MATHEMATICAL MODELS FOR QUALITY IN MULTISTAGE PRODUCTION SYSTEMS

BY

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كلية الدراسات العليا
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Dedicated to

My wife,

My sons Abdulrahman and Salih,

whose love, patience and perseverance

led to this accomplishment.

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Contents

List of Tables viii

List of Figures xi

Abstract (English) xiii

Abstract (Arabic) xv

Nomenclature xvi

1 Introduction 1

1.1 Overview 1

1.2 Statement of the problem 4

1.3 Cost of Variance 5

1.3.1 Relationship between Tolerance and Variance 6

1.3.2 Cost of Variance (Tolerance) 7

1.4 Proposed Work 9

1.5 Organization 10

2 Literature Review for Single and Multistage Production Systems 12

2.1 Introduction 12
2.2 Applications 13
2.3 Assumptions 14
2.4 The general approach 15
2.5 The literature survey of single stage production systems 17
2.5.1 Positive constant drift with linear trend 17
2.5.2 Positive constant drift with nonlinear trend 19
2.5.3 Negative constant drift with linear trend 19
2.5.4 Positive constant drift and positive or negative shift 19
2.5.5 Positive shift with no trend 20
2.5.6 Random drift 20
2.5.7 Quadratic loss function 22
2.6 The literature survey of multistage production systems 24
2.7 Limitations of the reviewed literature 25

3 Single Stage Production Systems Model (SSM) 27

3.1 Introduction 28
3.2 Problem Statement 29
3.3 The Notation 31
3.4 Assumptions 32
3.5 The Proposed Model 32
3.6 The Solution 35
3.7 Results and Discussion 40
3.8 Generalization of the drift function only for the SSM (GSSM1) 41
3.8.1 Special case 1: a positive shift (constant drift function) 43

3.8.2 Special case 2: Polynomial Drift Function 45

3.8.3 Special case 3: Exponential Drift Function 47

3.8.4 Example 3.2 50

3.9 Generalization of the pdf of x only for the SSM (GSSM2) 50

3.9.1 Example 3.3 52

3.10 Generalization of both the pdf of x and the drift function (GSSM3) . 53

3.10.1 Example 3.4 54

3.10.2 Reduction of GSSM3 to Previously Published Models 55

4 Variance Reduction and Sensitivity Analysis Studies of SSM 57

4.1 Variance Reduction Model for SSM (SSVRM) 58

4.1.1 Introduction 58

4.1.2 The Notation 61

4.1.3 The Proposed Variance Reduction Model (SSVRM) 62

4.1.4 Effect of variance on total cost 64

4.1.5 Optimization model for SSVRM 73

4.2 Sensitivity Analysis of SSM 75

4.2.1 Partial Derivatives of the Parameters 75

4.2.2 Design of Experiments 76

5 A Hybrid Tabu Search Algorithm for Function Minimization 83

5.1 Introduction 84

5.2 The Tabu Search Scheme 86

5.3 The Proposed Algorithm 87

5.3.1 Generation of random search directions 90

5.3.2 The Line Search Scheme 94
5.3.3 Storing in the Tabu List 95
5.4 Computational Results and Discussion 96
5.4.1 Parameter Setting 96
5.4.2 Test problems 97
5.4.3 Results and discussion 98

6 MultiStage Production Systems Without Buffers Model 103

6.1 Introduction 103
6.2 Problem Statement 105
6.3 The Notation 107
6.4 Assumptions 108
6.5 The Proposed Model 109
6.6 Results and Discussion 112
6.6.1 Process Improvement 114
6.7 Extensions to Multistage Lines without Buffers and with Nonzero Repair Times (MSM2) 117
6.8 Extensions of MSM2 120

7 Variance Reduction and Sensitivity Analysis Studies of MSM1 124

7.1 Variance Reduction of MSM1 (MSVRM) 124
7.1.1 Variance Reduction Model for MSM1 125
7.1.2 Solution Procedure (MSVRA) 126
7.1.3 Example 7.1 126
7.2 Sensitivity Analysis of MSM1 132

8 Models for MultiStage Production Systems With Buffers **137**

8.1 Introduction 138

8.2 Problem Statement 139

8.3 Notation 140

8.4 Assumptions 140

8.5 The Proposed Simulation Model for Multistage Systems with Buffers given μ_i's and T_i's (MSM3) 141

8.5.1 Simulation Model for Two-Stage Lines with a Buffer and with Nonzero Repair Times (TSM) 143

8.5.2 Simulation Model for Multistage Lines with Buffers and with Nonzero Repair Times (MSM3) 147

8.6 The Optimization Model for Multistage Production Systems With Buffers (MSM4) 152

8.6.1 The Objective Function 152

8.6.2 The Constraints 155

8.6.3 Problem formulation 156

8.6.4 Solution Methodology and Linkage between Simulation and Optimization 157

8.6.5 Results and Discussions of MSM4 162

9 Conclusions and Recommendations for Future Study **166**

9.1 Industrial Applications 166

9.2 Conclusions 167

9.3 Recommendations for future studies 170

Appendix 171

A Test Functions for The First Experiment of Testing TSFGO 171

B Test Functions for The Second Experiment of Testing TSFGO 175

C Experimental Design and Results of the Single Stage Model 176

D Fractions of the Experimental Design of the Multistage Model 180

E Experimental Design and Results of the Multistage Model 182

E.1 Experimental Design 182

E.2 Experimental Results 189

F *SLAM II* Model for Two-Stage Lines without a Buffer 196

G *SLAM II* Model for Two-Stage Lines with a Buffer 200

H *SLAM II* Model for Multistage Lines with Buffers 205

References 214

List of Tables

1.1 Cost-Tolerance functions 8

2.1 Summary of the literature. 23

3.1 Results of Example 3.2 with different drift functions 50

3.2 Results of Example 3.3 with different parameters 53

3.3 Results of Example 3.4 with different parameters 55

4.1 Input parameters and levels used in the experiment. 76

4.2 ANOVA for the single stage model. 82

5.1 Methods used for the first experiment. 97

5.2 Methods used for the second experiment. 98

5.3 Number of function evaluations of the first experiment. 99

5.4 Running time in units of standard time of the first experiment. 99

5.5 Number of function evaluations of the second experiment. 101

5.6 Final functional value of the second experiment. 102

6.1 Data for the example. 113

6.2 Results of Example 6.1 using Hooke and Jeeves algorithm. 113

6.3 Results of Example 6.1 using TSFGO algorithm. 114

6.4 Reduction of σ_i by 20% each. 115

6.5 Reduction of σ_i by 30% each. 115

6.6 Reduction of σ_i by 70% each. 115

6.7 Reduction of θ_i by 20% each. 116

6.8 Data for testing MSM2. 119

6.9 Comparison of MSM2 with *SLAM II* package. 120

6.10 Comparison of MSM2 with *Modification* with *SLAM II* package. . . . 122

7.1 Values of the parameters a_i and b_i for Example 7.1. 130

7.2 Current production system 130

7.3 Improved production system 131

7.4 Input parameters and levels used in the experiment. 132

7.5 ANOVA for the multistage model. 135

8.1 Possible states and actions for a two-stage line with buffer. 144

8.2 Comparison of TSMA with *SLAM II* package. 148

8.3 Cases used for testing MSM3A. 151

8.4 Comparison of MSM3A with *SLAM II* package. 151

8.5 Data of *Experiment 1* for testing MSM4. 163

8.6 Results of MSM4 for *Experiment 1*. 163

8.7 Data of *Experiment 2* for testing MSM4. 164

8.8 Results of MSM4 for *Experiment 2*. 164

8.9 Data of *Experiment 3* for testing MSM4. 165

8.10 Results of MSM4 for *Experiment 3*. 165

9.1 Summary of the developed models. 169

9.2 Summary of the developed algorithms. 169

A.1 H3 (n=3 and q=4). 172

A.2 H6 (n=6 and q=4). 173

A.3 S5, S7, S10. 174

C.1 Experimental design and results of the single stage model. 179

D.1 Fractions of the experimental design of the multistage model. 181

E.1 Experimental design of the multistage model. 188

E.2 Experimental results of the multistage model. 195

List of Figures

1.1 Typical cost-tolerance relationship. 8

3.1 The model. 30

3.2 Plot of ETCG =E(TC/unit good item) as a function of μ and T. . . 36

4.1 Raw material saving due to reduction of variance and shift of target (adopted from Twombly and Whiteman [1974]). 60

4.2 Flowchart for computing $\psi(\sigma)$ (SSVRA). 67

4.3 Plot of ETC =E(TC/good item) as a function of σ and T. 69

4.4 Plot of $\psi(\sigma)$. 70

4.5 Plot of $\frac{\partial ETC}{\partial \sigma}$. 72

5.1 Flowchart of TSFGO. 91

5.2 Flowchart of TSFGO (continued). 92

5.3 Flowchart of TSFGO (continued). 93

7.1 Flowchart of the algorithm MSVRA. 127

7.2 Flowchart of the algorithm MSVRA (continued). 128

7.3 Flowchart of the algorithm MSVRA (continued). 129

7.4 Defining relation of the fractional factorial design. 134

8.1 A multistage production system. 139
8.2 Flowchart of TSMA. 146
8.3 Flowchart of MSM3A. 149
8.4 Plot of $C(\theta)$. 154
8.5 Estimation process using MSM3A. 158
8.6 Flowchart for solving MSM4. 159
8.7 Flowchart for solving MSM4 (continued). 160
8.8 Flowchart for solving MSM4 (continued). 161
F.1 *SLAM II* Model for Two-Stage Lines without a Buffer. 199
G.1 *SLAM II* Model for Two-Stage Lines with a Buffer. 204
H.1 *SLAM II* Model for Multistage Lines with Buffers. 213

Abstract

Name: Mohammad A. Al-Fawzan

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In this dissertation, we consider multistage production systems in which the product has an upper specification limit (USL) and a lower specification limit (LSL) on its quality characteristic and the process deteriorates with time. That is, the mean setting of the production process drifts continuously with time, in either the positive (i.e. towards USL) or the negative (i.e. towards LSL) directions. This causes more defective items to be produced with time. We study this problem for both single and multistage production systems.

For single stage production systems, we develop a mathematical model which finds the optimal initial mean setting of the process and the optimal production cycle length when there are both USL and LSL on the quality characteristic of the product. We also study the effect of the variance reduction on the total cost of the model and conduct a sensitivity analysis to study the effect of changes in model parameters on its solution. Moreover, we develop a model for the single stage production system for general drift function and general probability density function of the quality characteristic of the product.

We extend the results of the single stage to multistage production systems. We develop a mathematical model for these systems to minimize the cost of processes adjustments, quality, and penalty for failing to deliver demanded items on time.

The model gives optimal initial mean settings for the processes and optimal production cycle lengths for every process in each stage. The parameters of this model are studied and analyzed to see their effects on the total cost by sensitivity analysis. We also study the effects of the variance of the process at every stage on the expected total cost per good item for the above model. We extend the multistage model to incorporate the work in process (WIP) inventory between stages and the maintenance of the stages through the reduction of the drift rate of each stage.

We develop a new global optimization algorithm for solving the above models. The algorithm is a hybrid approach which uses tabu search and Hooke and Jeeves schemes. The performance of the algorithm is tested on standard test functions and is compared with other global algorithms in the field. Results show that the algorithm outperforms all of these global algorithms which make it very useful for the optimization of the developed models in this dissertation.

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ملخص الأطروحة

الاسم : محمد عبدالرحمن الفوزان

العنوان : نماذج رياضية للجودة في أنظمة الانتاج المتسلسلة

التخصص : هندسة النظم

تاريخ الدرجة : ديسمبر ١٩٩٧ م

ندرس في هذه الأطروحة أنظمة الانتاج المتسلسلة حيث يكون للمنتج حد قياسي أعلى لجودة المنتج وحد قياسي أدنى، وتسوء عملية التصنيع مع الوقت أي أن متوسط عملية الانتاج ينحرف مع الوقت بإستمرار سواء في الاتجاه الموجب أي نحو الحد القياسي الأعلى أو في الإتجاه السالب أي نحو الحد القياسي الأدنى. هذا بالطبع يزيد من عدد المنتجات التالفة مع الوقت. ندرس هذه المشكلة في كل من أنظمة الانتاج ذات المرحلة الواحدة وذات المراحل المتعددة.

بالنسبة لأنظمة الانتاج ذات المرحلة الواحدة، سيتم تطوير نموذج رياضي لإيجاد القيم المثلى لكل من الوضع الأولي لمتوسط عملية الانتاج وطول فترة الانتاج آخذين في الإعتبار أن هناك حد قياسي أعلى لجودة المنتج وحد أدنى. كما سندرس تأثير تخفيض الانحراف المعياري على التكلفة الكلية وأثر عوامل النموذج على الحل. بالإضافة الى ذلك، سوف نقوم بتطوير نموذج رياضي عام لأي دالة انحراف ولأي دالة احتمالات.

بالنسبة لأنظمة الانتاج ذات المراحل المتعددة، سنطور نموذج رياضي لإيجاد القيم المثلى لكل من متوسطات العمليات وأطوال فترات الانتاج لكل مرحلة حيث أن الهدف هو تخفيض تكلفة الجودة والصيانة وغرامات تأخير الطلبات. كما سندرس ونحلل أثر عوامل النموذج الرياضي وكذلك أثر تخفيض الانحراف المعياري لكل عملية على التكلفة الكلية.

سوف نطور النموذج السابق الى نموذج رياضي آخر حيث سنأخذ في الاعتبار وجود مناطق تخزين بين المراحل المختلفة وامكانية اجراء صيانة لكل عملية عبر تخفيض معدل الانحراف.

أخيراً سوف نقوم بتطوير خوارزمية جديدة لإيجاد الحلول المثلى الشاملة للنماذج الرياضية السابقة. سيختبر أداء هذه الخوارزمية بدوال إختبارية قياسية وسيقارن بأداء أفضل الخوارزميات المنشورة. لقد أثبتت النتائج أن أداء الخوارزمية التي طورناها أفضل من أداء الخوارزميات الموجودة مما يجعلها مفيدة جداً في حل النماذج الرياضية المطورة في هذه الأطروحة.

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جامعة الملك فهد للبترول والمعادن

الظهران، المملكة العربية السعودية

ديسمبر ١٩٩٧ م

Nomenclature

$x(t)$	the random variate denoting the quality measurement of the product at time t with mean $\mu(t)$ and constant variance σ^2;
μ	the mean quality characteristic of the product when the process begins in an in-control state having variance σ^2;
μ^*	the optimal initial process mean;
τ	the elapsed time until the occurrence of the assignable cause.It is a random variable and is assumed to be exponentially distributed with a mean of $1/\lambda$ hours;
$g(\tau)$	$= \lambda e^{-\lambda\tau}, \lambda > 0, \tau \geq 0$, the density function of the occurrence time of the assignable cause;
θ	rate of drift in the process mean once occurrence of the assignable cause:
$\mu(t)$	the process mean at time t μ for $t \leq \tau$ $= \mu + (t - \tau)\theta$ for $t > \tau$;
$\phi(z)$	the probability density function (pdf) of the standardized normal variate z, the cumulative distribution being $\Phi(z)$;
USL	the upper specification limit for the quality characteristic;
LSL	the lower specification limit for the quality characteristic;
R	production rate in pieces per hour;
C_R	resetting cost;
T	cycle length (production run) in hours;
T^*	the optimal cycle length (production run) in hours;

Chapter 1

Introduction

1.1 Overview

In many production systems, the product has to pass through a number of processes performing different types of operation before it attains the desired final form. Such systems involving production activities in serial stages and holding inventories between successive stages are designated as multistage production systems (MPS). MPS are one of the most common environment in industry. MPS can be classified into four classes depending on the number of products, number of production stages, and number of machines at each stage (Goyal and Gunasekaran [1990]). The MPS classes are as follows:

1. Multistage systems with single machine at each stage and processing a single product.

2. Multistage systems with single machine at each stage and processing multiple products.

3. Multistage systems with multiple machines at each stage and processing a single product.

4. Multistage systems with multiple machines at each stage and processing multiple products.

We consider the first class in this dissertation.

Examples of multistage production systems may include the following:

- Production of rayon yarn (Gunasekaran et al. [1993]).
- Production of glass products (Imo and Das [1983]).
- Aluminum production systems (Farkas et al. [1993]).
- Soda ash production systems (Wagialla et al. [1992]).
- Iron and steel works (Hodgson and Wang [1991]).
- Production of condensors (Tsubone et al. [1991]).

Multistage production systems are characterized by the following:

- High dependence: The failure of one stage affects the operation of the others. This is known as blocking/starvation effect. The level of dependence between stages depends on the work-in-process (WIP) inventory. If the size of WIP inventory is infinity, the stages will be independent. On the other hand, if the size of the WIP inventory is zero, the stages are completely coupled.

- Expensive line stoppage: The production line may be stopped, either because of the failure of one of the stages (uncontrolled stoppage) or because of the maintenance work (controlled stoppage). When the line is stopped in either case, this may cause a delay in fulfilling the demand.

The elements of multistage production systems that we are going to study are listed below:

1. Quality: The traditional role of quality control was basically to eliminate from production systems those parts that do not conform to specifications, and to inspect and test finished products for defects. The increased emphasis on higher quality products at lower costs, combined with the worldwide competition has magnified the importance of quality control. Quality improvement has become an essential activity in most organizations, either to maintain existing customers and market share, or to make new products and technology more competitive.

2. Maintenance: In recent years, considerable attention has been devoted to maintenance role in production systems. The role of maintenance in production systems has been recognized as keeping the machines operating as long as possible, reducing the rate of defectives, minimizing the probability of machine breakdowns, minimizing lost sales due to breakdown periods, minimizing the periods on which the workers are idle, and many more.

3. WIP inventory: Buffers are installed between successive stages to keep the production line operating as long as possible. They also serve as a delay buffer for nonconforming items to pass to next stages.

4. Production schedule: Today, the competition in the market is very strong. Firms that do not fulfill their customers' demands on time, may find them

selves out of the market. Hence, production firms should look very closely to their production systems and take the necessary actions to meet their schedule (e.g., maintain the machines more frequently, increase WIP inventory, etc.).

An important special case of multistage production systems is a single stage production system which has many applications.

1.2 Statement of the problem

Multistage production systems are one of the most important types of production systems. In this dissertation, we consider single stage as well as multistage production systems in which the product has both an upper specification limit (USL) and a lower specification limit (LSL) on its quality characteristic and the process deteriorates with time. That is, the mean setting of the production process drifts continuously with time, in either the positive (i.e. towards USL) or negative (i.e. towards LSL) directions. This causes more defective items to be produced with time. Defective items can be reworked at different costs (or equivalently sold at a secondary market). Two decisions have to be made at the beginning of each production cycle. They are the initial mean setting of the process and the production cycle. Some of the cost elements that influence these decisions are the resetting cost and the cost of defective items. Clearly, if the process is reset too often, the resetting cost is more while the cost of producing defective items is less and vice versa. We study this problem for both single and multistage production systems.

For single stage production systems, we develop a mathematical model which finds the optimal initial mean setting of the process and the optimal production cycle length when there are both USL and LSL on the quality characteristic of the product. We also study the effect of the variance reduction on the total cost of the model and conduct a sensitivity analysis to study the effect of the change in model parameters on its solution. Moreover, we develop a model for the single stage production system for general drift function and general probability density function of the quality characteristic of the product.

We extend the results of the single stage model to multistage production systems. We develop a mathematical model for these systems to minimize the cost of processes adjustments, quality, and penalty for failing to deliver demanded items on time. The model gives optimal initial mean settings for the processes and optimal production cycle lengths for every process in each stage. The parameters of this model are studied and analyzed to see their effects on the total cost by sensitivity analysis. We also study the effects of the variance of the process at every stage on the expected total cost per good item for the above model. We extend the multistage model to incorporate the work in process (WIP) inventory between stages and the maintenance of the stages through the reduction of the drift rate of each stage.

We develop a new global optimization algorithm for solving the above models. The algorithm is a hybrid approach which uses tabu search and Hooke and Jeeves schemes.

1.3 Cost of Variance

In Chapter 4 and Chapter 7, we develop variance reduction models for the single stage and multistage production systems, respectively. A prerequisite to these models is a function which represents the cost of the variance. In this section, we present functions for the cost of the tolerance and its relationship to the variance.

One of the concepts that is used to evaluate quality of a manufactured product is *conformance to specification*. Tolerance is defined as the allowable variation within the design specification (Kapur et al. [1990]). Tolerance is needed because it is impossible to manufacture products at target due to process variability, material imperfections, human error, tool material, and other uncontrollable factors. Tolerancing plays a key role in design and manufacturing (Zhang and Wang [1993]). At the design stage, functionality performance and reliability are the major issues under consideration which implies that tolerances should be set as tight as possible. However, at the manufacturing stage, looser tolerances are desirable since tighter tolerances are usually associated with higher cost (Lee et al. [1993]).

There is a considerable amount of literature on tolerancing. Our purpose is not to study the tolerancing problem nor to review its literature. The aim here is to give an introduction for the following sections. For a literature review of the tolerancing problem, see the work by Wu et al. [1988] and a recent one by Abdel-Malek and Asadathorn [1994].

1.3.1 Relationship between Tolerance and Variance

As shown by Mansoor [1963], most manufacturing processes produce dimensions with normal distribution. Let μ and σ denote the mean and standard deviation for the normal distribution of the quality characteristic, x, of the product. Moreover, let tol denote the tolerance of x.

Many authors have used the following relationship between tolerance and standard deviation, i.e.

$$tol = 6\sigma \tag{1.1}$$

Among those are Speckhart [1972], Wu et al. [1988], Kapur et al. [1990], Lee et al. [1993], Gerth [1994], Krishnaswami and Mayne [1994], Nigam and Turner [1995], and Kusiak and Feng [1996].

1.3.2 Cost of Variance (Tolerance)

Cost of tolerance, $C(tol)$, is defined as the amount of expenditure needed to achieve certain levels of dimensional and geometrical accuracy (Abdel-Malek and Asadathorn [1994]). It is usually a function of design and manufacturing costs. Naturally, designs which require tighter tolerances have relatively higher costs. Also, machine tools with a small tolerance range are expensive to acquire and operate. Figure 1.1 shows a typical cost-tolerance relationship.

The curve shown in Figure 1.1 shows two well-known basic features, which are essential for a cost-tolerance relationship according to normal workshop experience (He [1991]). These two features are:

1. When tol=0, $C(tol) = \infty$.
2. $C(tol)$ should be a decreasing function of tol, tending to become flat as tol becomes large.

Several cost-tolerance functions appear in the literature. Table 1.1 shows some of the commonly used functions.

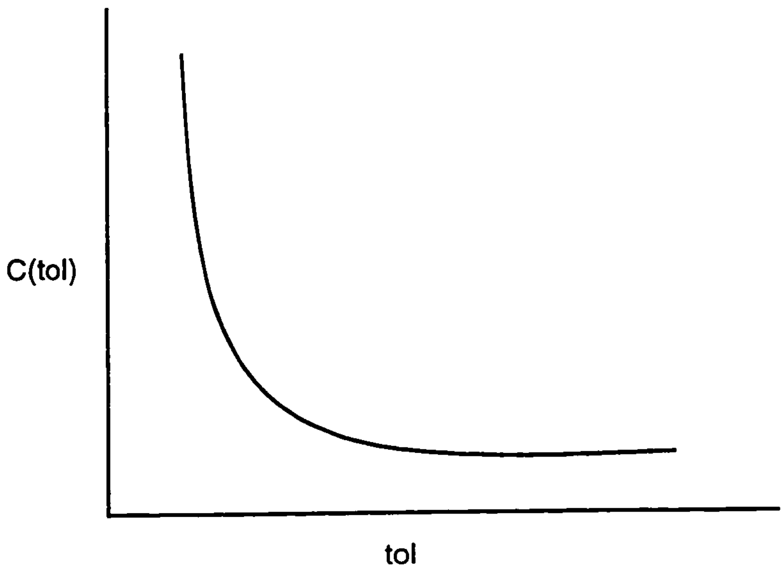


Figure 1.1: Typical cost-tolerance relationship.

Name	Function	Reference
Sutherland	$C(tol) = a(tol)^{-b}$	Sutherland and Roth [1975]
Reciprocal	$C(tol) = a/tol$	Chase and Greenwood [1988]
Reciprocal square	$C(tol) = a/(tol)^2$	Spotts [1973]
Exponential	$C(tol) = ae^{-b(tol)}$	Speckhart [1972]
Michael-Siddall	$C(tol) = a(tol)^{-b}e^{-d(tol)}$	Michael and Siddall [1981]

Table 1.1: Cost-Tolerance functions

In these functions, the parameters a, b, d can be estimated using a curve-fitting approach based on experimental data. The parameter a represents cost of producing a component, while b and d are constants which depend on the process.

Wu et al. [1988] reviewed and evaluated these functions and they found that the exponential function is the best for minimizing curve-fitting errors. The exponential function is also the most widely used in the literature (Kapur et al. [1990]). For example, it has been used by He [1991], Zhang and Wang [1993], Abdel-Malek and Asadathorn [1994], Krishnaswami and Mayne [1994].

One can use the exponential function, which represents the cost of tolerance, to represent the cost of variance by using equation (1.1). Hence, in this dissertation, the exponential function is going to be used to represent the cost of the variance in the variance reduction models that will be developed in Chapter 4 and Chapter 7 for the single stage and multistage production systems, respectively.

1.4 Proposed Work

The proposed work in this dissertation can be summarized as follows:

I. For single stage production system, the proposed work is as follows

- to do an extensive literature survey.
- to extend Rahim and Banerjee's [1988] model (SSM).
- to develop a Single Stage Variance Reduction Model (SSVRM).
- to conduct a sensitivity analysis of the (SSM) model .
- to generalize the single stage model (GSSM).

II. For multistage production system, the proposed work is as follows

- to develop a model for finding optimal production cycle and initial mean setting in multistage production systems without buffers (MSM1).
- to conduct a sensitivity analysis of the above model (MSM1).
- to develop a Variance Reduction Model for MultiStage production systems (MSVRM).
- to develop a model for Multistage Lines without Buffers and with Nonzero Repair Times (MSM2).
- to develop a simulation model for Multistage Systems with Buffers given μ_i's and T_i's (MSM3).
- to develop an optimization model for Multistage Lines with Buffers and with Nonzero Repair Times (MSM4).

III. Developing a hybrid tabu search algorithm for function minimization (TS-FGO).

1.5 Organization

In nomenclature, we give the notation that are used in common in all chapters. Notation required for a specific chapter will be introduced in that chapter. The dissertation is organized as follows: In chapter 2, we present the literature survey. Determination of the optimal production cycle and initial mean setting for single stage model (SSM) is proposed in chapter 3. We also give the generalizations of the single stage model (GSSM) in chapter 3. A sensitivity analysis and a variance reduction model for SSM is presented in chapter 4. In chapter 5, we develop a Hybrid Tabu Search Algorithm for Function Minimization. In chapter 6, we propose models for finding the optimal production cycle and initial mean setting in multistage production system (MSM1 and MSM2). A sensitivity analysis and a variance reduction model for MSM1 is presented in chapter 7. Extensions of MSM1 and MSM2 (MSM3, MSM4) to incorporate buffer storages and to take into consideration repair times are presented in chapter 8. Finally, we give conclusions and recommendations for future studies in chapter 9.

Chapter 2

Literature Review for Single and Multistage Production Systems

In this chapter, we review the literature in single as well as multistage production systems. This chapter is organized as follows: in section 2.1, we give an introduction. In section 2.2, we give some applications. We state the assumptions common to the reviewed models in section 2.3. We highlight the general approach in section 2.4. In section 2.5, we present the survey of single stage. A review of the literature of multistage production systems is given in section 2.6.

2.1 Introduction

The literature of single and multistage production systems in which the process deteriorates with time are reviewed in this chapter. Many production processes exhibit a trend or drift in the process mean during the course of operation. This problem has received little attention (Montgomery [1991a]). On the contrary, we have found a considerable literature and an increased interest in this problem. Examples of such operations include machining, drilling, grinding, milling, shaping and molding (Gibra [1967,1974]) and drawing (Hall and Eilon [1963]). The drift can be either positive or negative. That is, the drift can either be toward the upper specification limit or toward the lower specification limit of the measured quality characteristic. If the process mean drifts to one of the specification limits, the process is going to produce nonconforming items, since the product's measurable characteristic must lie within the specification limits to be considered acceptable. One would tend to think that when this happens, it may be more economical to stop the production and reset the process. An example of resetting the process is changing a worn tool.

However, there are cases where it is more economical to continue the production for some time and then reset the process. So, what is the optimal time to reset the process and at what level should the process mean be set? The optimal decision depends on the cost of resetting the process and the cost of producing nonconforming items.

2.2 Applications

The problem described above occurs in many areas. Some of those are listed below:

1. Optimal production run or production cycle (e.g., Hall and Elion [1963], Gibra [1967,1974], Rahim and Lashkari [1985], Rahim and Raouf [1988], Jeang and Yang [1992]).

2. Optimal tool replacement (e.g., Taha [1966], Rahim and Banerjee [1988], Drezner and Wesolosky [1989]).

3. Optimal maintenance policy (e.g., Schneider et al. [1990]).

4. Communications (e.g., Schneider et al. [1990]).

2.3 Assumptions

In this section, we state the common assumptions to the models discussed in this chapter. These include the following:

1. The measured quality characteristic is normally distributed.

2. The variance of the quality characteristic is constant throughout the process. Some authors relaxed this assumption (e.g. Albright and Collins [1977], Arcelus et al. [1981]).

3. There is a linear (or nonlinear) shift in the mean.

4. The drift can be either positive or negative.

5. The drift can be either deterministic or probabilisetic.

6. Nonconforming items are treated as worthless. Few authors relaxed this assumption (e.g. Arcelus and Banerjee [1987]).

7. Only one quality characteristic is considered. Few authors considered two quality characteristics (e.g. Rahim and Rouf [1988]).

8. The manufacturing system consists of only one production stage.

9. Production is continuous (i.e. transfer lines).

2.4 The general approach

In this problem, several costs are considered. The following is a list of the mostly considered types of costs in the literature.

C_s : cost of sampling.

C_I : cost of inspection.

C_r : cost associated with investigating an out of control signal.

C_c : cost of correcting any assignable cause found.

C_L : loss due to producing nonconforming items.

C_R : cost of reworking nonconforming items.

C_p : cost of production

C_a : cost of adjustment or resetting.

C_d : cost due to shutdown.

One way to build the cost model is to sum all cost elements. Hence, the cost model may be represented as

$$TC = C_s + C_I + C_r + C_c + C_L + C_R + C_p + C_a + C_d \tag{2.1}$$

Equation (2.1) is a general formula. However, one would rarely find an author who considers all of these costs in one model. Most researchers consider a subset of these costs in their model. Suppose that α is the vector to be optimized (i.e., TC is a function of α when one of the parameters in the vector α could be for example the production cycle length). Then one can optimize the function TC by setting the gradient of TC to zero as shown below

$$\nabla TC = 0 \tag{2.2}$$

Equation (2.2), of course, is just a necessary condition for optimality. However, in most quality control applications TC(α) is a convex function and, hence, (2.2) is also sufficient to guarantee global optimality.

It is usually difficult to get a closed form for α by solving equation (2.2). Hence, most authors use one or a combination of the following methods to optimize the function TC with respect to the parameters in α in (2.1):

1. *Numerical solution.* Some authors suggested solving equation (2.2) numerically using any numerical solution procedure (e.g. Newton's method).

2. *Optimization.* The function in (2.1) can be minimized using optimization techniques. These optimization techniques consist of two categories. The first category is derivative-free search procedures (e.g., the Hooke and Jeeves method). The second category is derivative-based search procedures (e.g., Newton's method). For more details see Bazaraa et al. [1993].

2.5 The literature survey of single stage production systems

2.5.1 Positive constant drift with linear trend

Hall and Eilon [1963] were the first to treat the trend in the process in an explicit manner. They assumed that the process mean is subject to a constant drift with time and it is moving towards the upper specification limit. Also, they assumed that the variance remains constant throughout the process. Their model objective was to maximize the production rate or to minimize the production cost per unit. Taha [1966] presented a procedure for determining the optimal cycle length for a cutting tool considering the wear of the tool with time which causes the machine to produce nonconforming items. He considered one measurable characteristic and he ignored the effect of the operator, machine, and the raw material. He assumed a linear trend of the mean with time. Gibra [1967] proposed models for determining the optimal production run for both stable and unstable processes. His assumptions are similar to those in Hall and Eilon [1963]. In his cost model, he included the resetting cost and a penalty for each nonconforming unit. He developed an equation which can be solved graphically.

Smith and Vemuganti [1968] generalized the model of Taha [1966]. They introduced two parameters in the linear function of the trend of the mean. The first is the initial mean and the second is the rate of wear of the tool per unit time. These two parameters are estimated initially from experience and as production continues they are updated using the sampling information. Kamat [1976] developed a smoothed Bayes control procedure for controlling the output quality characteristic when its basic underlying level is subject to systematic variation such as in tool wear. The variation is assumed to be linear and nonrandom. He used exponential smoothing to update the necessary parameter estimates.

Arcelus and Banerjee [1985] extended the work of Bisgaard et al. [1984] to consider the process in which there is a linear shift in the mean. Their objective is to select the initial setting of the mean and the run size that will maximize the expected profit per unit. Items that fail to meet the lower specification are sold as scrap. Hence, they did not consider the cost of reprocessing. Arcelus et al. [1985] considered the problem of determining the optimal schedule for producing a finite number of acceptable parts with a specified probability. The process is subject to a systematic increase in the process mean and it may be economical to change the tool and reset the machine after producing a certain number of parts. They struck a balance between the cost of resetting and the cost of producing nonconforming items in order to achieve their goal and minimize the total cost of production. They considered both specification limits.

Pugh [1988] presented methods for determining the optimal setting for a process mean and the number of parts produced before resetting where the shift in the process mean is uniformly distributed. His cost function consists of the cost of resetting, the cost of producing oversized parts, and the cost of producing undersized parts. Quesenberry [1988] proposed a statistical process control approach for adjusting a process which has a linear trend in its mean due to tool wear. He models this tool wear by a regression model over an interval of tool life. His approach determines the setting of the mean and the estimated wear since the last resetting. The objective is to maximize the expected mean square of deviations from nominal target value.

2.5.2 Positive constant drift with nonlinear trend

Gibra [1974] was the first to consider a nonlinear drift of the process mean in the positive direction. His optimal procedure establishes decision rules for resetting due to drift or the occurrence of an assignable cause. His objective is to minimize the resetting cost and the cost of producing nonconforming items. However, he did not consider the shutdown cost as well as the negative shift of the mean.

2.5.3 Negative constant drift with linear trend

Rahim and Lashkari [1985] have relaxed the assumption that the variance remains constant throughout the production period. They developed a cost function to determine the optimal length of the production run. The cost function consists of the cost of resetting the process, the cost of rejected items, the lost product cost due to shutdown, and the cost of sampling. Their objective is to determine the optimal production run by minimizing the cost function. They found that the optimal production run depends upon the magnitude and the direction of the shift and the drift. Rahim and Raouf [1988] considered the problem of determining the optimal production run for a continuous process having multi-tool machines where simultaneous gradual changes in the process mean and variance are experienced. They were the first to consider two measurable quality characteristics. Their work is an extension of Rahim and Lashkari [1985].

2.5.4 Positive constant drift and positive or negative shift

Albright and Collins [1977] presented a Bayesian model for an optimal on-line control of a process subject to continuous deterioration. This deterioration is reflected by an increase in the variance of the quality characteristic over the long run. They update the parameter estimates by sampling at each period through the use of an on-line measuring device. In their cost model, they considered the selling price, the materials cost, and the reprocessing cost. The model of Rahim and Lashkari [1985] described above is capable of handling both negative and positive constant drift. Rahim and Raouf [1988] considered both positive or negative drift and positive or negative shift. Arcelus et al. [1981] considered a nonnegative shift in both mean and variance. Arcelus and Banerjee [1987] extended the work of Arcelus et al. [1982] to include the possible rewards for nonconforming items.

2.5.5 Positive shift with no trend

In a recent paper, Chen and Chung [1996] considered the problem of determining both the optimal initial mean setting and the optimal production run for a process which shifts to an out-of-control state at a random point of time. They considered only one specification limit (i.e. lower specification limit) for the quality characteristic of the product. They developed a profit function for their model and it was optimized using Hooke and Jeeves algorithm. However, they assumed that the process mean shifts to an out-of-control state instantaneously (i.e. there is no trend). Also, they did not consider both specification limits.

2.5.6 Random drift

Rahim and Banerjee [1988] generalized the model of Gibra [1974] where they assumed that a positive drift starts at a random point of time (and not necessarily at the beginning). They have assumed that the quality characteristic under consideration has only an upper specification limit (USL) which is a multiple K of the standard deviation of the process mean (i.e. $USL = \mu + K\sigma$) (of course if there is only a lower specification limit (LSL) and the drift is negative then the foregoing argument is reversed). In their model, the proportion of defective increases as the mean drifts towards the upper specification limit. These defective items are, of course, sold at a lower price. At the end of the production cycle, the process is shut down for resetting. Clearly, if the production cycle is short, then the proportion of defective items produced by the process is less, but the frequency of resetting is more which makes the total cost of resetting more. On the other hand, if the production cycle is long, then the proportion of defective items produced by the process is more, but the frequency of resetting is less which makes the total cost of resetting less. Their model finds the optimal cycle length that strikes a balance between the costs of resetting and the cost due to defective items.

