2 + \int_{\mu+\Delta}^{\infty} f(y_2)dy_2\right\}
\end{aligned}$$

adding and subtracting $\delta\sigma$ from the mean squared deviations, we get

$$L_{out} = \frac{A}{\Delta^2}\left\{\int_{\mu-\Delta}^{\mu-k\sigma/\sqrt{n}} (y_2 - \mu - \delta\sigma + \delta\sigma)^2 f(y_2)dy_2 + \int_{\mu+k\sigma/\sqrt{n}}^{\mu+\Delta} (y_2 - \mu - \delta\sigma + \delta\sigma)^2 f(y_2)dy_2\right\}$$

$$+ A\left\{\int_{-\infty}^{\mu-\Delta} f(y_2)dy_2 + \int_{\mu+\Delta}^{\infty} f(y_2)dy_2\right\}$$

or,

$$L_{out} = \frac{A}{\Delta^2}\left\{\int_{\mu-\Delta}^{\mu-k\sigma/\sqrt{n}} \left((y_2 - \mu - \delta\sigma)^2 + 2\delta\sigma(y_2 - \mu - \delta\sigma) + \delta^2\sigma^2\right) f(y_2)dy_2\right.$$

$$
\begin{aligned}
&+ \int\limits_{\mu+k\sigma/\sqrt{n}}^{\mu+\Delta} \left((y_2-\mu-\delta\sigma)^2 + 2\delta\sigma(y_2-\mu-\delta\sigma) + \delta^2\sigma^2\right) f(y_2)dy_2 \Bigg\} \\
&+ A\left\{ \int\limits_{-\infty}^{\mu-\Delta} f(y_2)dy_2 + \int\limits_{\mu+\Delta}^{\infty} f(y_2)dy_2 \right\}
\end{aligned}
$$

transforming the above equation to standard normal, we get

$$
\begin{aligned}
L_{out} &= \frac{A}{\Delta^2}\left\{ \int\limits_{-\Delta\sqrt{n}/\sigma-\delta\sqrt{n}}^{-k-\delta\sqrt{n}} \left(\frac{\sigma^2}{n}z^2\phi(z) + \frac{2\delta\sigma^2}{\sqrt{n}}z\phi(z) + \delta^2\sigma^2\phi(z)\right) dz \right. \\
&+ \left. \int\limits_{\Delta\sqrt{n}/\sigma-\delta\sqrt{n}}^{k-\delta\sqrt{n}} \left(\frac{\sigma^2}{n}z^2\phi(z) + \frac{2\delta\sigma^2}{\sqrt{n}}z\phi(z) + \delta^2\sigma^2\phi(z)\right) dz \right\} \\
&+ A\left\{ \int\limits_{-\infty}^{-\Delta\sqrt{n}/\sigma-\delta\sqrt{n}} \phi(z)dz + \int\limits_{\Delta\sqrt{n}/\sigma-\delta\sqrt{n}}^{\infty} \phi(z)dz \right\}
\end{aligned}
$$

letting $z_1 = -\Delta\sqrt{n}/\sigma - \delta\sqrt{n}$, $z_2 = -k - \delta\sqrt{n}$, $z_3 = \Delta\sqrt{n}/\sigma - \delta\sqrt{n}$ and $z_4 = k - \delta\sqrt{n}$ and performing all of the above integrations, we get

$$
\begin{aligned}
L_{out} &= \frac{A}{\Delta^2}\left\{ \left[\frac{\sigma^2}{n}\frac{-z}{\sqrt{2\pi}}e^{-z^2/2} + \frac{\sigma^2}{n}\Phi(z) + \frac{-2\delta\sigma^2}{\sqrt{n}\sqrt{2\pi}}e^{-z^2/2} + \delta^2\sigma^2\Phi(z)\right]_{z_1}^{z_2} \right. \\
&+ \left. \left[\frac{\sigma^2}{n}\frac{-z}{\sqrt{2\pi}}e^{-z^2/2} + \frac{\sigma^2}{n}\Phi(z) + \frac{-2\delta\sigma^2}{\sqrt{n}\sqrt{2\pi}}e^{-z^2/2} + \delta^2\sigma^2\Phi(z)\right]_{z_3}^{z_4} \right\} \\
&+ A\left\{[\Phi(z)]_{-\infty}^{z_1} + [\Phi(z)]_{z_3}^{\infty}\right\}
\end{aligned}
$$

or simply,

$$
\begin{aligned}
L_{out} &= \frac{A\sigma^2}{n\Delta^2}\left\{ \left[\frac{-z}{\sqrt{2\pi}}e^{-z^2/2} + \Phi(z)(1+n\delta^2) + \frac{-2\delta\sqrt{n}}{\sqrt{2\pi}}e^{-z^2/2}\right]_{z_1}^{z_2} \right. \\
&+ \left. \left[\frac{-z}{\sqrt{2\pi}}e^{-z^2/2} + \Phi(z)(1+n\delta^2) + \frac{-2\delta\sqrt{n}}{\sqrt{2\pi}}e^{-z^2/2}\right]_{z_3}^{z_4} \right\} \\
&+ A\left\{[\Phi(z)]_{-\infty}^{z_1} + [\Phi(z)]_{z_3}^{\infty}\right\}
\end{aligned}
$$

evaluating the above expression at the integration limits, we get

$$
\begin{aligned}
L_{out} &= \frac{A\sigma^2}{n\Delta^2} \\
&\cdot \left\{ \left(\frac{-z_2}{\sqrt{2\pi}} e^{-z_2^2/2} + \Phi(z_2)(1+n\delta^2) + \frac{-2\delta\sqrt{n}}{\sqrt{2\pi}} e^{-z_2^2/2} \right) \right. \\
&- \left(\frac{-z_1}{\sqrt{2\pi}} e^{-z_1^2/2} + \Phi(z_1)(1+n\delta^2) + \frac{-2\delta\sqrt{n}}{\sqrt{2\pi}} e^{-z_1^2/2} \right) \\
&+ \left(\frac{-z_4}{\sqrt{2\pi}} e^{-z_4^2/2} + \Phi(z_4)(1+n\delta^2) + \frac{-2\delta\sqrt{n}}{\sqrt{2\pi}} e^{-z_4^2/2} \right) \\
&- \left. \left(\frac{-z_3}{\sqrt{2\pi}} e^{-z_3^2/2} + \Phi(z_3)(1+n\delta^2) + \frac{-2\delta\sqrt{n}}{\sqrt{2\pi}} e^{-z_3^2/2} \right) \right\} \\
&+ A\left\{1 + \Phi(z_1) - \Phi(z_3)\right\}.
\end{aligned}
$$

Further simplification of the above expression yields equation (4.4).

Note that both $L_{in}(k)$ and $L_{out}(k)$ represent the losses during the in control and out of control states per product item, respectively.

Ben Daya and Duffuaa have used the basic relationships of Duncan's model to develop the complete cost model for the expected total cost as follows:

$$
ETC(n,k,h) = \frac{(a+bn)E(T)/h + C_f \frac{\alpha}{h\lambda} + C_a + \frac{1}{\lambda} P L_{in}(n,k) + \left(E(T) - \frac{1}{\lambda}\right) P L_{out}(n,k)}{E(T)}
$$

where $E(T)$ is the expected length of the production cycle, P is the production rate and is λ the exponential parameter governing the in control period.

4.3.2 Generally Distributed Shifts to Failures Model

The generalized expected total cost model can be developed simply by modifying the expected quality control cost model (equation 3.2) developed in chapter 3 as follows:

$$\begin{aligned}
E(QC) &= (a+bn)\left\{1+\sum_{j=1}^{m-2}\bar{F}(y_j)+\beta\sum_{j=1}^{m-2}(F(y_j)-F(y_{j-1}))\right. \\
&\quad \cdot \left.\left[(1-\beta)\sum_{i=1}^{m-j-1} i\beta^{i-1}+(m-j-1)\beta^{m-j-1}\right]\right\} \\
&+ (C'_{in}-C'_{out})\int_0^{y_j} tf(t)dt+(C'_{out}-C'_{in})\sum_{j=1}^{m} y_j(F(y_j)-F(y_{j-1})) \\
&+ C'_{in}\sum_{j=1}^{m} h_j\bar{F}(y_{j-1})+C_a \\
&+ C'_{out}\beta\left[\sum_{j=1}^{m-1}(F(y_j)-F(y_{j-1}))\sum_{i=j+1}^{m} h_i\beta^{i-j-1}\right] \\
&+ \alpha C_f\sum_{j=1}^{m-1}\bar{F}(y_j)-\bar{F}(y_m)L(y_m). \qquad (4.5)
\end{aligned}$$

where C'_{in} and C'_{out} are given by:

$$C'_{in} = PL_{in}(n,k) \qquad (4.6)$$

and

$$C'_{out} = PL_{out}(n,k) \qquad (4.7)$$

Note that in chapter 3, these two costs were constants.

Hence, the expected total cost per unit time is given by:

$$ETC = \frac{E(QC)}{E(T)}$$

where $E(T)$ is the expected total length of production cycle and is given by equation (3.1).

4.4 Maintenance and Quality Model Using Taguchi's QLF

In this section we will develop an integrated quality and maintenance model. This will be done by introducing the preventive maintenance concept into the expected quality control and the expected total cost models developed in the previous section. The expected quality control cost model will be the same as equation 3.2 developed in chapter 3:

$$\begin{aligned} E(QC) &= (a+bn)\left\{1+\sum_{j=1}^{m-2}\bar{F}(w_j)+\beta\sum_{j=1}^{m-2}(F(y_j)-F(w_{j-1}))\right. \\ &\quad \cdot \left.\left[(1-\beta)\sum_{i=1}^{m-j-1} i\beta^{i-1}+(m-j-1)\beta^{m-j-1}\right]\right\} \\ &+ (C'_{in}-C'_{out})\sum_{j=1}^{m}\int_{w_{j-1}}^{y_j} tf(t)dt+(C'_{out}-C'_{in})\sum_{j=1}^{m} y_j(F(y_j)-F(w_{j-1})) \\ &+ C'_{in}\sum_{j=1}^{m} h_j\bar{F}(w_{j-1}) \\ &+ C_a[\sum_{j=1}^{m}(F(y_j)-F(w_{j-1}))+\bar{F}(y_m)] \\ &+ C'_{out}\beta\left[\sum_{j=1}^{m-1}(F(y_j)-F(w_{j-1}))\sum_{i=j+1}^{m} h_i\beta^{i-j-1}\right] \\ &+ \alpha C_f\sum_{j=1}^{m-1}\bar{F}(w_j)-\bar{F}(y_m)L(t_m). \end{aligned} \qquad (4.8)$$

where C'_{in} and C'_{out} are given by (4.6) and (4.7), respectively.