Schneider et al. [1990] considered the problem of determining the optimal starting level of the process mean and the lower point at which the process mean should be adjusted back to the starting level for a process subject to random deterioration. They assumed that the deterioration of the process mean in a given interval of time is a random variable with some mean and standard deviation. The goal is to minimize the long-run average production cost. Recently, Kubat and Lam [1992] presented a simple model for determining the optimal action limit in a slowly deteriorating repairable system by continuous monitoring. The deterioration is assumed to be approximated by a Wiener (Brownian) process with a positive drift. A repair or replacement order is initiated when the measured value of the parameter reaches the 'action time'. The optimal action limit is derived by minimizing the expected long run average total cost. Related work can be found in Hall and Eilon [1963]. Pate-Cornel et al. [1987], and Lee and Rosenblatt [1988].

2.5.7 Quadratic loss function

Drezner and Wesolowsky [1989] treated a problem which is similar to that in Gibra [1967]. They developed a simple procedure for determining the start and finish points for a tool-wear process where the rate of wear is linear and constant. They defined a quadratic loss function for the deviation from target value, not a step function as done by Gibra [1967]. They considered two cases: the first is when the quadratic loss function is symmetrical above and below the target value, and the second is when it is asymmetrical. Jeang and Yang [1992] considered the problem of selecting the optimal initial setting of the tool and the cycle of the tool replacement. Their work is an extension and generalization of the work of Drezner and Wesolowsky [1989]. They assumed that the trend in the mean (in their case the rate of wear) to be a monotone nonlinear function. The economic loss due to the deviation of the part dimension from its target value is assumed to be a quadratic function. The goal is achieved by minimizing the expected cost per unit.

As a summary, the literature surveyed in this chapter is summarized in Table 2.1. For each author, Table 2.1 shows the author(s)' name, model, objective of the model, solution methodology and the specification limits considered in the model.

Author	Year	Model	Objective	Solution	S.L.†
Hall and Eilon	1963	$\mu = \mu_0 + at$	max prod. rate min. total cost	Graphical	L&U
Taha	1966	$\mu = \mu_0 + at$	Opt. cyc. length	Numerical	L
Gibra	1967	$\mu = \mu_0 + at$	Opt. prod. run	Graphical	L&U
Smith and Vemuganti	1968	$\mu = \mu_0 + at$	Opt. tool adj.	Numerical	U
Gibra	1974	$\mu = \mu_0 + at^k, k \neq 0$	Opt. prod. run	Graphical	L&U
Kamat	1976	$\mu = \mu_0 + at$	Shift detect.	Bayesian	L&U
Arcelus et al.	1982	$\mu = \mu_0 + \Delta_j$ Δ_j:nonnegative shift	Min. total cost	Iterative method	L&U
Arcelus et al.	1985	$\mu = \mu_0 + \Delta_j$ Δ_j:nonnegative shift	Min. total cost	Iterative method	L&U
Arcelus and Banerjee	1985	$\mu = \mu_0 + (j-1)\delta$ δ:magnitude of shift	Max. unit profit	Numerical	U
Rahim and Lashkari	1985	$\mu = \mu_0 + at$ $\mu = \mu_0 - at + \delta\sigma$	Opt. prod. run	Numerical Graphical	L&U
Arcelus and Banerjee	1987	$\mu = \mu_0 + \Delta_j$	Max. unit profit	Numerical	L&U
Rahim and Raouf	1988	$\mu = \mu_0 + at$ $\mu = \mu_0 - at + \delta\sigma$	Opt. prod. run	Numerical Graphical	L&U
Rahim and Banerjee	1988	$\mu = \mu_0 + (t-\tau)\theta$ τ: random variable	Opt. prod. run	Numerical Graphical	U
Pugh	1988	$\mu = \mu_0 + (j-1)\delta$	Opt. prod. run	Numerical	U
Quesenberry	1988	$\mu = \mu_0 + aj$	Opt. tool adj.	Numerical	U
Drezner and Wesolowsky	1989	$\mu = \mu_0 + at$	Opt. cycle length	Numerical	U
Schneider et al.	1990	$\mu = \mu_0 - Y$ Y : random variable	Min total cost	Fibonacci	L
Jeang and Yang	1992	$\mu = \mu_0 + R(t)$ $R(t)$: nonlinear	Opt. cyc. length	Numerical	U
Chen and Chung	1996	$\mu = \mu_0 + \delta\sigma$	max. profit	Hooke & Jeeves	L

†S.L. : Specification limits considered.

U : Upper specification limit.

L : Lower specification limit.

Table 2.1: Summary of the literature.

2.6 The literature survey of multistage production systems

In this section, we review the related literature in multistage production systems.

Billatos and Kendall [1991] considered a variant of the multistage production system, where they restricted their attention to machining centers without considering specific demand requirement for produced items, and no explicit treatment of upper and lower specification limits. Their approach gives more emphasis on tool parameter optimization. However, they provide a nice practical example which requires 5 operations: 2 end milling, T-slot milling, drilling, and tap threading.

Agapiou [1992a] considered the problem of determining the optimum cutting conditions for multistage machining systems and utilizing the idle time at all stations as much as possible. He developed a mathematical model for this problem. Agapiou [1992b] developed an optimization procedure to determine the optimum machining parameters. Agapiou [1992c] investigated the problem of determining the optimum machining conditions for single-pass operation while reducing both the production cost and the production time. Agapiou [1992d] extended the work for multipass operations.

Alto et al. [1994] developed an expert system for tool replacement policies in metal cutting operations. Other work related to tool wear can be found in Waschkies et al. [1994] and Balazinski and Ennajimi [1994].

Gunasekaran et al. [1995] considered a multistage just in time (JIT) production system. Based on JIT philosophy, they stop the production once the process goes out of control, hence, zero defects. They developed a mathematical model to find the optimal lot sizing with respect to each stage, setup cost reduction, and the process shift reduction.

However, they did not consider the following points: (1) there is no explicit treatment of upper and lower specification limits for the quality characteristic of the product, (2) the size of the work in process is not optimized, (3) the initial setting of the process mean at each stage is not considered, (4) the process at each stage has a shift but not a drift.

2.7 Limitations of the reviewed literature

In this section, we highlight the limitations of the literature reviewed in the previous sections.

1. Limitations of the literature on single stage production systems

   - *Specification limits*: Most of the literature that we reviewed assume single specification limit. Few authors assumed double specification limits (e.g. Arcelus and Banerjee [1987]).
   - *Start of the drift*: Most of the reviewed models assume that the drift starts right from the beginning of the production cycle. To our knowledge, only Rahim and Banerjee [1988], and Chen and Chung [1996] have assumed that the drift starts at a random point of time, τ. Moreover, both models have assumed that τ follows an exponential distribution.
   - *Drift function*: In many models, the authors assume that the drift function is linear with time. Few others have assumed that the drift function is nonlinear (e.g. Gibra [1974], and Jeang and Yang [1992]).

- *Probability density function of the process*: Most of the literature that we reviewed assume that the production process follows a normal distribution. To the best of our knowledge, only Gibra [1974] relaxed this assumption, by considering a uniform distribution.
- *Joint optimization of the initial mean setting and the cycle length*: Most of the literature that we reviewed optimize either the initial mean setting or the cycle length but not both simultaneously. To our knowledge, only Arcelus and Banerjee [1987], Jeang and Yang [1992], and Chen and Chung [1996] who have considered the joint optimization of the initial mean setting and the cycle length.

2. Limitations of the literature on multistage production systems

- *Specification limits*: No explicit treatment of the specification limits either single or double.
- *Drift function*: The drift function is only considered to be constant (i.e. shift and no trend).
- *Buffer sizes*: Either no buffer is assumed or the buffer sizes are not restricted.
- *Initial setting of the process mean*: The initial setting of the process mean is not considered.
- *Budget limit*: When an investment is wanted to improve the performance of the multistage system, no constraint is set on the budget.

Chapter 3

Single Stage Production Systems Model (SSM)

In this chapter, we extend the model of Rahim and Banerjee [1988] by assuming that there are both upper and lower specification limits. Our model finds the optimal initial mean setting and the optimal cycle length when there are both USL and LSL on the quality characteristic under consideration. Later in this chapter, we present a generalized single stage model (GSSM) for general drift function and general probability distribution function of the quality characteristic of the product. This chapter is organized in the following way. An introduction is given in section 3.1. In section 3.2 we present the statement of the problem. Some new notations are introduced in Section 3.3. In Section 3.4, we state the assumptions. In section 3.5, we present our model and its solution is given in section 3.6. Then we discuss the results in section 3.7. In section 3.8, we give the generalization of the proposed model.

3.1 Introduction

In chapter 2, we have reviewed the work by Rahim and Banerjee [1988]. They have provided examples of positive as well as negative drift of the process mean. One example of a negative drift is when the diameter of a spray nozzle decreases due to clogging which reduces the amount of liquid that passes through the nozzle (Rahim and Banerjee [1988]). A tool wearout is an example of a positive drift. Many models have been developed for this problem where the general approach is to find the optimal production run (cycle length) for the process such that the total cost per good item is minimized. This total cost usually consists of the cost of producing defective items and the resetting cost. Clearly, as the cycle length increases, the former per good item increases, while the latter per good item decreases, and vice versa. The general approach is to find the cycle length that minimizes the sum of the two costs.

We assume that the resetting cost, C_R, is constant. However, it can be a function of the cycle length, $C_R(T)$, as considered by Lee and Rosenblatt [1989]. The proposed model can be easily modified by simply replacing C_R by $C_R(T)$.

Many models have been developed for the problem in which the process mean drifts with time which differ in the assumptions put on the process and quality characteristic under consideration. Most of these models are summarized in chapter 2.

In this chapter, we extend the model of Rahim and Banerjee [1988] by assuming that there are both upper and lower specification limits on the quality characteristic which are externally decided (i.e. not a function of the process standard deviation which is an internal parameter). We also assume that the initial mean setting is a controllable parameter, i.e., our model finds the optimal initial mean setting and the optimal cycle length when there are both USL and LSL on the quality characteristic under consideration.

3.2 Problem Statement

In this chapter, we consider a production process with known and constant variance. The quality characteristic of the product has both upper and lower specification limits, denoted by (USL) and (LSL), respectively. At a random point of time, τ, the process starts drifting either in the positive or negative direction with rate θ which will result in producing defective items (e.g., more oversized or undersized items, respectively) (see Figure 3.1). Oversized and undersized items can be reworked at different costs (or equivalently sold at a secondary market). The problem is to decide what should be the initial mean setting, μ^*, and the length of the cycle time, T^*, after which the process mean is reset to its initial setting, which can usually be done at a certain resetting cost (example of resetting the process is changing a wearing tool). Clearly, if the process is reset too often, the resetting cost is more while the cost of producing defective items is less and vice versa. Therefore, the goal is to find an initial mean setting, μ^*, and a cycle length, T^*, that strike a compromise between these two conflicting objectives.

The results of our model (μ^*,T^*) are helpful to engineers at the the shop floor. In metal cutting processes for example, the initial mean setting, μ^*, can be translated through some transformations to the machining parameters (e.g. cutting speed, feed rate, depth of cut). Taylor's equation (Conrad and McClamroch [1987], Iakovou et al. [1996]) is a necessary step in such transformations. Very few papers have

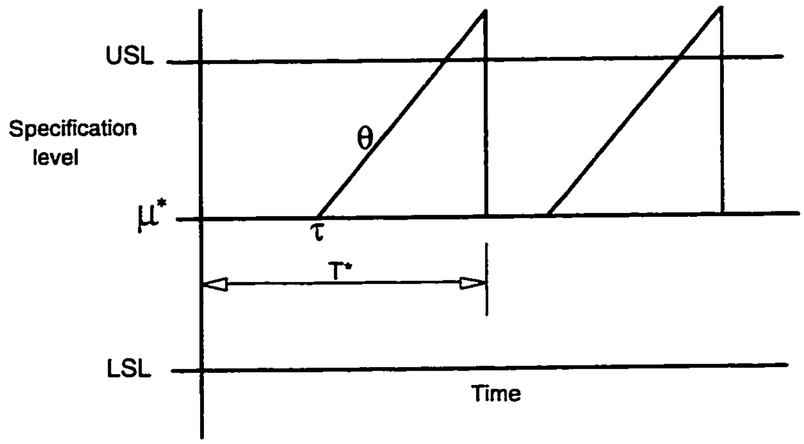


Figure 3.1: The model.

appeared in the literature which find the optimal cutting parameters and the cycle length. Conrad and McClamroch [1987] developed a stochastic model which finds the optimal feed rate and cycle length for the cutting tool. Iakovou et al. [1996] developed a model which finds the optimal cutting speed and the optimal tool replacement policy.

In the sequel, we give an example of our problem. Consider the production of shafts whose inner diameters have both USL and LSL. Shafts with inner diameters less than their LSL can be reworked to trim the excess material, and consequently transform them into good ones. But shafts with inner diameters greater than USL can not be reworked and thus should be scrapped or sold at a secondary market at a substantially reduced price. This makes the penalty for producing shafts with inner diameters less than LSL to be less than the penalty for those shafts with inner diameters greater than USL.

The above process with a tool-wear (e.g. turning operations), has a negative drift. That is, as the tool starts to wear out, its shift will be towards LSL, and the inner diameters of the shaft gets smaller with time.

The proposed model is different from that of Rahim and Banerjee [1988] in that both USL and LSL are considered (where only one of these limits is considered in their model), and that the initial mean setting is considered as a parameter to be optimized (while in their model, the initial mean setting is considered constant and given), which might have considerable reduction effect on total cost. The proposed model is also related to the work of Arcelus and Banerjee [1987] discussed in chapter 2 in that their work and the proposed model consider finding the optimal initial mean setting, μ^*, and the optimal cycle length, T^*. However, it is different in that they consider the process to start drifting right from the beginning, while in our model, we consider that the process starts drifting at a random point in time. In many realistic situations, the assumption that the drift happens right from the beginning is clearly unrealistic (e.g., sudden drop in the voltage might be the cause of the drift. This event usually happens randomly).

3.3 The Notation

We present below some new notation that are needed for this chapter.

$g(\tau)$	$= \lambda e^{-\lambda\tau}, \lambda > 0, \tau \geq 0$, the density function of the occurrence time of the assignable cause;
C_l	cost of producing an undersized item;
C_u	cost of producing an oversized item;
C_p	cost of producing a good item;
C_R	resetting cost;

R	production rate in pieces per hour;
$p_l(t)$	probability of producing an undersized item at time t (i.e.,$x(t) < LSL$);
$p_u(t)$	probability of producing an oversized item at time t (i.e.,$x(t) > USL$);
$D_l(T, \mu)$	average number of undersized items produced per unit time during T, given that the process is started at mean setting equals to μ;
$D_u(T, \mu)$	average number of oversized items produced per unit time during T, given that the process is started at mean setting equals to μ.

3.4 Assumptions

Before we develop our model we make the following assumptions:

1. The process begins in an in-control state having a normally distributed quality characteristic with mean μ and variance σ^2.

2. The process starts deteriorating at a random point of time, and deterioration is linear with time.

3. The process variance remains constant.

4. The material cost is either independent of the choice of μ and T (e.g. the process of producing inner holes in shafts), or their effect on cost of material can be assumed negligible. This assumption is implicitly made in most of the literature of this problem.

3.5 The Proposed Model

The probability of an oversized item at time t (i.e., $x(t) > USL$, $p_u(t)$) is given by

$$\begin{aligned} p_u(t) &= Pr[x(t) > USL \mid \mu(t), \sigma^2] \\ &= Pr[z \geq \frac{USL-\mu}{\sigma}] \cdot Pr[t < \tau] \\ &+ \int_0^t Pr[z \geq \frac{USL-(\mu+(t-\tau)\theta)}{\sigma}] g(\tau) d\tau \\ &= (1-\Phi(\frac{USL-\mu}{\sigma}))e^{-\lambda t} + \int_0^t (1-\Phi(\frac{USL-\mu(t)}{\sigma}))\lambda e^{-\lambda\tau} d\tau \end{aligned} \tag{3.1}$$

By integration by parts and after simplification,

$$\begin{aligned} p_u(t) = \; 1 \; & -\Phi(\frac{USL-\mu}{\sigma} - \frac{\theta t}{\sigma}) - [\Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\ - \; & \Phi(\frac{USL-\mu}{\sigma} - \frac{\theta t}{\sigma} + \frac{\lambda\sigma}{\theta})] \times \exp(-\lambda\{t - \frac{USL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \end{aligned} \tag{3.2}$$

Thus, the average number of oversized items per unit time during production cycle T, is

$$D_u(T,\mu) = \frac{R}{T}\int_0^T p_u(t)dt \tag{3.3}$$

$$\begin{aligned} D_u(T,\mu) &= R - \{\frac{R}{T}[\frac{\sigma}{\theta}(\frac{USL-\mu}{\sigma}\Phi(\frac{USL-\mu}{\sigma}) - (\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\ &\Phi\;(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}) + \phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma})) \\ &- \frac{1}{\lambda}\exp(-\lambda\{T - \frac{USL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \\ &\{\;\Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) - \Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma})\} \\ &+ \frac{1}{\lambda}\{\Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma})\}]\} \end{aligned} \tag{3.4}$$

Similarly, the probability of an undersized item at time t (i.e.,$x(t) < LSL$, $p_l(t)$) is given by

$$\begin{aligned} p_l(t) &= Pr[x(t) < LSL \mid \mu(t), \sigma^2] \\ &= Pr[z \leq \frac{LSL-\mu}{\sigma}] \cdot Pr[t < \tau] \\ &+ \int_0^t Pr[z \leq \frac{LSL-(\mu+(t-\tau)\theta)}{\sigma}] g(\tau) d\tau \end{aligned} \quad (3.5)$$

$$\begin{aligned} p_l(t) = \Phi & (\frac{LSL-\mu}{\sigma} - \frac{\theta t}{\sigma}) + [\Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\ - & \Phi(\frac{LSL-\mu}{\sigma} - \frac{\theta t}{\sigma} + \frac{\lambda\sigma}{\theta})] \times \exp(-\lambda\{t - \frac{LSL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \end{aligned} \quad (3.6)$$

Hence, the average number of undersized items per unit time during production cycle T, is

$$D_l(T,\mu) = \frac{R}{T}\int_0^T p_l(t)dt \quad (3.7)$$

$$\begin{aligned} D_l(T,\mu) &= \frac{R}{T}[\frac{\sigma}{\theta}(\frac{LSL-\mu}{\sigma}\Phi(\frac{LSL-\mu}{\sigma}) - (\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\ &\Phi \; (\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) + \phi(\frac{LSL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})) \\ &- \frac{1}{\lambda}\exp(-\lambda\{T - \frac{LSL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \\ &\{ \; \Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) - \Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma})\} \\ &+ \frac{1}{\lambda}\{\Phi(\frac{LSL-\mu}{\sigma}) - \Phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})\}] \end{aligned} \quad (3.8)$$

Thus, the expected total cost during the production cycle T, can be calculated as follows:

$$E(TC) = C_R + TC_lD_l(T,\mu) + TC_uD_u(T,\mu) + C_pT[R - D_l(T,\mu) - D_u(T,\mu)] \quad (3.9)$$

Then, the expected total cost per good item is given by

$$\begin{aligned} ETCG &= E(TC/\text{unit good item}) \quad (3.10) \\ &= \frac{C_R + TC_lD_l(T,\mu) + TC_uD_u(T,\mu)}{T[R - D_l(T,\mu) - D_u(T,\mu)]} + C_p \end{aligned}$$

Since C_p is a constant, one can see that minimizing (3.10) is equivalent to minimizing the following function

$$ETCG = E(TC/\text{unit good item}) = \frac{C_R + TC_lD_l(T,\mu) + TC_uD_u(T,\mu)}{T[R - D_l(T,\mu) - D_u(T,\mu)]} \quad (3.11)$$

A plot of a typical $E(TC/\text{unit good item})$ is depicted in figure 3.2.

It is interesting to note that our model reduces to the one of Rahim and Banerjee [1988] when: $C_l = 0$ (no LSL), $D_l(T,\mu) = 0$, $C_u = U$, $D_u(T,\mu) = R - W(T)$, $USL=\mu + K\sigma$, and $C_p = 0$.

It is clear that θ, λ, σ are internal parameters of the process while C_l, C_u, C_R are prices that are external in nature, and R is a production parameter.

3.6 The Solution

Clearly, $E(TC/\text{unit good item})$ is a function of two variables μ and T. One can differentiate the function partially with respect to μ and T as follows:

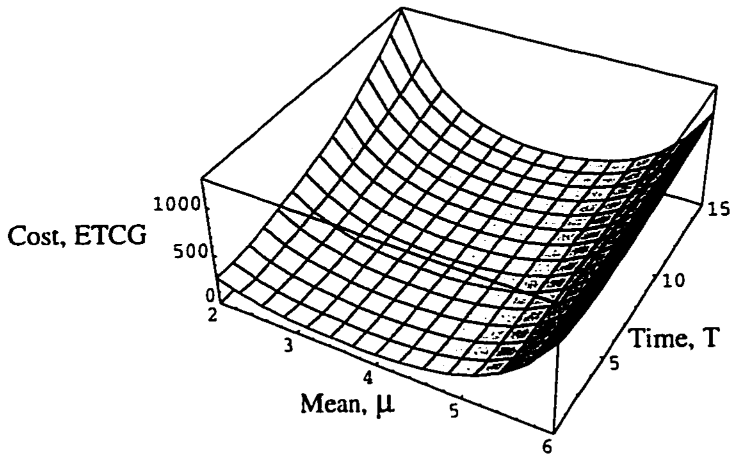


Figure 3.2: Plot of ETCG =E(TC/unit good item) as a function of μ and T.

The partial derivative of $ETCG$ (eq. (3.11)) with respect to μ is as follows:

$$\frac{\partial ETCG}{\partial \mu} = \frac{A - B}{(R - D_l(T, \mu) - D_u(T, \mu))^2} \tag{3.12}$$

where;

$A = (R - D_l(T, \mu) - D_u(T, \mu))(C_l \frac{\partial D_l(T,\mu)}{\partial \mu} + C_u \frac{\partial D_u(T,\mu)}{\partial \mu})$

$B = (C_R/T + C_l D_l(T, \mu) + C_u D_u(T, \mu))(R - \frac{\partial D_l(T,\mu)}{\partial \mu} - \frac{\partial D_u(T,\mu)}{\partial \mu})$

$$\begin{aligned}
\frac{\partial D_l(T, \mu)}{\partial \mu} &= \frac{R}{T}[-(\frac{LSL-\mu}{\sigma\theta})\phi(\frac{LSL-\mu}{\sigma}) - \frac{1}{\theta}\Phi(\frac{LSL-\mu}{\sigma}) \\
&+ (\frac{LSL-\mu-\theta T}{\theta\sigma})\phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) + \frac{1}{\theta}\Phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&+ (\frac{LSL-\mu}{\sigma\theta})\phi(\frac{LSL-\mu}{\sigma}) - \frac{1}{\theta}\phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&- \frac{1}{\lambda\sigma}\exp(-\lambda\{T - \frac{LSL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\})\{-\phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\
&+ \phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma}) - (\frac{\lambda\sigma}{\theta})\} \\
&- \frac{1}{\lambda\sigma}\{\phi(\frac{LSL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})\}]
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\frac{\partial D_u(T,\mu)}{\partial \mu} &= -\frac{R}{T}[-(\frac{USL-\mu}{\sigma\theta})\phi(\frac{USL-\mu}{\sigma}) - \frac{1}{\theta}\Phi(\frac{USL-\mu}{\sigma}) \\
&+ (\frac{USL-\mu-\theta T}{\theta\sigma})\phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}) + \frac{1}{\theta}\Phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&+ (\frac{USL-\mu}{\sigma\theta})\phi(\frac{USL-\mu}{\sigma}) - \frac{1}{\theta}\phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma})(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&- \frac{1}{\lambda\sigma}\exp(-\lambda\{T - \frac{USL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\})\{-\phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\
&+ \phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma}) - (\frac{\lambda\sigma}{\theta})\} \\
&- \frac{1}{\lambda\sigma}\{\phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma})\}]
\end{aligned} \tag{3.14}$$

The partial derivative of $ETCG$ (eq. (3.11)) with respect to T is as follows:

$$\frac{\partial ETCG}{\partial T} = \frac{C-D}{(R - D_l(T,\mu) - D_u(T,\mu))} \tag{3.15}$$

where;

$C = C_l\frac{\partial D_l(T,\mu)}{\partial T} + C_u\frac{\partial D_u(T,\mu)}{\partial T}$

$D = \frac{C_R}{T^2}(C_R/T + C_lD_l(T,\mu) + C_uD_u(T,\mu))(\frac{(\frac{\partial D_l(T,\mu)}{\partial T} + \frac{\partial D_u(T,\mu)}{\partial T})}{(R - D_l(T,\mu) - D_u(T,\mu))})$

$$\begin{aligned}
\frac{\partial D_l(T,\mu)}{\partial T} &= \frac{R}{T}[\phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})(\frac{\theta}{\lambda\sigma}) + \Phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&- \frac{1}{\lambda}\exp(-\lambda\{T - \frac{LSL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\})\{\phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma})(\frac{\theta}{\sigma}) \\
&- \lambda\Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) + \lambda\Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma})\}] \\
&- \frac{R}{T^2}[\frac{\sigma}{\theta}(\frac{LSL-\mu}{\sigma}\Phi(\frac{LSL-\mu}{\sigma}) - (\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})\Phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&+ \phi(\frac{LSL-\mu}{\sigma}) - \phi(\frac{LSL-\mu}{\sigma} - \frac{\theta T}{\sigma})) \\
&- \frac{1}{\lambda}\exp(-\lambda\{T - \frac{LSL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \\
&\{\ \Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) - \Phi(\frac{LSL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma})\}
\end{aligned}$$

$$\{\quad \Phi(\frac{LSL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta})-\Phi(\frac{LSL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta}-\frac{\theta T}{\sigma})\}$$
$$+\quad \frac{1}{\lambda}\{\Phi(\frac{LSL-\mu}{\sigma})-\Phi(\frac{LSL-\mu}{\sigma}-\frac{\theta T}{\sigma})\}]$$

(3.16)

$$\begin{aligned}\frac{\partial D_u(T,\mu)}{\partial T} &= -\frac{R}{T}[\phi(\frac{USL-\mu}{\sigma}-\frac{\theta T}{\sigma})(\frac{\theta}{\lambda\sigma})+\Phi(\frac{USL-\mu}{\sigma}-\frac{\theta T}{\sigma}) \qquad (3.17)\\
&- \frac{1}{\lambda}\exp(-\lambda\{T-\frac{USL-\mu}{\theta}-\frac{\lambda\sigma^2}{2\theta^2}\})\{\phi(\frac{USL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta}-\frac{\theta T}{\sigma})(\frac{\theta}{\sigma})\\
&- \lambda\Phi(\frac{USL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta})+\lambda\Phi(\frac{USL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta}-\frac{\theta T}{\sigma})\}]\\
&+ \frac{R}{T}-\frac{R}{T^2}[\frac{\sigma}{\theta}(\frac{USL-\mu}{\sigma}\Phi(\frac{USL-\mu}{\sigma})-(\frac{USL-\mu}{\sigma}-\frac{\theta T}{\sigma})\\
&\Phi\quad (\frac{USL-\mu}{\sigma}-\frac{\theta T}{\sigma})+\phi(\frac{USL-\mu}{\sigma})-\phi(\frac{USL-\mu}{\sigma}-\frac{\theta T}{\sigma}))\\
&- \frac{1}{\lambda}\exp(-\lambda\{T-\frac{USL-\mu}{\theta}-\frac{\lambda\sigma^2}{2\theta^2}\})\\
&\{\quad \Phi(\frac{USL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta})-\Phi(\frac{USL-\mu}{\sigma}+\frac{\lambda\sigma}{\theta}-\frac{\theta T}{\sigma})\}\\
&+ \frac{1}{\lambda}\{\Phi(\frac{USL-\mu}{\sigma})-\Phi(\frac{USL-\mu}{\sigma}-\frac{\theta T}{\sigma})\}]\end{aligned}$$

By equating each of the resulting partials (3.12 and 3.15) to zero, one gets two equations in μ and T, which can be solved numerically.

Another, and more direct approach, is to use multidimensional search algorithms, such as Hooke and Jeeves method, Newton's method, etc., (Bazaraa et al. [1993]) to optimize the function with respect to both μ and T.

One can see that both approaches are supposed to converge to a stationary point in general (which, of course, may not be a local minimum). We conjecture that the function $ETCG$ is convex, and therefore the stationary point obtained at termination of the two methods is actually a global (or best) minimum. We have plotted the cost functions for several examples and all the resulting plots show convexity which support the convexity claim. Moreover, the hybrid tabu search algorithm (TSFGO) which is developed in chapter 5 for global optimization was used to solve several examples of the above model and the results are identical for this global approach and the simple Hooke and Jeeves algorithm, i.e., the local minimum is also global supporting unimodality of the function, and strengthening our claim of convexity. Of course, the above argument is not a proof and the above conjecture remains to be proven.

Of course, to confirm this conjecture one has to get the hessian $H(\mu, T)$ of $ETCG$ which is the matrix of second partials of $ETCG$ with respect to both μ and T or

$$H(\mu, T) = \begin{bmatrix} \frac{\partial^2 ETCG}{\partial^2 \mu} & \frac{\partial^2 ETCG}{\partial \mu \partial T} \\ \\ \frac{\partial^2 ETCG}{\partial \mu \partial T} & \frac{\partial^2 ETCG}{\partial^2 T} \end{bmatrix} \tag{3.18}$$

and check the sign definiteness of $H(\mu, T)$. Then, if it is positive semidefinite, then $ETCG$ is actually convex. Looking at equation 3.12 and equation 3.15, it is clear that this job is formidable. However, we still conjecture that $H(\mu, T)$ is positive semidefinite, and therefore $ETCG$ is convex, and even if $ETCG$ is not convex, then the above methods converge to a point satisfying first order necessary conditions of optimality.

3.7 Results and Discussion

We used the optimization procedure of Hooke and Jeeves (Bazaraa et al. [1993]) to determine the optimal initial setting of the process mean, and the optimal cycle length using equation (3.11). Next, we present an example to illustrate our model.

Example 3.1:

This example has been adapted from Rahim and Banerjee [1988]. Consider a process which produces shafts. The inner diameter of the shafts have upper and lower specification limits USL=12 inches, LSL=10 inches, respectively. The process output can be described by a normal distribution with standard deviation, σ=1 inch (which characterizes the variation of the output of the process). The process mean drifts with rate θ=0.1 inch/hour at a random point of time that is exponentially distributed with λ=0.05 hour. The process produces shafts at a rate of R=500 units/hour. The resetting cost is C_R=\$300.

There are penalty costs for producing undersized or oversized shafts which are C_l=\$8 and C_u=\$8, respectively.

This example has been solved using Hooke and Jeeves algorithm. In this algorithm, we used the golden section method as a line search subroutine with a final interval of length 1×10^{-10}. The results are as follows:

optimal initial setting of process mean, $\mu^* = 10.96$ inches,

optimal cycle length, $T^* = 6.84$ hours,

ETCG = \$3.89.

The results of this model (SSM) are important and useful. The models in the literature lack the joint optimization of (1) initial mean setting, and (2) production cycle length when both specification limits are considered and the drift starts at a random point of time. The significance of our model is due to the linking of both the above two elements in one integrated model.

3.8 Generalization of the drift function only for the SSM (GSSM1)

In section 3.5, we have developed the SSM for a linear drift function. In this section, we generalize the SSM for a general drift function. The necessary changes are to adopt $D_u(T,\mu)$ (eq. 3.4) and $D_l(T,\mu)$ (eq. 3.8) for a general drift function.

Let :

$f_t(x) \sim N(\mu(t), \sigma^2)$

$F_t(x)$: CDF of $f_t(x)$

$R(t,\tau)$:drift function

The probability of an oversized item at time t, $x(t) > USL$, is

$$\begin{aligned}
p_u(t) &= \Pr[x(t) > USL|\mu(t), \sigma^2] \\
&= \Pr[z \geq \frac{USL-\mu}{\sigma}] \cdot \Pr[t < \tau] + \int_0^t \Pr[z \geq \frac{USL-\mu-R(t,\tau)}{\sigma}] \cdot g(\tau)d\tau \\
&= [1-\Phi(\frac{USL-\mu}{\sigma})] \cdot \int_t^\infty \lambda e^{-\lambda\tau}d\tau + \int_0^t [1-\Phi(\frac{USL-\mu-R(t,\tau)}{\sigma})] \cdot g(\tau)d\tau \\
&= [1-\Phi(\frac{USL-\mu}{\sigma})] \cdot e^{-\lambda t} + \int_0^t \lambda e^{-\lambda\tau}d\tau - \int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau \\
&= e^{-\lambda t} - e^{-\lambda t}\Phi(\frac{USL-\mu}{\sigma}) + 1 - e^{-\lambda t} - \int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau
\end{aligned}$$

$$= \quad 1 - e^{-\lambda t}\Phi(\frac{USL-\mu}{\sigma}) - \int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau \qquad (3.19)$$

The average number of oversized items per unit time during the cycle T.

$$\begin{aligned} D_u(T,\mu) &= \frac{r}{T}\int_0^T p_u(t)dt \\ &= \frac{R}{T}\int_0^T [1 - e^{-\lambda t}\Phi(\frac{USL-\mu}{\sigma}) - \int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau]dt \\ &= \frac{R}{T}[T - \int_0^T e^{-\lambda t}\Phi(\frac{USL-\mu}{\sigma})dt \\ &- \int_0^T\int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau dt] \\ &= \frac{R}{T}[T - \frac{1}{\lambda}(1-e^{-\lambda T})\Phi(\frac{USL-\mu}{\sigma}) \\ &- \int_0^T\int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau dt] \\ &= R - \frac{R}{\lambda T}(1-e^{-\lambda T})\Phi(\frac{USL-\mu}{\sigma}) \\ &- \frac{R}{T}\int_0^T\int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau dt \qquad (3.20) \end{aligned}$$

Similarly, the probability of an undersized item at time t, $x(t) < LSL$, is

$$\begin{aligned} p_l(t) &= \Pr[x(t) < LSL|\mu(t),\sigma^2] \\ &= e^{-\lambda t}\Phi(\frac{LSL-\mu}{\sigma}) + \int_0^t \Phi(\frac{LSL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau \qquad (3.21) \end{aligned}$$

The average number of undersized items per unit time during the cycle T.

$$
\begin{aligned}
D_l(T,\mu) &= \frac{R}{T}\int_0^T p_l(t)dt \\
&= \frac{R}{\lambda T}(1-e^{-\lambda T})\Phi(\frac{LSL-\mu}{\sigma}) \\
&+ \frac{R}{T}\int_0^T\int_0^t \Phi(\frac{LSL-\mu-R(t,\tau)}{\sigma})\cdot g(\tau)d\tau dt \qquad (3.22)
\end{aligned}
$$

In what follows, we present some special cases of the generalized model of the single stage model (GSSM1).

3.8.1 Special case 1: a positive shift (constant drift function)

In this section, we present a special case of the generalized model of the single stage model (GSSM1) where the drift function is constant. The constat drift function has been considered in the literature (for example, see Chen and Chung [1996]).