Assuming that the preventive maintenance tasks are performed according to Policy 1, the expected total cost will be as follows:

$$ETC = \frac{E(QC) + E(PM)}{E(T)}$$

where $E(PM)$ is given by (3.8).

4.5 Production, Quality and Maintenance Model Using Taguchi's QLF

This section presents the complete integrated production, quality and maintenance model using Taguchi's quadratic loss function. The mathematical derivations of the complete model is exactly as in chapter 3 and it will not be reproduced here again.

Assuming that the preventive maintenance tasks are performed according to Policy 1 where l is set to 1, the complete integrated production, quality and maintenance model is given as follows:

$$ETC = \frac{S_0 + E(HC) + E(QC) + E(PM)}{E(T)}$$

where S_0 is the set-up cost for each production cycle and $E(HC)$ is the expected inventory holding cost per inventory cycle and is given by (3.3).

The solution procedure discussed in section 3.8 will be still applicable for the above three models.

4.6 Computational Results

In this section, we present a preliminary analysis to see the effects of the scrap value A, process standard deviation σ, tolerance limit Δ, and PM level to illustrate some important aspects. We recognize that sensitivity analysis dealing with one parameter at a time may not reveal the interactions between those parameters. A more elaborate sensitivity analysis can be carried out through a proper design of experiment models. However, this is beyond the scope of this thesis.

In all examples, the process shift mechanism is assumed to follow a Weibull distribution with parameters $\lambda = 0.05$ and $\nu = 2$. The following data will be used as the basis for all examples unless specified otherwise: $Z_1^{max} = 0.1$, $Z_2 = 1.0$, $a = \$3.0$, $b = \$1.5$, $C_f = \$400$, $C_a = \$650$, $\delta = 0.5$, $L(t_m) = 1100e^{-t_m}$, $D = 1400$ units, $P = 1400$ units, $C_h = \$0.5$, $S_o = \$20$, $A = \$100$, $\sigma/\Delta = 0.1$ and $\eta = 0.99$.

4.6.1 Quality Cost Model

In this subsection, we present the effects of the scrap value A, process standard deviation σ and tolerance limit Δ. Tables 4.1 and 4.2 show the results of the expected total cost of the quality control cost model obtained for different values of the scrap value A and the process capability C_p, respectively.

A	m	n	k	h_1	α	$1-\beta$	ETC
100	2	60	1.84	3.43	0.0656	0.9787	216.0
300	2	82	1.95	2.13	0.0515	0.9951	305.1
500	2	91	1.85	1.74	0.0648	0.9983	357.5
1000	2	103	1.64	1.33	0.1001	0.9997	441.4

Table 4.1: Effect of the scrap value A on the quality control cost

σ/Δ	C_p	m	n	k	h_1	α	$1-\beta$	ETC
0.100	2.34	2	60	1.84	3.43	0.0656	0.9787	216.0
0.115	1.67	2	66	1.92	3.01	0.0548	0.9839	236.1
0.130	1.28	2	71	1.96	2.70	0.0504	0.9880	255.0
0.160	1.04	2	79	1.96	2.27	0.0498	0.9935	290.4

Table 4.2: Effect of the process capability C_p on the quality control cost

It is clear from the above tables that both of A and C_p have effects on both of the sampling intervals and the expected total cost. Table 4.1 demonstrates that the higher the scrap value is the more frequent the process should be inspected. This argument makes sense because at high values of A we expect that more quality control cost will be incurred during long intervals and therefore the process should not run for long time without frequent inspections. Also, Table 4.2 demonstrates that the less capable the process is the more frequent the process should be inspected. Again, this argument is valid too because when the process is less capable we expect that small process mean deviations from the target mean will lead to high percentages of items falling outside the specification tolerance and therefore less frequent inspections might be very costly. On the other hand, if the process is quite capable then small deviations from the mean will not lead to high percentages of non-conforming items as if the process is less capable and therefore frequent process inspections might be redundant task.

4.6.2 Integrated Quality and Maintenance Model

In this subsection, we present the effect of PM on the quality control cost. The following table shows two schemes results of the integrated quality and maintenance model obtained for different process capabilities without and with PM at $C^0_{pm} = C_{pm} = 200$.

Scheme	C_p	m	n	k	h_1	α	$1-\beta$	QC	PM	ETC
Without PM	2.34	2	60	1.84	3.43	0.0656	0.9787	216.0	0.0	216.0
	1.67	2	66	1.92	3.01	0.0548	0.9839	236.1	0.0	236.1
	1.28	2	71	1.96	2.70	0.0504	0.9880	255.0	0.0	255.0
	1.04	2	79	1.96	2.27	0.0498	0.9935	294.1	0.0	290.4
With PM	2.34	14	64	1.97	3.13	0.0486	0.9787	132.3	61.5	193.8
	1.67	14	65	1.92	2.80	0.0549	0.9826	149.3	68.5	217.7
	1.28	13	66	1.86	2.54	0.0629	0.9862	167.0	74.0	241.0
	1.04	12	68	1.73	2.18	0.0837	0.9917	202.0	84.2	286.3

Table 4.3: Effect of PM for different process capabilities

We can notice from Table 4.3 how PM affects both of the quality control and the expected total cost. For all of the four different process capabilities, both of the quality control and the expected total cost were reduced when PM was performed. However, if C^0_{pm} is too high to the point where PM costs are not compensated for by the reduction in the quality control costs then the same discussion provided in section 3.9.3 will be also true.

4.6.3 Integrated Production, Quality and Maintenance Model

This subsection provides some illustrative examples of the complete integrated production, quality and maintenance model. The following tables show the results obtained by performing PM at $C_{pm}^0 = C_{pm} = 200$ for different values of the scrap value A and the process capability C_p, respectively.

A	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
100	11	43	2.15	1.42	0.0312	0.8695	15.3	127.9	37.6	134.4	21360	301.1
500	11	63	1.48	1.42	0.1398	0.9936	15.1	263.6	36.6	132.4	21169	433.9
1000	12	62	1.05	1.39	0.2959	0.9981	16.2	375.4	34.9	135.5	22734	547.0

Table 4.4: Effect of the scrap value A

C_p	m	n	k	h_1	α	$1-\beta$	t_m	QC	HC	PM	Q^*	ETC
2.34	11	43	2.15	1.42	0.0312	0.8695	15.3	127.9	37.6	134.4	21360	301.1
1.67	11	47	2.09	1.42	0.0371	0.9103	15.2	142.8	37.6	133.8	21282	315.5
1.28	11	51	2.02	1.42	0.0434	0.9396	15.2	158.2	37.5	133.3	21234	330.4
1.04	11	57	1.86	1.42	0.0628	0.9722	15.1	190.3	37.5	132.7	21203	361.8

Table 4.5: Effect of the process capability C_p

We can notice from Tables 4.4 and 4.5 that both of the scrap value and the processes capability have direct effect on the quality control cost. The higher the scrap value is and the less capable the process is the more quality cost will be incurred. This is quite clear and has been already discussed in section 4.6.1. Also, one can notice that both of the preventive maintenance cost (PM) and the inventory holding cost (HC) were neither sensitive to the scrap value nor to the process capability.

The effects of PM level, frequency of PM and mean time to failure discussed in chapter 3 are also valid for the above examples.

Chapter 5

CONCLUSION

5.1 Summary

In this thesis we developed different integrated production, quality and maintenance models. Two techniques were used to model quality control cost in the developed models, namely, the classical quality control method and the Taguchi's quadratic loss function. In chapter 3, the classical quality control method was used to develop three integrated production, quality and maintenance models under various preventive maintenance policies. These policies are:

1. Perform PM at every inspection interval

2. Perform PM at every l inspection intervals

3. Perform PM if the process shift rate reaches a preset threshold

4. Perform PM if two consecutive observations in the warning zone

Three different FORTRAN programs have been developed to study and illustrate some important aspects of the developed integrated models. For the first policy, two different relationships between the PM level and the improvement in the age of the equipment have been used to illustrate the effect of PM level on the quality control cost. It was found that performing PM will always yield reductions in the quality control cost. Also, it was concluded that if the cost of performing PM is high to the point where it is not compensated for by the reductions in the quality control cost then performing PM will not be justifiable. Other influences imposed by other factors such as the frequency of PM and the mean time to failure have been also discussed and presented. Section 3.8.2 demonstrated that at low costs of PM, the optimal policy would be to perform PM at every inspection interval. This was reflected in Tables 3.3 and 3.4 by the optimal values of the PM frequency ($l = 1$) and the failure threshold setting ($r_{max} = 0.1$) at the low PM costs. Tables 3.5, 3.6 and 3.7 also demonstrated how the mean time to failure of the process can affect the optimal C_{pm} value for all PM policies.

In chapter 4, three different models have been developed using Taguchi's quadratic loss function. The first model has been developed by extending the economic design of control charts using Taguchi's quadratic loss function model developed by Ben Daya and Duffuaa to processes having general time to failure distribution. Imperfect preventive maintenance concept has been introduced to the generalized model to develop an integrated quality and maintenance model. Production has been also incorporated into the second model to develop the complete production, quality and maintenance model. Computational results of chapter 4 have presented and discussed the effects of the scrap value, the process standard deviation and the specification tolerance on the inspection intervals, quality control cost and the expected total cost. The effect of PM level has been also investigated.