Let :

$f_t(x) \sim N(\mu(t), \sigma^2)$

$R(t,\tau) = \delta\sigma$

The probability of an oversized item at time t, $x(t) > USL$, is

$$
\begin{aligned}
p_u(t) &= \Pr[x(t) > USL|\mu(t),\sigma^2] \\
&= \Pr[z \geq \frac{USL-\mu}{\sigma}]\cdot\Pr[t<\tau] + \int_0^t \Pr[z \geq \frac{USL-\mu-R(t,\tau)}{\sigma}]\cdot g(\tau)d\tau
\end{aligned}
$$

$$
\begin{aligned}
&= [1 - \Phi(\frac{USL - \mu}{\sigma})] \cdot \int_t^{\infty} \lambda e^{-\lambda\tau} d\tau + \int_0^t [1 - \Phi(\frac{USL - \mu - \delta\sigma}{\sigma})] \cdot g(\tau) d\tau \\
&= [1 - \Phi(\frac{USL - \mu}{\sigma})] e^{-\lambda t} + [1 - \Phi(\frac{USL - \mu}{\sigma} - \delta)](1 - e^{-\lambda t}) \\
&= 1 - \Phi(\frac{USL - \mu}{\sigma} - \delta) - e^{-\lambda t}[\Phi(\frac{USL - \mu}{\sigma}) - \Phi(\frac{USL - \mu}{\sigma} - \delta)] \qquad (3.23)
\end{aligned}
$$

The average number of oversized items per unit time during the cycle T,

$$
\begin{aligned}
D_u(T, \mu) &= \frac{R}{T} \int_0^T p_u(t) dt \\
&= \frac{R}{T} [\int_0^T 1 dt - \int_0^T \Phi(\frac{USL - \mu}{\sigma} - \delta) dt \\
&- \int_0^T e^{-\lambda t} [\Phi(\frac{USL - \mu}{\sigma}) - \Phi(\frac{USL - \mu}{\sigma} - \delta)] dt] \\
&= R - R\Phi(\frac{USL - \mu}{\sigma} - \delta) - \frac{R}{T}[\Phi(\frac{USL - \mu}{\sigma}) - \Phi(\frac{USL - \mu}{\sigma} - \delta)] \\
&(\; T - \frac{1}{\lambda}(1 - e^{-\lambda T})) \qquad (3.24)
\end{aligned}
$$

The probability of an undersized item at time t, $x(t) > USL$, is

$$
\begin{aligned}
p_l(t) &= \Pr[x(t) < LSL | \mu(t), \sigma^2] \\
&= \Pr[z \leq \frac{LSL - \mu}{\sigma}] \cdot \Pr[t < \tau] + \int_0^t \Pr[z \leq \frac{LSL - \mu - R(t, \tau)}{\sigma}] \cdot g(\tau) d\tau \\
&= \Phi(\frac{LSL - \mu}{\sigma}) \cdot \int_t^{\infty} \lambda e^{-\lambda\tau} d\tau + \int_0^t \Phi(\frac{LSL - \mu - \delta\sigma}{\sigma}) \cdot g(\tau) d\tau \\
&= \Phi(\frac{LSL - \mu}{\sigma}) e^{-\lambda t} + \Phi(\frac{LSL - \mu}{\sigma} - \delta)(1 - e^{-\lambda t}) \qquad (3.25)
\end{aligned}
$$

The average number of undersized items per unit time during the cycle T.

$$
\begin{aligned}
D_u(T,\mu) &= \frac{R}{T}\int_0^T p_l(t)dt \\
&= \frac{R}{T}[\int_0^T \Phi(\frac{LSL-\mu}{\sigma})e^{-\lambda t}dt + \int_0^T \Phi(\frac{LSL-\mu}{\sigma}-\delta)(1-e^{-\lambda t})dt] \\
&= \frac{R}{T}\Phi(\frac{LSL-\mu}{\sigma})(\frac{1}{\lambda})(1-e^{-\lambda T}) + \frac{R}{T}\Phi(\frac{LSL-\mu}{\sigma}-\delta) \\
&\quad (\quad T - \frac{1}{\lambda}(1-e^{-\lambda T}))
\end{aligned}
\tag{3.26}
$$

3.8.2 Special case 2: Polynomial Drift Function

In this section, we present a special case of the generalized model of the single stage model (GSSM1) where the drift function is polynomial. The polynomial drift function has been considered in the literature (for example, see Jeang and Yanng [1992]).

Let :

$f_t(x) \sim N(\mu(t), \sigma^2)$

$R(t,\tau) = a + b(t-\tau) + c(t-\tau)^2$

The probability of an oversized item at time t, $x(t) > USL$, is

$$
\begin{aligned}
p_u(t) &= \Pr[x(t) > USL|\mu(t),\sigma^2] \\
&= \Pr[z \geq \frac{USL-\mu}{\sigma}]\cdot \Pr[t<\tau] + \int_0^t \Pr[z \geq \frac{USL-\mu-R(t,\tau)}{\sigma}]\cdot g(\tau)d\tau \\
&= [1-\Phi(\frac{USL-\mu}{\sigma})]\cdot \int_t^\infty \lambda e^{-\lambda\tau}d\tau + \int_0^t [1-\Phi(\frac{USL-\mu-R(t,\tau)}{\sigma})]\cdot g(\tau)d\tau
\end{aligned}
$$

$$
\begin{aligned}
&= [1 - \Phi(\frac{USL - \mu}{\sigma})] \cdot e^{-\lambda t} + \int_0^t \lambda e^{-\lambda\tau} d\tau - \int_0^t \Phi(\frac{USL - \mu - R(t,\tau)}{\sigma}) \cdot g(\tau) d\tau \\
&= e^{-\lambda t} - e^{-\lambda t}\Phi(\frac{USL - \mu}{\sigma}) + 1 - e^{-\lambda t} - \int_0^t \Phi(\frac{USL - \mu - R(t,\tau)}{\sigma}) \cdot g(\tau) d\tau \\
&= 1 - e^{-\lambda t}\Phi(\frac{USL - \mu}{\sigma}) - \int_0^t \Phi(\gamma)) \cdot \lambda e^{-\lambda\tau} d\tau \qquad (3.27)
\end{aligned}
$$

where $\gamma = \frac{USL-\mu-a-b(t-\tau)-c(t-\tau)^2}{\sigma}$

Let $A1 = \int_0^t \Phi(\gamma)) \cdot \lambda e^{-\lambda\tau} d\tau$

Integrating by parts

$$
\begin{aligned}
A1 &= [-e^{-\lambda\tau}\Phi(\gamma)]_0^t + \underbrace{\int_0^t (\frac{b + 2c(t-\tau)}{\sigma}) \cdot \phi(\gamma)) \cdot e^{-\lambda\tau} d\tau}_{B1} \qquad (3.28) \\
&= [\Phi(\frac{USL - \mu - a - bt - ct^2}{\sigma}) - e^{-\lambda t}\Phi(\frac{USL - \mu - a}{\sigma})] + B1
\end{aligned}
$$

where

$$
B1 = \int_0^t (\frac{b + 2c(t-\tau)}{\sigma}) \cdot \phi(\gamma)) \cdot e^{-\lambda\tau} d\tau \qquad (3.29)
$$

There is no closed form solution for the integral in (3.29).

Thus,

$$
\begin{aligned}
p_u(t) &= 1 - e^{-\lambda t}[\Phi(\frac{USL - \mu}{\sigma}) - \Phi(\frac{USL - \mu - a}{\sigma})] \qquad (3.30) \\
&- \Phi(\frac{USL - \mu - a - bt - ct^2}{\sigma}) - B1
\end{aligned}
$$

The average number of oversized items per unit time during the cycle T,

$$\begin{aligned}
D_u(T,\mu) &= \frac{R}{T}\int_0^T p_u(t)dt \\
&= \frac{R}{T}[T - \frac{1}{\lambda}(1-e^{-\lambda T})\Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{USL-\mu-a}{\sigma}) \\
&- \int_0^T \Phi(\frac{USL-\mu-a-bt-ct^2}{\sigma})dt - \int_0^T B1dt] \\
&= R - \frac{R}{\lambda T}(1-e^{-\lambda T})\Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{USL-\mu-a}{\sigma}) \\
&- \frac{R}{T}\int_0^T \int_{-\infty}^{\frac{USL-\mu-a-bt-ct^2}{\sigma}} \frac{1}{\sqrt{2\pi}}e^{-0.5x^2}dydt \\
&- \frac{R}{T}\int_0^T\int_0^t \frac{1}{\sqrt{2\pi}}(\frac{b+2c(t-\tau)}{\sigma})e^{-\lambda\tau} \\
&\quad e^{-0.5(\frac{USL-\mu-a-b(t-\tau)-c(t-\tau)^2}{\sigma})^2}d\tau dt
\end{aligned} \tag{3.31}$$

The average number of defectives due to LSL can be obtained in a similar way. In order to evaluate $D_u(T,\mu)$ at different values of T and μ, one has to resort to numerical integration.

3.8.3 Special case 3: Exponential Drift Function

In this section, we present a special case of the generalized model of the single stage model (GSSM1) where the drift function is exponential. The exponential drift function has been considered in the literature (for example, see Jeang and Yanng [1992]).

Let :

$f_t(x) \sim N(\mu(t), \sigma^2)$

$R(t,\tau) = ae^{b(t-\tau)}$

The probability of an oversized item at time t, $x(t) > USL$, is

$$\begin{aligned}
p_u(t) &= \Pr[x(t) > USL|\mu(t),\sigma^2] \\
&= \Pr[z \geq \frac{USL-\mu}{\sigma}] \cdot \Pr[t < \tau] + \int_0^t \Pr[z \geq \frac{USL-\mu-R(t,\tau)}{\sigma}] \cdot g(\tau)d\tau \\
&= [1-\Phi(\frac{USL-\mu}{\sigma})] \cdot \int_t^\infty \lambda e^{-\lambda\tau}d\tau + \int_0^t [1-\Phi(\frac{USL-\mu-R(t,\tau)}{\sigma})] \cdot g(\tau)d\tau \\
&= [1-\Phi(\frac{USL-\mu}{\sigma})] \cdot e^{-\lambda t} + \int_0^t \lambda e^{-\lambda\tau}d\tau - \int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau \\
&= e^{-\lambda t} - e^{-\lambda t}\Phi(\frac{USL-\mu}{\sigma}) + 1 - e^{-\lambda t} - \int_0^t \Phi(\frac{USL-\mu-R(t,\tau)}{\sigma}) \cdot g(\tau)d\tau \\
&= 1 - e^{-\lambda t}\Phi(\frac{USL-\mu}{\sigma}) - \int_0^t \Phi(\zeta)) \cdot \lambda e^{-\lambda\tau}d\tau \qquad (3.32)
\end{aligned}$$

where $\zeta = \frac{USL-\mu-ae^{b(t-\tau)}}{\sigma}$

Let $A2 = \int_0^t \Phi(\zeta)) \cdot \lambda e^{-\lambda\tau}d\tau$

Integrating by parts

$$\begin{aligned}
A2 &= [-e^{-\lambda\tau}\Phi(\zeta)]_0^t + \underbrace{\int_0^t (abe^{b(t-\tau)}) \cdot \phi(\zeta)) \cdot e^{-\lambda\tau}d\tau}_{B2} \qquad (3.33) \\
&= [\Phi(\frac{USL-\mu-ae^{bt}}{\sigma}) - e^{-\lambda t}\Phi(\frac{USL-\mu-a}{\sigma})] + B2
\end{aligned}$$

where

$$B2 = \int_0^t (abe^{b(t-\tau)}) \cdot \phi(\zeta)) \cdot e^{-\lambda\tau} d\tau \tag{3.34}$$

There is no closed form solution for the integral in (3.34).

Thus,

$$\begin{aligned} p_u(t) &= 1 - e^{-\lambda t}[\Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{USL-\mu-a}{\sigma})] \\ &- \Phi(\frac{USL-\mu-ae^{bt}}{\sigma}) - B2 \end{aligned} \tag{3.35}$$

The average number of oversized items per unit time during the cycle T,

$$\begin{aligned} D_u(T,\mu) &= \frac{R}{T}\int_0^T p_u(t)dt \\ &= \frac{R}{T}[T - \frac{1}{\lambda}(1-e^{-\lambda T})\Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{USL-\mu-a)}{\sigma}) \\ &- \int_0^T \Phi(\frac{USL-\mu-ae^{bt}}{\sigma})dt - \int_0^T B2dt] \\ &= R - \frac{R}{\lambda T}(1-e^{-\lambda T})\Phi(\frac{USL-\mu}{\sigma}) - \Phi(\frac{USL-\mu-a)}{\sigma}) \\ &- \frac{R}{T}\int_0^T \int_{-\infty}^{\frac{USL-\mu-ae^{bt}}{\sigma}} \frac{1}{\sqrt{2\pi}}e^{-0.5x^2}dydt \\ &- \frac{R}{T}\int_0^T\int_0^t \frac{ab}{\sqrt{2\pi}}e^{b(t-\tau)-\lambda\tau}e^{-0.5(\frac{USL-\mu-ae^{b(t-\tau)}}{\sigma})^2}d\tau dt \end{aligned} \tag{3.36}$$

The average number of defectives due to LSL can be obtained in a similar way. In order to evaluate $D_u(T,\mu)$ at different values of T and μ, one has to resort to numerical integration.

3.8.4 Example 3.2

We present here a numerical example for GSSM1. We consider Example 3.1 presented in section 3.7.

Let: LSL=10, USL=12, σ=1, λ=0.05, R=500, C_R=300, C_l=8, C_u=8.

Table 3.1 shows the results of solving the generalized single stage model (GSSM1) with different drift functions.

Drift function	μ^*	T^*	$ETCG$
Linear: $0.1(t-\tau)$	10.96	6.84	3.89
Polynomial: $0.01+0.0001(t-\tau)+0.001(t-\tau)^2$	10.97	18.12	3.77
Exponential: $0.5e^{0.03(t-\tau)}$	10.92	6.56	4.00

Table 3.1: Results of Example 3.2 with different drift functions

Two observations are in order. First, we have observed that the smaller the value of the parameters of any type of drift function (i.e. a and b for exponential), the longer the duration of the production cycle (i.e. T).

Second, the polynomial drift function reduces to linear when $a = 0$ and $c = 0$. Also, the exponential drift function reduces to linear when $a = 1$ and b=$ln\ \theta$.

3.9 Generalization of the pdf of x only for the SSM (GSSM2)

In section 3.5, we have developed the SSM for a normal probability density function of the quality characteristic. In this section, we generalize the SSM for a general probability density function. The necessary changes are to adopt $D_u(T,\mu)$ (eq. 3.4) and $D_l(T,\mu)$ (eq. 3.8) for a general probability density function.

Let :

$f_t(x)$:p.d.f. of the quality characteristic x at time t where $a < x < b$ (e.g. for normal distribution $a = -\infty$ and $b = \infty$).

$F_t(x)$:CDF of $f_t(x)$

$R(t,\tau) = \theta(t-\tau)$: drift function

The probability of an oversized item at time t, $x(t) > USL$, is

$$\begin{aligned} p_u(t) &= \Pr[x(t) > USL|\mu(t), \sigma^2] \\ &= (\int_{USL}^{b} f_t(x)dx) \cdot \Pr[t < \tau] + \int_0^t \int_{USL}^{b} f_t(x)dx \cdot g(\tau)d\tau \\ &= (1 - F_t(USL)) \int_t^{\infty} g(\tau)d\tau + \int_0^t (1 - F_t(USL))g(\tau)d\tau \\ &= \int_t^{\infty} g(\tau)d\tau - F_t(USL) \int_t^{\infty} g(\tau)d\tau + \int_0^t g(\tau)d\tau - \int_0^t F_t(USL)f(\tau)d\tau \\ &= 1 - F_t(USL) \int_t^{\infty} g(\tau)d\tau - \int_0^t F_t(USL)g(\tau)d\tau \end{aligned} \quad (3.37)$$

The average number of oversized items per unit time during the cycle T,

$$\begin{aligned} D_u(T,\mu) &= \frac{R}{T}\int_0^T p_u(t)dt \\ &= \frac{R}{T}\int_0^T 1dt - \frac{R}{T}\int_0^T F_t(USL)\int_t^{\infty} g(\tau)d\tau dt - \frac{R}{T}\int_0^T\int_0^t F_t(USL)g(\tau)d\tau dt \\ &= R - \frac{R}{T}\int_0^T\int_t^{\infty} F_T(USL)g(\tau)d\tau dt - \frac{R}{T}\int_0^T\int_0^t F_t(USL)g(\tau)d\tau dt \end{aligned} \quad (3.38)$$

The probability of an undersized item at time t, $x(t) < LSL$, is

$$
\begin{aligned}
p_l(t) &= \Pr[x(t) < LSL|\mu(t), \sigma^2] \\
&= (\int_a^{LSL} f_t(x)dx) \cdot \Pr[t < \tau] + \int_0^t \int_a^{LSL} f_t(x)dx \cdot g(\tau)d\tau \\
&= F_t(LSL) \int_t^{\infty} g(\tau)d\tau + \int_0^t F_t(LSL)g(\tau)d\tau \qquad (3.39)
\end{aligned}
$$

The average number of undersized items per unit time during the cycle T,

$$
\begin{aligned}
D_l(T,\mu) &= \frac{R}{T}\int_0^T p_l(t)dt \\
&= \frac{R}{T}\int_0^T F_t(LSL)\int_t^{\infty} g(\tau)d\tau dt + \frac{R}{T}\int_0^T\int_0^t F_t(LSL)g(\tau)d\tau dt \\
&= \frac{R}{T}\int_0^T\int_t^{\infty} F_T(LSL)g(\tau)d\tau dt + \frac{R}{T}\int_0^T\int_0^t F_t(LSL)g(\tau)d\tau dt \qquad (3.40)
\end{aligned}
$$

3.9.1 Example 3.3

We present here a numerical example for GSSM2. We consider Example 3.1 presented in section 3.7. We consider the case where the quality characteristic of the product follows a uniform distribution (Gibra [1974]) and a linear drift (Rahim and Banerjee [1988]).

Let :

$f_t(x)$: U(c,d) where $c < x < d$ (i.e. uniform distribution).

$R(t,\tau) = 0.1(t-\tau)$: drift function

Let: LSL=10, USL=12, σ=1, λ=0.05, R=500, C_R=300, C_l=8, C_u=8.

Table 3.2 shows the results of solving the generalized single stage model (GSSM2) with different parameters.

c	d	c^*	d^*	T^*	$ETCG$
10.1	11.9	10	11.8	5.59	0.137
9.8	12.2	9.6	12	7.22	1.72

Table 3.2: Results of Example 3.3 with different parameters

3.10 Generalization of both the pdf of x and the drift function (GSSM3)

In section 3.5, we have developed the SSM for a normal probability density function of the quality characteristic and a linear drift function. In this section, we generalize the SSM for both a general probability density function and a general drift function. The necessary changes are to adopt $D_u(T,\mu)$ (eq. 3.4) and $D_l(T,\mu)$ (eq. 3.8) for a general probability density function and a general drift function. Let $f_t(x)$ be the p.d.f. of the quality characteristic x at time t where $a < x < b$ (e.g. for normal distribution $a = -\infty$ and $b = \infty$).

The probability of an oversized item at time t, $x(t) > USL$, is

$$\begin{aligned} p_u(t) &= \Pr[x(t) > USL|\mu(t),\sigma^2] \\ &= (\int_{USL}^{b} f_t(x)dx) \cdot \int_{t}^{\infty} g(\tau)d\tau + \int_{0}^{t}\int_{USL}^{b} f_t(x)dx \cdot g(\tau)d\tau \end{aligned} \qquad (3.41)$$

The average number of oversized items per unit time during the cycle T,

$$\begin{aligned} D_u(T,\mu) &= \frac{R}{T}\int_0^T p_u(t)dt \\ &= \frac{R}{T}[\int_0^T (\int_{USL}^{b} f_t(x)dx)\cdot\int_t^{\infty} g(\tau)d\tau dt \\ &+ \int_0^T\int_0^t\int_{USL}^{b} f_t(x)dx\cdot g(\tau)d\tau dt] \end{aligned} \quad (3.42)$$

The probability of an undersized item at time t, $x(t) < LSL$, is

$$\begin{aligned} p_l(t) &= \Pr[x(t) < LSL|\mu(t),\sigma^2] \\ &= (\int_a^{LSL} f_t(x)dx)\cdot\int_t^{\infty} g(\tau)d\tau + \int_0^t\int_a^{LSL} f_t(x)dx\cdot g(\tau)d\tau \end{aligned} \quad (3.43)$$

The average number of undersized items per unit time during the cycle T,

$$\begin{aligned} D_l(T,\mu) &= \frac{R}{T}\int_0^T p_u(t)dt \\ &= \frac{R}{T}[\int_0^T (\int_a^{LSL} f_t(x)dx)\cdot\int_t^{\infty} g(\tau)d\tau dt \\ &+ \int_0^T\int_0^t\int_a^{LSL} f_t(x)dx\cdot g(\tau)d\tau dt] \end{aligned} \quad (3.44)$$

3.10.1 Example 3.4

We present here a numerical example for GSSM2. We consider Example 3.1 presented in section 3.7. We consider the case where the quality characteristic of the product follows a uniform distribution (Gibra [1974]) and an exponential drift (Jeang and Yang [1992]).

Let :

$f_t(x)$: U(c,d) where $c < x < d$ (i.e. uniform distribution).

$R(t,\tau)$=0.7 $e^{0.05(t-\tau)}$: drift function

Let: LSL=10, USL=12, σ=1, λ=0.05, R=500, C_R=300, C_l=8, C_u=8.

Table 3.3 shows the results of solving the generalized single stage model (GSSM3) with different parameters.

c	d	c^*	d^*	T^*	$ETCG$
10.1	11.9	10	11.8	3.13	0.378
9.8	12.2	9.6	12	4.02	1.94

Table 3.3: Results of Example 3.4 with different parameters

The results of this model (GSSM) are important and useful. The models in the literature lack the joint optimization of (1) initial mean setting, and (2) production cycle length for a general distribution of the quality characteristic and a general drift function. The significance of our model came from linking both of the above two elements in one general integrated model. Thus, the stated objective in chapter 1 has been accomplished.

3.10.2 Reduction of GSSM3 to Previously Published Models

1. When $\tau = 0, R(t,\tau) = \theta t, f_t(x) \sim N(\mu(t), \sigma^2)$

   GSSM3 reduces to the models of Hall and Eilon [1963], Gibra [1967], and Taha [1966].

2. When $\tau = 0, R(t,\tau) = \theta_1 t^{\theta_2}, f_t(x) \sim N(\mu(t), \sigma^2)$

GSSM3 reduces to the model of Gibra [1974].

3. When $g(\tau) \sim Exponential(\lambda), R(t,\tau) = \theta(t-\tau), f_t(x) \sim N(\mu(t), \sigma^2)$

   GSSM3 reduces to the model of Rahim and Banerjee [1988].

Chapter 4

Variance Reduction and Sensitivity Analysis Studies of SSM

In this chapter, we study the effect of reducing the variance on the total cost of the single stage production system model (SSM). We also present a sensitivity analysis of the SSM parameters. This chapter is organized as follows: In section 4.1, we present the variance reduction model for the SSM. In section 4.2, we present the sensitivity analysis of the SSM parameters.

4.1 Variance Reduction Model for SSM (SSVRM)

4.1.1 Introduction

In most of the models reviewed in chapter 2, the variance of the process is assumed to be either constant or increasing with time, but in both cases uncontrollable. In many industrial processes, it has been found that controlling the production run duration alone (which is achieved by the above mentioned models) is not enough to reduce the expected cost per good item to an acceptable level, and therefore, one has to turn to reduction in variance in order to achieve the required reduction in the expected total cost per good item, which is needed in today's global competitive market. A prerequisite for reducing the variance is to the see how much the expected total cost per good item at the current level of the variance compares with that at level zero and decide whether to reduce the variance or not. Of course, this decision also needs information on the cost of achieving the variance reduction.

The review of literature in chapter 2 covers only the dynamic case (i.e. the mean drifts with time). For the static case, the objective is to find the best mean setting that minimizes a cost function (e.g. expected total cost per good item). For this problem, there has been an enormous amount of work (see Al-Sultan and Rahim [1994] for a survey). One of the more relevant studies is that of Golhar and Pollock [1992] in which they developed a procedure for studying the cost savings due to variance reduction in Golhar and Pollock's model [1988].

In this section, we use the same approach of Golhar and Pollock to study the effect of variance on the expected total cost per good item for a modified version of the Rahim and Banerjee [1988] model.

The proposed analysis could be of use to the practicing production manager or production engineer, as illustrated below :

1. Bisgaard, Hunter, and Pallesen [1984] stated the following

   > "In principle, the variance of any process can be reduced by discovering better ways to operate the present process or by modifying it in some way, perhaps by the incorporation of better machinery. But such changes cannot be continued indefinitely because eventually a point of diminishing returns will be reached."

   Clearly the benefit from reducing the variance of a process (like the one described in this section) is quantified by the proposed model, and therefore if the cost of reducing the variance (i.e. cost of the better machinery, costs of implementation of better ways to operate the process by modifying it) is less than the benefit gained from doing so, then one can go ahead with the variance reduction project; otherwise, it is not advisable to entertain it.

2. Golhar and Pollock [1992] mentioned that machine precision (i.e. the inverse of process standard deviation) can often be improved at a cost. They give an example where there is a choice among different filling machines that vary in cost and precision. One can use the proposed model in this chapter to compare these machines and pick the best one.

3. One can also use the proposed model when studying the reduction of the variance when considering an automatic control system to replace a manual one in a plastic coating industry (Twombly and Whiteman [1974]). Reduced variability of an automatic control system will save raw material (Figure 4.1).

This saving will have to be compared with the cost of adopting the costly automatic control system. Therefore, one can use the proposed model to evaluate the benefit gained from reducing the variance and compare it with the cost to be incurred to attain that reduction, and then decide whether the automatic control system should be adopted or not.

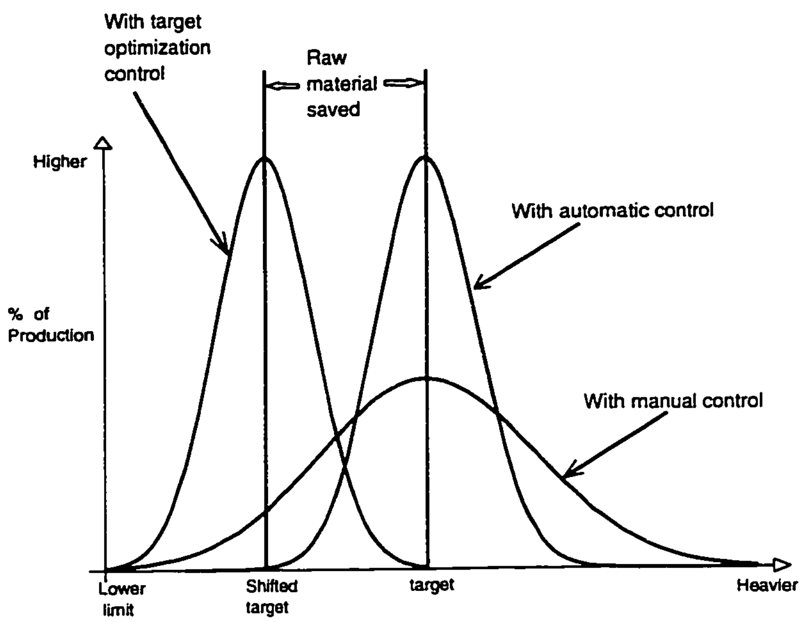


Figure 4.1: Raw material saving due to reduction of variance and shift of target (adopted from Twombly and Whiteman [1974]).

4. There are other situations where the reduction of variance of the process can be done by training operators, or purchasing more uniform raw material. The proposed model can assist in deciding whether to go ahead with the variance reduction project or not, depending on the comparison between the cost of the variance reduction and the saving due to this reduction. However, one has to remember that the reduction in the variance due to operator training or

more uniform material is more difficult to quantify compared to the above two cases. Nonetheless, one can use historical data and expert opinions to come up with a rough estimate.

In summary, the production engineer or production manager can use the proposed model in this chapter to do the following:

1. Evaluate alternative machines with different costs and precisions.
2. Make cost-benefit analysis when entertaining changing the process from manual to automatic, or when improving the process performance by other means.
3. Evaluate the benefits obtained from training programs for operators (which usually help to reduce their variability), and from having more uniform raw materials, although it is more difficult in these cases to quantify the improvement (reduction) in the process variance.
4. Identify how much extra cost (beyond ideal but not achievable variance of zero) is attributed to variance, and whether it is sizable to the point that it is worth investigating to reduce part of it.

4.1.2 The Notation

The following notation are needed in this chapter.

$\mu + K\sigma$	the upper specification limit for the quality characteristic;
$p(t)$	probability of producing a defective item at time $t = Pr(x(t) \geq \mu + K\sigma)$;
$W(T)$	average number of non-defective items produced per unit time

	during a production period of length T;
U	penalty cost per defective item;
$T(\sigma)$	the optimal production run in hours at variance = σ^2;
$T(0)$	the optimal production run in hours at variance =0;
$\psi(\sigma)$	the excess expected total cost per good item due to the variance.

4.1.3 The Proposed Variance Reduction Model (SSVRM)

In this section, we study the effect of the process variance on the optimal production run and the expected total cost per good item for the model of Rahim and Banerjee [1988]. A closer look at their model reveals that they consider the upper specification limit (USL) to be equal to $\mu + K\sigma$, i.e. it is a function of σ. Normally, USL is an external requirement on the production dedicated by the customer while σ is an internal process parameter. In Rahim and Banerjee's model reducing σ will reduce USL by a proportional amount, and hence, exercising control over σ would be of no help. Therefore, we propose the following refinement for Rahim and Banerjee's model. We assume that USL is given while σ is a controllable parameter.

The derivation of $p(t)$ and $W(T)$ follows immediately from Rahim and Banerjee [1988] by substituting $K = \frac{USL-\mu}{\sigma}$. Hence, the probability of a defective item at time t is given by

$$\begin{aligned} p(t) &= Pr[x(t) > USL \mid \mu(t), \sigma^2] \\ &= Pr[z \geq \frac{USL-\mu}{\sigma}] \cdot Pr[t < \tau] \\ &+ \int_0^t Pr[z \geq \frac{USL-(\mu+(t-\tau)\theta)}{\sigma}] g(\tau) d\tau \end{aligned} \tag{4.1}$$

$$= (1 - \Phi(\frac{USL - \mu}{\sigma})e^{-\lambda t} + \int_0^t (1 - \Phi(\frac{USL - \mu(t)}{\sigma})\lambda e^{-\lambda\tau} d\tau$$

After integration by parts and simplification,

$$\begin{aligned} p(t) = \ & 1 \quad -\Phi(\frac{USL - \mu}{\sigma} - \frac{\theta t}{\sigma}) - [\Phi(\frac{USL - \mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\ & - \quad \Phi(\frac{USL - \mu}{\sigma} - \frac{\theta t}{\sigma} + \frac{\lambda\sigma}{\theta})] \times \exp(-\lambda\{t - \frac{USL - \mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \end{aligned} \quad (4.2)$$

The average number of good items per unit time during production cycle T is

$$W(T(\sigma)) = \frac{R}{T(\sigma)} \int_0^{T(\sigma)} (1 - p(t))\, dt \quad (4.3)$$

$$\begin{aligned} W(T(\sigma)) \ = \ & \frac{R}{T(\sigma)}[\frac{\sigma}{\theta}(\frac{USL - \mu}{\sigma}\Phi(\frac{USL - \mu}{\sigma}) - (\frac{USL - \mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma}) \\ \Phi \ & (\frac{USL - \mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma}) + \phi(\frac{USL - \mu}{\sigma}) - \phi(\frac{USL - \mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma})) \\ - \ & \frac{1}{\lambda}\exp(-\lambda\{T(\sigma) - \frac{USL - \mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) \\ \{ \ & \Phi(\frac{USL - \mu}{\sigma} + \frac{\lambda\sigma}{\theta}) - \Phi(\frac{USL - \mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T(\sigma)}{\sigma})\} \\ + \ & \frac{1}{\lambda}\{\Phi(\frac{USL - \mu}{\sigma}) - \Phi(\frac{USL - \mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma})\}] \end{aligned} \quad (4.4)$$

The expected cost per good item is given by

$$ETC = E(TC/\text{unit good product}) = \frac{C_R + RUT(\sigma)}{T(\sigma) \cdot W(T(\sigma))} - U \quad (4.5)$$

Our model which can be obtained from theirs by substituting $K = \frac{USL-\mu}{\sigma}$, has the advantage that USL is independent of σ. This is desirable as control over σ

does not change USL.

4.1.4 Effect of variance on total cost

We will use the approach by Golhar and Pollock [1992] to study the effect of σ on the expected total cost per good item. Let us define the following

$\psi(\sigma)$: is the excess expected total cost per good item due to the variance.

$E(TC/\text{unit good item}|\sigma > 0)$: is the expected total cost per good item, for a certain value of $\sigma > 0$

$E(TC/\text{unit good item}|\sigma = 0)$: is the expected total cost per good item when $\sigma = 0$.

Thus, the function $\psi(\sigma)$ can be defined as follows

$$\psi(\sigma) = E(TC/\text{unit good item}|\sigma > 0) - E(TC/\text{unit good item}|\sigma = 0) \quad (4.6)$$

$E(TC/\text{unit good item}|\sigma = 0)$ is derived as follows:

The probability of a defective item at time t given that $\sigma = 0$ is as follows

$$p(t) = \begin{cases} 0 & t \leq \tau + \frac{USL-\mu}{\theta} \\ 1 & t > \tau + \frac{USL-\mu}{\theta} \end{cases} \quad (4.7)$$

$$\begin{aligned} P(t) &= \int_{\frac{USL-\mu}{\theta}}^{t} 1 \cdot g(\tau)d\tau \quad (4.8) \\ &= e^{-\lambda(\frac{USL-\mu}{\theta})} - e^{-\lambda t} \end{aligned}$$

Let

$T(\sigma)$: is the optimal production run at variance $= \sigma^2$, and

$T(0)$: is the optimal production run at variance $=0$.

$T(\sigma)$ and $T(0)$ are the minimizers of $E(TC/$ unit good item$|\sigma > 0)$ and $E(TC/$unit good item$|\sigma = 0)$ respectively.

The average number of good items per unit time during production cycle T given that $\sigma = 0$ is

$$\begin{aligned} W(T(0)) &= \frac{R}{T(0)}\int_0^{T(0)}(1-P(t))dt \qquad (4.9) \\ &= \frac{R}{T(0)}\int_0^{\frac{USL-\mu}{\theta}} 1dt + \frac{r}{T(0)}\int_{\frac{USL-\mu}{\theta}}^{T(0)}(1-P(t))dt \\ &= \frac{R}{T(0)}(\frac{USL-\mu}{\theta}) + \frac{R}{T(0)}[1-e^{-\lambda(\frac{USL-\mu}{\theta})}][T(0) - \frac{USL-\mu}{\theta}] \\ &+ \frac{R}{\lambda T(0)}[e^{-\lambda(\frac{USL-\mu}{\theta})} - e^{-\lambda T(0)}] \end{aligned}$$

$$\begin{aligned} E(TC \quad / \quad \text{unit good item} \mid \sigma = 0) &= \frac{C_R + URT(0)}{T(0)\cdot W(T(0))} - U \qquad (4.10) \\ &= \frac{C_R + URT(0)}{R(\frac{USL-\mu}{\theta}) + R[1-e^{-\lambda(\frac{USL-\mu}{\theta})}][T(0) - (\frac{USL-\mu}{\theta})] + \frac{R}{\lambda}[e^{-\lambda(\frac{USL-\mu}{\theta})} - e^{-\lambda T(0)}]} \end{aligned}$$

$$ETC = E(TC/\text{unit good item} \mid \sigma > 0) = \frac{C_R + URT(\sigma)}{T(\sigma)\cdot W(T(\sigma))} \qquad (4.11)$$

Therefore, $\psi(\sigma)$ is the excess cost that one expects to pay per good item due to the increase in the value of σ from $\sigma = 0$ to its current value. $\psi(\sigma)$ is given as follows

$$\psi(\sigma) = \frac{C_R + URT(\sigma)}{T(\sigma) \cdot W(T(\sigma))} - \frac{C_R + URT(0)}{R(\frac{USL-\mu}{\theta}) + R[1 - e^{-\lambda(\frac{USL-\mu}{\theta})}][T(0) - \frac{USL-\mu}{\theta}] + \frac{R}{\lambda}[e^{-\lambda(\frac{USL-\mu}{\theta})} - e^{-\lambda T(0)}]} \tag{4.12}$$

An algorithm has been developed to compute $\psi(\sigma)$ and is called SSVRA. A flowchart of SSVRA is provided in Figure 4.2. $\psi(\sigma)$ represents the difference between the current performance (at the current σ), with the ideal best performance (at $\sigma = 0$).

Analysis of $\psi(\sigma)$

In the sequel, we study and analyze the function $\psi(\sigma)$ as it is given in (4.12). One can note from (4.12) that the function $\psi(\sigma)$ is not defined when either $T(0)$=0, or $T(\sigma)$=0.

We study each of the above cases separately.

Case 1: $T(0)$=0

When $\sigma = 0$, the minimum value of $T(0)$ is $\frac{USL-\mu}{\theta}$. Hence, $T(0) = \frac{USL-\mu}{\theta} \neq 0$.

Case 2: $T(\sigma)$=0

When $\sigma > 0$, we consider two subcases:

Case 2.1

If we make the assumption that $T(\sigma) > 0$, then the function will be defined.

If not , then we have to study the function when $T(\sigma) \to 0$ which is discussed next.

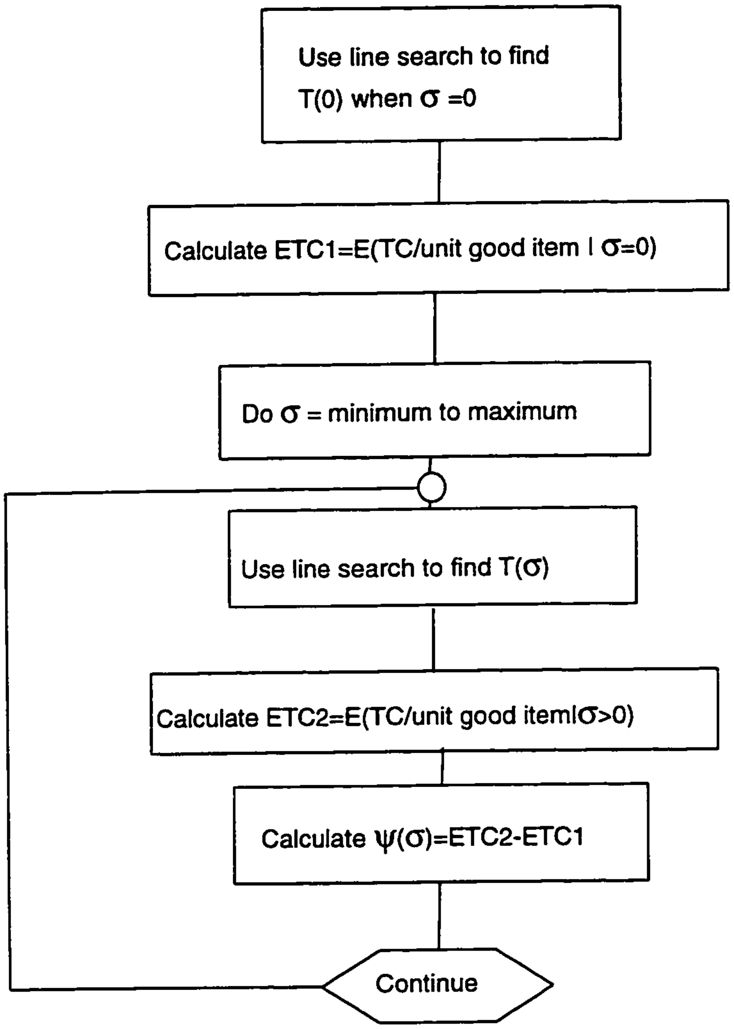


Figure 4.2: Flowchart for computing $\psi(\sigma)$ (SSVRA).