5.2 Contributions

The major contributions of this thesis are:

1. Model development for the three PM policies:

    a) PM at every l sampling interval

    b) PM if the process shift rate reaches a preset threshold

    c) PM if two consecutive observations in the warning zone.

2. Extension of Ben Daya and Duffuaa's economic design of control charts using Taguchi's quadratic loss function model to processes having general shift to failure distribution.

3. Development of an integrated production, quality and maintenance model including Taguchi's quadratic loss function.

4. Incorporation of imperfectness factor in the process age improvement.

5. Studying both linear and non-linear relationships between PM level and process age improvement.

6. Development of computer software for all models.

5.3 Future Research

In all of the above models, it was implicitly assumed that the quality of the production items is perfect. However, in practice this might not be true for some kind of production items. So, it might be a worthwhile investigating production processes with items that are subjected to deterioration.

Another interesting area would be also investigating the impact of errors that may occur during performing preventive maintenance activities. As we noticed above, it was assumed that preventive maintenance always yields a reduction in the age of the process. However, this might not be the case if faulty procedures were followed or non-experienced technicians improperly maintained the process.

A third issue of interest is to consider cases for which available resources are limited. This can be done by putting constraints on some of the resources such as the inventory holding space, budget and lot sizes.

Appendix A

Proofs

A.1 Proof of Theorem 3.1

In this appendix, we present the proof of Theorem 1. This proof is provided in Ben Daya's [6] model, but we regenerate it here again so that this thesis is self contained.

As in [6], let $E(T_j), j = 0, 1, 2, ..., m-1$, be the expected residual time in the cycle beyond time t_j given that the process was in control at time t_j, $E(T_0) = E(T)$. Let $E(R_j)$ be the expected residual time in the cycle beyond time t_j given that the process was in the out of control state at time t_j and a true alarm has not been triggered so far. Clearly,

$$E(R_m) - Z_2 = 0$$
$$E(R_{m-1}) - Z_2 = h_m,$$

and for $j = 1, 2, ..., m-2$

$$E(R_j) - Z_2 = h_{j+1} + Z_{1_j} + \beta\{E(R_{j+1}) - Z_2\} = \sum_{i=j+1}^{m} \beta^{i-j-1}(h_i + Z_{1_j}) + \beta^{m-j-1} h_m. \quad \text{(A.1)}$$

Further let $p_0 = 0$ and $p_j (j = 1, 2, ..., m)$ be the conditional probability that the process shifts to the out of control state during the time interval (t_{j-1}, t_j) given that the process was at the in control state at time t_{j-1}. Equivalently,

$$p_j = \frac{\Delta F(t_j)}{\overline{F}(w_{j-1})} = \frac{F(y_j) - F(w_{j-1})}{\overline{F}(w_{j-1})}. \tag{A.2}$$

It can be shown that:

$$\Delta F(t_j) = p_j \Pi_{i=0}^{j-1}(1 - p_i), \text{ and}$$
$$\overline{F}(w_j) = \Pi_{i=1}^{j}(1 - p_i).$$

In order to find the expression of $E(T)$, consider the possible states of the process after the first PM. For each possible state the expected residual time in the cycle, and the associated probabilities are presented in Table (A.1).

State	Probability	Expected Residual Time
In-Control and no alarm	$(1-p_1)(1-\alpha)$	$E(T_1)$
In-Control and a false alarm	$(1-p_1)\alpha$	$E(T_1)$
Out-of-Control but no alarm	$p_1\beta$	$E(R_1)$
Out-of-Control and true alarm	$p_1(1-\beta)$	Z_2

Table A.1: Expected residual time

Consequently,

$$E(T) = h_1 + Z_{1_1} + (1-\beta)Z_2 p_1 + \beta p_1 E(R_1) + (1-p_1)E(T_1).$$

Similarly, for $j = 0, 1, 2, ..., (m-2)$ we have

$$E(T_j) = h_{j+1} + Z_{1_j} + (1-\beta)Z_2 p_{j+1} + \beta p_{j+1} E(R_{j+1}) + (1-p_{j+1})E(T_{j+1}).$$

Note that there is no sampling or charting during the last sampling interval, consequently the expression for $E(T_{m-1})$ is given by:

$$E(T_{m-1}) = h_m + Z_2,$$

therefore,

$$E(T) = \sum_{j=1}^{m} h_j \overline{F}(w_{j-1}) + Z_{1_j} \sum_{j=1}^{m-1} \bar{F}(w_{j-1}) + Z_2 \left[\sum_{j=1}^{m-1} \Delta F(t_j) + \overline{F}(w_{m-1}) \right] \quad (A.3)$$

$$+ \quad \beta \sum_{j=1}^{m-1} \Delta F(t_j)[E(R_j) - Z_2]. \quad (A.4)$$

Simplification of this equation leads to (3.1).

Now, in order to develop the expression of $E(QC)$, let us define $E(QC_j)$; $j = 1, 2, ..., m-2$ to be the expected residual cost beyond time t_j given that the process is at the in control state at time t_j. Further, let τ_j the conditional expected in control duration within the time interval (t_{j-1}, t_j) given that the shift to the out of control state occurred during the sampling interval (t_{j-1}, t_j), i.e.

$$\tau_j = \int_{w_{j-1}}^{y_j} (x - w_{j-1}) f(x) dx / \Delta F(t_j).$$

Also, let N_j be the expected number of sampling and charting conducted after time t_j given that the process is at an out of control state at time t_j. Clearly

$N_{m-1} = 0$
$N_{m-2} = 1$ and for $j = 1, 2, ..., m-3$
$N_j = \sum_{i=1}^{m-1-j} i(1-\beta)\beta^{i-1} + (m-1-j)\beta^{m-1-j}$.

In order to compute an expression for $E(QC)$, we determine the expected residual cost beyond time h_1 for each possible state of the process after the first PM. These values are presented in Table (A.2).

State	Probability	Current cost	Expected residual cost
In-Control and no alarm	$(1-p_1)(1-\alpha)$	$(a+bn)+C_{in}h_1$	$E(QC_1)$
In-Control and a false alarm	$(1-p_1)\alpha$	$(a+bn)+C_f+C_{in}h_1$	$E(QC_1)$
Out-of-Control but no alarm	$p_1\beta$	$(a+bn)+C_{in}\tau_1+C_{out}(h_1-\tau_1)$	$C_a+N_1(a+bn)$ $+C_{out}[E(R_1)-Z_2]$
Out-of-Control and true alarm	$p_1(1-\beta)$	$(a+bn)+C_{in}\tau_1+C_{out}(h_1-\tau_1)$	C_a

Table A.2: Expected residual costs

Similar to $E(T)$, this yields the following recursive system:

$$\begin{aligned}
E(QC) &= a+bn+p_1[C_{in}\tau_1+C_{out}(h_1-\tau_1)]+p_1(1-\beta)C_a \\
&\quad +p_1\beta[C_a+C_{out}\{E(R_1)-Z_2\}+(a+bn)N_1]+(1-p_1)E(QC_1) \\
&\quad +(1-p_1)\alpha C_f+(1-p_1)C_{in}h_1,
\end{aligned}$$

and for $j=1,2,...,m-2$

$$\begin{aligned}
E(QC_j) &= a+bn+p_{j+1}[D_0\tau_{j+1}+C_{out}(h_{j+1}-\tau_{j+1})+(1-\beta)C_a p_{j+1} \\
&\quad +p_{j+1}\beta[C_a+C_{out}\{E(R_{j+1})-Z_2\}+(a+bn)N_{j+1}]+(1-p_{j+1})E(C_{j+1}) \\
&\quad +(1-p_{j+1})\alpha C_f+C_{in}(1-p_{j+1})h_{j+1},
\end{aligned}$$

and

$$
\begin{aligned}
E(QC_{m-1}) &= p_m[C_{in}\tau_m + C_{out}(h_m - \tau_m) + C_a] \\
&\quad +(1-p_m)[C_{in}h_m + C_a - L(t_m)].
\end{aligned}
$$

Equivalently,

$$
\begin{aligned}
E(QC) &= (a+bn)[1 + \sum_{j=1}^{m-2} \Pi_{j=1}^{j}(1-p_i) + \beta \sum_{j=1}^{m-1} p_j N_j \Pi_{i=0}^{j-1}(1-p_i)] \\
&\quad +(C_{in} - C_{out})[\sum_{j=1}^{m} p_j(\tau_j - h_j)\Pi_{i=0}^{j-1}(1-p_i)] \\
&\quad +C_{in}[\sum_{j=1}^{m} h_j \Pi_{i=0}^{j-1}(1-p_i)] \\
&\quad +C_a[\sum_{j=1}^{m} p_j \Pi_{i=0}^{j-1}(1-p_i) + (1-p_m)\Pi_{i=1}^{m-1}(1-p_i)] \\
&\quad +\beta C_{out}[\sum_{j=1}^{m-1} p_j\{E(R_j) - Z_2\}\Pi_{i=0}^{j-1}(1-p_i)] \\
&\quad +\alpha C_f[\sum_{j=1}^{m-1} \Pi_{i=1}^{j}(1-p_i)] - L(t_m)\Pi_{i=1}^{m}(1-p_i).
\end{aligned}
$$

Equation (3.2) is a simplification of the above expression.