<u>Case 2.2</u>

To study the function when $T(\sigma) \to 0$, we investigate the following limit:

$$\lim_{T(\sigma)\to 0} \frac{C_R + URT(\sigma)}{T(\sigma)\cdot W(T(\sigma))} \tag{4.13}$$

by L'Hôpital's rule,

$$\begin{aligned}\lim_{T(\sigma)\to 0} \frac{C_R + URT(\sigma)}{T(\sigma)\cdot W(T(\sigma))} &= \lim_{T(\sigma)\to 0} \frac{UR}{T(\sigma)\cdot W'(T(\sigma)) + W(T(\sigma))} \\ &= \frac{UR}{0 + \lim_{T(\sigma)\to 0} W(T(\sigma))}\end{aligned} \tag{4.14}$$

Hence, the limit exists if

$$\lim_{T(\sigma)\to 0} W(T(\sigma)) \neq 0 \tag{4.15}$$

by L'Hôpital's rule,

$$\begin{aligned}\lim_{T(\sigma)\to 0} W(T(\sigma)) &= \lim_{T(\sigma)\to 0} \frac{R\int_0^{T(\sigma)} (1-p(t))\,dt}{T(\sigma)} \\ &= \lim_{T(\sigma)\to 0} \frac{R(1-p(T(\sigma)))}{1} \\ &= R\Phi(\frac{USL-\mu}{\sigma}) \neq 0\end{aligned} \tag{4.16}$$

Plugging (4.16) into (4.14), we get

$$\lim_{T(\sigma)\to 0} \frac{C_R + URT(\sigma)}{T(\sigma)\cdot W(T(\sigma))} = \frac{U}{\Phi(\frac{USL-\mu}{\sigma})} \tag{4.17}$$

Therefore, $\psi(\sigma)$ is well-defined.

Example 4.1

We consider the example given by Rahim and Banerjee [1988] where U = \$1.00, R = 500/hr, C_R = \$ 340, USL = 0.208, θ = 0.01.

Solution

The plot of the total cost per unit good item as a function of T and σ is shown in Figure 4.3. $\psi(\sigma)$ is plotted in Figure 4.4. It is clear that as σ increases from 0, the expected total cost per unit good product increases sharply, and later it levels off in *a diminishing return fashion.* This is expected, since the effect of the change in σ becomes less important as σ increases in value.

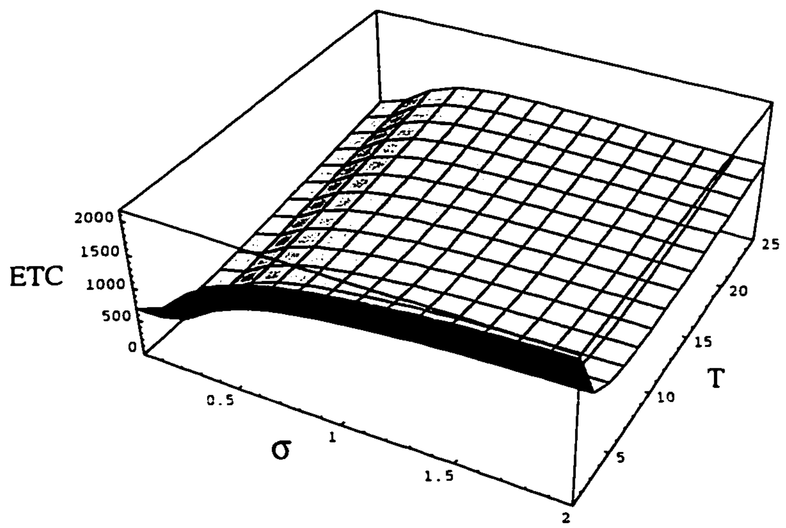


Figure 4.3: Plot of ETC =E(TC/good item) as a function of σ and T.

Another way of studying the effect of σ on the expected total cost per unit good item can be done by taking the partial derivative of ETC with respect to σ.

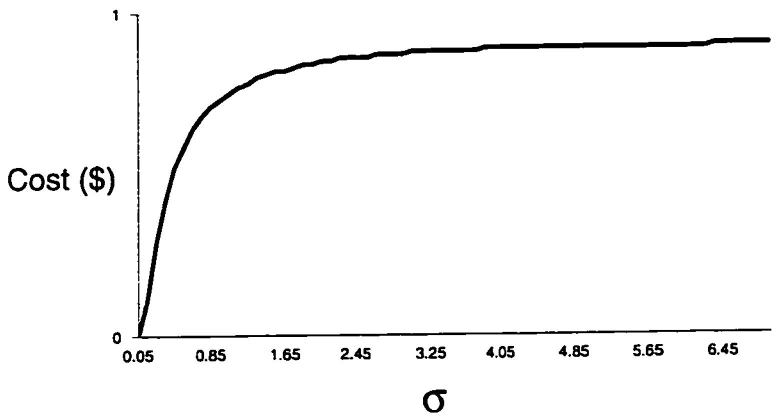


Figure 4.4: Plot of $\psi(\sigma)$.

Differentiating (4.11) with respect to σ yields

$$\frac{\partial ETC}{\partial \sigma} = \frac{\partial ETC}{\partial T(\sigma)} \cdot \frac{\partial T(\sigma)}{\partial \sigma} \tag{4.18}$$

$$\frac{\partial ETC}{\partial T(\sigma)} = \frac{RU[T(\sigma)W(T(\sigma))] - (C_R + RUT(\sigma))(W(T(\sigma)) + T(\sigma)W'(T(\sigma)))}{(T(\sigma) \cdot W(T(\sigma)))^2} \tag{4.19}$$

where

$$\begin{aligned} W'(T(\sigma)) &= \frac{\partial W(T(\sigma))}{\partial T(\sigma)} \\ &= \frac{R}{T(\sigma)}[\Phi(\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma})(\frac{\theta}{\sigma}) + (\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma}) \\ &\times \phi(\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma})(\frac{\theta}{\sigma}) - \{\Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\ &- \Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T(\sigma)}{\sigma})\} \times \exp(-\lambda\{T(\sigma) \end{aligned} \tag{4.20}$$

$$
\begin{aligned}
&- \frac{USL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\}) - \frac{1}{\lambda}\exp(-\lambda\{T(\sigma) - \frac{USL-\mu}{\theta} \\
&- \frac{\lambda\sigma^2}{2\theta^2}\})\{\phi(\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma} + \frac{\lambda\sigma}{\theta})(\frac{\theta}{\sigma})\}] \\
&- \frac{R}{T(\sigma)^2}[\frac{\sigma}{\theta}\{(\frac{USL-\mu}{\sigma})\Phi(\frac{USL-\mu}{\sigma}) - (\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma}) \\
&\times \Phi(\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma}) + \phi(\frac{USL-\mu}{\sigma}) - \phi(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}) \\
&- \frac{1}{\lambda}\exp(-\lambda\{T(\sigma) - \frac{USL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\})\{\Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) \\
&- \Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T(\sigma)}{\sigma})\} + \frac{1}{\lambda}\{\Phi(\frac{USL-\mu}{\sigma}) \\
&- \Phi(\frac{USL-\mu}{\sigma} - \frac{\theta T(\sigma)}{\sigma})\}]
\end{aligned}
$$

If we equate (4.19) to zero, we will get

$$
T^2(\sigma)W^2(T(\sigma)) = -C_R W(T(\sigma)) - C_R T(\sigma)W'(T(\sigma)) - RUT^2(\sigma)W'(T(\sigma)) \quad (4.21)
$$

Now, differentiating (4.22) with respect to σ, we get

$$
\begin{aligned}
\frac{\partial W(T(\sigma))}{\partial\sigma}(2\ T\ (\sigma)^2 W(T(\sigma)) + 2C_R + 2RUT(\sigma)) \quad (4.22) \\
= \frac{\partial T(\sigma)}{\partial\sigma}(-2T(\sigma)W^2(T(\sigma))) + \frac{\partial W'(T(\sigma))}{\partial\sigma}(-C_R - RUT^2(\sigma))
\end{aligned}
$$

Solving for $\frac{\partial T(\sigma)}{\partial\sigma}$, we get

$$
\begin{aligned}
\frac{\partial T(\sigma)}{\partial\sigma} &= -\frac{1}{2T(\sigma)W^2(T(\sigma))}[\frac{\partial W(T(\sigma))}{\partial\sigma}(2T(\sigma)^2 W(T(\sigma)) + 2C_R + 2RUT(\sigma))] \\
&+ \frac{\partial W'(T(\sigma))}{\partial\sigma}(-R - rUT^2(\sigma)) \quad (4.23)
\end{aligned}
$$

To complete (4.23), we need to get $\frac{\partial W(T(\sigma))}{\partial \sigma}$ and $\frac{\partial W'(T(\sigma))}{\partial \sigma}$. We can get $\frac{\partial W(T(\sigma))}{\partial \sigma}$ by differentiating (4.4) with respect to σ. Similarly we can get $\frac{\partial W'(T(\sigma))}{\partial \sigma}$ by differentiating (4.20) with respect to σ.

Although the expression is very involved, we have numerically evaluated $\frac{\partial ETC}{\partial \sigma}$ for different values of σ. We have plotted $\frac{\partial ETC}{\partial \sigma}$ versus σ and it is shown in Figure 4.5. Figure 4.5 shows that it is decreasing as σ increases, that is ETC is a concave function, for this example. We conjecture that it is true in general. Rigorous proof requires demonstrating that either $\frac{\partial ETC^2}{\partial \sigma^2} < 0$ or $\frac{\partial ETC}{\partial \sigma}$ decreases as σ increases, both of which are beyond the scope of this dissertation.

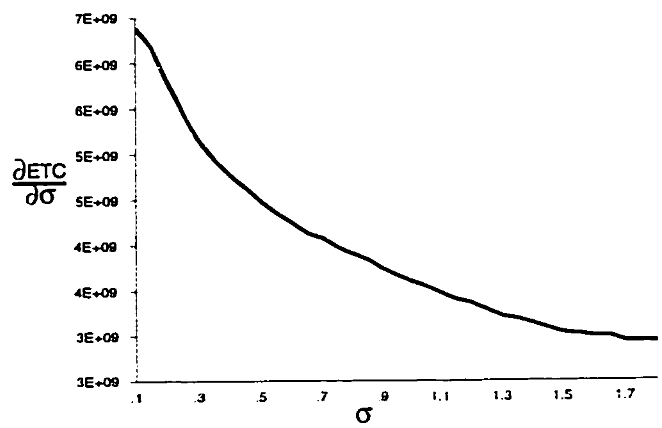


Figure 4.5: Plot of $\frac{\partial ETC}{\partial \sigma}$.

Similarly, we have numerically evaluated $\frac{\partial T(\sigma)}{\partial \sigma}$ for different values of σ. We have plotted $\frac{\partial T(\sigma)}{\partial \sigma}$ versus σ and the figure was similar to Figure 4.5. Again, $\frac{\partial T(\sigma)}{\partial \sigma}$ decreases for this example. One interpretation is that the higher the variance, the more likely that more items are produced with $X < USL$ (when the mean of the process increases beyond USL).

4.1.5 Optimization model for SSVRM

In section 4.1.4, we have proposed a function, $\psi(\sigma)$, that represents the excess cost per unit good itme due to the increase in the value of σ form $\sigma = 0$ to its current value. In this section, we use this function in an optimization model to find the optimal percent reduction in the variance.

In this model, we use the exponential function (Speckhart [1972]) to represent the cost of the variance as discussed in section 1.3.

Next, we present our mathematical model for variance reduction in a single stage production system. But first, we introduce the following notation.

α	percentage of reducing the variance;
a, b	parameters of the cost of the variance;
B	limited budget allocated for the variance reduction program.

$$\max \ \psi(\sigma) - \psi((1-\alpha)\sigma) - (ae^{-b(1-\alpha)\sigma} - ae^{-b\sigma}) \tag{4.24}$$

subject to

$$(ae^{-b(1-\alpha)\sigma} - ae^{-b\sigma}) \leq B \tag{4.25}$$

$$0 \leq \alpha \leq 1 \tag{4.26}$$

In the above model, the objective function is simply the net saving which results from the variance reduction program. The net saving can be calculated as the reduction in the total cost due to the variance reduction program minus the cost of applying the variance reduction program. The first constraint makes sure that the cost of reducing the variance does not exceed the available budget for the variance reduction program. The second constraint sets a lower and an upper bound on the decision variable α.

The mathematical model will find the optimal value of α which in turn tell us how much reduction is going to be made to the current variance.

Example 4.2

We consider the example given by Rahim and Banerjee [1988]. Let the parameters of the exponential function which represents the cost of the variance be $a = 10$ and $b = 0.1$. Let the available budget for the variance reduction program be B =\$50. The current system is operated with σ=0.1 and T = 15.058 with ETC =\$0.0831 per unit good item.

After solving the model, the optimal percent reduction in the variance, α^*=14.75%. That is, the value of the reduced variance is σ=0.0852. The new production cycle length is, T=15.697, and the new ETC=\$0.0658 per unit good item. The reduction in the variance costs \$9.91.

The results of this model (SSVRM) are important and useful. The models in the literature lack the optimization in the variance reduction. The significance of our model comes from optimizing the reduction in the variance.

4.2 Sensitivity Analysis of SSM

4.2.1 Partial Derivatives of the Parameters

Sensitivity of the optimal solution of SSM model developed in chapter 3 to values of its parameters can be analyzed by taking partial derivatives of $ETCG$ with respect to various parameters. However, the resulting expressions are formidable to handle, and therefore may not be of practical use. For example, the rate of change of θ is as follows:

$$\frac{\partial ETCG}{\partial \theta} = \frac{A - B}{(R - D_l(T, \mu) - D_u(T, \mu))^2} \tag{4.27}$$

where;

$$A = (R - D_l(T, \mu) - D_u(T, \mu))(C_l \tfrac{\partial D_l(T,\mu)}{\partial \theta} + C_u \tfrac{\partial D_u(T,\mu)}{\partial \theta})$$

$$B = (C_R/T + C_l D_l(T, \mu) + C_u D_u(T, \mu))(R - \tfrac{\partial D_l(T,\mu)}{\partial \theta} + \tfrac{\partial D_u(T,\mu)}{\partial \theta})$$

$$\begin{aligned}
\tfrac{\partial D_u(T,\mu)}{\partial \theta} = \tfrac{R}{\theta}[&\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) \phi\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) + \Phi\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) + \phi\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) \\
&(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma})] + \tfrac{\sigma R}{T\theta^2}\{\tfrac{LSL-\mu}{\sigma}\Phi\left(\tfrac{LSL-\mu}{\sigma}\right) - \left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right)\Phi\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) \\
&+\phi\left(\tfrac{LSL-\mu}{\sigma}\right) - \phi\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right)\} + \tfrac{R}{\lambda T}\exp\left(-\lambda\left\{T - \tfrac{LSL-\mu}{\theta} - \tfrac{\lambda\sigma^2}{2\theta^2}\right\}\right) \\
&[\left\{\phi\left(\tfrac{LSL-\mu}{\sigma} + \tfrac{\lambda\sigma}{\theta}\right)\left(\tfrac{-\lambda\sigma}{\theta^2}\right) - \phi\left(\tfrac{LSL-\mu}{\sigma} + \tfrac{\lambda\sigma}{\theta} - \tfrac{\theta T}{\sigma}\right)\left(\tfrac{-\lambda\sigma}{\theta^2} - \tfrac{T}{\sigma}\right)\right\} \\
&+\Phi(\tfrac{LSL-\mu}{\sigma} + \tfrac{\lambda\sigma}{\theta}) - \Phi(\tfrac{LSL-\mu}{\sigma} + \tfrac{\lambda\sigma}{\theta} - \tfrac{\theta T}{\sigma})(\tfrac{-\lambda^2\sigma^2}{\theta^3} - \tfrac{\lambda(LSL-\mu)}{\theta^2})] \\
&-\tfrac{R}{\lambda\sigma}\phi\left(\tfrac{LSL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right)
\end{aligned}$$

$$\begin{aligned}
\tfrac{\partial D_u(T,\mu)}{\partial \theta} = \tfrac{r}{\theta}[&\left(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) \phi\left(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) + \Phi\left(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) + \phi\left(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right) \\
&(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma})] + \tfrac{\sigma R}{T\theta^2}\{\tfrac{USL-\mu}{\sigma}\Phi\left(\tfrac{USL-\mu}{\sigma}\right) - \left(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right)\Phi\left(\tfrac{USL-\mu}{\sigma} - \tfrac{\theta T}{\sigma}\right)
\end{aligned}$$

$$+\phi\left(\frac{USL-\mu}{\sigma}\right) - \phi\left(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}\right)\} + \frac{R}{\lambda T}\exp\left(-\lambda\left\{T - \frac{USL-\mu}{\theta} - \frac{\lambda\sigma^2}{2\theta^2}\right\}\right)$$
$$[\left\{\phi\left(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}\right)\left(\frac{-\lambda\sigma}{\theta^2}\right) - \phi\left(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma}\right)\left(\frac{-\lambda\sigma}{\theta^2} - \frac{T}{\sigma}\right)\right\}$$
$$+\Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta}) - \Phi(\frac{USL-\mu}{\sigma} + \frac{\lambda\sigma}{\theta} - \frac{\theta T}{\sigma})(\frac{-\lambda^2\sigma^2}{\theta^3} - \frac{\lambda(USL-\mu)}{\theta^2})]$$
$$-\frac{R}{\lambda\sigma}\phi\left(\frac{USL-\mu}{\sigma} - \frac{\theta T}{\sigma}\right)$$

Therefore, we use design of experiments to do the sensitivity analysis.

4.2.2 Design of Experiments

In this section, we conduct a factorial experiment to study the effects of the input parameters on the total cost of the single stage model (SSM) presented on chapter 3.

There are 7 input parameters for the single stage model. We assign two levels to each input parameter in the factorial experimental design. Table 4.1 shows the input parameters and their assigned levels. Note that the low level values of the input parameters are exactly the same as in the example of the single stage model presented in section 3.7. This example has been taken from Rahim and Banerjee [1988].

		Levels	
Parameter	**Factor**	**Low**	**High**
C_l	A	8	29
C_u	B	8	28
C_R	C	300	5000
R	D	500	8000
λ	E	0.05	8.4
θ	F	0.1	6.5
σ	G	1.0	1.4

Table 4.1: Input parameters and levels used in the experiment.

It should be noted that if the levels of the input parameters are varied over different ranges, the experimental results may be different. However, the levels used for this study are not unrealistic.

The factorial experimental design used for the sensitivity analysis is 2^k design. A total of 2^7 or 128 runs are required to conduct the experiment. A test for curvature has been conducted (Montgomery [1991b]). The result of the test shows no evidence of curvature in the response surface over the selected ranges of the parameters (see Table 4.1). Hence, the 2^k design that is going to be used is justified.

Since the solution procedure (i.e. Hooke and Jeeves algorithm) used to solve the single stage model is deterministic, this experiment can not be replicated. Thus, there is no estimate of error.

One approach to solve this problem is to assume that certain high-order interactions are negligible and combine their sum of squares to estimate the error. This approach is justified by the fact that most systems are dominated by some of the main effects and low-order interactions and most high-order interactions are negligible.

Therefore, before we construct the ANOVA table, we assume that 4-order interactions and higher are negligible to get an estimate of error.

The results of all the runs of this experiment are given in Appendix C. Table 4.2 shows the ANOVA table. It can be noted that the main effects (i.e. A–G) and interactions AB, AG, BG, CD, CE, CF, DE, DF, EF, CDE, CDF, CEF, and DEF are significant at level 1%.

It can be noted from Table 4.2 that the most significant factor is θ which is the drift rate. The second most significant is σ, followed by C_u, C_l, C_R, R, and λ.

Hence, when one is interested in improving the production system, he should look very carefully to θ and σ. The most significant interaction is between θ and C_R. For a fixed value of θ, as C_R increases, $ETCG$ increases. However, for a fixed value of C_R, as θ increases, $ETCG$ decreases. The second most significant interaction is between θ and R, followed by θ and λ, C_R and R, C_R and λ, R and λ, C_l and σ, C_u and σ, C_l and C_u. The most insignificant interaction is between θ and λ. This is because when θ increases, $ETCG$ increases for a fixed value of λ and when λ increases, $ETCG$ increases for a fixed value of θ.

Next, we discuss the effect of the parameters of the model on μ^*, T^*, and $ETCG^*$.

1. *Effect of the parameters on* μ^*

   - As C_l increases, more penalty is put on rejecting an undersized item and hence μ^* increases to make the probability of producing this kind of defective items less. As expected, the opposite is true for C_u, i.e., μ^* decreases as C_u increases to make the probability of producing oversized items less.

   - As the drift rate θ increases, μ^* decreases which is expected to counteract that increase in the drift rate if θ is positive. Of course, the fact that the optimal cycle length T^* gets decreased makes the effect of θ on μ^* much less than the previous case.

     If the drift is negative, then the above stated effects of C_l, C_u, and θ on μ^* will be reversed.

   - For the other parameters, namely λ, σ, C_R, and R, the effects of their values on μ^* are minimal due to the adjustments made to T^* which offset their effects.

2. *Effect of the parameters on T^**

   - Clearly, as C_l or C_u increases, more penalty is incurred when producing defective (undersized or oversized) items, and hence optimal cycle length, T^*, becomes smaller to guard against this (i.e. to reduce the number of defectives).

   - As θ or λ increases, the probability of producing defective items increases, and hence T^* becomes smaller to guard against producing defectives.

   - As the resetting cost per cycle, C_R, gets higher, clearly one would like to avoid resetting frequently and hence T^* increases.

   - The standard deviation σ effect is subtle, since as it increases, the dispersion of the quality characteristic around μ^* is higher, and hence the probability of producing good items is higher when μ^* is way above USL (or way below LSL in case of negative drift) and hence T^* increases.

   - With the same probability of producing defectives, as the production rate R increases, more defective items are produced per unit time and hence as R increases, T^* decreases.

3. *Effect of the parameters on $ETCG^*$*

   - Clearly, as C_l or C_u (which are costs assigned to producing undersized and oversized items, respectively) increases and as C_R increases, the cost per good item, $ETCG^*$, increases.

   - As λ, σ, or θ increases, the performance of the system deteriorates which makes the cost per good item, $ETCG^*$, increase.

- As the production rate, R, increases, one would be able to use shorter cycle time T^*, which makes the probability of producing defective items less, which in turn, makes $ETCG^*$ decrease.

Our results are in line with Rahim and Banerjee [1988] conclusions for their model. One should note that the above observations apply only when the parameter levels are as in Table 4.1. It is not clear what the results would be if different levels are used.

Source	SS	df	MS	F-ratio
Main Effects:				
A	1825.207	1	1825.207	222.397
B	1984.244	1	1984.244	241.775
C	1678.531	1	1678.531	204.525
D	1652.821	1	1652.821	201.392
E	1271.954	1	1271.954	154.984
F	2083.704	1	2083.704	253.894
G	2012.742	1	2012.742	245.248
Interactions:				
AB	124.8939	1	124.8939	15.218
AC	2.9712	1	2.9712	0.362
AD	2.9103	1	2.9103	0.355
AE	5.0216	1	5.0216	0.612
AF	3.3849	1	3.3849	0.412
AG	175.0778	1	175.0778	21.333
BC	15.136	1	15.136	1.844
BD	14.8342	1	14.8342	1.808
BE	2.2156	1	2.2156	0.27
BF	21.3146	1	21.3146	2.597
BG	167.2129	1	167.2129	20.374
CD	981.2639	1	981.2639	119.565
CE	817.3949	1	817.3949	99.598
CF	1235.872	1	1235.872	150.588
CG	4.0107	1	4.0107	0.489
DE	806.2843	1	806.2843	98.244
DF	1217.154	1	1217.154	148.307
DG	3.9314	1	3.9314	0.479
EF	1190.305	1	1190.305	145.036
EG	0.7508	1	0.7508	0.091
FG	5.6619	1	5.6619	0.69
ABC	0.3092	1	0.3092	0.038
ABD	0.3022	1	0.3022	0.037
ABE	0.12	1	0.12	0.015
ABF	0.4477	1	0.4477	0.055
ABG	10.0831	1	10.0831	1.229
ACD	1.3049	1	1.3049	0.159
ACE	2.5161	1	2.5161	0.307
ACF	1.5257	1	1.5257	0.186

ACG	0.0469	1	0.0469	0.006
ADE	2.4635	1	2.4635	0.3
ADF	1.4925	1	1.4925	0.182
ADG	0.0461	1	0.0461	0.006
AEF	4.017	1	4.017	0.489
AEG	0.0685	1	0.0685	0.008
AFG	0.0614	1	0.0614	0.007
BCD	7.2455	1	7.2455	0.883
BCE	1.1156	1	1.1156	0.136
BCF	11.0381	1	11.0381	1.345
BCG	0.074	1	0.074	0.009
BDE	1.0943	1	1.0943	0.133
BDF	10.8108	1	10.8108	1.317
BDG	0.0725	1	0.0725	0.009
BEF	1.901	1	1.901	0.232
BEG	0.001	1	0.001	0
BFG	0.1015	1	0.1015	0.012
CDE	526.9485	1	526.9485	64.207
CDF	727.5236	1	727.5236	88.647
CDG	1.8444	1	1.8444	0.225
CEF	775.8567	1	775.8567	94.536
CEG	0.3164	1	0.3164	0.039
CFG	2.7431	1	2.7431	0.334
DEF	765.4759	1	765.4759	93.271
DEG	0.3096	1	0.3096	0.038
DFG	2.6871	1	2.6871	0.327
EFG	0.7495	1	0.7495	0.091
Residual	525.2464	64	8.206975	
Total	22694.76	127		

Table 4.2: ANOVA for the single stage model.

Chapter 5

A Hybrid Tabu Search Algorithm for Function Minimization

In chapter 6, 7 and 8, we develop mathematical models for multistage production systems. The objective functions in these models are generally nonconvex. Hence, in order to find the global optimal solution of these models, we need an algorithm for global optimization.

In this chapter, we present a new tabu search algorithm for finding the global optimal solution. We call it 'TSFGO'. This chapter is organized as follows: an introduction is given in section 5.1. In section 5.2, we give a brief introduction to tabu search. We present and state the proposed algorithm in section 5.3. Computational results and discussion are presented in section 5.4.

5.1 Introduction

Global optimization problems arise in many practical engineering problems. These optimization problems have the feature that the objective function may not necessarily be convex and therefore may possess many local minima in the region of interest. For applications of global optimization, see Törn and Žilinskas [1989].

A standard global optimization problem can be defined as follows:

Given a continuous function $f : R^n \longrightarrow R$*, find a point* $x^* \in R^n$ *satisfying* $f(x^*) \leq f(x)$ *for all* $x \in R^n$.

Classical nonlinear programming algorithms including derivative-based (e.g., the steepest descent method, Newton's method, the method of conjugate gradients) and derivative-free methods (e.g., Rosenbrock's method, Hooke and Jeeves' method, Nelder and Mead's method) have not been successful in solving these problems. For more details on these methods, see Bazaraa et al. [1993] and Rekalitis et al. [1983]. In general, these methods converge to a stationary point for which there is no guarantee of even local optimality.

There have been attempts by several researchers to develop global algorithms for nonconvex function minimization problems. For example, Corana et al. [1987] have developed a simulated annealing-based algorithm which handles multimodal functions, and provides the global minimum irrespective of the initial point in most of the cases. However, this algorithm is very costly in terms of the number of function evaluations needed to obtain the solution even though it performs well in terms of the quality of the solution. Aluffi-Pentini et al. [1985] presented a simulated annealing algorithm which follows the paths of a system of stochastic differential equations. This method found the global minimum for all test functions that were used. However, both methods require a large number of function evaluations. Recently, Dekkers and Aarts [1991] proposed a simulated annealing algorithm which outperformed the one of Aluffi-Pentini et al. [1985].

Genetic algorithms have also been proposed in the literature for global optimization. For example, see Hajela [1990] and Pham and Yang [1993]. These algorithms use genetic techniques on the solution space of the problem. This is done by defining an injective mapping from a pre-defined set to the solution space in R^n. The pre-defined set is a collection of binary vectors, each one representing a point in the solution space, and the genetic algorithm is used to go from one point to another in the solution space. A complete representation of the solution space is not possible using these algorithms, as all real variables cannot be represented completely by a pre-defined set of binary vectors. Hussien and Al-Sultan [1994] have solved this problem, and proposed generating search directions (rather than solution points) using genetic algorithms. These directions are used in Hooke and Jeeves' algorithm. Their algorithm is very efficient.

There are very few algorithms that use tabu search. Hu [1992] developed a tabu search algorithm particullary suited for optimal engineering design. His algorithm assumes that each variable of the problem is bounded by a known closed interval. A set H of different steps of each interval is computed. Then the algorithm will generate random neighbors from the current point which are contained in a ball of radius $h_i \in H$. The best of these neighbors is selected and its corresponding h_i stored in the tabu list. In his experiments, Hu tested his algorithm with one- and two-dimensional problems only. He compared it with random search and with the genetic algorithm of Hajela [1990]. Hu's algorithm requires 2^n possible random moves to be examined for the general problems of n variables.

In this chapter, we present a new tabu search based algorithm for solving the above discussed problem which is more efficient than the most competitive algorithms in the literature, i.e., it requires less function evaluations. This algorithm has two features. First, the algorithm resembles Hooke and Jeeves' algorithm (Bazaraa et al. [1993]) in the sense that it goes through an exploratory search and pattern search. Second, the algorithm generates random search directions and performs a line search on each direction and the direction of the best point is stored in the tabu list. That is why we call it a hybrid tabu search algorithm. The algorithm shares some spirit with that of Hussien and Al-Sultan [1994] in the sense that it generates directions and uses them in an optimization algorithm. However, we use tabu search rather than genetic algorithm.

5.2 The Tabu Search Scheme

Tabu search is a metaheuristic that guides local heuristic search procedures to explore the solution space beyond local optimality. It was introduced by Glover [1986, 1989, 1990] specifically for combinatorial problems. Its basic ideas have also been also proposed by Hansen [1986] and Hansen and Jaumard [1987] with another name "steepest ascent mildest descent". Since then, tabu search has been applied to a wide range of problem settings in which it has consistently found better solutions than methods previously applied to these problems. For example, tabu search has been applied to flow shop scheduling (Widmer and Hertz [1989], Taillard [1990]), architectural design (Bland and Dawson [1991], time tabling problem (Hertz [1991]), among others.

The tabu search starts at some initial point and then moves successively among neighboring points. At each iteration, a move is made to the best point in the neighborhood of the current point which may not be an improving solution. The method forbids (makes tabu) points with certain attributes with the goals of preventing cycling and guiding the search towards unexplored regions of the solution space. This is done using an important feature of the tabu search method called *tabu list*. A tabu list consists of the latest moves made so that recently visited points are not generated again. The size of the tabu list can be either fixed or variable. In its simplest form, tabu search requires the following ingredients:

- Initial point
- Mechanism for generating some neighborhood of the current point
- Tabu list
- Aspiration criterion
- Stopping criterion

For more complete description of this method, see Glover [1989,1990].

5.3 The Proposed Algorithm

As explained in section 5.1, our algorithm uses the optimization technique of Hooke and Jeeves where the directions are generated randomly and a line search is performed along each generated direction to determine the optimal step length. Then the best nontabu improving point (or best nontabu if no improving point is found) is selected and its associated direction is stored in the tabu list. This procedure is repeated and controlled by tabu search.

Specifically, the algorithm starts at some point, say x_1, and it goes through several iterations. At each iteration k, the algorithm goes through m exploratory searches (cycles) and one pattern search. The point x_k of each iteration becomes the starting point z_1 for the m cycles resulting in the points $z_2, z_3, ..., z_{m+1}$. In each cycle, r directions are generated randomly, and a line search is performed along each direction. The direction that gives the minimum functional value is selected provided that it is nontabu or it is tabu and it improves on the best solution found so far. Then, this direction is stored in the *tabu list*. A direction is said to be tabu if it is the negative of any direction in the *tabu list*, otherwise it is said to be nontabu. After m cycles, the algorithm performs a line search along the direction $z_{m+1} - z_1$ to generate the next point x_{k+1} and this constitutes the pattern step. If $k = ITERMAX$, where $ITERMAX$ is the maximum number of nonimproving iterations, or the improvement between two consecutive iterations is less than a predetermined value, the algorithm stops; otherwise the algorithm goes through iteration $k+1$ starting from the point x_{k+1}. Next, we formally present the proposed algorithm. A flowchart of the algorithm TSFGO is depicted in Figure 5.1.

Statement of the Algorithm (TSFGO)

Initialization Step

Choose r (the number of random search directions to be used in each cycle). Choose m (the number of cycles to be performed in each iteration). Choose a suitable size (TLS) for the *tabu list*, TL. Choose $ITERMAX$ (the maximum number of nonimproving iterations). Choose ϵ (the desired accuracy of the percentage of improvement in the objective function between two consecutive iterations). Choose a starting point x_1. Let $z_1 = x_1$.

Let $TL = \phi$, $BFV = f(x_1)$.

Let $k = j = 1$, and go to the main step.

Main Step

1. *Perform m cycles*

   1.1 Generate r different random directions, $d_1, d_2, ..., d_r$ (see Section 5.3.1). Let λ^* and d^* be such that

   $$f(z_j + \lambda^* d^*) = \min_{1 \leq i \leq r} f(z_j + \lambda_i d_i)$$

   (or d^* is the best direction in this cycle with respect to the generated directions, and λ^* is the corresponding optimal step length as discussed in Section 5.3.2).

   1.2 *Check Tabu Status*

   1.2.1 l=1

   1.2.2 If ($d^* \notin TL$) or ($d^* \in TL$ and $f(z_j + \lambda^* d^*) < BFV$) then go to step 1.3; otherwise, replace l by $l + 1$. Let the l^{th} best direction (step length) be d^* (λ^*) (i.e. this is the best direction among all generated directions in this cycle excluding those considered earlier in this step) and repeat this step.

   1.3 *Update Current Point*

   Let $z_{j+1} = z_j + \lambda^* d^*$

Store d^* in TL and update it accordingly (see Section 5.3.3).

If $f(z_j + \lambda^* d^*) < BFV$ then $BFV = f(z_j + \lambda^* d^*)$.

1.4 If $j = m$ go to step 2; otherwise, replace j by $j + 1$ and go to step 1.1.

2. *Perform pattern search*

   Let $d = z_{m+1} - z_1$. Let $\overline{\lambda}$ be an optimal solution to the problem

$$\min_{\lambda \in R} f(z_{m+1} + \lambda d)$$

   (see Section 5.3.2)

   Let $x_{k+1} = z_{m+1} + \overline{\lambda} d$, and go to step 3.

3. *Check stopping criterion*

   If either $f(x_k)$ or $f(x_{k+1})$ is equal to zero, let *improv*=1, *k*=0. If both are equal to zero, let *improv*=0.

   If neither $f(x_k)$ nor $f(x_{k+1})$ is equal to zero then let $improv = \left| \frac{f(x_{k+1}) - f(x_k)}{f(x_k)} \right|$.

   If $k = ITERMAX$ or $improv \leq \epsilon$, stop; otherwise, let $j = 1$, and $z_1 = x_{k+1}$, and replace k by $k + 1$, and go to step 1.

5.3.1 Generation of random search directions

In our experiments, we used the following scheme for generating the random search directions.

Let d_i be the i^{th} component of the direction d, where $1 \leq i \leq n$ and n is the dimension of the problem. To generate a direction, perform the following steps:

1. $i = 1$

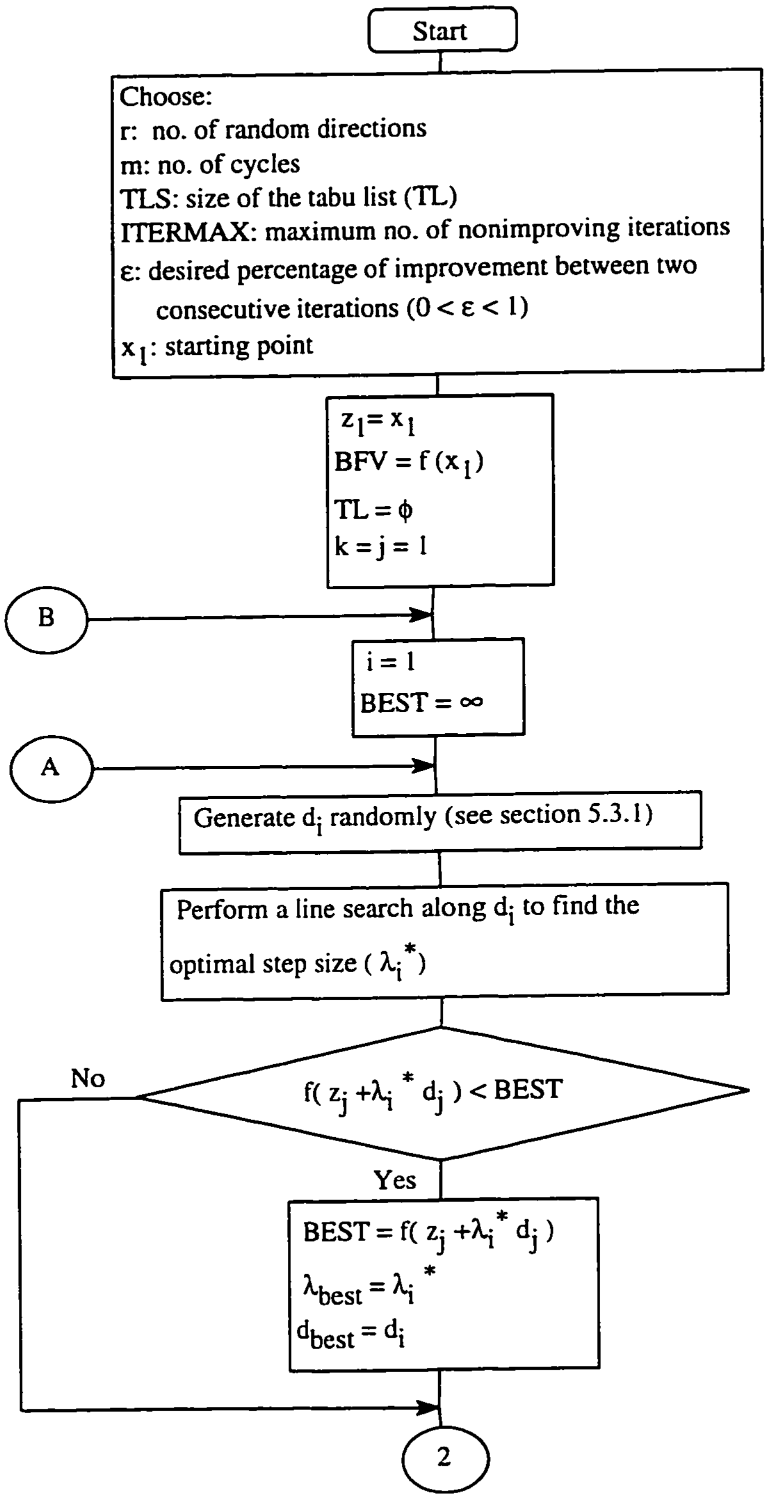


Figure 5.1: Flowchart of TSFGO.

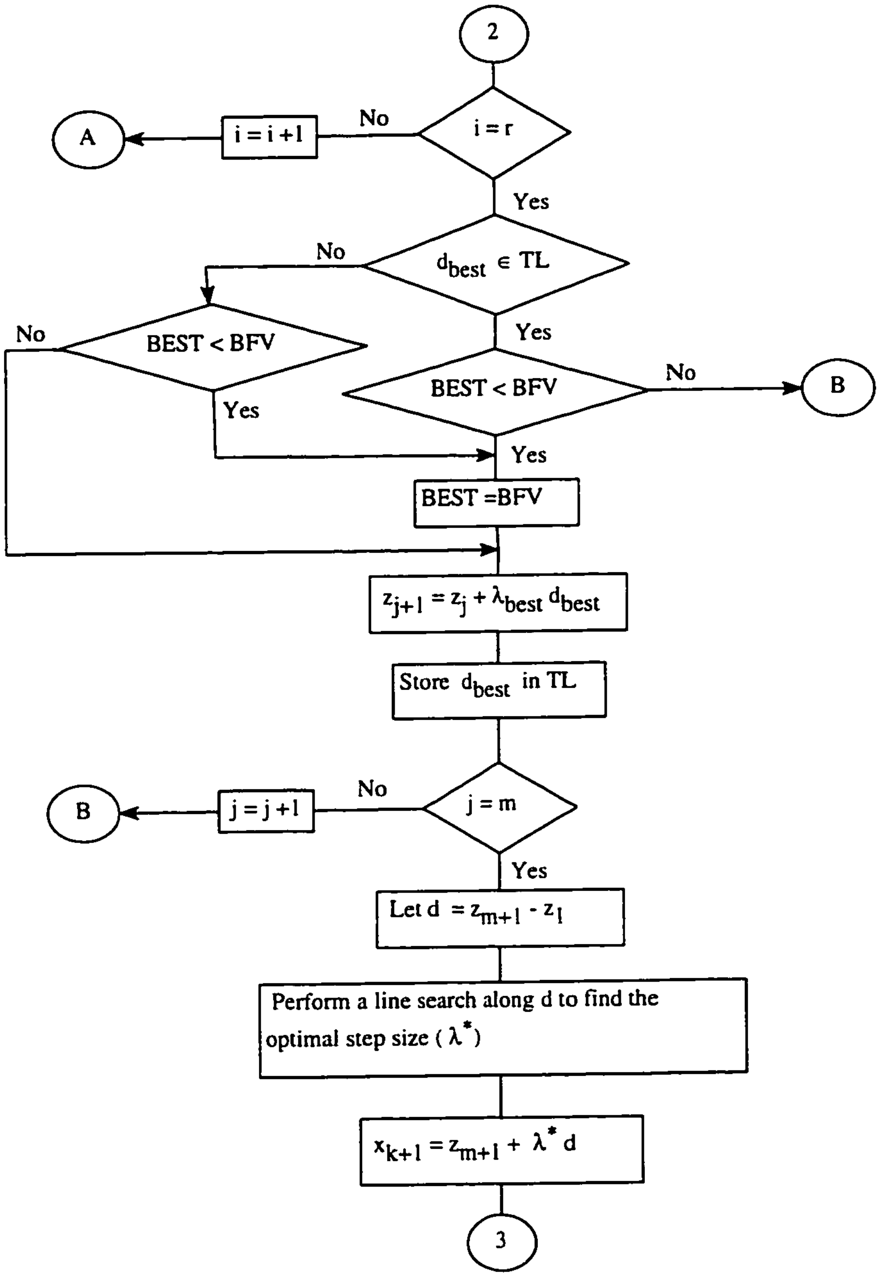


Figure 5.2: Flowchart of TSFGO (continued).

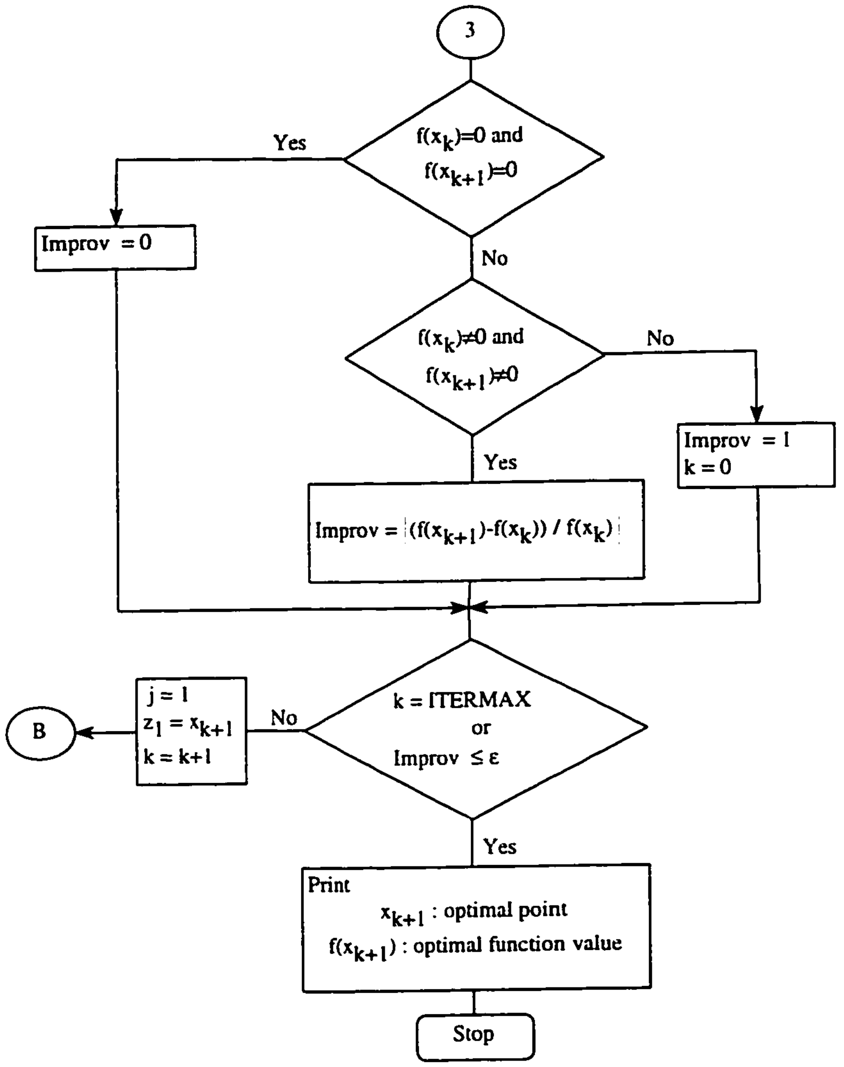


Figure 5.3: Flowchart of TSFGO (continued).

2. Let $rand \sim U[0,1]$. $rand$ is a random number uniformly distributed between 0 and 1.

3. $d_i = \begin{cases} -1 & \text{if } 0 \leq rand \leq \frac{1}{3} \\ 0 & \text{if } \frac{1}{3} < rand \leq \frac{2}{3} \\ 1 & \text{if } \frac{2}{3} < rand \leq 1 \end{cases}$

4. If $i = n$, stop; otherwise let $i = i + 1$ and go to step 2.

5.3.2 The Line Search Scheme

Many line search schemes exist in the literature. However, all of them assume that the function is unimodal which is not usually encountered in global optimization problems. We have tried many of these, but unfortunately they failed. Therefore, we propose to use an exhaustive line search scheme which decides first on the general location of the minimum along the direction and then do a more fine search in the vicinity of the minimum.

We used the following line search scheme in our experiments. Let $[a, b]$ be the interval of uncertainty on which the line search is to be performed. Let x and d be the current point and direction respectively. Given the parameters of the scheme δ, k, s, and δ_f, perform the following steps:

1. $\lambda^* = \lambda = a$, $min = f(x + \lambda d)$.

2. Let $\lambda = \lambda + k$. If $\lambda > b$, go to step 4; otherwise go to step 3.

3. If $f(x + \lambda d) < min$, then $min = f(x + \lambda d)$, and $\lambda^* = \lambda$. Go to step 2.

4. $a = \lambda^* - \delta$, $b = \lambda^* + \delta$, $min = f(x + \lambda^* d)$, $\lambda^* = \lambda = a$, $k = k/s$.

5. Let $\lambda = \lambda + k$. If $\lambda > b$, go to step 7; otherwise go to step 6.

6. If $f(x + \lambda d) < min$, then $min = f(x + \lambda d)$, and $\lambda^* = \lambda$. Go to step 5.

7. Let $\delta = \delta / s$. If $\delta \leq \delta_f$, stop; otherwise go to step 4.

λ^* is the optimal step length. However, one has to qualify this statement by the fact that the accuracy and reliability of the line search scheme are controlled by the constants δ, k, s, and δ_f. Both reliability and accuracy can be enhanced by finding the best values of these parameters by parameteric study which could be at the expense of more function evaluations. Hence, a balance between reliability and accuracy on one hand and the number of function evaluations on the other hand is sought.

5.3.3 Storing in the Tabu List

In our algorithm, the chosen direction, (d_{best}), has to be stored in the tabu list, TL, in step 1.3. We store in TL the negative of the chosen direction, (d_{best}) as follows: The tabu list (TL) is a two dimensional array, say $TL(i, j)$, where $1 \leq i \leq TLS$, $1 \leq j \leq n$, and TLS is the tabu list size as defined in the initialization step and n is the problem dimension. For example, let d_{best}=(1,0,-1) and i=1. Then, d_{best} is stored in TL as follows: $TL(1,1)$=-1, $TL(1,2)$=0, $TL(1,3)$=1.

We illustrate how we check whether a direction, d, is stored in TL or not by an example. Suppose that TL has one element and it is given as in the above example. Suppose that we want to check whether the direction d=(1,1,1) is in TL or not. Then, we go and check if d_j=$TL(1, j)$ for j=1,2,3. Clearly, d_1=1 is not equal to $TL(1,1)$=-1. Hence, d is not in TL.

TL has a fixed size, TLS. Each time a new element needs to be stored in TL, the new element is appended at the end of TL, and the oldest element is removed form TL.

5.4 Computational Results and Discussion

In this section, we discuss various implementation details and results of our computational experiments.

5.4.1 Parameter Setting

Tabu search is a parameter sensitive technique similar to simulated annealing and genetic algorithms. We have conducted an extensive parametric study for the proposed algorithm on the test problems in section 5.4.2. The results of this study show that the following two sets of parameters give the best performance:

- *Set 1*: A compromise between solution quality and number of function evaluations.

  $r = 2n$

  $m = 4$

  Tabu list size = 20

  $ITERMAX = 2$

  $\epsilon = 10^{-4}$.

- *Set 2*: A very good solution quality with probably more function evaluations and more computation time.

  $r = n^2$

$m = 10$

Tabu list size = 25

$ITERMAX = 5$

$\epsilon = 10^{-10}$.

In our experiments we used *Set 1*.

We have also conducted an extensive parameteric study for the line search scheme on the test problems in section 5.4.2 and we have found that $k = 1$, $\delta = 0.5$, $s = 10$, and $\delta_f = 0.05$ work very well for the test problems given in section 5.4.2.

5.4.2 Test problems

We test our algorithm using a set of test functions known from the literature. These test functions are taken from Dixon and Szegö [1978] (see Appendix A). We perform two experiments. In the first experiment, we compare our algorithm with the methods shown in Table 5.1.

In the second experiment, we compare it with the methods shown in Table 5.2 using the Rosenbrock test function in 2 and 4 dimensions (see Appendix B).

Method	Name	Reference
A	Multistart	Rinnooy Kan and Timmer [1984]
B	Controlled Random Search	Price [1978]
C	Density Clustering	Törn [1978]
D	Clustering with dist. function	De Biase and Frontini [1978]
E	Multi Level Single Linkage	Rinnooy Kan and Timmer [1987]
F	Simulated Annealing	Dekkers and Aarts [1991]
G	Simulated Annealing based on stochastic differential equations	Aluffi-Pentini et al. [1985]
TS	Tabu Search	This dissertation

Table 5.1: Methods used for the first experiment.

Since all the algorithms shown in Table 5.1 are tested on different machines, we use the standard unit of time as shown in Dixon and Szegö [1978] which make comparisons machine-independent. One unit of standard time is equivalent to the running time needed for 1000 evaluations of the Shekel 5 function using the point (4,4,4,4).

Method	Name	Reference
Simplex	Simplex method	Nelder and Mead [1964]
ARS	Adaptive Random Search	Masri et al. [1980]
SA	Simulated Annealing	Corana et al. [1987]
GA	Genetic Algorithm	Hussien and Al-Sultan [1996]

Table 5.2: Methods used for the second experiment.

5.4.3 Results and discussion

Table 5.3 shows the number of function evaluations for each method. In Table 5.4, the computation times in units of standard time for each method are given. For methods A-G, numbers of function evaluations and computation times are taken from Dekkers and Aarts [1991]. Table 5.4 shows no results for method G, since the running time available is only in absolute computer time.

Note that the number of function evaluations and running time used in generating the initial random sample are not counted in many methods. For example, for the initial random sample, the Multi Level Single Linkage method (Rinnooy Kan and Timmer [1987]) uses 1000 function evaluations and the Simulated Annealing method (Dekkers and Aarts [1991]) uses $10n$ function evaluations.

Tables 5.3 and 5.4 show that the results of our algorithm are very encouraging. Note that our algorithm never failed in finding the global minimum in all the tested

Function Method	GP	BR	H3	H6	S5	S7	S10
A	4400	1600	2500	6000	6500	9300	11000
B	2500	1800	2400	7600	3800	4900	4400
C	2499	1558	2584	3447	3649	3606	3874
D	378	597	732	807	620	788	1160
E	148	206	197	487	404	432[a]	564
F[c]	563	505	1459	4648	365[a]	558	797
G[c]	5349	2700	3416	3975	2446	4759	4741
TS[b]	281	398	578	2125	753	755	1203

a:The global minimum was not found in one of four runs.
b:The average number of function evaluations of four runs.
c:The number of function evaluations of the initial sampling are not included.

Table 5.3: Number of function evaluations of the first experiment.

Function Method	GP	BR	H3	H6	S5	S7	S10
A	4.5	2	7	22	13	21	32
B	3	4	8	46	14	20	20
C	4	4	8	16	10	13	15
D	15	14	16	21	23	20	30
E[c]	0.15	0.25	0.5	2	1	1[a]	2
F[c]	0.9	0.9	5	20	0.8[a]	1.5	2.7
TS[b]	0.22	0.3	1.02	7.2	1.47	1.58	3.34

a:The global minimum was not found in one of four runs.
b:The average running time of four runs.
c:The running time of the initial sampling was not counted.

Table 5.4: Running time in units of standard time of the first experiment.

problems, while methods E and F failed in problems S7 and S5, respectively, and our algorithm is still competitive in terms of efficiency.

We observe that the quality of the solution, number of function evaluations, and running time are very much dependent on the initial starting point. Thus, the comparisons provided above are not entirley fair since the different methods may start at a different initial point. In addition, the type of language used to code each method, the data structure used, and the machine on which each method is tested, are also influencing factors for execution time.

Another important point is that the first and second derivatives of each of the tested functions can be easily obtained. Hence, methods which utilize these tools have an advantage over other methods. Unlike other methods, our algorithm does not require the derivatives of the function. Moreover, it can handle functions that are not differentiable and functions that are not even explicit.

For the second experiment, Tables 5.5 and 5.6 show comparisons of our algorithm with the algorithms shown in Table 5.2 using the Rosenbrock function in 2 and 4 dimensions. Results are taken from Corana et al. [1987] and Hussien and Al-Sultan [1994]. Computation times are not reported in Corana et al. [1987], and we relied on function evaluations to measure efficiency. Our algorithm outperformed previous methods in terms of number of function evaluations except the Simplex method which fails to get the global solution in some cases. Table 5.6 shows that our algorithm is reliable.

The results of this algorithm (TSFGO) are important and useful. Our algorithm outperforms the algorithms in the literature in both efficiency and reliability. Thus, the stated objective in chapter 1 has been accomplished.

Method Starting Point	Simplex	ARS	SA	GA	TS
2 dimensions:					
1001,1001	993	3411	500001	2389	1954
1001,-999	276	131841	508001	2214	1762
-999,-999	730	15141	524001	3254	2483
-999,1001	907	3802	484001	3412	2671
1443,1	907	181280	492001	2115	1616
1,1443	924	2629	512001	5781	4121
1.2,1	161	6630	488001	1548	1195
4 dimensions:					
101,101,101,101	1869	519632	1288001	228534	16528
101,101,101,-99	784	194720	1328001	213422	27940
101,101,-99,-99	973	183608	1264001	264521	13995
101,-99,-99,-99	1079	195902	1296001	299321	11443
-99,-99,-99,-99	859	190737	1304001	44567	14007
-99,101,-99,101	967	4172290	1280001	234512	16572
101,-99,101,-99	870	53878	1272001	193134	11471
201,0,0,0	1419	209415	1288001	182131	25413
1,201,1,1	1077	215116	1304001	283946	8884
1,1,1,201	1265	29069006	1272001	214312	34083

Table 5.5: Number of function evaluations of the second experiment.

Method Starting Point	Simplex	ARS	SA	GA	TS
2 dimensions:					
1001,1001	4.9E-10	1586.4	1.8E-10	1.2E-12	7.1E-11
1001,-999	7.4E-10	8.6E-9	2.6E-9	2.3E-10	1.5E-9
-999,-999	2.7E-10	1.2E-8	1.2E-9	4.4E-12	4.2E-9
-999,1001	9.2E-10	583.2	4.2E-8	3.4E-10	6.1E-9
1443,1	5.4E-11	4.7E-10	1.5E-8	3.3E-10	7.2E-9
1,1443	2.2E-10	1468.9	1.6E-9	1.2E-10	3.4E-9
1.2,1	2.4E-10	5.5E-7	2.0E-8	2.2E-18	2.6E-10
4 dimensions:					
101,101,101,101	3.70	1.9E-6	5.0E-7	4.8E-9	5.1E-8
101,101,101,-99	5.46E-17	1.7E-6	1.8E-7	2.1E-9	1.6E-7
101,101,-99,-99	9.8E-18	3.8E-6	5.9E-7	2.2E-8	8.3E-8
101,-99,-99,-99	3.4E-17	2.3E-6	7.4E-8	3.0E-9	1.4E-7
-99,-99,-99,-99	8.3E-18	2.7E-6	3.3E-7	5.7E-9	6.3E-7
-99,101,-99,101	1.2E-17	2.6E-6	2.8E-7	3.9E-8	9.5E-7
101,-99,101,-99	6.0E-18	3.7	2.3E-7	4.1E-8	4.5E-7
201,0,0,0	3.70	1.1E-6	7.5E-7	3.0E-8	5.7E-7
1,201,1,1	9.4E-18	1.2E-6	4.6E-7	5.8E-8	7.2E-7
1,1,1,201	3.9E-17	2.2E-6	5.2E-7	4.3E-8	6.1E-7

Table 5.6: Final functional value of the second experiment.

Chapter 6

MultiStage Production Systems Without Buffers Model

In this chapter, we extend the models developed in chapter 3 to multistage production systems. This chapter is organized as follows: in section 6.1, we give an introduction. In section 6.2, statement of the problem is presented, followed by notation in section 6.3. Assumptions are given in section 6.4. The proposed model and its solution are given in section 6.5. Results and discussion are presented in section 6.6. Extended models are given in sections 6.7 and 6.8.

6.1 Introduction

In chapter 3, we have presented models for a single stage production system. In this chapter, we consider a multistage production system where processing at each stage is performed by a process that deteriorates with time. We assume that processes are subject to random deterioration. Items produced are required to conform to the given specifications for the quality characteristic related to the process at every stage. If a product does not conform to these specifications, then there is a penalty incurred which depends on the stage at which the product does not conform to specifications, and whether the lack of conformity to specification is due to the quality characteristic under consideration is above the given upper specification limit, or below the given lower specification limit. It is also assumed that there is a certain demand per unit time for finished items, and a penalty is incurred for not delivering demanded items.

From the above, it is clear that the costs involved include cost of maintenance (processes' adjustments), cost of rejected items due to lack of conformance to either upper or lower specification limits, and costs of failing to deliver demanded items. The decisions to be made are: finding the optimal initial setting of the process mean, and the optimal production cycle time for every process. The general approach is to build a model to find the optimal values of these decision variables such that the total cost is minimized.

A literature review of the models of multistage production systems have been presented in chapter 2.

In this chapter, we develop a mathematical model to minimize the cost of maintenance (processes' adjustments), quality, and penalty for failing to deliver demanded items. The model gives optimal initial settings and optimal cycle lengths for every process in each stage. We use the hybrid tabu search algorithm (TSFGO) to optimize the model.

6.2 Problem Statement

In this chapter, we consider a production system with n stages where each stage consists of a process with known and constant variance. The quality characteristic produced by the process at each stage i has both upper and lower specification limits, denoted by (USL_i) and (LSL_i), respectively. At a random point of time, process i starts drifting either in the positive or negative direction which will result in producing more defective items (e.g., more oversized or undersized items, respectively). Oversized and undersized items can be reworked at different costs (or equivalently sold at a secondary market). The problem is to decide for every stage what should be the initial mean setting, and the length of the cycle time after which the process mean is reset to its initial setting, which can usually be done at a certain resetting cost (an example of resetting a process could be sharpening or changing a wearing tool). Clearly, if process i is reset too often, the resetting cost is more while the cost of producing defective items is less and vice versa. Therefore, the goal is to find initial mean settings and cycle lengths for processes 1,2,...,n, that strike a compromise between these two conflicting objectives.

As mentioned in chapter 2, processes such as machining, drilling, grinding, drawing, stamping, and moulding (Hall and Eilon [1963] and Gibra [1967,1974]), are some examples of processes that deteriorate with time. Below we give some examples of multistage production systems that have deteriorating processes.

As a first example, consider a production system of two stages. This production system will produce shafts where the first stage makes the outer diameters of the shafts while the second stage makes their inner diameters. The outer diameters have upper and lower specification limits as USL_1 and LSL_1, respectively while the inner diameters have upper and lower specification limits as USL_2 and LSL_2, respectively. At the first stage, raw materials are processed to produce outer diameters of the shafts. The process of the first stage has a positive drift towards the USL_1. The process of the second stage will produce the inner diameters of the shafts. The process of the second stage has a negative drift. That is, as the tool starts to wear out, its shift will be towards LSL_2, and the inner diameter of the shaft gets smaller with time. Shafts with outer (inner) diameters greater (less) than their USL_1 (LSL_2) can be reworked to trim the excess material, and consequently transform them into good ones. But shafts with outer (inner) diameters less (greater) than LSL_1 (USL_2) can not be reworked and thus should be scrapped or sold at a secondary market at a substantially reduced price. This makes the penalty for producing shafts with outer (inner) diameters greater (less) than USL_1 (LSL_2) to be less than the penalty for those shafts with outer (inner) diameters less (greater) than LSL_1 (USL_2). Moreover, at the end unfulfilled demand will be penalized.

As a second example, consider the production system with three stages which produces the part considered by Egbelu [1993]. The three stages consists of turning, milling, and drilling, operations respectively. Egbelu [1996] showed another example of three stages.

The third example can be found in Park and Steudel [1991]. They considered the production of 98 types of gears. Their production system consists of six stages. Other examples can be found in Raz [1986], Abdou and Cheng [1993], and Billatos and Kendall [1991].

6.3 The Notation

The following notation are needed in this chapter.

$x_i(t)$	the random variate denoting the quality measurement of the product characteristic i at time t with mean $\mu(t)$ and constant variance σ^2;
μ_i	the mean quality characteristic i of the product when the process begins in an in-control state having variance σ_i^2;
μ_i^*	the optimal initial mean for process i;
τ_i	the elapsed time until the occurrence of the assignable cause for process i is a random variable and is assumed to be exponentially distributed with a mean of $1/\lambda_i$ hours;
$g_i(\tau_i)$	$\lambda_i e^{-\lambda_i \tau_i}, \lambda_i > 0, \tau_i \geq 0$, the density function of the occurrence time of the assignable cause for process i;
θ_i	rate of drift in the mean of process i;
$\mu_i(t)$	the process mean at time t for process i $=\mu_i$ for $t \leq \tau_i$ $= \mu_i + (t - \tau_i)\theta_i$ for $t > \tau_i$;
USL_i	the upper specification limit for the quality characteristic i;
LSL_i	the lower specification limit for the quality characteristic i;
R_i	the arrival rate of nondefective items at stage i;
C_R^i	resetting cost for process i;
T_i	cycle length (production run) in time units for process i;
C_l^i	cost of producing an undersized item for quality characteristic i;
C_u^i	cost of producing an oversized item for quality characteristic i;

$pr_l^i(t)$	probability of producing an undersized item at time t ; (i.e.,$x_i(t) < LSL_i$) for process i;
$pr_u^i(t)$	probability of producing an oversized item at time t ; (i.e.,$x_i(t) > USL_i$) for process i;
$P_l^i(T_i, \mu_i)$	percentage of undersized items produced per unit time during T_i, given that process i is started at mean setting equals to μ_i;
$P_u^i(T_i, \mu_i)$	percentage of oversized items produced per unit time during T_i, given that process i is started at mean setting equals to μ_i;
Q	demand/unit time;
W	penalty for unfullfilled demand/item;
n	number of stages;
R	production rate for process at the first stage;
R_{eff}	effective (actual) production rate of the production system.

6.4 Assumptions

We make the following assumptions:

1. Process i, i=1,2,...,n begins in an in-control state having a normally distributed quality characteristic with mean μ_i and variance σ_i^2.
2. Process i, i=1,2,...,n starts deteriorating at a random point of time, and deterioration is linear with time.
3. Variance of process i, i=1,2,...,n remains constant.

4. The material cost is either independent of the choice of μ_i and T_i, i=1,2,....,n (e.g. the process of producing inner holes in shafts), or their effect on cost of material can be assumed negligible. This assumption is implicitly made in most of the literature of this problem.

5. There is enough supply for raw material at the first stage, and the production rate for every process is less than or equal to the succeeding process.

6. Resetting (repair) time for each stage is negligible (i.e. instantaneous).

7. There is no buffer storage between stages.

6.5 The Proposed Model

The probability of an oversized item at time t (i.e., $x_i(t) > USL_i$), $pr_u^i(t)$ at process i, is given by

$$\begin{aligned} pr_u^i(t) &= Pr[x_i(t) > USL_i \mid \mu_i(t), \sigma_i^2] \\ &= Pr[z \geq \frac{USL_i - \mu_i}{\sigma_i}] \cdot Pr[t < \tau_i] \\ &+ \int_0^t Pr[z \geq \frac{USL_i - (\mu_i + (t - \tau_i)\theta_i)}{\sigma_i}] g_i(\tau_i) d\tau_i \end{aligned} \quad (6.1)$$

$$\begin{aligned} pr_u^i(t) = \; & 1 \; - \Phi(\frac{USL_i - \mu_i}{\sigma_i} - \frac{\theta_i t}{\sigma_i}) - [\Phi(\frac{USL_i - \mu_i}{\sigma_i} + \frac{\lambda_i \sigma_i}{\theta_i}) \\ & - \; \Phi(\frac{USL_i - \mu_i}{\sigma_i} - \frac{\theta_i t}{\sigma_i} + \frac{\lambda_i \sigma_i}{\theta_i})] \\ & \times \; \exp(-\lambda_i\{t - \frac{USL_i - \mu_i}{\theta_i} - \frac{\lambda_i \sigma_i^2}{2\theta_i^2}\}) \end{aligned} \quad (6.2)$$

Similarly, the probability of an undersized item at time t (i.e.,$x_i(t) < LSL_i$), $pr_l^i(t)$, at process i is given by

$$\begin{aligned} pr_l^i(t) &= Pr[x_i(t) < LSL_i \mid \mu_i(t), \sigma_i^2] \\ &= Pr[z \leq \frac{LSL_i - \mu_i}{\sigma_i}] \cdot Pr[t < \tau_i] \\ &+ \int_0^t Pr[z \leq \frac{LSL_i - (\mu_i + (t - \tau_i)\theta_i)}{\sigma_i}] g_i(\tau_i) d\tau_i \end{aligned} \tag{6.3}$$

$$\begin{aligned} pr_l^i(t) = \; & \Phi \; (\frac{LSL_i - \mu_i}{\sigma_i} - \frac{\theta_i t}{\sigma_i}) + [\Phi(\frac{LSL_i - \mu_i}{\sigma_i} + \frac{\lambda_i \sigma_i}{\theta_i}) \\ - \; & \Phi(\frac{LSL_i - \mu_i}{\sigma_i} - \frac{\theta_i t}{\sigma_i} + \frac{\lambda_i \sigma_i}{\theta_i})] \\ \times \; & \exp(-\lambda_i \{t - \frac{LSL_i - \mu_i}{\theta_i} - \frac{\lambda_i \sigma_i^2}{2\theta_i^2}\}) \end{aligned} \tag{6.4}$$

The percentage of undersized items during time T_i for stage i is given by

$$P_l^i(T_i, \mu_i) = \frac{1}{T_i} \int_0^{T_i} pr_l^i(t) dt \tag{6.5}$$

$$\begin{aligned} P_l^i(T_i, \mu_i) &= \frac{1}{T_i} [\frac{\sigma_i}{\theta_i} (\frac{LSL_i - \mu_i}{\sigma_i} \Phi(\frac{LSL_i - \mu_i}{\sigma_i}) - (\frac{LSL_i - \mu_i}{\sigma_i} - \frac{\theta_i T_i}{\sigma_i}) \\ & \Phi \; (\frac{LSL_i - \mu_i}{\sigma_i} - \frac{\theta_i T_i}{\sigma_i}) + \phi(\frac{LSL_i - \mu_i}{\sigma_i}) - \phi(\frac{LSL_i - \mu_i}{\sigma_i} - \frac{\theta_i T_i}{\sigma_i})) \\ &- \frac{1}{\lambda_i} \exp(-\lambda_i \{T_i - \frac{LSL_i - \mu_i}{\theta_i} - \frac{\lambda_i \sigma_i^2}{2\theta_i^2}\}) \\ & \{ \; \Phi(\frac{LSL_i - \mu_i}{\sigma_i} + \frac{\lambda_i \sigma_i}{\theta_i}) - \Phi(\frac{LSL_i - \mu_i}{\sigma_i} + \frac{\lambda_i \sigma_i}{\theta_i} - \frac{\theta_i T_i}{\sigma_i})\} \\ &+ \frac{1}{\lambda_i} \{\Phi(\frac{LSL_i - \mu_i}{\sigma_i}) - \Phi(\frac{LSL_i - \mu_i}{\sigma_i} - \frac{\theta_i T_i}{\sigma_i})\}] \end{aligned} \tag{6.6}$$

Similarly, the percentage of oversized items during time T_i for stage i is given by

$$P_u(T_i,\mu_i) = \frac{1}{T_i}\int_0^{T_i} pr_u^i(t)dt \tag{6.7}$$

$$\begin{aligned}
P_u^i(T_i,\mu_i) &= 1-\{\frac{1}{T_i}[\frac{\sigma_i}{\theta_i}(\frac{USL_i-\mu_i}{\sigma_i}\Phi(\frac{USL_i-\mu_i}{\sigma_i}) - (\frac{USL_i-\mu_i}{\sigma_i}-\frac{\theta_i T_i}{\sigma_i}) \\
\Phi & \ (\frac{USL_i-\mu_i}{\sigma_i}-\frac{\theta_i T_i}{\sigma_i}) + \phi(\frac{USL_i-\mu_i}{\sigma_i}) - \phi(\frac{USL_i-\mu_i}{\sigma_i}-\frac{\theta_i T_i}{\sigma_i})) \\
- & \ \frac{1}{\lambda_i}\exp(-\lambda_i\{T_i - \frac{USL_i-\mu_i}{\theta_i} - \frac{\lambda_i\sigma_i^2}{2\theta_i^2}\}) \\
\{ & \ \Phi(\frac{USL_i-\mu_i}{\sigma_i}+\frac{\lambda_i\sigma_i}{\theta_i}) - \Phi(\frac{USL_i-\mu_i}{\sigma_i}+\frac{\lambda_i\sigma_i}{\theta_i}-\frac{\theta_i T_i}{\sigma_i})\} \\
+ & \ \frac{1}{\lambda_i}\{\Phi(\frac{USL_i-\mu_i}{\sigma_i}) - \Phi(\frac{USL_i-\mu_i}{\sigma_i}-\frac{\theta_i T_i}{\sigma_i})\}]\}
\end{aligned} \tag{6.8}$$

The percentage of defectives for stage i is given by:

$$P_d^i = P_l^i(T_i,\mu_i) + P_u^i(T_i,\mu_i), i = 1,\ldots,n \tag{6.9}$$

The production rate for stage $i+1$ can be expressed as:

$$R_{i+1} = R\prod_{j=1}^{i}(1-P_d^j), i = 1,\ldots,n \tag{6.10}$$

where $R_1 = R$ and R is the production rate for the first stage.

The effective (actual) production rate of the production system or the rate of delivering nondefective finished items is given by:

$$R_{eff} = R_{n+1}$$

$$= R\prod_{j=1}^{n}(1 - P_d^j) \quad (6.11)$$

Thus, the expected total cost can be calculated as follows:

$$E(TC) = \sum_{i=1}^{n}[R_i(C_l^i P_l^i(T_i, \mu_i) + C_u^i P_u^i(T_i, \mu_i)) + \frac{C_R^i}{T_i}] + W \cdot max(0, Q - R_{eff}) \quad (6.12)$$

The model described above can be posed as a nonlinear programming (NLP) problem. This NLP problem may be written as follows:

$min\ \sum_{i=1}^{n}[R_i(C_l^i P_l^i(T_i, \mu_i) + C_u^i P_u^i(T_i, \mu_i)) + \frac{C_R^i}{T_i}] + W \cdot max(0, Q - R_{eff})$

subject to

$P_d^i = P_l^i(T_i, \mu_i) + P_u^i(T_i, \mu_i), i = 1, ..., n$

$R_{i+1} = R\prod_{j=1}^{i}(1 - P_d^j), i = 1, ..., n$

$R_1 = R$

$R_{eff} = R\prod_{j=1}^{n}(1 - P_d^j)$

Clearly, by using proper substitution, the above NLP can be transformed to an unconstrained NLP.