A.2 Proof of Theorem 3.2

The proof of Theorem 2 uses an approach similar to the one used in Appendix A.1 to derive $E(T)$, the expected length of the production cycle. Let B_j be the area under $I(t)$ beyond t_j if the process is out of control at time t_j, $j = 1, 2, ..., m-1$ and an alarm has been triggered or $j = m$. Let $E(A_j), j = 0, 1, 2, ..., m-1$, be the expected residual area under $I(t)$ beyond time $t_j, E(A_0) = E(A)$, given that the process was in control at time t_j. Let $E(Q_j)$ be the expected residual area under $I(t)$ beyond time t_j given that the process was out of control at time t_j and a true alarm has not been triggered so far. Clearly,

$$E(Q_m) = B_m, \tag{A.5}$$

and for $j = 1, 2, ..., m-1$

$$E(Q_j) = U_{j+1} + \beta E(Q_{j+1}) = \sum_{i=j+1}^{m} \beta^{i-j-1} U_i + \beta^{m-j-1} B_m, \tag{A.6}$$

where U_j is given by (3.4). Note from Figure 1 that

$$B_j = \frac{I_j^2}{2D}, \quad j = 1, 2, ..., m.$$

An expression for $E(A)$ may be developed as follows. Consider the possible states of the process at the end of the first sampling and PM interval. For each possible state the expected residual area under $I(t)$, and the associated probabilities are presented in Table (A.3).

State	Probability	Expected Residual Area
In-Control and no alarm	$(1-p_1)(1-\alpha)$	$E(A_1)$
In-Control and a false alarm	$(1-p_1)\alpha$	$E(A_1)$
Out-of-Control but no alarm	$p_1\beta$	$E(Q_1)$
Out-of-Control and true alarm	$p_1(1-\beta)$	B_1

Table A.3: Expected residual Area

Consequently,

$$E(A) = U_1 + (1-\beta)B_1 p_1 + \beta p_1 E(Q_1) + (1-p_1)E(A_1).$$

Proceeding in a similar fashion, we have for $j = 1, 2, ..., m-2$

$$E(A_j) = U_{j+1} + (1-\beta)B_{j+1}p_{j+1} + \beta p_{j+1}E(Q_{j+1}) + (1-p_{j+1})E(A_{j+1}).$$

The expression for $E(A_{m-1})$ is given by the following:

$$E(A_{m-1}) = U_m + B_m,$$

therefore,

$$\begin{aligned} E(A) &= \sum_{j=1}^{m} U_j \Pi_{i=0}^{j-1}(1-p_i) + (1-\beta)\sum_{j=1}^{m-1} B_j p_j \Pi_{i=0}^{j-1}(1-p_i) \\ &+ \beta \sum_{j=1}^{m-1} E(Q_j) p_j \Pi_{i=0}^{j-1}(1-p_i) + \Pi_{i=1}^{m-1}(1-p_i)B_m. \end{aligned} \tag{A.7}$$

Further simplification will lead to expression (3.5).

A.3 Proof of Theorem 3.3

In order to compute an expression for $E(PM)$, we determine the expected residual maintenance cost beyond time h_1 for each possible state of the process after the first PM. These values are presented in Table (A.4).

State	Probability	Current cost	Expected residual cost
In-Control and no alarm	$(1-p_1)(1-\alpha)$	C_{pm_1}	$E(PM_1)$
In-Control and a false alarm	$(1-p_1)\alpha$	C_{pm_1}	$E(PM_1)$
Out-of-Control but no alarm	$p_1\beta$	C_{pm_1}	$N_1 C_{pm}$
Out-of-Control and true alarm	$p_1(1-\beta)$	0	0

Table A.4: Expected residual maintenance cost

Note that if at the end of the first interval the process is out of control and there is a true alarm, then no PM is performed. Consequently, both the current and residual costs are zero. Similar to $E(QC)$, the above table yields the following recursive system:

$$E(PM) = C_{pm_1}[1 - p_1 + \beta p_1(1 + N_1)] + (1 - p_1)E(PM_1),$$

and for $j = 1, 2, ..., m-2$

$$E(PM_j) = C_{pm_j}[1 - p_{j+1} + \beta p_{j+1}(1 + N_{j+1})] + (1 - p_{j+1})E(PM_{j+1}),$$

and

$$E(PM_{m-1}) = 0.$$

Equivalently,

$$E(PM) = \sum_{j=1}^{m-1} C_{pm_j}\Pi_{i=1}^{j}(1-p_i) + \beta\sum_{j=1}^{m-1} C_{pm_j}p_j\Pi_{i=0}^{j-1}(1-p_i) + \beta\sum_{j=1}^{m-1} C_{pm_j}p_jN_j\Pi_{i=0}^{j-1}(1-p_i).$$

Further simplification yields expression (3.8).