6.6 Results and Discussion

One can find the optimal process means (i.e. μ_i's) and the optimal cycle lengths (i.e. T_i's) by minimizing $E(TC)$ using any unconstrained optimization procedure (e.g. Hooke and Jeeves method (Bazaraa et al. [1993])). However, the result will be at best a local minimum. Therefore, we have used the hybrid tabu search algorithm

(TSFGO) developed in chapter 5 to find the global minimum of the proposed model.

Next, we present an example to illustrate our model.

Example 6.1:

We consider the example given by Billatos and Kendall [1991] with some additions needed by our problem statement. The data of the example is given in Table 6.1. Also, let $W = 3$, $R = 110$ and $Q = 100$.

This example was solved using both Hooke and Jeeves algorithm, and TSFGO algorithm. The results are given in Table 6.2 and Table 6.3. Notice that the percent

Stage, i	σ_i	C_R^i	C_l^i	C_u^i	USL_i	LSL_i	λ_i	θ_i
1	0.49	100	0.1	0.1	14	10	0.5	0.1
2	0.663	100	0.15	0.15	13	11	0.4	-0.2
3	2.13	100	0.2	0.2	24	15	0.3	0.1
4	2.15	100	0.25	0.25	22	14	0.2	0.15
5	2.29	100	0.3	0.3	25.5	15	0.1	-0.25

Table 6.1: Data for the example.

Stage, i	μ_i	T_i	R_i
1	11.96	12.80	110
2	12.20	4.88	109.94
3	20.08	12.26	92.35
4	18.47	7.27	87.78
5	20.88	13.61	83.66

$E(TC)$ = \$125.17 and R_{eff}=81.03

Table 6.2: Results of Example 6.1 using Hooke and Jeeves algorithm.

reduction in the expected total cost by using TSFGO algorithm over Hooke and Jeeves algorithm is 10.7%. For other examples, this percent reduction ranges from 4% to 38%.

The results of this model (MSM) are important and useful. The models in the literature lack the joint optimization of (1) initial means settings, and (2) production

Stage, i	μ_i	T_i	R_i
1	11.15	20.87	110
2	12.20	4.88	109.78
3	18.75	21.38	92.21
4	17.65	12.73	88.56
5	20.85	13.59	83.95

$E(TC)$ = \$111.81 and R_{eff}=81.32

Table 6.3: Results of Example 6.1 using TSFGO algorithm.

cycle lengths when considering multistage production systems. The significance of our model came from linking both of the above two elements in one integrated model. Thus, the stated objective in chapter 1 has been accomplished.

Next, we discuss ways of improving the performance of the above system, which consequently reduces the total cost.

6.6.1 Process Improvement

For the given example, one can see that this system is not capable of meeting the demand. One approach to solve this problem is by improving the performance of the process. This may include variance reduction and drift rate alleviation. Possible ways of reducing the variance include training of operators, regulating the current, etc. We have examined reengineering the production system of the example by reducing the variance of each process at each stage by 20%, 30%, and 70% (for simplicity, we assume that all variances are reduced at the same rate). Results are shown in Table 6.4, 6.5, and 6.6 respectively.

One can see that by reducing the variances of the processes, the total cost per unit time is reduced. However, this can only be done at a certain cost (which includes operator retraining, better raw material, homogeneity, regulating the current, etc.). Let us assume that the net gain from the above reduction in the variance

Stage, i	μ_i	T_i	R_i
1	11.59	18.14	110
2	12.19	4.65	109.69
3	19.71	15.84	100.17
4	18.71	9.49	98.70
5	21.10	15.27	93.94

$E(TC)$ = \$ 78.23 and R_{eff}=92.88.

Table 6.4: Reduction of σ_i by 20% each.

Stage, i	μ_i	T_i	R_i
1	11.37	20.38	110
2	12.23	5.10	109.87
3	19.79	17.57	102.98
4	18.98	9.10	102.19
5	21.36	16.97	98.09

$E(TC)$ = \$ 59.47 and R_{eff}=97.47.

Table 6.5: Reduction of σ_i by 30% each.

Stage, i	μ_i	T_i	R_i
1	10.58	35.39	110
2	12.69	9.80	108.18
3	17.55	55.41	101.66
4	16.58	35.39	101.48
5	23.42	30.53	100.102

$E(TC)$ = \$ 24.07 and R_{eff}=100.

Table 6.6: Reduction of σ_i by 70% each.

(i.e., reduction in $E(TC)$ - cost of variance reduction) is computed, and that 30% reduction was found to be the best.

Further reduction of the total cost can be done by considering the maintenance of the system. Proper maintenance of the system (e.g. use lubricants more frequently) will reduce the drift rate, hence improving the capability of the system. Suppose that, we have been able to reduce the drift rate by 20% for all processes (e.g. sharpening the cutting tools) (again for simplicity, we assume reduction at the same rate). Like the variance reduction above, one has to weigh benefits and costs in considering drift rate reduction. Result of this reduction is shown in Table 6.7.

Stage, i	μ_i	T_i	R_i
1	11.37	30.09	110
2	12.28	6.56	108.66
3	19.69	20.30	101.39
4	18.75	8.33	100.65
5	21.34	19.90	100.07

$E(TC) = \$\ 45.35$ and R_{eff}=99.50.

Table 6.7: Reduction of θ_i by 20% each.

Hence, one can see that the production system can be improved by reducing the variances and the drift rates of the processes. However, this has been shown by an example using the reduction of both variances and drift rates by fixed amounts in all stages (e.g. reduction of θ_i by 20% each). To find the optimal percent reduction in the variance of the process at each stage, one needs to develop a model. This topic is discussed in chapter 7.

6.7 Extensions to Multistage Lines without Buffers and with Nonzero Repair Times (MSM2)

In section 6.5, we have assumed zero repair times (i.e. repair times are negligible) and zero buffers. In this section, we relax this assumption (assumption 6) while keeping the remaining as they are. In sectoin 6.8, we treat the case when assumptions 5 and 6 stated in section 6.4 are relaxed.

In this section, we develop a model (MSM2) which finds the effective production rate when repair times (downtimes) are nonzero as was the case in section 6.5. In order to develop MSM2, we need to define the availability of the line.

We define the availability of the line (A) as the percentage of time the line is up (i.e. working). That is

$$A = \frac{\text{Total time the line is up}}{\text{Total time}} \tag{6.13}$$

where total time includes total time the line is up and total time the line is down.

Our model is motivated by the following observation: One can note that the availability of the line using MSM1 is unity since we assumed instantaneous repair times. By the inclusion of repair times, the availability of the line may decrease and hence the effective production rate of the line, R_{eff}, may also decrease. Using this observation, we propose the following formula for estimating R_{eff} for a multistage line with nonzero repair times.

$$R_{eff} = A \ R^0_{eff} \tag{6.14}$$

where R^0_{eff} is the effective production rate using MSM1. Thus, the only requirement is to estimate A which is developed hereunder.

Let:

T_i be the cycle time (uptime) of stage i.

D_i be the repair time (downtime) of stage i.

R_i be the production rate of stage i.

P^i_d be the percentage of defectives for stage i.

The percentage of defectives for stage i has been presented in section 6.5 and it is given by:

$$P^i_d = P^i_l(T_i, \mu_i) + P^i_u(T_i, \mu_i) \tag{6.15}$$

We assume that D_i's are deterministic and known. Also, we assume that the line satisfy assumption 5. In section 6.8, we show how to treat the case when assumption 5 is relaxed.

For a series system with units that have uptimes (T_i) and downtimes (D_i), Barlow and Proschan [1975] have proved the following result

$$A = \frac{1}{1 + \sum_{i=1}^{n} \frac{D_i}{T_i}} \tag{6.16}$$

However, the above formula is valid for a system with no defectives. For a system with defectives, the above formula is modified as follows:

$$A = \frac{1}{1 + \sum_{i=1}^{n} \frac{D_i}{T_i}} \tag{6.17}$$

where

$$\hat{T}_i = \frac{T_i}{1 - P_d^{i-1}} \tag{6.18}$$

and P_d^0=0.

Now, in order to calculate the effective production rate for a multistage line with nonzero repair times, one has to do the following steps:

1. Find R_{eff}^0 using MSM1.

2. Find A using equation (6.17).

3. Calculate R_{eff} using equation (6.14).

Next, we demonstrate the performance of MSM2. The performance of MSM2 has been verified and compared with simulation. The simulation model has been developed using *SLAM II* package and is shown in Appendix F. We performed two experiments. The input parameters for the two experiments are shown in Table 8.8.

Stage, i	μ_i	λ_i	θ_i	σ_i	R_i	USL_i	LSL_i
1	10.7	0.5	0.1	1	110	12	10
2	10.7	0.5	0.1	1	110	12	10

Table 6.8: Data for testing MSM2.

The first step is to solve the problem using MSM1 to get R_{eff}^0 by assuming $D_1 = D_2 = 0$. The solution came out to be $R_{eff}^0 = 48.24$ with $\mu_1 = \mu_2 = 10.7$ and $T_1 = T_2 = 9.42$. For the second experiment, we fix $T_1 = 9$ and $T_2 = 5$ while keeping $\mu_1 = \mu_2 = 10.7$. $R_{eff}^0 = 48.07$ for the second experiment.

For estimating A and R_{eff} in *SLAM II*, we have used the replication/deletion method (Law and Kelton [1991]) with 10 replications. We have calculated the confidence interval (CI) for A and R_{eff} for each case. In Table 6.9, we show the half length of the confidence interval. We have also used the method of Common Random Numbers (CRN) (Law and Kelton [1991]) to reduce the variance of the simulation output.

Table 6.9 shows the results for different repair times for both experiments. As can be seen, MSM2 performed very well in estimating the availability and the effective production rate of the line.

	Stage 1		Stage 2		MSM2		*SLAM II*			
Exp.	T_1	D_1	T_2	D_2	A	R_{eff}	A	CI	R_{eff}	CI
1	9.42	1	9.42	1	0.848	40.90	0.854	0.0003	41.19	0.21
	9.42	2	9.42	2	0.737	35.55	0.740	0.0003	36.14	0.21
	9.42	3	9.42	3	0.651	31.40	0.668	0.0008	32.20	0.24
	9.42	4	9.42	4	0.583	28.15	0.592	0.001	29.06	0.08
	9.42	1	9.42	4	0.716	34.58	0.723	0.0001	35.36	0.22
	9.42	2	9.42	4	0.666	32.13	0.671	0.001	32.99	0.26
	9.42	3	9.42	4	0.622	30.02	0.627	0.0007	30.60	0.14
	9.42	4	9.42	1	0.668	32.23	0.673	0.0004	32.91	0.20
	9.42	4	9.42	2	0.637	30.75	0.631	0.001	31.35	0.20
	9.42	4	9.42	3	0.609	29.40	0.618	0.0007	30.22	0.13
2	9	4	5	3	0.547	26.29	0.543	0.001	26.83	0.12
	9	2	5	3	0.617	29.65	0.618	0.0002	30.48	0.21
	9	1	5	3	0.663	31.87	0.663	0.001	32.76	0.21
	9	4	5	2	0.588	28.26	0.588	0.001	29.05	0.14
	9	4	5	1	0.634	30.47	0.634	0.001	31.35	0.20
	9	4	5	4	0.507	24.37	0.508	0.002	25.14	0.16

Table 6.9: Comparison of MSM2 with *SLAM II* package.

6.8 Extensions of MSM2

In section 6.7, we have presented an extended model for production lines without buffers and with nonzero repair times with the following assumption:

Assumption 5: Production rate for every stage is less than or equal to the succeeding stage.

However, when the production rates are not equal and do not satisfy assumption 5, one needs to modify the production rates in order to satisfy this assumption. We assume that the production rate for every process is adjustable. In this section, we present two approaches for dealing with the above case.

First Approach: Modification

The basic idea of this approach is to keep or modify the production rate of every stage so that assumption 5 is satisfied.

The *Modification* approach that we are proposing consists of the following steps:

Step 1: Let $R_i^M = R_i$, $i=1, ..., n$.

Step 2: Let $i=n$.

Step 3: If $R_{i-1}^M > R_i^M$ then set $R_{i-1}^M = R_i^M$; otherwise continue.

Step 4: Let $i = i - 1$.

Step 5: If $i = 1$, stop; otherwise go to step 3.

where n is number of stages, R_i^M is the new production rate of stage i for the modified line. Upon applying the *Modification* approach, one can use MSM2 model which is developed in section 6.7.

To test the performance of the *Modification* approach, we performed the following experiment. We consider a line of two stages without a buffer. Table 6.10 shows the results of this experiment. As can be seen from Table 6.10, the *Modification* approach is very effective.

		Cases		MSM2 with *Modification*		*SLAM II*	
R_1	R_2	σ_1	σ_2	A	R_{eff}	A	R_{eff}
110	100	1	1	0.59	28.49	0.59	29.09
		2	2	0.63	10.06	0.63	10.10
		1	0.5	0.59	39.37	0.59	39.98
		0.5	1	0.55	37.45	0.55	38.23
220	110	1	1	0.57	27.77	0.57	28.57
		2	2	0.66	10.55	0.66	10.56
		1	0.5	0.57	38.46	0.57	39.65
		0.5	1	0.51	34.48	0.51	35.09
330	120	1	1	0.60	28.90	0.60	29.42
		2	2	0.70	11.14	0.70	11.17
		1	0.5	0.60	39.84	0.60	41.17
		0.5	1	0.53	35.84	0.54	36.63

Table 6.10: Comparison of MSM2 with *Modification* with *SLAM II* package.

Second Approach: Homogenization

The basic idea of this approach is to convert a nonhomogeneous line into a homogeneous one. In the literature, this approach is called *Homogenization*. This approach has the advantage that models developed for homogeneous lines can be utilized even with nonhomogeneous lines.

We adopt this approach to our problem. The *Homogenization* approach that we are proposing consists of the following steps:

Step 1: Let

$$R_{min} = \min_{1 \le i \le n} \{R_i\}$$

Step 2: Let $R_i^H = R_{min}$, $i = 1, ..., n$.

where n is number of stages, R_i^H is the new production rate of stage i for the homogenized line. Of course, when the production rates are equal (i.e. homogenous line), assumption 5 is satisfied. Upon applying the *Homogenization* approach, one can use MSM2 model which is developed in section 6.7.

Clearly, the production rate of every stage using the *Modification* approach is greater than or equal to the production rate of the corresponding stage using the *Homogenization* approach.

Chapter 7

Variance Reduction and Sensitivity Analysis Studies of MSM1

In chapter 6, we have proposed a model for multistage production system (MSM1). In this chapter, we study the effect of reducing the variance on the total cost of MSM1. We also present a sensitivity analysis for the same model. This chapter is organized as follows: the variance reduction model for MSM1 is presented in section 7.1. Sensitivity analysis of MSM1 is given in section 7.2.

7.1 Variance Reduction of MSM1 (MSVRM)

In chapter 1, we have discussed various cost functions for the variance and we have suggested to use the exponential function to represent the cost of the variance. Hence, in the model covered in this chapter we are going to represent the cost of the variance by an exponential function.

7.1.1 Variance Reduction Model for MSM1

In this section, we present a mathematical model for variance reduction in multistage production systems. First, we introduce the following notation.

$E[TC\|\sigma_1, ..., \sigma_n]$	expected total cost given $\sigma_1, ..., \sigma_n$;
$E[TC\|(1-\alpha_1)\sigma_1, ..., (1-\alpha_n)\sigma_n]$	expected total cost given $(1-\alpha_1)\sigma_1, ..., (1-\alpha_n)\sigma_n$;
n	number of stages;
α_i	percentage of reducing the variance at stage i;
α_i^*	optimal percentage of reducing the variance at stage i;
a_i, b_i	parameters of the cost of the variance at stage i;
B	limited budget allocated for the variance reduction program.

$$\max NS = E[TC|\sigma_1, ..., \sigma_n] - E[TC|(1-\alpha_1)\sigma_1, ..., (1-\alpha_n)\sigma_n] \\ -(\sum_{i=1}^{n} a_i e^{-b_i(1-\alpha_i)\sigma_i} - \sum_{i=1}^{n} a_i e^{-b_i\sigma_i}) \tag{7.1}$$

subject to

$$(\sum_{i=1}^{n} a_i e^{-b_i(1-\alpha_i)\sigma_i} - \sum_{i=1}^{n} a_i e^{-b_i\sigma_i}) \leq B \tag{7.2}$$

$$0 \leq \alpha_i \leq 1, \quad i = 1, ..., n \tag{7.3}$$

In the above model, the objective function is simply the net saving (NS) which results from the variance reduction program. The net saving can be calculated as the difference between the reduction in the total cost due to the variance reduction program plus the cost of applying the variance reduction program. The first constraint (i.e. 7.2) makes sure that the cost of reducing the variances does not exceed the available budget for the variance reduction program. The second set of constraints (i.e. 7.3) sets lower and upper bounds for the decision variables (α_i's).

The mathematical model will find the optimal values of the α_i's which in turn tell us how much reduction is going to be made to the current variance at each stage.

One could look at the above model as a capital budgeting problem where one is given a budget of B and is interested in spending it for cost reduction through reduction of variances of different stages and the question is how much to invest at each stage to give the maximum saving.

7.1.2 Solution Procedure (MSVRA)

In order to solve the mathematical model of the variance reduction in the multistage production system, the scheme depicted in the flowchart of Figure 7.1 is proposed. In this scheme, we use the algorithm 'TSFGO' which was developed in chapter 5.

7.1.3 Example 7.1

Consider Example 6.1 of the multistage production system model presented in section 6.6 which has been taken from Billatos and Kendall [1991]. This example was chosen in particular to see the improvement on the multistage production system when the variance reduction program is implemented. Table 7.1 shows the values

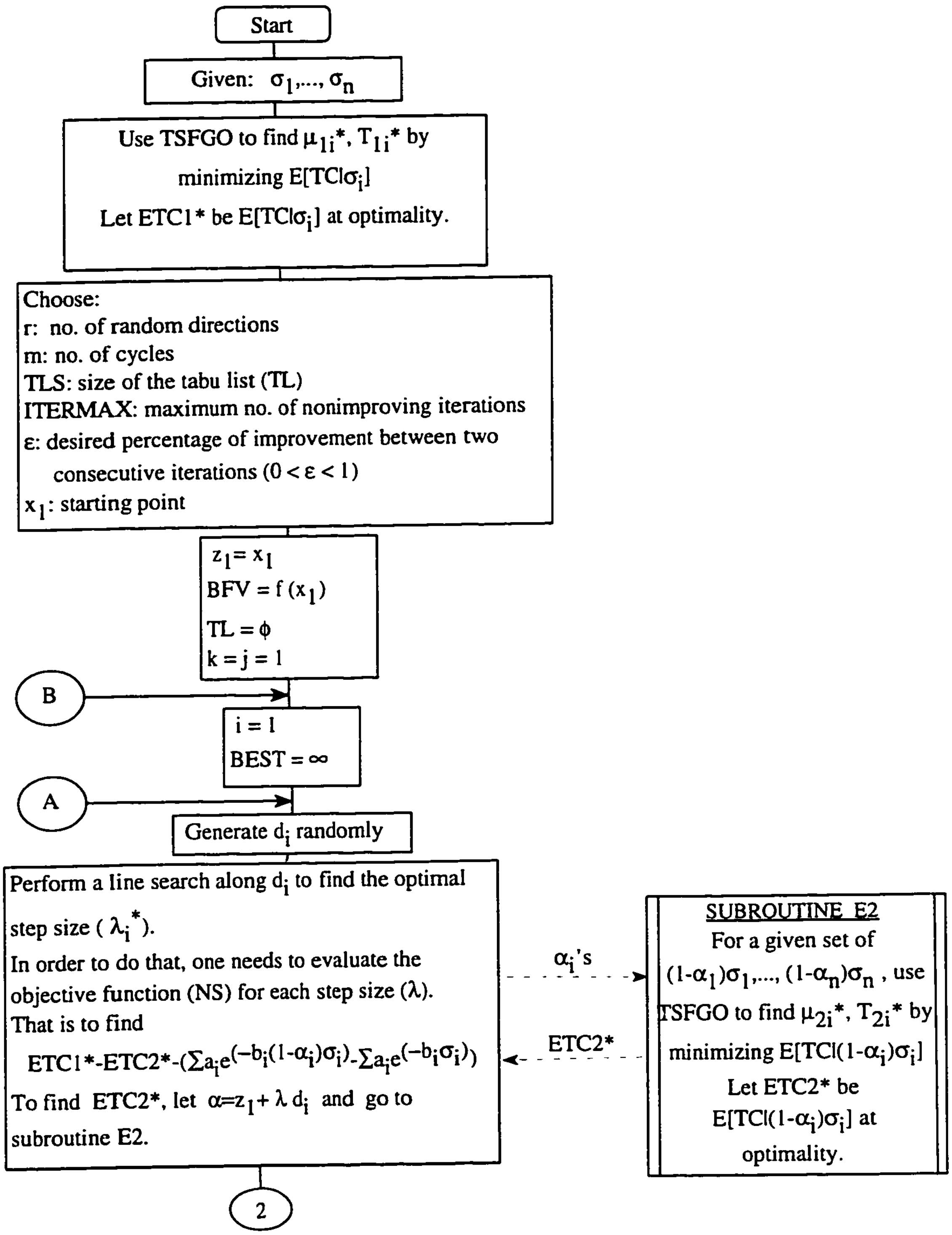


Figure 7.1: Flowchart of the algorithm MSVRA.

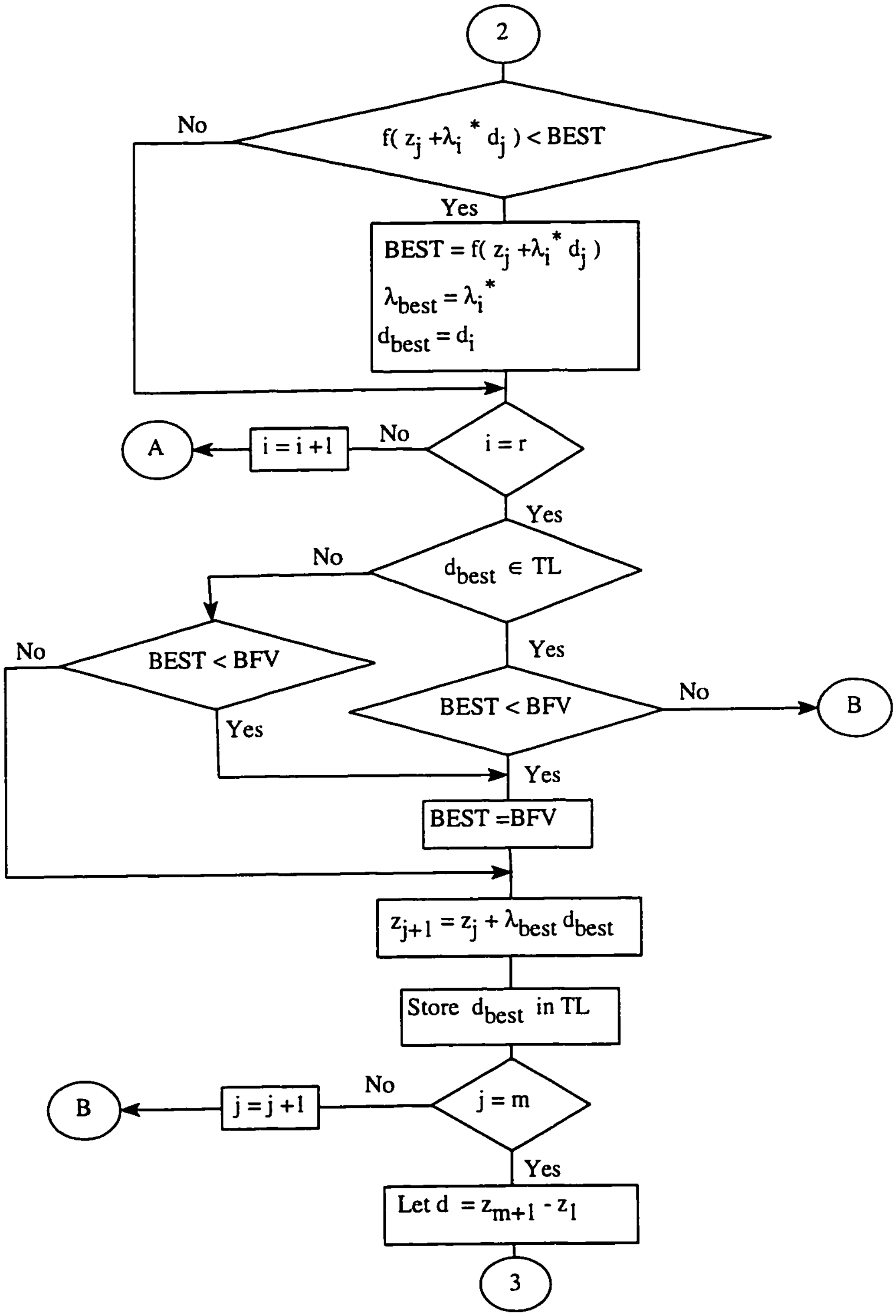


Figure 7.2: Flowchart of the algorithm MSVRA (continued).

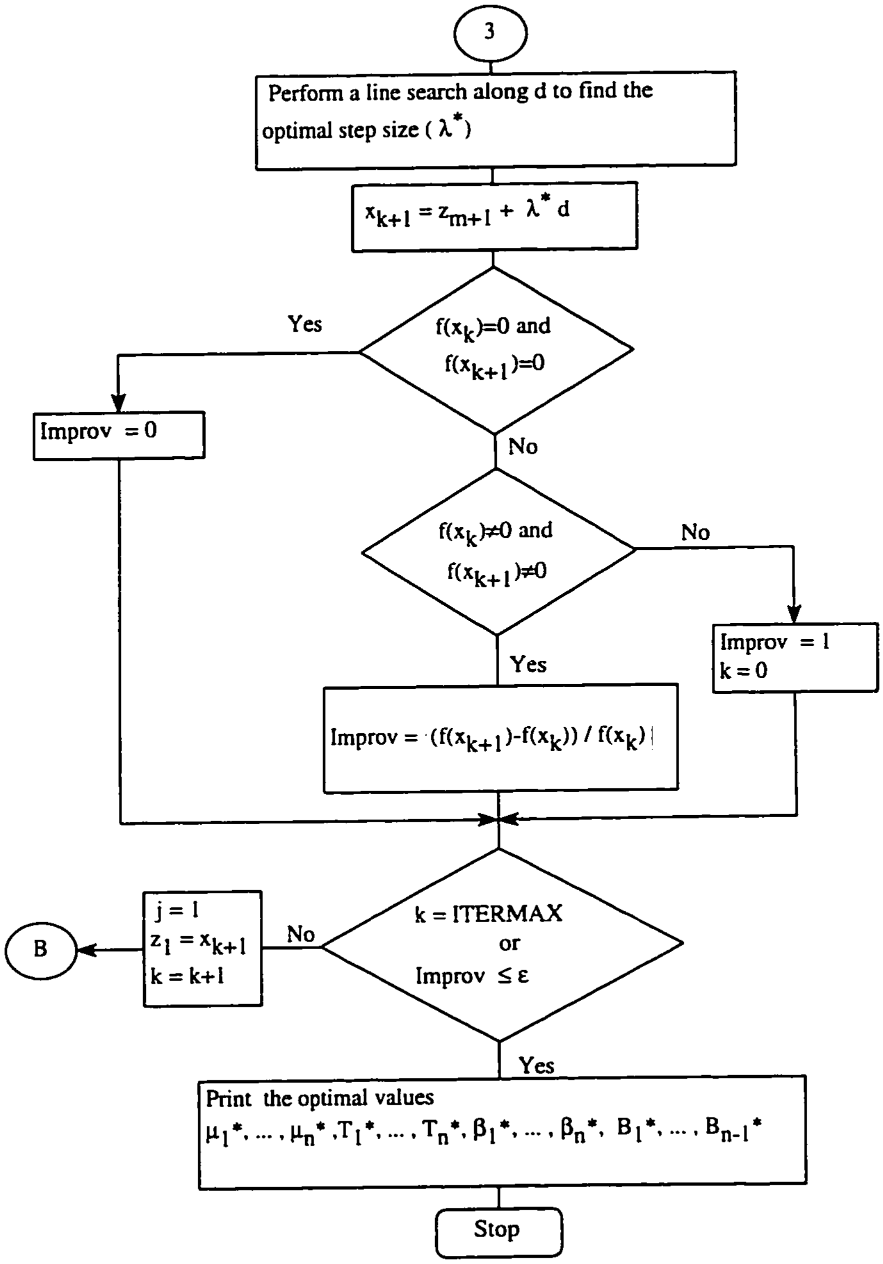


Figure 7.3: Flowchart of the algorithm MSVRA (continued).

of the parmeters a_i and b_i of the exponential function, which represents the cost of the variance, appeared in the model in (7.1) and (7.2). The budeget allocated for the variance reduction program is B=\$100.

The example is solved using the algorithm (MSVRA) proposed in the previous section. Table 7.2 shows the current production system before applying the variance reduction program. Its expected total cost is $ETC1$=\$111.81 and its effective production rate is R_{eff}=81.32.

Stage, i	a_i	b_i
1	10	0.1
2	19	0.7
3	12	0.4
4	20	0.3
5	14	0.1

Table 7.1: Values of the parameters a_i and b_i for Example 7.1.

Stage, i	σ_i	μ_i^*	T_i^*
1	0.49	11.15	20.87
2	0.663	12.20	4.88
3	2.13	18.75	21.38
4	2.15	17.65	12.73
5	2.29	20.85	13.59

Table 7.2: Current production system

Table 7.3 shows the optimal percentage of reduction of the variance at each stage. The new expected total cost is $ETC2$=\$24.56 and its effective production rate is R_{eff}=101.13.

The amount of money needed for applying the variance reduction program is as

Stage, i	α_i^*	σ_i^*	μ_i^*	T_i^*
1	0.67	0.16	10.25	36.82
2	0.87	0.09	12.85	10.03
3	0.25	1.59	17.74	43.67
4	0.36	1.37	16.55	28.96
5	0.84	0.36	24.56	35.65

Table 7.3: Improved production system

follows

$$\text{Amount of money needed} = (\sum_{i=1}^{5} a_i e^{-b_i(1-\alpha_i)\sigma_i} - \sum_{i=1}^{5} a_i e^{-b_i\sigma_i}) = \$12.58$$

The net saving due to applying the variance reduction program is as follows

$$\text{Net Saving} = 111.81 - 24.56 - 12.58 = \$74.67$$

The percentage reduction in total cost is

$$\text{Percentage reduction in total cost} = (\frac{111.81 - (24.56 + 12.58)}{111.81}) \times 100 = 66.78$$

The percentage increase in R_{eff} is

$$\text{Percentage increase in } R_{eff} = (\frac{101.13 - 81.32}{81.32}) \times 100 = 24.36$$

The results of this model (MSVRM) are important and useful. The models in the literature do not consider variances reduction in a multistage production system.

7.2 Sensitivity Analysis of MSM1

In this section, we conduct a fractional factorial experiment to study the effects of the input parameters on the total cost of the multistage model (MSM1).

There are 15 ($3+6n$, where n is number of stages) input parameters for the multistage model. We assign two levels to each input parameter in the fractional factorial experimental design. Table 7.4 shows the input parameters and their assigned levels. Notice that the values of the low level of the input parameters are taken from Example 6.1 of the multistage model presented in section 6.6. This example is based on the example of Billatos and Kendall [1991]. For ease of exposition, we consider only the first two stages.

	Parameter	Factor	Levels Low	Levels High
Common	W	A	3	5
	Q	B	100	150
	R	C	110	200
Stage 1	σ_1	D	0.49	0.6
	θ_1	E	0.1	0.2
	λ_1	F	0.5	0.6
	C_l^1	G	0.1	0.2
	C_u^1	H	0.1	0.2
	C_R^1	J	100	150
Stage 2	σ_2	K	0.663	0.8
	θ_2	L	-0.2	-0.1
	λ_2	M	0.4	0.5
	C_l^2	N	0.15	0.3
	C_u^2	O	0.15	0.3
	C_R^2	P	100	150

Table 7.4: Input parameters and levels used in the experiment.

Since the number of runs for a full factorial design of this experiment is huge (i.e. 2^{15}), a fractional factorial design is going to be used. The fractional factorial experimental design used for this experiment is 2_V^{15-7} design. This design is of resolution V, since the minimum number of letters in a word appearing in the defining relation (Figure 7.4) is 5 (Montgomery [1991b]). A total of 2^8 or 256 runs are required to conduct the experiment. This fractional design has been taken from standard fractional designs proposed by National Bureau of Standards (NBS), Applied Mathematics Series, Vol. 48, which appeared in the book of McLean and Anderson [1984]. This design corresponds to Plan 128.15.8 in that book. In this design, the defining relation as proposed by NBS (McLean and Anderson [1984]) is given in Figure 7.4. The complete set of fractions are given in Appendix D. Experimental design and results of this experiment are given in Appendix E.

Based on the results of the sensitivity analysis of the single stage, some of the two-order interactions are important and the rest are not important. Thus they are eligible for pooling for error. We have used those results here in this experiment. Table 7.5 shows the ANOVA table. It can be noted that Factors A, B, C, E, J, K, L, N, O, P, AB, AC, BC, LM and MP are significant at level 1%. That is, the parameters W, Q, R, θ_1, C_R^1, σ_2, θ_2, C_l^2, C_u^2, and C_R^2 (i.e. all main factors except σ_1, λ_1, C_l^1, C_u^1, and λ_2), and the interactions between W and Q, W and R, Q and R, θ_2 and λ_2, and λ_2 and C_R^2 are significant.

One can see from Table 7.5 that the most significant factor is R which is the production rate of the first stage. This is expected since the effective production rate of the multistage system is dependent on R (see equation (6.11)). As expected, the second and third most significant factors are Q and W. The order of the remaining significant factors is as follows: θ_2, σ_2, θ_1, C_R^1, C_u^2, C_R^2, and C_l^2. Therefore, for one to improve a multistage production system, he should concentrate on the production

Plan 128.15.8. 1/128 replication of 15 factors.
Factors: A, B, C, D, E, F, G, H, J, K, L, M, N, O, P.

I = ABEGN = ACEFNP = BCFGP = DEFGO = ABDFNO = ACDGNOP = BCDEOP
= ADHKO = BDEGHKNO = CDEFHKNOP = ABCDFGHKOP = AEFGHK = BFHKN
= CGHKNP = ABCEHKP = BCHJNOP = ACEGHJOP = ABEFHJO = FGHJNO
= BCDEFGHJNP = ACDFHJP = ABDGHJ = DEHJN = ABCDJKNP = CDEGJKP
= BDEFJK = ADFGJKN = ABCEFGJKNOP = CFJKOP = BGJKO = AEJKNO
= ABKLOP = EGKLNOP = BCEFKLNO = ACFGKLO = ABDEFGKLP = DFKLNP
= BCDGKLN = ACDEKL = BDHLP = ADEGHLNP = ABCDEFHLN
= CDFGHL = BEFGHLOP = AFHLNOP = ABCGHLNO = CEHLO = ACHJKLN
= BCEGHJKL = EFHJKLP = ABFGHJKLNP = ACDEFGHJKLNO = BCDFHJKLO
= DGHJKLOP = ABDEHJKLNOP = CDJLNO = ABCDEGJLO = ADEFJLOP
= BDFGJLNOP = CEFGJLN = ABCFJL = AGJLP = BEJLNP = CDGHJMO
= ABCDEHJMNO = ADEFGHJMNOP = BDFHJMOP = CEFHJM = ABCFGHJMN
= AHJMNP = BEGHJMP = ACGJKM = BCEJKMN = EFGJKMNP = ABFJKMP
= ACDEFJKMO = BCDFGJKMNO = DJKMNOP = ABDEGJKMOP = BDGMNP
= ADEMP = ABCDEFGM = CDFMN = BEFMNOP = AFGMOP = ABCMO
= CEGMNO = ABGHKMNOP = EHKMOP = BCEFGHKMO = ACFHKMNO
= ABDEFHKMNP = DFGHKMP = BCDHKM = ACDEGHKMN = ABCDGHJKLMP
= CDEHJKLMNP = BDEFGHJKLMN = ADFHJKLM = ABCEFHJKLMOP
= CFGHJKLMNOP = BHJKLMNO = AEGHJKLMO = BCGJLMOP = ACEJLMNOP
= ABEFGJLMNO = FJLMO = BCDEFJLMP = ACDFGJLMNP = ABDJLMN
= DEGJLM = ADGKLMNO = BDEKLMO = CDEFGKLMOP = ABCDFKLMNOP
= AEFKLMN = BFGKLM = CKLMP = ABCEGKLMNP = GHLMN = ABEHLM
= ACEFGHLMP = BCFHLMNP = DEFHLMNO = ABDFGHLMO = ACDHLMOP
= BCDEGHLMNOP.

Figure 7.4: Defining relation of the fractional factorial design.

Source	SS	df	MS	F-ratio
Main Effects:				
A	100445.3	1	100445.3	286.534
B	695927.3	1	695927.3	1985.228
C	1371034	1	1371034	3911.063
D	16.82313	1	16.82313	0.04799
E	2423.034	1	2423.034	6.912038
F	153.8696	1	153.8696	0.438934
G	141.6636	1	141.6636	0.404115
H	596.8544	1	596.8544	1.702609
J	1519.488	1	1519.488	4.334548
K	30398.25	1	30398.25	86.71519
L	41146.17	1	41146.17	117.3751
M	1.493181	1	1.493181	0.00426
N	1404.258	1	1404.258	4.005839
O	2033.59	1	2033.59	5.801095
P	1994.08	1	1994.08	5.688387
Interactions:				
AB	45314.84	1	45314.84	129.2668
AC	88170.7	1	88170.7	251.519
BC	561158.9	1	561158.9	1600.783
DG	153.4117	1	153.4117	0.437628
DH	89.28947	1	89.28947	0.25471
EF	114.4512	1	114.4512	0.326488
EJ	292.0274	1	292.0274	0.833048
FJ	17.79304	1	17.79304	0.050757
GH	5.325238	1	5.325238	0.015191
KO	9.89044	1	9.89044	0.028214
LM	1240.552	1	1240.552	3.538845
LP	15.62352	1	15.62352	0.044568
MP	834.1794	1	834.1794	2.379611
NO	15.60881	1	15.60881	0.044526
Residual	79224.93	226	350.5528	
Total	3025894	255		

Table 7.5: ANOVA for the multistage model.

rate of the first stage (i.e. R). The most significant interaction is between Q and R. As Q increases, $E(TC)$ increases, when R is fixed. However, as R increases, $E(TC)$ decreases, when Q is fixed. The second most significant interaction is between W and R, followed by W and Q, λ_2 and θ_2, λ_2 and C_R^2.

An interesting observation is that all the parameters of the first stage and their interactions are not significant except θ_1 and C_R^1. All the conclusions of the sensitivity analysis of the single stage model regarding the effect of the parameters on μ_i's, T_i's, and $E(TC)$ which were preseted in section 6.5 are also applicable to the multistage model.

One has to remeber that the above observations apply only for the given levels of the input parameters (see Table 7.4). It is not clear what would be the the results if these levels were changed.

Chapter 8

Models for MultiStage Production Systems With Buffers

In chapter 6, we have presented models for multistage production systems without buffers (MSM1 and MSM2). In this chapter, we extend these models to incorporate buffer storages between stages and the maintenance of the stages through the reduction of the drift rate of each stage. This chapter is organized as follows: an introduction is given in section 8.1. In section 8.2 we present the statement of the problem. Some new notation are introduced in Section 8.3. In Section 8.4, we state the assumptions. A simulation model for multistage systems with buffers (MSM3) is developed in section 8.5. In section 8.6, the optimization model for multistage production systems with buffers (MSM4) is presented.

8.1 Introduction

In chapter 6, we have developed models for finding the the initial mean settings of the processes and the cycle times in multistage production systems without buffers (MSM1 and MSM2). In those models, we assumed that no buffer is allowed between stages. Moreover, we assumed that the drift rate of each process is uncontrollable.

These assumptions if relaxed might reduce the expected total cost. However, they make the model complicated from the mathematical point of view which necessitates the use of simulation in analyzing the model.

In this chapter, we relax the above assumptions. The relaxed assumptions are more realistic and may increase the effective production rate which may result in reducing the expected total cost.

In order to remove confusion about the various models, we highlight below the proposed models in this chapter and the differences between them:

- First model (Simulation), MSM3. This is a simulation model for multistage lines with buffers for estimating the effective production rate of the line and the WIP (Work In Process) in each buffer, for fixed values of μ_i's and T_i's.

- Second model (Optimization), MSM4. This is a model for finding the optimal initial mean setting of the processes, the optimal cycle lengths, the optimal buffers sizes, and the optimal percent reduction in drift rates of the processes.

The system that is going to be studied in this chapter is fully described in the next section. A literature review of multistage production systems has been given in chapter 2.

8.2 Problem Statement

Consider a multistage production line which consists of n stages and $n-1$ interstage buffers, $(B_1, ..., B_{n-1})$. We consider discrete parts production. A part flows from stage 1 to B_1, then to stage 2, and so on until it reaches stage n, after which it leaves the system (see Figure 8.1). It is assumed that there are always parts available at the input of the line and that spaces are available at the output of the line. Each buffer, B_i, has a finite capacity, K_i.

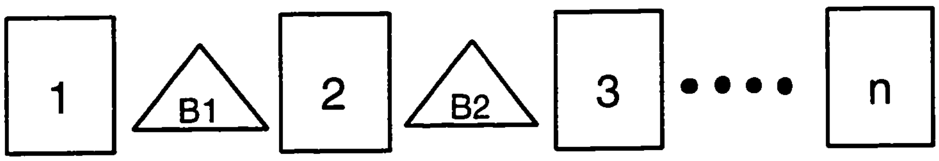


Figure 8.1: A multistage production system.

On each stage i, the process mean (μ_i) starts drifting at a random point of time, τ_i. We assume that τ_i is exponentially distributed and that the drift is linear with time. Every time process i, completes a cycle time, T_i (a decision variable), process i is stopped for repair. The repair time (D_i) is assumed to be deterministic and known. Process i resumes its operation after the repair is completed. When process i is under repair, the parts in buffer B_{i-1} tends= to increase while the parts in B_i tend to decrease. After some time, B_{i-1} becomes full and process $i-1$ is blocked and thus it is forced to stop. This is known as *blocking*. On the other hand, B_i becomes empty and process $i+1$ is starved and thus it is forced to stop. This is known as *starvation*.

In this chapter, we develop a mathematical model for finding the optimal initial settings of the processes means (μ_i^*'s), the production cycle times (T_i^*'s), the optimal percent reduction in drift rates (β_i^*'s), and the optimal buffer sizes (B_i^*'s) for the above described problem.

8.3 Notation

We present below some new notation that are needed for this chapter.

R_i	production rate of stage i;
P_d^i	percentage of defective items at stage i;
Q	demand/unit time;
W	penalty for unfulfilled demand ($/item);
R_{eff}	effective (actual) production rate of the production system (item/unit time);
β_i	percentage of reduction in drift rate at stage i;
B_i	buffer size after stage i;
WIP_i	average number of parts in buffer i;
K_i	maximum capacity of buffer i;
H	inventory holding cost per unit time per item;
Z	total available budget for investment in drift rate reduction.

8.4 Assumptions

Before we develop our model we make the following assumptions:

1. The process at each stage begins in an in-control state having a normally distributed quality characteristic with mean μ_i and variance σ_i^2.

2. The process at each stage starts deteriorating at a random point of time, which is exponentially distributed, and deterioration is linear with time.

3. The process variance at each stage remains constant.

4. The material cost is either independent of the choice of μ_i and T_i (e.g. the process of producing inner holes in shafts), or their effect on cost of material can be assumed negligible. This assumption is implicitly made in most of the literature of this kind of problem.

5. Demand per unit time is deterministic and known.

6. The investment in drift rate reduction leads to favorable results.

7. Parts are transported between stages in discrete fashion (i.e. individually).

8.5 The Proposed Simulation Model for Multistage Systems with Buffers given μ_i's and T_i's (MSM3)

In this section, we develop a simulation model which estimates the effective production rate and the *WIP* at each buffer of multistage lines with buffers and stages have nonzero repair times (downtimes).

We are going to develop the simulation model of multistage production systems with buffers (MSM3) using our own modelling. Therefore, we will develop our own code for MSM3. This enables us to link it with our global optimization algorithm

(TSFGO). The linkage between the simulation model (MSM3) and the optimization algorithm TSFGO is shown in section 8.6.4 as in the second model.

One may think that developing MSM3 using *SLAM II* package will serve the purpose. This is true if one is only interested in estimating the effective production rate and the WIP at each buffer for a specific multistage line, and he is not interested in optimizing the line. Suppose that he is intereseted in finding the optimal buffer sizes, then he has to run (manually) his *SLAM II* model for each possible alternative (combination). The number of alternatives grows very fast with the number of buffers and maximum capacity of each buffer. One can see how much effort and time are needed to optimize the multistage line using *SLAM II* model. In order to save time and effort, and to link the simulation model with our optimization algorithm (TSFGO), we have automated the process by developing our own simulation code for MSM3.

Before we present our simulation model for multistage production systems with buffers (MSM3), we present a simulation model for two-stage lines with buffers (TSM). In addition, repair times are treated as nonzero. This step is needed since the building block for MSM3 is the model for two-stage line with buffer (TSM). Hence, first, we develop a simulation model for two-stage lines with buffers and nonzero repair times (TSM) in section 8.5.1. Secondly, we develop a simulation model for multistage lines with buffers and nonzero repair times (MSM3) in sectoin 8.5.2.

8.5.1 Simulation Model for Two-Stage Lines with a Buffer and with Nonzero Repair Times (TSM)

In this section, we develop a simulation model for two-stage lines with a buffer and with nonzero repair times. The availability of the line is similarly defined as in equation (6.13), that is

$$A = \frac{\text{Total time the line is up}}{\text{Total time}} \tag{8.1}$$

However, total time now includes total time the line is up, total time the line is down for repair, total time the line is down because of blocking, and total time the line is down because of starvation.

Equation (6.14) proposed in section 6.7 can not be applied here since we do not have a model for estimating R_{eff} for a line with buffers and with zero repair times. Hence, we will develop a simulation model for estimating R_{eff} and average content of the buffer (WIP) for a two-stage line with a finite buffer and with nonzero repair times.

One can note that the possible states of a stage are four, namely, up, down, blocked, and starved. Since we have two stages, the overall possible states is 16. However, since the state {first stage is blocked, second stage is starved} never happens, the overall possible states becomes 15. These states are shown in Table 8.1. Corresponding to each state, there is a corresponding action that should be taken. These actions are also shown in Table 8.1.

When the production line consists of only two stages and one buffer, one should remember that states 4, 8, 11, 12, 13, 14, and 15 are not applicable since we assume

State	Status		Action
	Stage 1	Stage 2	
1	up	up	Advance the uptime of both stages and change the content of the buffer if necessary.
2	up	down	Advance the uptime of stage 1, advance the downtime of stage 2, and change the content of the buffer if necessary.
3	up	starved	Advance the uptime of stage 1 and change the content of the buffer if necessary.
4	up	blocked	Advance the uptime of stage 1 and change the content of the buffer if necessary.
5	down	down	Advance the downtime of both stages.
6	down	up	Advance the downtime of stage 1, advance the uptime of stage 2, and change the content of the buffer if necessary.
7	down	starved	Advance the downtime of stage 1.
8	down	blocked	Advance the downtime of stage 1.
9	blocked	up	Advance the uptime of stage 2 and change the content of the buffer if necessary.
10	blocked	down	Advance the downtime of stage 2.
11	blocked	blocked	Advance the clock time.
12	starved	up	Advance the uptime of stage 2 and change the content of the buffer if necessary.
13	starved	down	Advance the downtime of stage 2.
14	starved	blocked	Advance the clock time.
15	starved	starved	Advance the clock time.

Table 8.1: Possible states and actions for a two-stage line with buffer.

that the first stage in the line is never starved and the last stage is never blocked. However, they are shown here since two-stage model (TSM) is going to be used as a building block for the model of multistage line with buffers.

Utilizing Table 8.1, we have developed the algorithm TSMA. This algorithm consists of 3 modules. The first module is for checking the status of each stage and update them (if necessary) according to the present situation. The second module is for generating a random variable, representing quality characteristic i at stage i, from a normal distribution with a mean μ_i and a variance σ_i^2. This random variable is checked against the specification limits of the product to determine whether or not the current item is defective. The third module is for taking the necessary actions according to the current state of the line with the help of Table 8.1. Figure 8.2 shows the flowchart of TSMA.

To demonstrate the performance of the algorithm TSMA, we have performed the following experiments. The input parameters for the following experiments are the same as in Table 6.8. We consider $T_1 = 9$, $D_1 = 4$, $T_2 = 5$, $D_2 = 4$. For each buffer level, we consider four cases:

- case 1 ($\sigma_1 = 1$, $\sigma_2 = 1$): current system.
- case 2 ($\sigma_1 = 2$, $\sigma_2 = 2$): stage 1 worsen, stage 2 worsen.
- case 3 ($\sigma_1 = 1$, $\sigma_2 = 0.5$): stage 1 the same, stage 2 improved.
- case 4 ($\sigma_1 = 0.5$, $\sigma_2 = 1$): stage 1 improved, stage 2 the same.

We have verified and compared the performance of TSMA with *SLAM II* package. The simulation model using *SLAM II* package is shown in Appendix G.

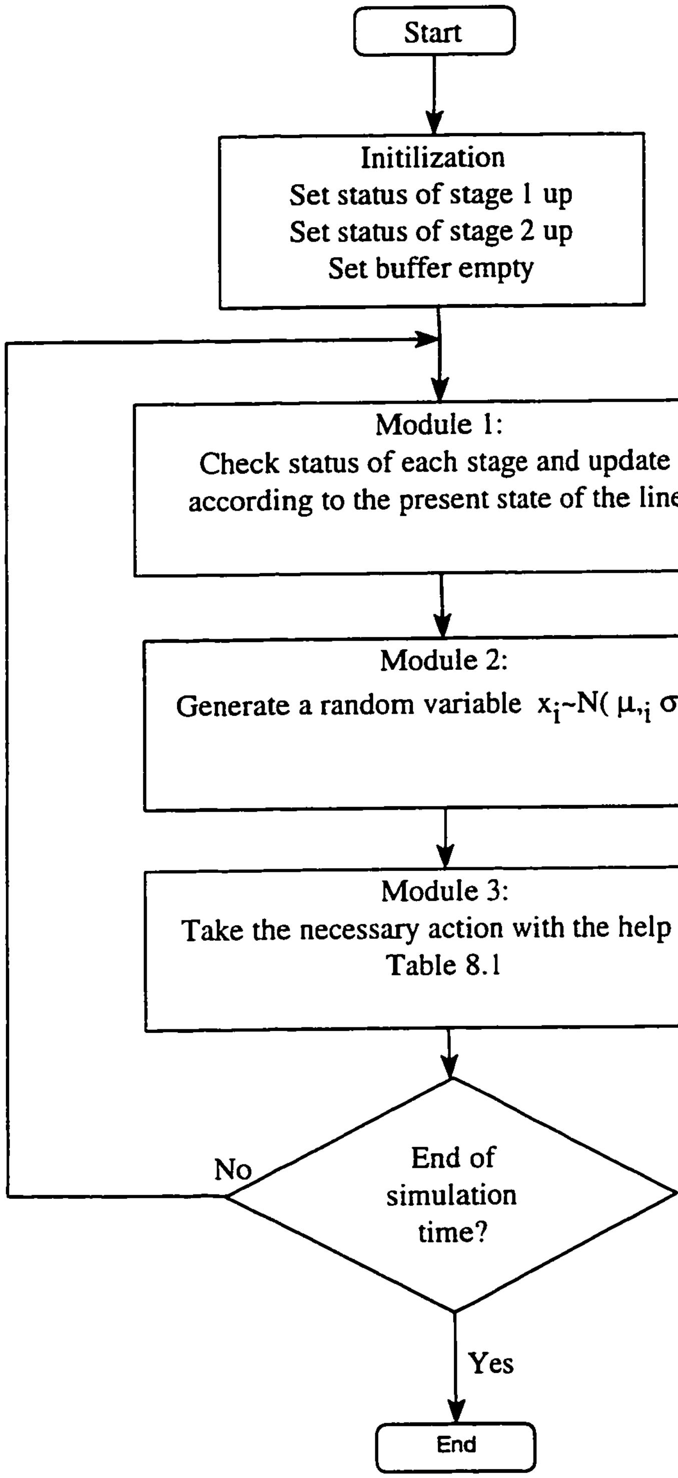


Figure 8.2: Flowchart of TSMA.

For estimating R_{eff} and average WIP in *SLAM II*, we have used the replication/deletion method (Law and Kelton [1991]) with 10 replications. We have calculated the 95% confidence interval (CI) for R_{eff} and average WIP for each case. In Table 8.2, we show the half length of the confidence interval. We have also used the method of Common Random Numbers (CRN) (Law and Kelton [1991]) to reduce the variance of the simulation output.

Table 8.2 shows the complete results. Note that when the buffer level is 1000, there are no results for *SLAM II*. This is due to the limitations of the *SLAM II* package. Hence, one of the advantages of our simulation algorithm, TSMA, is that it can handle larger buffer levels.

As it is shown in Table 8.2, the performance of TSMA is very good. The estimation of R_{eff} and the average WIP is quite accurate. As expected, increasing the capacity of the buffer increases both R_{eff} and the average WIP. Notice that, when stage 1 has improved (i.e. case 4), the average WIP has increased. This is because more good items are now produced by stage 1.

8.5.2 Simulation Model for Multistage Lines with Buffers and with Nonzero Repair Times (MSM3)

In sections 8.5.1, we have presented a model for two-stage lines with buffers. In this section, we develop a model for multistage production lines (MSM3) by generalizing and extending the two-stage model (TSM).

As we said before, the building block for the multistage model (MSM3) is TSM. The basic idea behind MSM3 is to divide the production line into two-stage line subsystems in such a way that will reflect the behavior of the whole line. More

Buffer	Cases σ_1	σ_2	TSMA R_{eff}	*Avg. WIP*	*SLAM II* R_{eff}	*CI*	*Avg. WIP*	*CI*
10	1	1	24.87	2.73	25.56	0.26	2.69	0.04
	2	2	9.17	1.69	9.33	0.06	1.67	0.05
	1	0.5	34.48	2.74	35.40	0.23	2.71	0.03
	0.5	1	31.34	3.72	31.88	0.11	3.80	0.03
100	1	1	27.62	31.10	28.48	0.20	31.16	0.96
	2	2	10.43	13.09	10.50	0.21	14.06	0.71
	1	0.5	38.48	31.33	39.33	0.31	31.47	0.69
	0.5	1	34.96	55.30	34.86	0.25	57.87	1.15
560	1	1	33.89	93.55	34.26	0.20	94.74	0.87
	2	2	10.94	19.81	11.01	0.09	20.50	1.42
	1	0.5	46.29	96.70	47.81	0.30	95.52	1.37
	0.5	1	40.81	456.2	41.34	0.19	459.59	1.61
1000	1	1	34.44	94.68				
	2	2	11.06	21.87				
	1	0.5	47.86	99.94				
	0.5	1	91.32	812.45				

Table 8.2: Comparison of TSMA with *SLAM II* package.

specifically, we first consider the first and second stages with the buffer between them as a separate subsystem and analyze its behavior, then we consider the second and the third stages with the buffer between them as a separate subsystem and analyze its behavior, and so on until we reach the final stage. Hence, we have $n - 1$ two-stage subsystems where n is number of stages. The aforementioned division of the production line into two-stage subsystems is known in the literature as the *Decomposition* principle. We have developed an algorithm that implements the idea above and we call it MSM3A. Figure 8.3 shows the flowchart of MSM3A.

An experiment has been performed to test the performance of MSM3A. We consider a line of three stages and two buffers. We select $T_1 = 9$, $D_1 = 4$, $T_2 = 5$, $D_2 = 4$, $T_3 = 6$, $D_3 = 2$. Table 8.3 shows the cases used in the experiment. The performance of MSM3A has been verified and compared with *SLAM II* package.

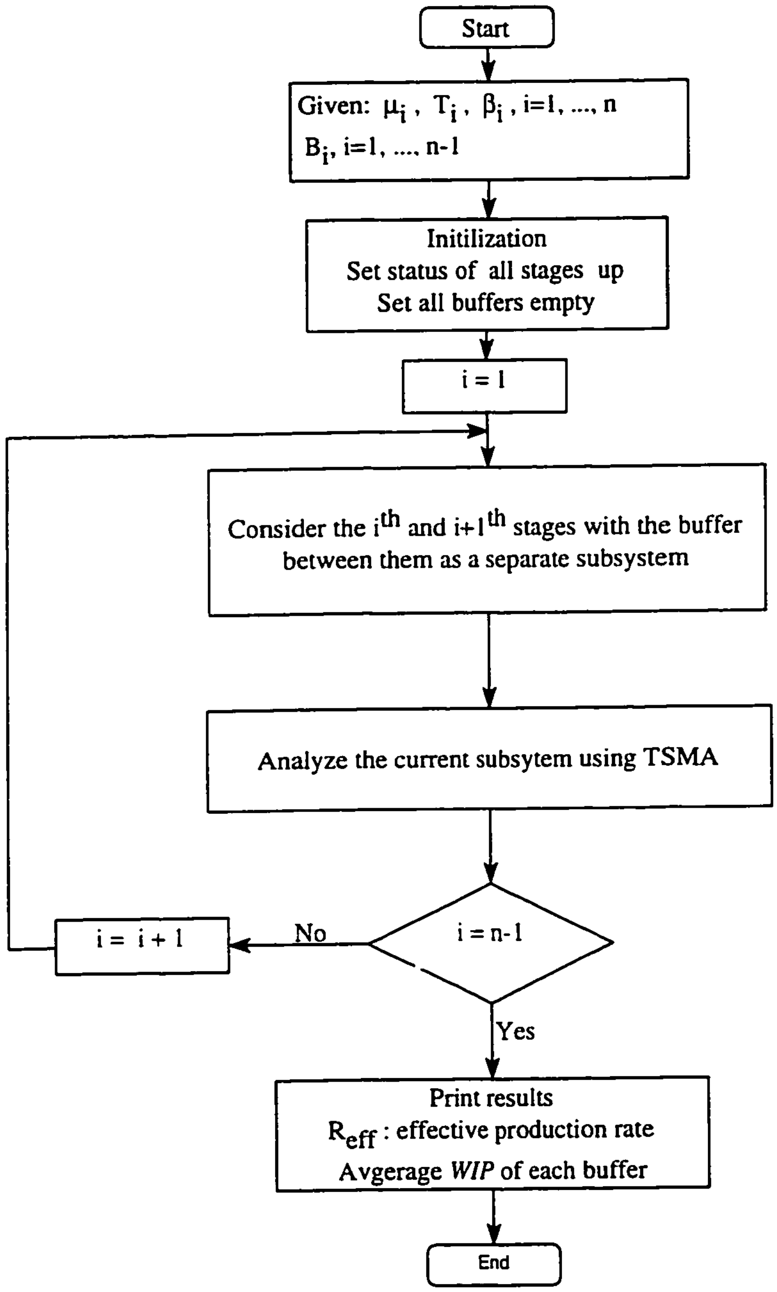


Figure 8.3: Flowchart of MSM3A.

The simulation model that has been developed using *SLAM II* package is shown in Appendix H.

For estimating R_{eff} and average WIP_i's in *SLAM II*, we have used the replication/deletion method (Law and Kelton [1991]) with 10 replications. We have calculated the 95% confidence interval (CI) for R_{eff} and average WIP_i's for each case. In Table 8.4, we show the half length of the confidence interval. We have also used the method of Common Random Numbers (CRN) (Law and Kelton [1991]) to reduce the variance of the simulation output.

Table 8.4 shows the results of this experiment. As can be seen from Table 8.4, MSM3A is fairly close to *SLAM II* package. The average percentage of absolute deviation from *SLAM II* package is about 2%. From Table 8.4, the algorithm MSM3A found R_{eff} within the confidence interval in all the cases except for five cases where it is overestimated. The estimates of average WIP found by MSM3A are within the confidence interval in all the cases except for four cases where it is underestimated and one case where it is overestimated. However, for those cases where the estimates are beyond the confidence interval, the estimates are not substantially far from the confidence limits.

Two observations from case 1 and 2 are in order. First, since stage 1 and stage 3 have been improved, the effective production rate of the line has increased. Second, the average WIP of the first buffer after stage 1 has increased. This is because more good items are now produced from stage 1.

Case	Buffer 1	Buffer 2	Production Rates R_1	R_2	R_3	Variances σ_1	σ_2	σ_3
1	10	10	110	110	110	1	1	1
2	10	10	110	110	110	0.5	1	0.5
3	10	10	220	220	220	1	1	1
4	10	10	220	220	220	0.5	1	0.5
5	10	10	330	110	220	1	1	1
6	50	50	110	110	110	1	1	1
7	50	200	110	110	110	1	1	1
8	100	100	110	110	110	1	1	1
9	100	100	110	110	110	0.5	1	0.5
10	200	100	110	110	110	1	1	1
11	200	100	110	220	330	1	0.5	1
12	200	100	330	220	110	1	1	1
13	200	100	330	220	110	1	0.5	1

Table 8.3: Cases used for testing MSM3A.

Case	MSM3A R_{eff}	MSM3A Avg. WIP_1	MSM3A Avg. WIP_2	*SLAM II* R_{eff}	CI	Avg. WIP_1	CI	Avg. WIP_2	CI
1	17.08	2.75	0.46	16.13	0.18	3.13	0.05	0.67	0.03
2	28.24	3.96	0.62	27.82	0.24	4.41	0.11	0.87	0.02
3	34.31	2.72	0.65	33.04	0.63	3.18	0.03	0.69	0.02
4	55.37	3.54	0.78	55.19	0.28	4.24	0.03	0.87	0.01
5	20.38	3.27	0.29	19.71	0.25	3.35	0.16	0.34	0.05
6	18.05	15.64	3.91	17.95	0.18	15.98	0.26	3.53	0.21
7	18.09	14.42	5.07	18.10	0.24	14.47	0.23	4.99	0.53
8	19.15	31.98	6.11	19.17	0.18	31.96	1.12	5.59	0.59
9	32.31	61.05	9.17	32.14	0.35	61.89	0.92	8.83	0.44
10	21.30	69.55	6.77	21.53	0.74	68.57	1.35	7.61	1.21
11	30.74	24.11	3.09	30.59	0.24	23.96	0.75	2.99	0.16
12	26.97	35.06	14.91	26.79	0.18	34.32	0.78	12.96	0.66
13	37.70	41.38	28.19	36.03	1.06	38.06	0.99	27.21	0.88

Table 8.4: Comparison of MSM3A with *SLAM II* package.

8.6 The Optimization Model for Multistage Production Systems With Buffers (MSM4)

In this section, we present an optimization model for the multistage production systems with buffers that was described in section 8.2. We present first the cost elements of the expected total cost (the objective function). Later, we present the constraints.

8.6.1 The Objective Function

The objective function is the sum of the following costs: (a) cost of producing defectives, (b) cost of unfulfilled demand, (c) cost of restoration, (d) cost of investment in drift rate reduction program, and (e) cost of inventory between stages. These cost elements are derived hereunder.

(a) Cost of producing defectives

The percentages of undersized and oversized items at stage i (P_l^i, P_u^i, respectively) have been presented in chapter 6.

The percentage of defectives for stage i is given by:

$$P_d^i = P_l^i(T_i, \mu_i) + P_u^i(T_i, \mu_i) \qquad , i = 1, ..., n \tag{8.2}$$

Hence, the expected cost of producing defectives at stage i is as follows

$$\sum_{i=1}^{n} R_i[C_l^i P_l^i(T_i, \mu_i) + C_u^i P_u^i(T_i, \mu_i))] \tag{8.3}$$

(b) Cost of unfulfilled demand

Given a demand per unit time Q, the cost of unfulfilled demand can be obtained as

$$W \cdot max(0, Q - R_{eff}) \tag{8.4}$$

where R_{eff} is the effective production rate of the production system or the rate of delivering nondefective finished items, which is estimated using the algorithm MSM3A.

(c) Cost of restoration

This cost is incurred when restoring the process at each stage and it is given by

$$\sum_{i=1}^{n} \frac{C_R^i}{T_i} \tag{8.5}$$

(d) Cost of investment in drift rate reduction program

For a fixed cycle time, T, as the drift rate, θ, decreases, the percentage of defectives decreases. To study the relationship between the cost and the drift rate, we have conducted many simulation experiments. A typical plot of the cost versus the drift rate is shown in Figure 8.4. One can see that the relationship between them can be empirically approximated by an exponential function.

Hence, in this model we are going to represent the relationship between the cost and the drift rate by an exponential function as follows

$$C(\theta_i) = y_1^i \ e^{y_2^i \theta_i} \tag{8.6}$$

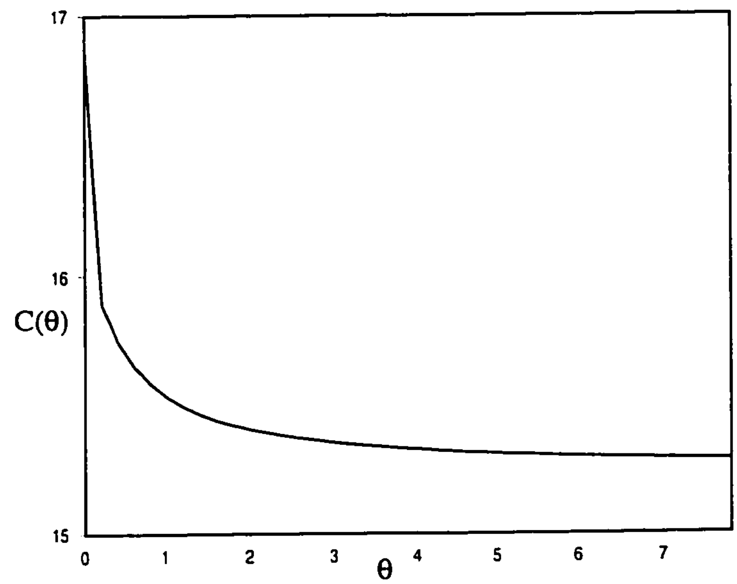


Figure 8.4: Plot of $C(\theta)$.

where $C(\theta_i)$ is the cost of attaining θ_i, and y_1^i and y_2^i are constants.

Thus, the amount of money needed for the investment in the drift rate reduction program is given by

$$(\sum_{i=1}^{n} y_1^i \quad e^{y_2^i(1-\beta_i)\theta_i} - \sum_{i=1}^{n} y_1^i \quad e^{y_2^i\theta_i}) \tag{8.7}$$

(d) Cost of inventory between stages

This cost is due to the waiting items in each buffer and it is given by

$$\sum_{i=1}^{n-1} WIP_i \quad H \tag{8.8}$$

where WIP_i is the average content in buffer i, which is estimated using the algorithm MSM3A.

8.6.2 The Constraints

The constraints of the proposed model composed of (a) constraints on buffer sizes, (b) a constraint on budget of investment, and (c) constraints on β_i's. These constraints are given hereunder.

(a) Constraints on buffer sizes

The following constraints ensure that each buffer size does not exceed its capacity

$$0 \leq B_i \leq K_i \qquad , i = 1, ..., n-1 \tag{8.9}$$

(b) A constraint on budget of investment

The following constraint makes sure that the investment on drift rate reduction does not exceed the available budget

$$(\sum_{i=1}^{n} y_1^i \quad e^{y_2^i(1-\beta_i)\theta_i} - \sum_{i=1}^{n} y_1^i \quad e^{y_2^i\theta_i}) \leq Z \tag{8.10}$$

(c) Constraints on β_i's

The following set of constraints sets lower and upper bounds on the decision variable (β_i)

$$0 \leq \beta_i \leq 1 \qquad , i = 1, ..., n \tag{8.11}$$

8.6.3 Problem formulation

The expected total cost per unit time can be obtained by summing the cost elements (8.3), (8.4), (8.5), (8.7), and (8.8). The formulation of the problem of multistage production systems with buffers (MSM4) can be given as

$$\min \quad ETC = \sum_{i=1}^{n} R_i \, [C_l^i P_l^i(T_i, \mu_i) + C_u^i P_u^i(T_i, \mu_i))] + W \cdot max(0, Q - R_{eff})$$
$$+ \sum_{i=1}^{n} \frac{C_R^i}{T_i} + (\sum_{i=1}^{n} y_1^i \; e^{y_2^i(1-\beta_i)\theta_i} - \sum_{i=1}^{n} y_1^i \; e^{y_2^i \theta_i}) + \sum_{i=1}^{n-1} WIP_i \; H \qquad (8.12)$$

subject to

$$0 \leq B_i \leq K_i, \quad i = 1, ..., n-1, \text{ and integers} \qquad (8.13)$$
$$(\sum_{i=1}^{n} y_1^i \; e^{y_2^i(1-\beta_i)\theta_i} - \sum_{i=1}^{n} y_1^i \; e^{y_2^i \theta_i}) \leq Z \qquad (8.14)$$
$$0 \leq \beta_i \leq 1, \quad i = 1, ..., n \qquad (8.15)$$

The decision variables are:

1. μ_i, i=1, ... , n.
2. T_i, i=1, ... , n.
3. β_i, i=1, ... , n.
4. B_i, i=1, ... , $n-1$.

8.6.4 Solution Methodology and Linkage between Simulation and Optimization

The solution methodology for the above model (MSM4) is our hybrid tabu search algorithm (TSFGO). TSFGO was designed for unconstrained optimization problems. However, one can still use TSFGO for constrained optimization problems by using a feasibility check.

The feasibility check can be explained as follows: For a given direction, trial points are generated by varying the step size (λ) over the given range. Each trial point that satisfies the constraints is going to be inserted in a list called *candidate list*. Hence, the *candidate list* contains only feasible points. The point in the *candidate list* that has the best objective function is accepted as a legitimate neighbor for the current point. If the *candidate list* is empty, a new random direction is generated. A flowchart of the solution procedure for solving MSM4 is shown in Figure 8.6.

In the sequel, we show how we link the simulation algorithm MSM3A to the optimization algorithm TSFGO. The simulation algorithm MSM3A estimates R_{eff} and WIP_i (i=1, ..., $n-1$) when the following decision variables (μ_i, T_i, β_i, i=1, ... , n) and (B_i, i=1, ... , $n-1$) are given. For ease of notation, we combine them together in one parenthesis (μ_i, T_i, β_i, B_i). The estimation process of R_{eff} and WIP_i using MSM3A is shown schematically in Figure 8.5.

The objective function, ETC, of the model MSM4 consists of the following cost elements:

1. $\sum_{i=1}^{n} R_i\ [C_l^i P_l^i(T_i, \mu_i) + C_u^i P_u^i(T_i, \mu_i))]$

2. $W \cdot max(0, Q - R_{eff})$

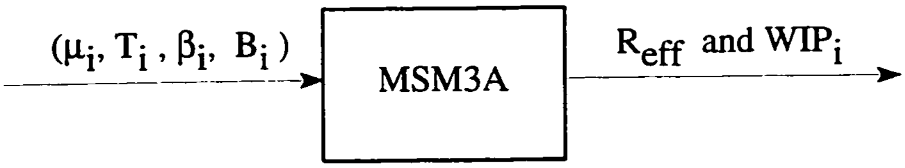


Figure 8.5: Estimation process using MSM3A.

3. $\sum_{i=1}^{n} \frac{C_R^i}{T_i}$

4. $(\sum_{i=1}^{n} y_1^i \; e^{y_2^i(1-\beta_i)\theta_i} - \sum_{i=1}^{n} y_1^i \; e^{y_2^i\theta_i})$

5. $\sum_{i=1}^{n-1} WIP_i \; H$

Hence, each time the objective function has to be evaluated, we have to use the simulation algorithm MSM3A in order to evaluate the second and the fifth cost elements.

In the optimization algorithm TSFGO, the objective function has to be evaluated for each trial point during the optimization search. The calls for evaluating the objective function are explicitly made in the line search subroutine when a direction is given and an optimal step size is sought. Each time there is a call for evaluating the objective function, the subroutine which contains the simulation algorithm MSM3A is called. Figure 8.6 shows the link between MSM3A and TSFGO.