Appendix B

Programs Listing

```
C NAME:              General PM Model with Gama=(1-frac*(Cpm/Cnew))
C
C DESCRIPTION:       This program represents the code listing of the
C                    first PM policy (perform PM at every 1 inspection
C                    interval). Also, it represents the basis of all
C                    other models programs. To develop the necessary
C                    programs needed to simulate the other PM policies
C                    you simply need to change the expected production
C                    cycle E(T), the expected inventory holding cost
C                    E(HC), the expected quality cost and the expected
C                    PM cost formula accordingly.
C                    For Taguchi's quadratic loss function models, you
C                    need only to select scheme 2 to compute the in-
C                    control and out-of-control costs dynamically.
C
C USAGE:             Economic design of x-bar control charts,
C                    determination of the economic production
C                    quantity and imperfect preventive maintenance
C                    cost model under truncated Weibull shock models
C                    using a non-uniform sampling scheme.
C
C OVERVIEW:          This program subjects the production process to
C                    the occurrence of a non-Markovian shock having an
C                    increasing failure rate. Then, it employs Hook &
C                    Jeevs method to determine the optimal control
C                    chart design and the preventive maintenance level
C                    which minimize the objective function. Once these
```

```
C                           parameters optimized, the optimal Production
C                           Quantity and the Inspection Schedule are
C                           determined.
C
C INPUT/OUTPUT:             None.
C
C RESTRICTIONS:             None.
C
C AUTHOR:                   1.0  Mohammed A. Makhdoum; December 25, 1995.
C
C REVISIONS:                None.
C
      COMMON NMAX(95),NMIN(95),NOPT(95),NPT(95,100)
      COMMON ZM,Z1,Z2,A,B,D0,D1,S0,CF,CA,XLAMDA,DELTA,XNU
      COMMON H,ALPHA,BETA,POWER,XK,CPM,CNEW
      COMMON X0(2),XM(2),XX(2),XT(2)
      COMMON XKPT(95,100),HPT(95,100),ALFPT(95,100)
      COMMON PWRPT(95,100),ETPPT(95,100),FXBPT(95,100)
      COMMON ETIPT(95,100),ECQPT(95,100)
      COMMON ECHPT(95,100),ECMPT(95,100)
      COMMON ENPMPT(95,100),IQSPT(95,100)
      COMMON PP,D,C0,CH
      CHARACTER*1 ANS
      CHARACTER*54 FLNAME
      IFMT=6
C
C***********************************************************************
C***********************************************************************
C************************** M A I N   B O D Y **************************
C***********************************************************************
C***********************************************************************
      WRITE(*,805)
      READ*, ZM
      WRITE(*,810)
      READ*, Z2
      WRITE(*,815)
      READ*, D0
      WRITE(*,820)
      READ*, D1
      WRITE(*,825)
      READ*, CA
      WRITE(*,830)
      READ*, CF
      WRITE(*,835)
      READ*, A
```

```
      WRITE(*,840)
      READ*, B
      WRITE(*,845)
      READ*, S0
      WRITE(*,850)
      READ*, DELTA
      WRITE(*,855)
      READ*, XLAMDA,XNU
      WRITE(*,860)
      READ*, XK
      WRITE(*,865)
      READ*, H
      WRITE(*,866)
      READ*, D
      WRITE(*,867)
      READ*, PP
      WRITE(*,868)
      READ*, CH
      WRITE(*,869)
      READ*, C0
      WRITE(*,880)
      READ*, CPM
      WRITE(*,881)
      READ*, CNEW
      WRITE(*,800)
      READ*, MDL
      WRITE(*,870)
      READ(*,200) ANS
      IF (CPM.GT.CNEW) STOP '<<< ERROR >>>: Cpm > Cnew.'
      IF (MDL.NE.1 .AND. MDL.NE.2) STOP '<<< ERROR >>>: Unknown Scheme.'
      IF (MDL.EQ.2) THEN
        WRITE(*,*)'Please enter Scrap value, Sigma, Tolerance: '
        READ(*,*)SCR,SEG,TOL
      ENDIF
C
      IF (ANS.EQ.'F' .OR. ANS.EQ.'f') THEN
         IFMT=8
         WRITE(*,875)
         READ(*,210) FLNAME
         OPEN (UNIT=IFMT,FILE=FLNAME, STATUS='UNKNOWN')
         WRITE(IFMT,*)
         WRITE(IFMT,*) '  PROGRAM INPUT:'
         WRITE(IFMT,*)
         WRITE(IFMT,805)
         WRITE(IFMT,230) ZM
```

```
        WRITE(IFMT,810)
        WRITE(IFMT,230) Z2
        WRITE(IFMT,815)
        WRITE(IFMT,230) D0
        WRITE(IFMT,820)
        WRITE(IFMT,230) D1
        WRITE(IFMT,825)
        WRITE(IFMT,230) CA
        WRITE(IFMT,830)
        WRITE(IFMT,230) CF
        WRITE(IFMT,835)
        WRITE(IFMT,230) A
        WRITE(IFMT,840)
        WRITE(IFMT,230) B
        WRITE(IFMT,845)
        WRITE(IFMT,230) S0
        WRITE(IFMT,850)
        WRITE(IFMT,230) DELTA
        WRITE(IFMT,855)
        WRITE(IFMT,240) XLAMDA,XNU
        WRITE(IFMT,860)
        WRITE(IFMT,230) XK
        WRITE(IFMT,865)
        WRITE(IFMT,230) H
        WRITE(IFMT,866)
        WRITE(IFMT,230) D
        WRITE(IFMT,867)
        WRITE(IFMT,230) PP
        WRITE(IFMT,868)
        WRITE(IFMT,230) CH
        WRITE(IFMT,869)
        WRITE(IFMT,230) CO
        WRITE(IFMT,880)
        WRITE(IFMT,230) CPM
        WRITE(IFMT,881)
        WRITE(IFMT,230) CNEW
        WRITE(IFMT,800)
        WRITE(IFMT,220) MDL
        WRITE(IFMT,870)
        WRITE(IFMT,205) ANS
      ENDIF
      WRITE(IFMT,2)
2     FORMAT(//,T3,'PROGRAM OUTPUT:',/)
      IF (MDL.EQ.1) THEN
        WRITE(IFMT,650)
```

```
      ELSE
        WRITE(IFMT,651)
      ENDIF
      WRITE(IFMT,*)
   3  FORMAT('-',T12,'N',T20,'OPTIMUM K',T33,
     *'OPTIMUM H',T45,'ALPHA',T55,'POWER',T67,'COST')
C
      N=60
      MMIN=5
      MMAX=7
      MMIN1=MMIN
      MMAX1=MMAX
C
C  Finding the optimum inspection interval MOPT
C
      CALL MWISE(MDL,MMIN,MMAX,N)
      IF (MMIN .EQ. MMAX) THEN
         MOPT=MMIN
         GO TO 180
      ENDIF
      IF (FXBPT(MMIN,NOPT(MMIN)).LT.FXBPT(MMIN+1,NOPT(MMIN+1))) THEN
         M=MMIN
 510     M=M-1
C
         IF (M.LT.1) THEN
           MOPT=M+1
           GOTO 180
         ENDIF
C
         MMIN=M
         MMAX=M
         CALL MWISE(MDL,MMIN,MMAX,N)
         IF (FXBPT(M,NOPT(M)).LE.FXBPT(M+1,NOPT(M+1))) GOTO 510
         MOPT=M+1
         GO TO 550
      ENDIF
      DO 520 M=MMIN,MMAX
         IF (FXBPT(M+1,NOPT(M+1)).GT.FXBPT(M,NOPT(M))) THEN
            MOPT=M
            GO TO 550
         ENDIF
         IF (M .EQ. MMAX-1) THEN
            IF (FXBPT(M,NOPT(M)).LT.FXBPT(M+1,NOPT(M+1))) THEN
               MOPT=M
               GO TO 550
```

```
          ELSE
             GO TO 530
          ENDIF
       ENDIF
 520  CONTINUE
 530  M=MMAX
 540  M=M+1
      MMIN=M
      MMAX=M
      CALL MWISE(MDL,MMIN,MMAX,N)
      IF (FXBPT(M,NOPT(M)).LT.FXBPT(M-1,NOPT(M-1))) THEN
         GOTO 540
      ELSE
         M=M-1
         GOTO 280
      ENDIF
C
 280  CONTINUE
      IF (M-1.LT.1) THEN
         MOPT=M+1
         GOTO 180
      ENDIF
C
      MOPT=M
C
 550  IF (MMIN .NE. MMIN1) THEN
         MMAX=MMAX1
         GO TO 180
      ENDIF
      IF (MMAX .NE. MMAX1) MMIN=MMIN1
 180  DO 190 I=NMIN(MOPT),NMAX(MOPT)
         WRITE(IFMT,10) NPT(MOPT,I),XKPT(MOPT,I),
     *                  HPT(MOPT,I),ALFPT(MOPT,I),
     *                  PWRPT(MOPT,I),FXBPT(MOPT,I)
 190  CONTINUE
      WRITE(IFMT,940) MOPT
      WRITE(IFMT,950) NOPT(MOPT)
      WRITE(IFMT,960) HPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,970) XKPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,980) ALFPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,990) PWRPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1000) FXBPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1090) ETIPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1190) ETPPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1195) ECQPT(MOPT,NOPT(MOPT))
```

```
      WRITE(IFMT,1196) ECHPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1197) ECMPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1198) ENPMPT(MOPT,NOPT(MOPT))
      WRITE(IFMT,1199) IQSPT(MOPT,NOPT(MOPT))
C
   10 FORMAT(' ',T10,I4,T20,2(F7.2,6X),T44,2(F7.4,3X),T63,F10.2)
  200 FORMAT(A1)
  205 FORMAT(1X,A1)
  210 FORMAT(A54)
  220 FORMAT(1X,I10)
  230 FORMAT(F8.2)
  240 FORMAT(F8.5,/,F5.2)
  650 FORMAT(/' Non-uniform sampling scheme under a'
     *,' truncated weibull shock model')
  651 FORMAT(/' Uniform sampling scheme under a'
     *,' truncated weibull shock model')
  800 FORMAT(/' Select your scheme: ',/,
     * '      (1) using classical quality control,/,
     *'      (2) using Taguchi''s quadratic loss funciotn',/,
     *5X,'Enter 1 or 2:')
  805 FORMAT(/' Enter the maximum expected time ',
     *'associated with a full PM: ')
  810 FORMAT(/' Enter the expected search and repair',
     *' time if a failure is detected: ')
  815 FORMAT(/' Enter the expected cost per hour of '
     *,'producing nonconforming items'/4x,'When the '
     *,'process is in control: ')
  820 FORMAT(4x,'When the process is out of ',
     *'control: ')
  825 FORMAT(/' Enter the expected cost to locate and'
     *,' repair an assignable cause: ')
  830 FORMAT(/' Enter the expected cost per false ',
     *'alarm: ')
  835 FORMAT(/' Enter the fixed cost of sampling: ')
  840 FORMAT(/' Enter the expected variable cost of ',
     *'sampling: ')
  845 FORMAT(/' Enter the salvage value of the ',
     *'machine at age zero: ')
  850 FORMAT(/' Enter the shift (delta) at which you '
     *,'wish to obtain the power: ')
  855 FORMAT(/' Enter the parameters (lambda, nu) for'
     *,' the Weibull shock model: ')
  860 FORMAT(/' Enter the initial value of the ',
     *'parameter k: ')
  865 FORMAT(/' Enter the initial value of the ',
```

```
     *'parameter h1: ')
  866 FORMAT(/' Enter the demand rate: ')
  867 FORMAT(/' Enter the production rate: ')
  868 FORMAT(/' Enter the holding cost per unit per',
     *' unit of time: ')
  869 FORMAT(/' Enter the set-up cost for each',
     *' production cycle: ')
  870 FORMAT(/' Do you want output sent to the screen'
     *,' (S) or to a file (F)? ')
  875 FORMAT(/' Enter name of data file: ')
  880 FORMAT(/' Enter the Preventive Maintenance cost: ')
  881 FORMAT(/' Enter the cost of a new system: ')
C
  940 FORMAT(/,T5,'The optimal no. of inspection intervals is ',I4)
  950 FORMAT(T5,'The optimal SAMPLE SIZE is',I3)
  960 FORMAT(T5,'The optimal SAMPLING INTERVAL is',F6.2)
  970 FORMAT(T5,'The optimal CONTROL LIMIT',1X,
     *'COEFFICIENT of X-Bar chart is',F6.2)
  980 FORMAT(T5,'The optimal value of ALPHA is ',F7.4)
  990 FORMAT(T5,'The optimal value of POWER is ',F7.4)
 1000 FORMAT(T5,'The expected minimum cost of',1X,
     *'this DESIGN is ',F12.2,' per unit time')
 1090 FORMAT(T5,'The expected duration of',1X,
     *'INVENTORY CYCLE is ',F10.4)
 1190 FORMAT(T5,'The expected duration of',1X,
     *'a PRODUCTION CYCLE is ',F10.4)
 1195 FORMAT(T5,'The expected QUALITY control',1X,
     *'cost per unit time is ',F10.4)
 1196 FORMAT(T5,'The expected HOLDING',1X,
     *'cost per unit time is ',F10.4)
 1197 FORMAT(T5,'The expected PM',1X,
     *'cost per unit time is ',F10.4)
 1198 FORMAT(T5,'The expected no. of Preventive Maintenance is ',F6.2)
 1199 FORMAT(T5,'Optimal Production Quantity is ',I8)
 2200 CLOSE(IFMT)
      STOP '******************** DONE ********************'
      END
C
C**********************************************************************
C**********************************************************************
C*********************** SUBROUTINES & FUNCTIONS **********************
C**********************************************************************
C**********************************************************************
      SUBROUTINE MWISE(MDL,MMIN,MMAX,N)
C  Subroutine of M-wise iteration, where M is the
```

```
C  number of inspection interval.
C
      COMMON NMAX(95),NMIN(95),NOPT(95),NPT(95,100)
      COMMON ZM,Z1,Z2,A,B,D0,D1,S0,CF,CA,XLAMDA,DELTA,XNU
      COMMON H,ALPHA,BETA,POWER,XK,CPM,CNEW
      COMMON X0(2),XM(2),XX(2),XT(2)
      COMMON XKPT(95,100),HPT(95,100),ALFPT(95,100)
      COMMON PWRPT(95,100),ETPPT(95,100),FXBPT(95,100)
      COMMON ETIPT(95,100),ECQPT(95,100)
      COMMON ECHPT(95,100),ECMPT(95,100)
      COMMON ENPMPT(95,100),IQSPT(95,100)
      COMMON PP,D,C0,CH
      DO 610 M=MMIN,MMAX
         CALL NWISE(MDL,M,N)
         WRITE(*,731)M,FXBPT(M,NOPT(M))
 731     FORMAT(1X,'For M=',I2,' the best value of the OBJ is=',F7.2)
 610  CONTINUE
      RETURN
      END
C
C***********************************************************************
      SUBROUTINE NWISE(MDL,M,N)
C  Subroutine of N-wise iteration, where N is the
C  sample size.
C
      COMMON NMAX(95),NMIN(95),NOPT(95),NPT(95,100)
      COMMON ZM,Z1,Z2,A,B,D0,D1,S0,CF,CA,XLAMDA,DELTA,XNU
      COMMON H,ALPHA,BETA,POWER,XK,CPM,CNEW
      COMMON X0(2),XM(2),XX(2),XT(2)
      COMMON XKPT(95,100),HPT(95,100),ALFPT(95,100)
      COMMON PWRPT(95,100),ETPPT(95,100),FXBPT(95,100)
      COMMON ETIPT(95,100),ECQPT(95,100)
      COMMON ECHPT(95,100),ECMPT(95,100)
      COMMON ENPMPT(95,100),IQSPT(95,100)
      COMMON PP,D,C0,CH
      NMIN(M)=N-1
      NMAX(M)=N+1
      NN=2
      IF (NMIN(M) .LE. 0) NMIN(M)=2
      X0(1)=XK
      X0(2)=H
      DO 610 I=NMIN(M),NMAX(M)
         CALL NCALC(MDL,M,I,NN)
 610  CONTINUE
C
```

```
C  Finding the optimum sample size, NOPT
C
      IF (FXBPT(M,NMIN(M)).LT.FXBPT(M,NMIN(M)+1)) THEN
         I=NMIN(M)
 620     I=I-1
         CALL NCALC(MDL,M,I,NN)
         IF ((FXBPT(M,I) .LE. FXBPT(M,I+1)) .AND.
     *                                 (I.GT.2)) GO TO 620
         NOPT(M)=I+1
         NMIN(M)=I
         GO TO 700
      ENDIF
      DO 630 I=NMIN(M),NMAX(M)
         IF ((FXBPT(M,I+1) .GT. FXBPT(M,I)) .AND.
     *                        (I .LT. NMAX(M)-1)) THEN
            NOPT(M)=I
            GO TO 700
         ENDIF
         IF (I.EQ.NMAX(M)-1) THEN
            IF (FXBPT(M,I) .LT. FXBPT(M,I+1)) THEN
               NOPT(M)=I
               GO TO 700
            ELSE
               GO TO 640
            ENDIF
         ENDIF
 630  CONTINUE
 640  I=NMAX(M)
 650  I=I+1
      CALL NCALC(MDL,M,I,NN)
      IF (FXBPT(M,I) .LE. FXBPT(M,I-1)) GO TO 650
      NOPT(M)=I-1
      NMAX(M)=I
 700  RETURN
      END
C
C***********************************************************************
      SUBROUTINE NCALC(MDL,M,I,NN)
C  Subroutine for finding the value of sample size N
C  and control limit coefficient for X-bar chart.
C
      COMMON NMAX(95),NMIN(95),NOPT(95),NPT(95,100)
      COMMON ZM,Z1,Z2,A,B,D0,D1,S0,CF,CA,XLAMDA,DELTA,XNU
      COMMON H,ALPHA,BETA,POWER,XK,CPM,CNEW
      COMMON X0(2),XM(2),XX(2),XT(2)
```

```
      COMMON XKPT(95,100),HPT(95,100),ALFPT(95,100)
      COMMON PWRPT(95,100),ETPPT(95,100),FXBPT(95,100)
      COMMON ETIPT(95,100),ECQPT(95,100)
      COMMON ECHPT(95,100),ECMPT(95,100)
      COMMON ENPMPT(95,100),IQSPT(95,100)
      COMMON PP,D,C0,CH
      REAL FXB,FXE,FXT,XN
      XN=I
      N=I
      ICOUNT=0
C
C   Find optimal decision variables using pattern
C   search technique of Hooke and Jeeves (1961).
C   Journal of the Association for Computing
C   Machinery 8, pp. 212-229.
C
      XM(1)=X0(1)
      XM(2)=X0(2)
      XK=XM(1)
      H=XM(2)
      STEP=.05
      CALL OBJF(MDL,XK,CPM,CNEW,XN,H,DELTA,XLAMDA,A,B,D0,D1,
     *S0,CF,CA,XNU,ALPHA,BETA,POWER,FXB,ZM,Z1,Z2,M,ECQ,ECH,ECM,
     *ENPM,IQS,ETI,ETP,PP,D,C0,CH)
      ICOUNT=ICOUNT+1
 60   KKK=0
      DO 80 II=1,NN
         TEMP=XM(II)
         XM(II)=XM(II)+STEP
         XK=XM(1)
         H=XM(2)
         IF ((XK.LT.0.).OR.(H.LT.0.)) THEN
            FXE=9000000.
         ELSE
            CALL OBJF(MDL,XK,CPM,CNEW,XN,H,DELTA,XLAMDA,A,B,
     *      D0,D1,S0,CF,CA,XNU,ALPHA,BETA,POWER,FXE,ZM,Z1,
     *      Z2,M,ECQ,ECH,ECM,ENPM,IQS,ETI,ETP,PP,D,C0,CH)
            ICOUNT=ICOUNT+1
         ENDIF
         IF (FXE.LT.FXB) GO TO 70
         XM(II)=TEMP
         XM(II)=XM(II)-STEP
         XK=XM(1)
         H=XM(2)
         IF ((XK.LT.0.).OR.(H.LT.0.)) THEN
```

```
            FXE=9000000.
         ELSE
            CALL OBJF(MDL,XK,CPM,CNEW,XN,H,DELTA,XLAMDA,A,B,
     *      D0,D1,S0,CF,CA,XNU,ALPHA,BETA,POWER,FXE,ZM,Z1,
     *      Z2,M,ECQ,ECH,ECM,ENPM,IQS,ETI,ETP,PP,D,C0,CH)
            ICOUNT=ICOUNT+1
         ENDIF
         ICOUNT=ICOUNT+1
         IF (FXE.LT.FXB) GO TO 70
         XM(II)=TEMP
         GO TO 80
 70      FXB=FXE
         KKK=1
 80   CONTINUE
      IF(KKK.EQ.0) GO TO 140
      DO 90 JJ=1,NN
 90      XX(JJ)=XM(JJ)
      DO 100 JJ=1,NN
100      XT(JJ)=2.*XX(JJ)-X0(JJ)
      XK=XT(1)
      H=XT(2)
      IF ((XK.LT.0.).OR.(H.LT.0.)) THEN
         FXT=9000000.
      ELSE
         CALL OBJF(MDL,XK,CPM,CNEW,XN,H,DELTA,XLAMDA,A,B,
     *   D0,D1,S0,CF,CA,XNU,ALPHA,BETA,POWER,FXT,ZM,Z1,Z2,
     *   M,ECQ,ECH,ECM,ENPM,IQS,ETI,ETP,PP,D,C0,CH)
         ICOUNT=ICOUNT+1
      ENDIF
      DO 110 MM=1,NN
110      X0(MM)=XX(MM)
      IF (FXT.LE.FXB) GO TO 120
      GO TO 60
120   FXB=FXT
      DO 130 II=1,NN
130      XM(II)=XT(II)
      GO TO 60
140   IF((STEP.LT..001).OR.(ICOUNT.GT.9999))
     *                                            GO TO 150
      STEP=STEP/2.
      GO TO 60
150   CONTINUE
      IF ((XK.LT.0.) .OR. (H .LT. 0.).OR.(FXB.LT.0.))
     *                                              GO TO 160
      NPT(M,I)=N
```

```
      XKPT(M,I)=XK
      HPT(M,I)=H
      ALFPT(M,I)=ALPHA
      PWRPT(M,I)=POWER
      ETIPT(M,I)=ETI
      ETPPT(M,I)=ETP
      FXBPT(M,I)=FXB
      ECQPT(M,I)=ECQ
      ECHPT(M,I)=ECH
      ECMPT(M,I)=ECM
      ENPMPT(M,I)=ENPM
      IQSPT(M,I)=IQS
      WRITE(*,111)M,N,XK,H,ECQ,ECH,ECM,FXB
 111  FORMAT(1X,'M=',I2,' N=',I3,' K=',F4.2,' H=',F5.2,'   ECQ=',F6.1,
     *      '   ECH=',F6.1,'   ECM=',F6.1,'   OBJ=',F7.2)
 160  RETURN
      END
C
C***************************************************************************
      SUBROUTINE OBJF(MDL,XK,CPM,CNEW,XN,H,DELTA,XLAMDA,A,B,
     *D0,D1,S0,CF,CA,XNU,ALPHA,BETA,POWER,FX,ZM,Z1,Z2,
     *M,ECQ,ECH,ECM,ENPM,IQS,ETI,ETP,PP,D,C0,CH)
C  Subroutine for evaluation of cost function
C
      PARAMETER (MM=95)
      DOUBLE PRECISION WJ(MM),YJ(MM),TJ(MM),HJ(MM)
      DOUBLE PRECISION FJNOT(MM),FJDEL(MM),P(MM)
      REAL XK,CPM,CNEW,XN,H,DELTA,XLAMDA,A,B
      REAL D0,D1,S0,CF,CA,XNU,ALPHA,BETA,POWER,FX,ZM,Z1,Z2
      REAL PHI,FIX1,FIX2,ETP,ETI,ECQ,ECH,ECM
      REAL GAMMLN,GAMMP,AIGB,XIGB,XMEAN
      REAL XI(MM),XU(MM),XB(MM)
      REAL INTG1,INTG2
      INTEGER IQS
      EXTERNAL GAMMLN,GAMMP
      FIX=PHI(-XK)
      ALPHA=2.*FIX
      FIX1=DELTA*SQRT(XN)-XK
      FIX2=-DELTA*SQRT(XN)-XK
      POWER=PHI(FIX1)+PHI(FIX2)
      BETA=1.-POWER
      H1=H
      GAMA=(1.-CPM/CNEW)
      Z1=ZM*(CPM/CNEW)
      RAT=CPM/CNEW
```

```
      FRAC=0.99
C
C
C     ************************************************************
C     Computing the conditional probility Pj's that the process
C     shifts to out-of-control state during interval (tj-1,tj)
C     ************************************************************
      DO 119 J=1,M
        IF (J.LT.M) THEN
          IF (J.EQ.1) THEN
            RAT=1.*RAT
          ELSE
            RAT=FRAC*RAT
          ENDIF
          GAMA=(1.-RAT)
        ELSE
          GAMA=1.
        ENDIF
        IF (J.EQ.1) THEN
          HJ(J)=H1
          TJ(J)=HJ(J)
          YJ(J)=HJ(J)
          WJ(J)=GAMA*YJ(J)
          FJNOT(J)=EXP(-XLAMDA*WJ(J)**XNU)
          FJDEL(J)=1.0-EXP(-XLAMDA*YJ(J)**XNU)
          P(J)=FJDEL(J)
        ELSEIF ((J.GT.1).AND.(J.LT.M)) THEN
          IF (MDL.EQ.1) THEN
            HJ(J)=(H1**XNU+WJ(J-1)**XNU)**(1./XNU)-WJ(J-1)
          ELSE
            HJ(J)=H1
          ENDIF
          TJ(J)=TJ(J-1)+HJ(J)
          YJ(J)=WJ(J-1)+HJ(J)
          WJ(J)=GAMA*YJ(J)
          FJNOT(J)=EXP(-XLAMDA*WJ(J)**XNU)
          FJDEL(J)=(1-EXP(-XLAMDA*YJ(J)**XNU))-
     *    (1-EXP(-XLAMDA*WJ(J-1)**XNU))
          P(J)=FJDEL(J)/FJNOT(J-1)
        ELSE
          IF (MDL.EQ.1) THEN
            HJ(J)=(H1**XNU+WJ(J-1)**XNU)**(1./XNU)-WJ(J-1)
          ELSE
            HJ(J)=H1
          ENDIF
```

```
        TJ(J)=TJ(J-1)+HJ(J)
        YJ(J)=WJ(J-1)+HJ(J)
        WJ(J)=GAMA*YJ(J)
        FJNOT(J)=EXP(-XLAMDA*YJ(J)**XNU)
        FJDEL(J)=(1-EXP(-XLAMDA*YJ(J)**XNU))-
     *   (1-EXP(-XLAMDA*WJ(J-1)**XNU))
        P(J)=FJDEL(J)/FJNOT(J-1)
      ENDIF
      IF ((P(J).LT.0.0).OR.(P(J).GT.1.0)) THEN
        DO I=1,J
          WRITE(*,*)'P(',I,')=',P(I)
        ENDDO
        STOP ' *** ERROR *** :Invalid probability was calculated'
      ENDIF
 119  CONTINUE
C
C     ********************************************
C     Computing the expected cycle length E(T)=ETT
C     ********************************************
      SUMT1=0.
      DO J=1,M-1
        SUMT1=SUMT1+FJDEL(J)
      ENDDO
      SUMT1=SUMT1+FJNOT(M-1)
      SUMT1=Z2*SUMT1
C
      SUMT2=0.
      DO J=1,M
        IF (J .EQ. 1) THEN
           SUMT2=SUMT2+H1*1.
        ELSE
          SUMT2=SUMT2+HJ(J)*FJNOT(J-1)
        ENDIF
      ENDDO
C
      SUMT3=0.
      DO J=1,M-1
        IF (J.EQ.1) THEN
          SUMT3=SUMT3+1.
        ELSE
          SUMT3=SUMT3+FJNOT(J-1)
        ENDIF
      ENDDO
      SUMT3=Z1*SUMT3
C
```

```
      SUMT4=0.
      DO J=1,M-1
        SUMTX=0.
        DO I=J+1,M
           IF (BETA .EQ. 0.) THEN
              SUMTX=SUMTX
           ELSE
              SUMTX=SUMTX+(HJ(I)+Z1)*(BETA**(I-J-1))
           ENDIF
        ENDDO
        SUMT4=SUMT4+FJDEL(J)*(SUMTX+HJ(M)*BETA**(M-J-1))
      ENDDO
      SUMT4=BETA*SUMT4
C
      ETT=SUMT1+SUMT2+SUMT3+SUMT4
C
C     **************************************************************
C     Computing the expected preventive maintenance cost E(MC)=EM
C     **************************************************************
      SUMM1=0.
      DO J=1,M-1
        SUMM1=SUMM1+FJNOT(J)
      ENDDO
C
      SUMM2=0.
      DO J=1,M-1
        SUMM2=SUMM2+FJDEL(J)
      ENDDO
      SUMM2=BETA*SUMM2
C
      SUMM3=0.
      DO J=1,M-1
        SUMM3X=0.
        DO I=1,M-J-1
          IF (BETA .EQ. 0.) THEN
            SUMM3X=SUMM3X
          ELSE
            SUMM3X=SUMM3X+I*(1.-BETA)*BETA**(I-1)
          ENDIF
        ENDDO
        SUMM3=SUMM3+FJDEL(J)*(SUMM3X+(M-J-1)*BETA**(M-J-1))
      ENDDO
      SUMM3=BETA*SUMM3
C
      ENPM=SUMM1+SUMM2+SUMM3
```

```
C
      EM=CPM*ENPM
C
C     ******************************************************
C     Computing the expected inventory holding cost E(HC)=EH
C     ******************************************************
      PD=PP-D
      DO J=1,M
        IF (J.EQ.1) THEN
          XI(J)=PD*HJ(J)-D*Z1
          IF (XI(J).LE.0.) THEN
            XI(J)=0.
            XU(J)=(PD*HJ(J)**2)/2.+((PD*HJ(J))**2)/(2.*D)
          ELSE
            XU(J)=(PD*HJ(J)**2)/2.+(XI(J)+PD*HJ(J))*Z1/2.
          ENDIF
          XB(J)=(XI(J)**2)/(2.*D)
        ELSEIF (J.EQ.M) THEN
          XI(M)=XI(M-1)+PD*HJ(M)
          IF (XI(M).LE.0.) THEN
            XI(M)=0.
            XU(M)=(2.*XI(M-1)+PD*HJ(M))*HJ(M)/2.+
     *                ((XI(M-1)+PD*HJ(M))**2)/(2.*D)
          ELSE
            XU(M)=(2.*XI(M-1)+PD*HJ(M))*HJ(M)/2.
          ENDIF
          XB(M)=(XI(M)**2)/(2.*D)
        ELSE
          XI(J)=XI(J-1)+PD*HJ(J)-D*Z1
          IF (XI(J).LE.0.) THEN
            XI(J)=0.
            XU(J)=(2.*XI(J-1)+PD*HJ(J))*HJ(J)/2.+
     *                ((XI(J-1)+PD*HJ(J))**2)/(2.*D)
          ELSE
            XU(J)=(2.*XI(J-1)+PD*HJ(J))*HJ(J)/2.+
     *                (XI(J-1)+XI(J)+PD*HJ(J))*Z1/2.
          ENDIF
          XB(J)=(XI(J)**2)/(2.*D)
        ENDIF
      ENDDO
C
      SUMH1=0.
      DO J=1,M
        IF (J .EQ. 1) THEN
          SUMH1=SUMH1+XU(1)
```

```
          ELSE
            SUMH1=SUMH1+XU(J)*FJNOT(J-1)
          ENDIF
        ENDDO
C
        SUMH2=0.
        DO J=1,M-1
          SUMH2=SUMH2+XB(J)*FJDEL(J)
        ENDDO
        SUMH2=(1-BETA)*SUMH2
C
        SUMH3=0.
        DO J=1,M-1
          SUMHX=0.
          DO I=J+1,M
            IF (BETA .EQ. 0.) THEN
              SUMHX=SUMHX
            ELSE
              SUMHX=SUMHX+(BETA**(I-J-1))*XU(I)
            ENDIF
          ENDDO
          SUMH3=SUMH3+FJDEL(J)*(SUMHX+(BETA**(M-J-1))*XB(M))
        ENDDO
        SUMH3=BETA*SUMH3
C
        EA=SUMH1+SUMH2+SUMH3+XB(M)*FJNOT(M-1)

        EH=CH*EA
C       ******************************************************
C       Computing the expected quality control cost E(QC)=EQ
C       ******************************************************
        IF (MDL.EQ.2) THEN
          IF (XK .GT. ((TOL/SEG)*SQRT(XN))) THEN
            FX=999999.
            RETURN
          ENDIF
          DSN=DELTA*SQRT(XN)
          XZ1=(-TOL*SQRT(XN))/SEG -DSN
          XZ2=(-XK -DSN)
          XZ3=( TOL*SQRT(XN))/SEG -DSN
          XZ4=( XK -DSN)
          EZ1=EXP(-.5*XZ1**2)
          EZ2=EXP(-.5*XZ2**2)
          EZ3=EXP(-.5*XZ3**2)
          EZ4=EXP(-.5*XZ4**2)
```

```
      PZ1=PHI(XZ1)
      PZ2=PHI(XZ2)
      PZ3=PHI(XZ3)
      PZ4=PHI(XZ4)
      D0=(SCR/XN)*(SEG/TOL)**2*
   &     (1.-(2.*XK/SQRT(2.*PI))*EXP(-.5*XK**2)-2.*PHI(-XK))
      D1=-(SCR/XN)*(SEG/TOL)**2*
   &     ((1./SQRT(2.*PI))*(XZ1*EZ1-XZ2*EZ2+XZ3*EZ3-XZ4*EZ4)+
   &      (1.+XN*DELTA**2)*(PZ2-PZ1+PZ4-PZ3)+
   &      ((2.*DELTA*SQRT(XN))/SQRT(2.*PI))*(EZ1-EZ2+EZ3-EZ4))+
   &     SCR*(1+PZ1-PZ3)

      D0=D0*PP
      D1=D1*PP
     ENDIF
C
     SUMQ1=0.
     AIGB=1.+1./XNU
     XIGB=XLAMDA*WJ(1)**XNU
     XMEAN=EXP(GAMMLN(AIGB))
     SUMQ1=XMEAN*GAMMP(AIGB,XIGB)/XLAMDA**(1./XNU)
     DO J=2,M
       AIGB=1.+1./XNU
       XIGB=XLAMDA*WJ(J-1)**XNU
       INTG1=XMEAN*GAMMP(AIGB,XIGB)/XLAMDA**(1./XNU)
       XMEAN=EXP(GAMMLN(AIGB))
       AIGB=1.+1./XNU
       XIGB=XLAMDA*YJ(J)**XNU
       XMEAN=EXP(GAMMLN(AIGB))
       INTG2=XMEAN*GAMMP(AIGB,XIGB)/XLAMDA**(1./XNU)
       SUMQ1=SUMQ1+INTG2-INTG1
     ENDDO
     SUMQ1=(D0-D1)*SUMQ1
C
     SUMQ2=0.
     DO J=1,M
       SUMQ2=SUMQ2+YJ(J)*FJDEL(J)
     ENDDO
     SUMQ2=(D1-D0)*SUMQ2
C
     SUMQ3=0.
     DO J=1,M
       IF (J .EQ. 1) THEN
          SUMQ3=SUMQ3+H1*1.
       ELSE
```

```
          SUMQ3=SUMQ3+HJ(J)*FJNOT(J-1)
        ENDIF
      ENDDO
      SUMQ3=D0*SUMQ3
C
      SUMQ4=0.
      DO J=1,M
        SUMQ4=SUMQ4+FJNOT(J)
      ENDDO
      SUMQ4=ALPHA*CF*SUMQ4
C
      SUMQ5=0.
      DO J=1,M-1
        SUMQX=0.
        DO I=J+1,M
           IF (BETA .EQ. 0.) THEN
              SUMQX=SUMQX
           ELSE
              SUMQX=SUMQX+HJ(I)*(BETA**(I-J-1))
           ENDIF
        ENDDO
        SUMQ5=SUMQ5+FJDEL(J)*SUMQX
      ENDDO
      SUMQ5=D1*BETA*SUMQ5
C
      SUMQ6=0.
      SUMQ6=CA
C
      SUMQ7=0.
      DO J=1,M-2
        SUMQ7=SUMQ7+FJNOT(J)
      ENDDO
      SUMQ7=(A+B*XN)*(SUMQ7+1+BETA/(1-BETA))
C
      EC=SUMQ1+SUMQ2+SUMQ3+SUMQ4+SUMQ5+SUMQ6+SUMQ7
C
C     *************************************************
C     Computing the expected total cost per unit time=FX
C     *************************************************
      ETP=ETT-Z2
      IQS=INT((ETP-ENPM*Z1)*PP)
      ETI=(PP/D)*(ETP-ENPM*Z1)
      ETC=C0+EM+EH+EC
      ECM=EM/ETI
      ECH=EH/ETI
```

```
      ECQ=EC/ETI
      FX=ETC/ETI
      RETURN
      END
C
C***********************************************************************
      FUNCTION PHI(X)
C  Compute the cumulative distribution function of a
C  random variable X having a standard normal distri-
C  bution (see reference: Abramowitz, M., and Stegun,
C  I.A., Handbook of Mathematical Functions, Dover
C  Publications, Inc., 1966, p. 297).
      REAL X,ZX,ROTRA,CON,ERFC
      DATA CON/.7978846/
      IF (ABS(X) .LE. 18.7) GO TO 630
      IF (X .LT. -18.7) PHI=0.
      IF (X .GT. 18.7) PHI=1.
      GO TO 905
 630  ZX=0.39894228*EXP(-X*X*0.5)
C
C  The quantity ROTRA is called Mill's ratio.
C
      ROTRA=CON*EXP(-X*X*0.5)
      ROTRA=ROTRA/ERFC(.7071068*X)
      PHI=1.-ZX/ROTRA
 905  RETURN
      END
C
C***********************************************************************
      FUNCTION ERFC(X)
C  Calculation of the complementary error function
C  erfc(X) (see: William H. Press, Brian P. Flannery,
C  Saul A. Teukolsky and William T. Vetterling, Nume-
C  rical Recipes, FORTRAN Version, Cambridge Univer-
C  sity Press, 1990, pp. 156-164).
      REAL X,GAMMP,GAMMQ
      IF (X.LT.0.) THEN
         ERFC=1.+GAMMP(.5,X**2)
      ELSE
         ERFC=GAMMQ(.5,X**2)
      ENDIF
      RETURN
      END
C
C
```

```
C***********************************************************************
      FUNCTION GAMMLN(XX)
C  Calculation of the logarithm value of gamma func-
C  tion of XX, for XX > 0 (see: William H. Press,
C  Brian P. Flannery, Saul A. Teukolsky and William T.
C  Vetterling, Numerical Recipes, FORTRAN Version,
C  Cambridge University Press, 1990, pp. 156-164).
      REAL XX
      REAL COF(6),STP,HALF,ONE,FPF,X,TMP,SER
      DATA COF,STP/76.18009173D0,-86.50532033D0,
     *     24.01409822D0,-1.231739516D0,.120858003D-2,
     *     -.536382D-5,2.50662827465D0/
      DATA HALF,ONE,FPF/0.5D0,1.0D0,5.5D0/
      X=XX-ONE
      TMP=X+FPF
      TMP=(X+HALF)*LOG(TMP)-TMP
      SER=ONE
      DO 11 J=1,6
         X=X+ONE
         SER=SER+COF(J)/X
 11   CONTINUE
      GAMMLN=TMP+LOG(STP*SER)
      RETURN
      END
C
C***********************************************************************
      FUNCTION GAMMP(A,X)
C  Calculation of the incomplete gamma function P(a,x)
C  (see: William H. Press, Brian P. Flannery, Saul A.
C  Teukolsky and William T. Vetterling, Numerical Re-
C  cipes, FORTRAN Version, Cambridge University Press,
C  1990, pp. 156-164).
      REAL A,X,GAMSER,GAMMCF,GLN
      IF (X.LT.0. .OR. A.LE.0.) PAUSE
      IF (X.LT.A+1.) THEN
         CALL GSER(GAMSER,A,X,GLN)
         GAMMP=GAMSER
      ELSE
         CALL GCF(GAMMCF,A,X,GLN)
         GAMMP=1.-GAMMCF
      ENDIF
      RETURN
      END
C
C
```

```
C***********************************************************************
      FUNCTION GAMMQ(A,X)
C  Calculation of complementary incomplete gamma
C  function Q(a,x)=1 - P(a,x), (see: William H. Press,
C  Brian P. Flannery, Saul A. Teukolsky and William T.
C  Vetterling, Numerical Recipes, FORTRAN Version,
C  Cambridge University Press, 1990, pp. 156-164).
      REAL A,X,GAMSER,GAMMCF,GLN
      IF (X.LT.0. .OR. A.LE.0.) PAUSE
      IF (X.LT.A+1.) THEN
         CALL GSER(GAMSER,A,X,GLN)
         GAMMQ=1.-GAMSER
      ELSE
         CALL GCF(GAMMCF,A,X,GLN)
         GAMMQ=GAMMCF
      ENDIF
      RETURN
      END
C
C***********************************************************************
      SUBROUTINE GSER(GAMSER,A,X,GLN)
C  Subroutine to calculate the incomplete gamma
C  function P(a,x) evaluated by its series represent-
C  ation as GAMSER. (see: William H. Press, Brian P.
C  Flannery, Saul A. Teukolsky and William T. Vetter-
C  ling, Numerical Recipes, FORTRAN Version, Cam-
C  bridge University Press, 1990, pp. 156-164).
      PARAMETER (ITMAX=100,EPS=3.E-7)
      REAL GAMSER,GAMMLN,A,X,GLN
      REAL AP,DEL,SUM
      GLN=GAMMLN(A)
      IF (X.LE.0.) THEN
         IF (X.LT.0.) PAUSE
         GAMSER=0.
         RETURN
      ENDIF
      AP=A
      SUM=1./A
      DEL=SUM
      DO 11 N=1,ITMAX
         AP=AP+1.
         DEL=DEL*X/AP
         SUM=SUM+DEL
         IF (ABS(DEL) .LT. ABS(SUM)*EPS) GOTO 1
 11   CONTINUE
```

```
      PAUSE 'GSER: A too large, ITMAX too small'
 1    GAMSER=SUM*EXP(-X+A*LOG(X)-GLN)
      RETURN
      END
C
C*************************************************************************
      SUBROUTINE GCF(GAMMCF,A,X,GLN)
C  Subroutine to calculate the incomplete gamma
C  function Q(a,x) evaluated by its continued fraction
C  representation as GAMMCF. (see: William H. Press,
C  Brian P. Flannery, Saul A. Teukolsky and William T.
C  Vetterling, Numerical Recipes, FORTRAN Version,
C  Cambridge University Press, 1990, pp. 156-164).
      REAL GAMMCF,GAMMLN,A,X,GLN
      REAL G,GOLD,A0,A1,B0,B1,FAC,AN,ANA
      PARAMETER (ITMAX=100,EPS=3.E-7)
      GLN=GAMMLN(A)
      GOLD=0.
      A0=1.
      A1=X
      B0=0.
      B1=1.
      FAC=1.
      DO 11 N=1,ITMAX
         AN=N
         ANA=AN-A
         A0=(A1+A0*ANA)*FAC
         B0=(B1+B0*ANA)*FAC
         ANF=AN*FAC
         A1=X*A0+ANF*A1
         B1=X*B0+ANF*B1
         IF (A1.NE.0.) THEN
            FAC=1./A1
            G=B1*FAC
            IF (ABS((G-GOLD)/G) .LT. EPS) GOTO 1
            GOLD=G
         ENDIF
11    CONTINUE
      PAUSE 'GCF: A too large, ITMAX too small'
1     GAMMCF=EXP(-X+A*ALOG(X)-GLN)*G
      RETURN
      END
```

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Vita

Mohammed Akram Abdul-Hai Makhdoum was born and grew up in Makkah. In 1984, Mohammed Akram attended King Fahd University of Petroleum and Minerals (KFUPM). In 1991, he was awarded a Bachelor of science degree in Systems Engineering major with Automation minor. In the same year, he was admitted by the graduate studies collage of KFUPM as a full-time graduate student majoring in Systems Engineering. After one year, he changed his academic status to part-time graduate student and joined Saudi Aramco company as an Industrial Computer Engineer responsible for gas networks leak detection modelling system. Meanwhile, Mohammed Akram is working as a Supervisory Control and Data Acquisition (SCADA) engineer at the same company.