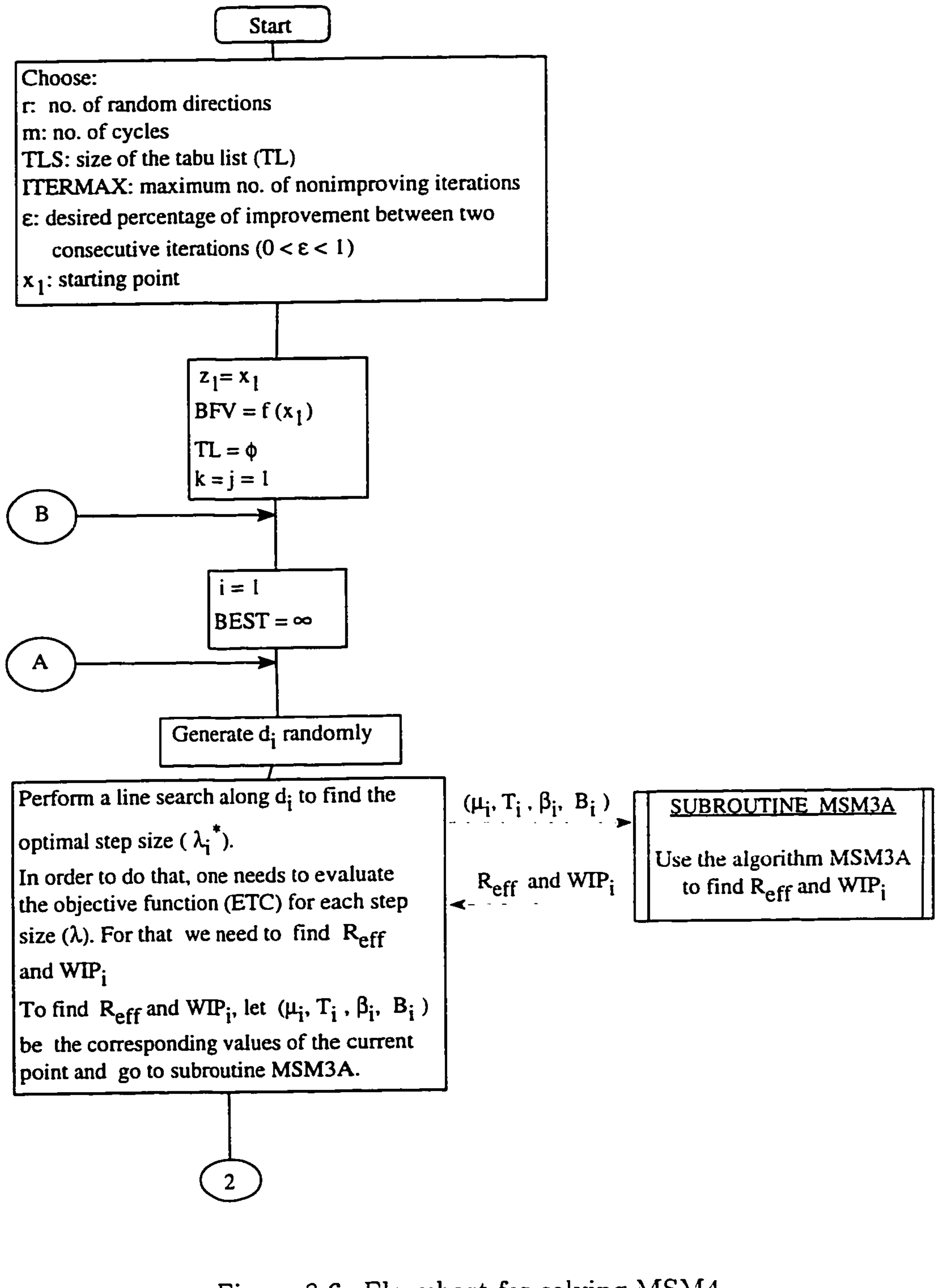


Figure 8.6: Flowchart for solving MSM4.

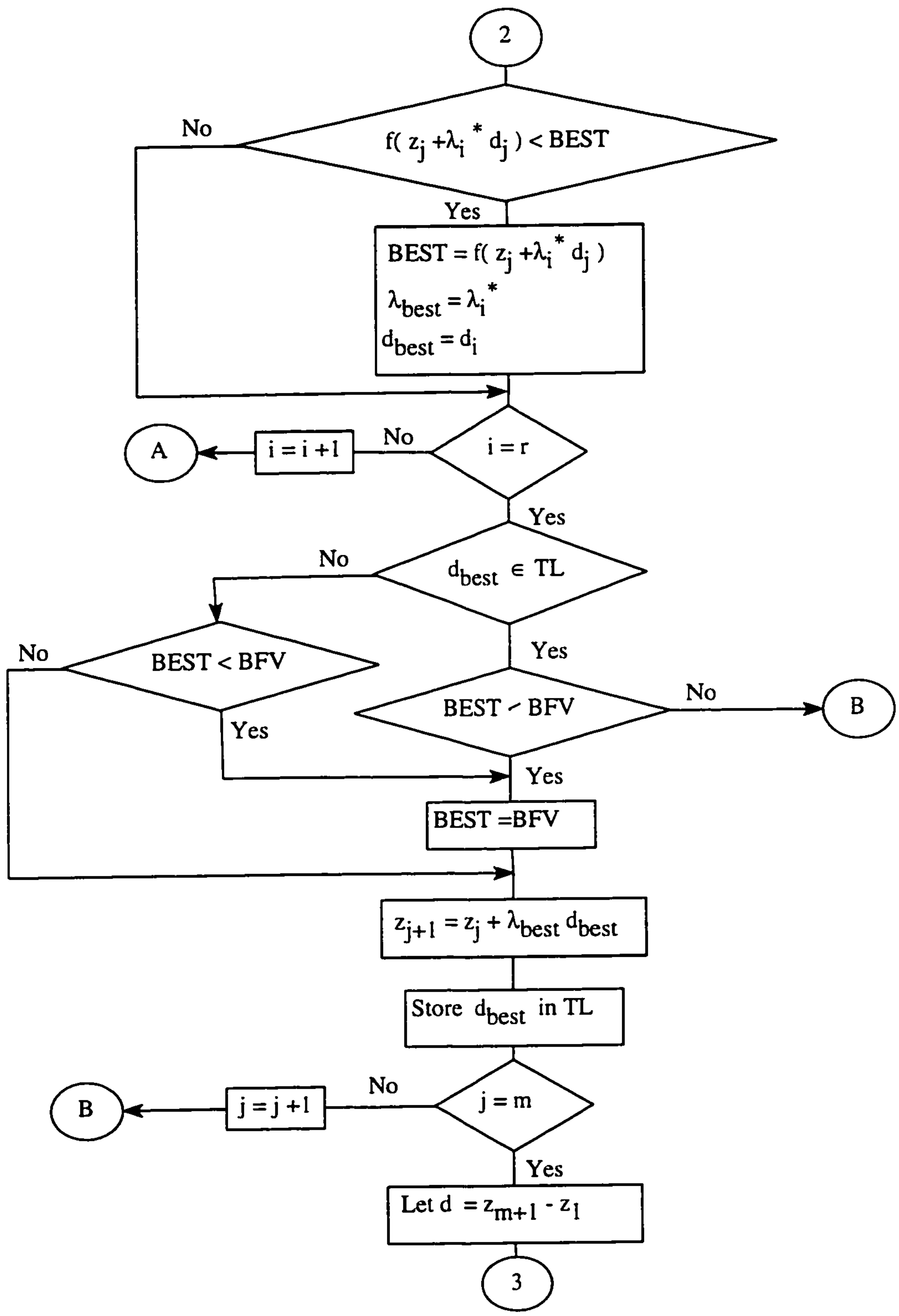


Figure 8.7: Flowchart for solving MSM4 (continued).

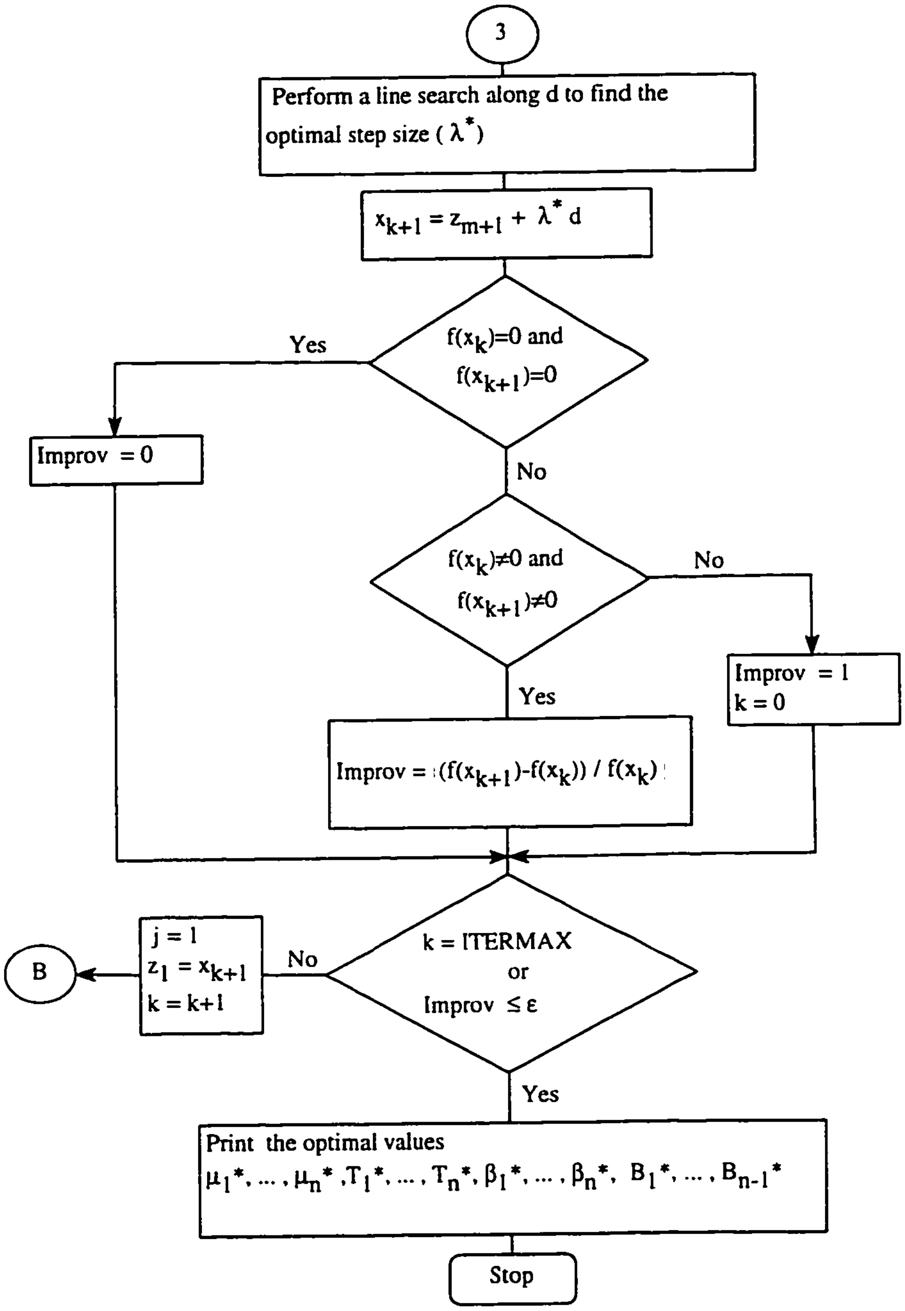


Figure 8.8: Flowchart for solving MSM4 (continued).

8.6.5 Results and Discussions of MSM4

Before we present our results, we make note of the following experiments:

- *Experiment 1*: μ_i, T_i are optimized, and B_i are fixed. No investment on the drift rate reduction program is allowed.
- *Experiment 2*: μ_i, T_i, and B_i are optimized. No investment on the drift rate reduction program is allowed.
- *Experiment 3*: μ_i, T_i, and B_i are optimized. There is an investment on the drift rate reduction program.

We have used the hybrid tabu search algorithm (TSFGO) for solving the MSM4 model. Table 8.5 shows the data of *Experiment 1*. Table 8.6 shows the results of this experiment. The expected total cost, ETC, is \$181.59 and R_{eff} is 69.31. The data of *Experiment 2* are shown in Table 8.7. Table 8.8 shows the results of this experiment. The expected total cost, ETC, is \$173.74 and R_{eff} is 73.89. Note that ETC has decreased and R_{eff} has increased. This is due to the optimization in buffer sizes, B_i's. Table 8.9 shows the data of *Experiment 3*. Table 8.10 shows the results of this experiment. The expected total cost, ETC, is \$126.09 and R_{eff} is 87.33. Further reduction in ETC have been made possible when all the decision variables are optimized. Note that the buffer sizes have increased for the last experiment. One possible reason is that the processes are producing more good items since their drift rates have been reduced.

The results of this model (MSM4) are important and useful. The models in the literature lack the joint optimization of (1) initial means settings, (2) production cycle lengths, (3) buffer sizes, and (4) percent reduction in drift rates for a multistage production system with buffers. The significance of our model is due to the linking of all the above four elements in one integrated model.

Parameter	**Stage 1**	**Stage 2**	**Stage 3**
LSL_i	12	10	13
USL_i	18	16	19
σ_i	0.41	0.59	0.25
λ_i	0.40	0.50	0.30
θ_i	0.80	0.60	0.90
R_i	140	210	180
C_l^i	1.20	1.40	1.70
C_u^i	1.50	2.20	2.40
C_R^i	150	190	120
D_i	3	2	4

Parameter	**Buffer 1**	**Buffer 2**
B_i	10	10

$$Q = 100$$
$$W = 2.00$$
$$H = 2.50$$

Table 8.5: Data of *Experiment 1* for testing MSM4.

	μ_i	T_i	**Avg.** WIP_i
Stage 1	12.21	7.38	
Stage 2	10.54	9.27	
Stage 3	14.03	5.18	
Buffer 1			3.10
Buffer 2			3.25

Table 8.6: Results of MSM4 for *Experiment 1*.

Parameter	Stage 1	Stage 2	Stage 3
LSL_i	12	10	13
USL_i	18	16	19
σ_i	0.41	0.59	0.25
λ_i	0.40	0.50	0.30
θ_i	0.80	0.60	0.90
R_i	140	210	180
C_l^i	1.20	1.40	1.70
C_u^i	1.50	2.20	2.40
C_R^i	150	190	120
D_i	3	2	4

Parameter	Buffer 1	Buffer 2
K_i	200	200

Q	$= 100$
W	$= 2.00$
H	$= 2.50$

Table 8.7: Data of *Experiment 2* for testing MSM4.

	μ_i	T_i	B_i	**Avg.** WIP_i
Stage 1	12.20	7.39		
Stage 2	10.43	9.30		
Stage 3	13.75	5.19		
Buffer 1			8	2.14
Buffer 2			17	4.74

Table 8.8: Results of MSM4 for *Experiment 2*.

Parameter	Stage 1	Stage 2	Stage 3
LSL_i	12	10	13
USL_i	18	16	19
σ_i	0.41	0.59	0.25
λ_i	0.40	0.50	0.30
θ_i	0.80	0.60	0.90
R_i	140	210	180
C_l^i	1.20	1.40	1.70
C_u^i	1.50	2.20	2.40
C_R^i	150	190	120
D_i	3	2	4
e_1^i	25	40	35
e_2^i	0.10	0.20	0.10

Parameter	Buffer 1	Buffer 2
K_i	200	200

$Z = 200$
$Q = 100$
$W = 2.00$
$H = 2.50$

Table 8.9: Data of *Experiment 3* for testing MSM4.

	μ_i	T_i	β_i	B_i	**Avg.** WIP_i
Stage 1	12.84	8.63	0.83		
Stage 2	12.22	9.85	0.64		
Stage 3	14.27	6.68	0.91		
Buffer 1				15	6.08
Buffer 2				22	8.95

Table 8.10: Results of MSM4 for *Experiment 3*.

Chapter 9

Conclusions and Recommendations for Future Study

In this chapter, we highlight the main conclusions and we give some directions for further research. This chapter is organized as follows: In section 9.1, we give some practical applications to industry. We list the main conclusions in section 9.2. In section 9.3, we provide some extension for future study.

9.1 Industrial Applications

In this dissertation, we have considered practical problems which may rise in many industrial environments. We have developed several mathematical models to solve those kind of problems. The proposed models in this dissertation can be applied to different kind of industries. For example:

1. Pulp and paper industry (Arcelus and Rahim [1994]).
2. Glass industry (Arcelus and Rahim [1991]).
3. Pharmaceutical industry (Golhar [1987], Gupta and Golhar [1991]).
4. Canning industry (Golhar and Pollock [1988]).
5. Rubber industry (Albright and Collins [1977]).
6. Industries having metal cutting operations (Hall and Eilon [1963], Gibra [1967,1974], Quesenberry [1988]).
7. Shafts production (Arcelus et al. [1982]).
8. Communication (Schneider et al. [1990]).

9.2 Conclusions

In this dissertation, we have developed the following models :

- A model for finding the optimal initial setting of the process mean and the optimal production cycle length of a single stage production system when the quality characteristic has a normal distribution function and the drift function is linear.
- A model for finding the optimal initial setting of the process mean and the optimal production cycle length of a single stage production system when the quality characteristic has a general distribution function and the drift function is general.

- A model for studying the effect of variance reduction on the model of single stage production system.
- A model for finding the optimal initial setting of the process mean and the optimal production cycle length for every process at each stage for a multistage production system without buffers and with zero repair times.
- A model for studying the effect of reducing the variance of every process at each stage on the model of multistage production system.
- A model for finding the effective production rate for a multistage production system without buffers and with nonzero repair times.
- A simulation model for estimating the effective production rate and the WIP for a two-stage production system with a buffer and with nonzero repair times.
- A simulation model for estimating the effective production rate of the line and the WIP for each buffer for a multistage production system with buffers and with nonzero repair times.
- A model for finding the optimal initial setting of the process mean, the optimal production cycle length, the optimal percent reduction in drift rate, for every process at each stage, and the optimal buffer sizes for a multistage production system with buffers and with nonzero repair times.

As a summary, the models developed in this dissertation are listed in Table 9.1. Similarly, the algorithms are summarized in Table 9.2.

Model	Description	Section
SSM	Single stage production system model	3.5
GSSM1	Generalized drift function for the single stage production system model	3.8
GSSM2	Generalized probability density function of the quality characteristic for the single stage production system model	3.9
GSSM3	Generalized drift function and probability density function of the quality characteristic for the single stage production system model	3.10
SSVRM	Single stage variance reduction model	4.1
MSM1	Multistage production system model 1	6.5
MSVRM	Multistage variance reduction model	7.1
MSM2	Multistage production system model 2	6.7
TSM	Two-stage lines with a buffer and with nonzero repair times model	8.5.1
MSM3	Multistage production system model 3	8.5.2
MSM4	Multistage production system model 4	8.6

Table 9.1: Summary of the developed models.

Algorithm	Description	Section
SSVRA	Single stage variance reduction algorithm	4.1.4
TSFGO	Tabu search algorithm for global optimization	5.3
MSVRA	Multistage variance reduction algorithm	7.1.2
TSMA	Algorithm for two-stage lines with a buffer and with nonzero repair times	8.5.1
MSM3A	Algorithm for multistage lines with buffers and with nonzero repair times	8.5.2

Table 9.2: Summary of the developed algorithms.

9.3 Recommendations for future studies

Some aspects of the problem need further investigations. These may include the following:

1. For the models SSM, MSM1, and MSM2, one may extend the work by considering:
   - A shock model having a decreasing or an increasing hazard rate.
   - An attribute quality characteristic.
   - More than one quality characteristic.
   - Doing preventive maintenance actions before the complete resetting.
   - The defective items are reworkable.
2. For the model MSM3, one may extend the work by considering:
   - The uptimes and downtimes of each stage to be random variables.
3. For the model MSM4, one may extend the work by considering:
   - The reduction in the variance of each process.
   - A deadline for delivering the demand.
   - One of the extensions in 1.

Appendix A

Test Functions for The First Experiment of Testing TSFGO

The following test functions are taken from Dixon and Szegö [1978].

GP (Goldstein and Price).

$$\begin{aligned} f(x_1, x_2) &= [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \\ &\times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 \\ &+ 48x_2 - 36x_1x_2 + 27x_2^2)] \end{aligned} \tag{A.1}$$

$$S = \{x \in R^2 | -2 \leq x_i \leq 2, i = 1, 2\}, x_{min} = (0, -1), f(x_{min}) = 3. \tag{A.2}$$

There are 4 local minima.

BR (Branin).

$$f(x_1, x_2) = a(x_2 - bx_1^2 + cx_1 - d)^2 + e(1-f)\cos(x_1) + e \tag{A.3}$$

where $a = 1, b = 5.1/(4\pi^2), c = 5/\pi, d = 6, e = 10, f = 1/(8\pi)$.

$S = \{x \in R^2 | -5 \leq x_1 \leq 10 \text{ and } 0 \leq x_2 \leq 15\}$,

$x_{min} = (-\pi, 12.275); (\pi, 2.275); (3\pi, 2.475), f(x_{min}) = 5/(4\pi)$.

There are no more minima.

H3 and H6 (Hartmann's family).

$$f(x) = -\sum_{i=1}^{q} c_i \exp(-\sum_{j=1}^{n} a_{ij}(x_j - p_{ij})^2). \tag{A.4}$$

i	a_{ij}			c_i	p_{ij}		
1	3	10	30	1	0.3689	0.1170	0.2673
2	0.1	10	35	1.2	0.4699	0.4387	0.7470
3	3	10	30	3	0.1091	0.8732	0.5547
4	0.1	10	35	3.2	0.03815	0.5743	0.8828

Table A.1: H3 (n=3 and q=4).

$$S = \{x \in R^n | 0 \leq x_j \leq 1, 1 \leq j \leq n\}. \tag{A.5}$$

These functions both have four local minima, $x_{loc} \approx (p_{i1}, ..., p_{in}), f(x_{loc}) \approx -c_i$.

i	a_{ij}						c_i
1	10	3	17	3.5	1.7	8	1
2	0.05	10	17	0.1	8	14	1.2
3	3	3.5	1.7	10	17	8	3
4	17	8	0.05	10	0.1	14	3.2

i	p_{ij}					
1	0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2	0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3	0.2348	0.1451	0.3522	0.2883	0.3047	0.6550
4	0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

Table A.2: H6 (n=6 and q=4).

S5, S7 and S10 (Shekel's family).

$$f(x) = -\sum_{i=1}^{q}((x-a_i)^T(x-a_i)+c_i)^{-1} \tag{A.6}$$

with n=4, q=5, 7, 10 for S5, S7, S10, respectively, $x = (x_1, ..., x_n)^T$ and $a_i = (a_{i1}, ..., a_{in})^T$.

$$S = \{x \in R^4 | 0 \leq x_i \leq 1, 1 \leq i \leq 4\}. \tag{A.7}$$

These functions have 5, 7 and 10 local minima for S5 and S7 and S10, respectively, $x_{loc} \approx (1/c_i, \dots, 1/c_q)$.

i	a_i				c_i
1	4	4	4	4	0.1
2	1	1	1	1	0.2
3	8	8	8	8	0.2
4	6	6	6	6	0.4
5	3	7	3	7	0.4
6	2	9	2	9	0.6
7	5	5	3	3	0.3
8	8	1	8	1	0.7
9	6	2	6	2	0.5
10	7	3.6	7	3.6	0.5

Table A.3: S5, S7, S10.

Appendix B

Test Functions for The Second Experiment of Testing TSFGO

The following Rosenbrock test function in 2 and 4 dimensions is taken from Rosenbrock [1960] and Corana et al. [1987].

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 - (1 - x_1)^2 \tag{B.1}$$

$$f(x_1, x_2, x_3, x_4) = \sum_{i=1}^{3} 100(x_{i+1} - x_i^2)^2 - (1 - x_i)^2 \tag{B.2}$$

Appendix C

Experimental Design and Results of the Single Stage Model

Run	Input parameters							Output		
	C_l	C_u	R	r	λ	θ	σ	μ^*	T^*	ETC^*
1	8	8	300	500	.05	.1	1	10.96528	6.848591	3.892789
2	29	8	300	500	.05	.1	1	11.39918	6.124778	7.234078
3	8	28	300	500	.05	.1	1	10.55125	5.6911	7.123296
4	29	28	300	500	.05	.1	1	10.9951	4.881759	13.48709
5	8	8	5000	500	.05	.1	1	10.86556	14.82629	5.102528
6	29	8	5000	500	.05	.1	1	11.31747	13.33445	8.67459
7	8	28	5000	500	.05	.1	1	10.49247	11.89227	8.691729
8	29	28	5000	500	.05	.1	1	10.93776	10.42447	15.17985
9	8	8	300	8000	.05	.1	1	10.9909	3.335264	3.740678
10	29	8	300	8000	.05	.1	1	11.43066	2.902217	7.048868
11	8	28	300	8000	.05	.1	1	10.56785	2.798358	6.930168
12	29	28	300	8000	.05	.1	1	11.00811	2.375064	13.27538
13	8	8	5000	8000	.05	.1	1	10.96485	6.929901	3.898129
14	29	8	5000	8000	.05	.1	1	11.40901	6.087684	7.24055
15	8	28	5000	8000	.05	.1	1	10.55119	5.751163	7.130127
16	29	28	5000	8000	.05	.1	1	10.99286	4.9435	13.49454
17	8	8	300	500	8.4	.1	1	10.762	5.006712	3.981472
18	29	8	300	500	8.4	.1	1	11.2432	4.11411	7.363197

19	8	28	300	500	8.4	.1	1	10.38277	4.108641	7.24114
20	29	28	300	500	8.4	.1	1	10.86039	3.279335	13.64397
21	8	8	5000	500	8.4	.1	1	10.38638	12.54373	5.488385
22	29	8	5000	500	8.4	.1	1	10.92699	10.37705	9.298858
23	8	28	5000	500	8.4	.1	1	10.06824	10.42784	9.161037
24	29	28	5000	500	8.4	.1	1	10.60717	8.328611	15.89519
25	8	8	300	8000	8.4	.1	1	10.91256	1.989052	3.759121
26	29	8	300	8000	8.4	.1	1	11.36771	1.628221	7.07414
27	8	28	300	8000	8.4	.1	1	10.50505	1.628048	6.954513
28	29	28	300	8000	8.4	.1	1	10.95893	1.308446	13.30556
29	8	8	5000	8000	8.4	.1	1	10.7588	5.075849	3.988788
30	29	8	5000	8000	8.4	.1	1	11.23995	4.171801	7.372691
31	8	28	5000	8000	8.4	.1	1	10.3763	4.206682	7.250552
32	29	28	5000	8000	8.4	.1	1	10.85815	3.321326	13.65508
33	8	8	300	500	.05	6.5	1	10.99118	1.729906	4.631314
34	29	8	300	500	.05	6.5	1	11.41352	1.720319	8.015055
35	8	28	300	500	.05	6.5	1	10.58837	.964884	8.454625
36	29	28	300	500	.05	6.5	1	11.00502	.9515721	14.76523
37	8	8	5000	500	.05	6.5	1	10.98972	6.68051	8.183702
38	29	8	5000	500	.05	6.5	1	11.35431	6.679236	11.7687
39	8	28	5000	500	.05	6.5	1	10.66885	3.67904	14.92167
40	29	28	5000	500	.05	6.5	1	11	3.669703	20.9622
41	8	8	300	8000	.05	6.5	1	10.99358	.4993633	3.880338
42	29	8	300	8000	.05	6.5	1	11.42646	.4821245	7.205804
43	8	28	300	8000	.05	6.5	1	10.58314	.3501298	7.13135
44	29	28	300	8000	.05	6.5	1	11.00776	.3238261	13.48072
45	8	8	5000	8000	.05	6.5	1	10.99003	1.763418	4.653016
46	29	8	5000	8000	.05	6.5	1	11.4125	1.754529	8.03823
47	8	28	5000	8000	.05	6.5	1	10.58914	.9860441	8.495077
48	29	28	5000	8000	.05	6.5	1	11.00502	.9745136	14.80389
49	8	8	300	500	8.4	6.5	1	10.5632	.2983256	8.102164
50	29	8	300	500	8.4	6.5	1	11.06611	.2608653	12.30366
51	8	28	300	500	8.4	6.5	1	10.29005	.238457	12.51761
52	29	28	300	500	8.4	6.5	1	10.76774	.204447	19.42562
53	8	8	5000	500	8.4	6.5	1	9.857007	.6268779	44.03499
54	29	8	5000	500	8.4	6.5	1	10.32699	.5717363	51.82003
55	8	28	5000	500	8.4	6.5	1	9.770086	.51331	54.32648
56	29	28	5000	500	8.4	6.5	1	10.21354	.4617918	63.64792

57	8	8	300	8000	8.4	6.5	1	10.88258	.1312395	4.304945
58	29	8	300	8000	8.4	6.5	1	11.34522	.1136628	7.742653
59	8	28	300	8000	8.4	6.5	1	10.49846	.1076232	7.651269
60	29	28	300	8000	8.4	6.5	1	10.94893	9.150998E-02	14.07514
61	8	8	5000	8000	8.4	6.5	1	10.55559	.3020234	8.23625
62	29	8	5000	8000	8.4	6.5	1	11.05935	.2640535	12.46192
63	8	28	5000	8000	8.4	6.5	1	10.28617	.2410625	12.68721
64	29	28	5000	8000	8.4	6.5	1	10.76313	.2068403	19.61086
65	8	8	300	500	.05	.1	1.4	10.95507	7.828424	7.438627
66	29	8	300	500	.05	.1	1.4	11.62185	6.832294	14.16098
67	8	28	300	500	.05	.1	1.4	10.3501	6.403545	13.92007
68	29	28	300	500	.05	.1	1.4	10.99428	5.566342	26.06426
69	8	8	5000	500	.05	.1	1.4	10.82262	16.98081	8.811105
70	29	8	5000	500	.05	.1	1.4	11.49877	15.11547	15.84859
71	8	28	5000	500	.05	.1	1.4	10.25303	13.74014	15.73082
72	29	28	5000	500	.05	.1	1.4	10.9188	11.9189	27.99117
73	8	8	300	8000	.05	.1	1.4	10.98907	3.740512	7.265059
74	29	8	300	8000	.05	.1	1.4	11.64868	3.27708	13.94348
75	8	28	300	8000	.05	.1	1.4	10.35103	3.188049	13.69508
76	29	28	300	8000	.05	.1	1.4	11.01189	2.715993	25.82232
77	8	8	5000	8000	.05	.1	1.4	10.95436	7.88109	7.444709
78	29	8	5000	8000	.05	.1	1.4	11.62096	6.931613	14.16858
79	8	28	5000	8000	.05	.1	1.4	10.32868	6.593647	13.92799
80	29	28	5000	8000	.05	.1	1.4	10.99657	5.610994	26.07277
81	8	8	300	500	8.4	.1	1.4	10.71673	5.914537	7.528946
82	29	8	300	500	8.4	.1	1.4	11.42544	4.861386	14.29673
83	8	28	300	500	8.4	.1	1.4	10.13287	4.844192	14.04413
84	29	28	300	500	8.4	.1	1.4	10.8363	3.878978	26.22703
85	8	8	5000	500	8.4	.1	1.4	10.26547	14.92264	9.176109
86	29	8	5000	500	8.4	.1	1.4	11.04428	12.30195	16.46589
87	8	28	5000	500	8.4	.1	1.4	9.763812	12.36324	16.19289
88	29	28	5000	500	8.4	.1	1.4	10.53672	9.860761	28.69679
89	8	8	300	8000	8.4	.1	1.4	10.89423	2.360646	7.284559
90	29	8	300	8000	8.4	.1	1.4	11.57838	1.89401	13.97108
91	8	28	300	8000	8.4	.1	1.4	10.27398	1.933708	13.72162
92	29	28	300	8000	8.4	.1	1.4	10.9576	1.495075	25.85489
93	8	8	5000	8000	8.4	.1	1.4	10.71245	5.991581	7.536977
94	29	8	5000	8000	8.4	.1	1.4	11.42294	4.934473	14.30741
95	8	28	5000	8000	8.4	.1	1.4	10.12502	4.957761	14.05471
96	29	28	5000	8000	8.4	.1	1.4	10.8453	3.81291	26.23925

97	8	8	300	500	.05	6.5	1.4	10.98327	1.739744	8.381848
98	29	8	300	500	.05	6.5	1.4	11.62209	1.730483	15.19272
99	8	28	300	500	.05	6.5	1.4	10.38227	.9970675	15.57819
100	29	28	300	500	.05	6.5	1.4	11.00661	.9789494	27.64244
101	8	8	5000	500	.05	6.5	1.4	10.98151	6.684698	12.97888
102	29	8	5000	500	.05	6.5	1.4	11.53487	6.686662	20.15302
103	8	28	5000	500	.05	6.5	1.4	10.49772	3.688083	24.04323
104	29	28	5000	500	.05	6.5	1.4	10.99924	3.677753	35.62937
105	8	8	300	8000	.05	6.5	1.4	10.99054	.5363185	7.430631
106	29	8	300	8000	.05	6.5	1.4	11.6491	.5106406	14.13649
107	8	28	300	8000	.05	6.5	1.4	10.35988	.3880324	13.9322
108	29	28	300	8000	.05	6.5	1.4	11.01432	.36028	26.06153
109	8	8	5000	8000	.05	6.5	1.4	10.98325	1.771849	8.409777
110	29	8	5000	8000	.05	6.5	1.4	11.62414	1.768055	15.22333
111	8	28	5000	8000	.05	6.5	1.4	10.37785	1.018791	15.63008
112	29	28	5000	8000	.05	6.5	1.4	11.00395	.9956863	27.69125
113	8	8	300	500	8.4	6.5	1.4	10.41727	.3533873	12.07893
114	29	8	300	500	8.4	6.5	1.4	11.16065	.3044949	19.94366
115	8	28	300	500	8.4	6.5	1.4	9.975616	.2846234	19.98705
116	29	28	300	500	8.4	6.5	1.4	10.69756	.2390527	32.6926
117	8	8	5000	500	8.4	6.5	1.4	9.387155	.765023	49.86227
118	29	8	5000	500	8.4	6.5	1.4	10.12512	.6804668	62.74227
119	8	28	5000	500	8.4	6.5	1.4	9.22167	.6345288	64.57121
120	29	28	5000	500	8.4	6.5	1.4	9.930051	.5548984	80.27138
121	8	8	300	8000	8.4	6.5	1.4	10.84705	.1521346	7.900771
122	29	8	300	8000	8.4	6.5	1.4	11.54066	.1305248	14.74932
123	8	28	300	8000	8.4	6.5	1.4	10.2563	.1255341	14.52558
124	29	28	300	8000	8.4	6.5	1.4	10.93756	.1050632	26.72815
125	8	8	5000	8000	8.4	6.5	1.4	10.40754	.357697	12.22466
126	29	8	5000	8000	8.4	6.5	1.4	11.15021	.3085633	20.12179
127	8	28	5000	8000	8.4	6.5	1.4	9.983944	.2861713	20.17479
128	29	28	5000	8000	8.4	6.5	1.4	10.69141	.2421572	32.89734

Table C.1: Experimental design and results of the single stage model.

Appendix D

Fractions of the Experimental Design of the Multistage Model

(1)	ajkmn	bjmnp	abkp	clmp
acjklnp	bcjln	abcklm	djlmo	adklno
bdlnop	abdjklmop	cdjop	acdkmnop	bcdmno
abcdjko	ekmno	aejo	bejkop	abemnop
ceklnop	acejlmop	bcejklmo	abcelno	dejkln
adelm	bdeklmp	abdejlnp	cdejkmnp	acdep
bcdek	abcdejmn	fklmop	afjlnop	bfjklno
abflmo	cfko	acfjmno	bcfjkmnop	abcfop
dfjkp	adfmnp	bdfkmn	abdfj	cdfjklm
acdfln	bcdfklnp	abcdfjlmp	eflnp	aefjklmp
befjlm	abefkln	cefmn	acefjk	bcefjp
abcefkmnp	defjmnop	adefkop	bdefo	abdefjkmno
cdefjlno	acdefklmo	bcdeflmop	abcdefjklnop	gjklmnop
aglop	bgklo	abgjlmno	cgjkno	acgmo
bcgkmop	abcgjnop	dgknp	adgjmp	bdgjkm

abdgn	cdgklmn	acdgjl	bcdgjklp	abcdglmnp
egjlp	aegklmnp	beglmn	abegjkl	cegjm
acegkn	bcegnp	abcegjkmp	degmop	adegjknop
bdegjno	abdegkmo	cdeglo	acdegjklmno	bcdegjlmnop
abcdegklop	fgjn	afgkm	bfgmp	abfgjknp
cfgjlmnp	acfgklp	bcfgl	abcfgjklmn	dfglmno
adfgjklo	bdfgjlop	abdfgklmnop	cdfgnop	acdfgjkmop
bcdfgjmo	abcdfgkno	efgjkmo	aefgno	befgknop
abefgjmop	cefgjklop	acefglmnop	bcefgklmno	abcefgjlo
defgkl	adefgjlmn	bdefgjklmnp	abdefglp	cdefgkmp
acdefgjnp	bcdefgjkn	abcdefgm	hjkl	ahlmn
bhklmnp	abhjlp	chjkmp	achnp	bchkn
abchjm	dhkmo	adhjno	bdhjknop	abdhmop
cdhklop	acdhjlmnop	bcdhjklmno	abcdhlo	ehjlmno
aehklo	behlop	abehjklmnop	cehjnop	acehkmop
bcehmo	abcehjkno	dehn	adehjkm	bdehjmp
abdehknp	cdehlmnp	acdehjklp	bcdehjl	abcdehklmn
fhjmop	afhknop	bfhno	abfhjkmo	cfhjlo
acfhklmno	bcfhlmnop	abcfhjklop	dfhlp	adfhjklmnp
bdfhjlmn	abdfhkl	cdfhm	acdfhjkn	bcdfhjnp
abcdfhkmp	efhjknp	aefhmp	befhkm	abefhjn
cefhjklmn	acefhl	bcefhklp	abcefhjlmnp	defhklmnop
adefhjlop	bdefhjklo	abdefhlmno	cdefhkno	acdefhjmo
bcdefhjkmop	abcdefhnop	ghmnop	aghjkop	bghjo
abghkmno	cghlno	acghjklmo	bcghjlmop	abcghklnop
dghjlnp	adghklmp	bdghlm	abdghjkln	cdghjmn
acdghk	bcdghp	abcdghjkmnp	eghkp	aeghjmnp
beghjkmn	abegh	ceghklm	aceghjln	bceghjklnp
abceghlmp	deghjklmop	adeghlnop	bdeghklno	abdeghjlmo
cdeghjko	acdeghmno	bcdeghkmnop	abcdeghjop	fghkln
afghjlm	bfghjklmp	abfghlnp	cfghkmnp	acfghjp
bcfghjk	abcfghmn	dfghjkmno	adfgho	bdfghkop
abdfghjmnop	cdfghjklnop	acdfghlmop	bcdfghklmo	abcdfghjlno
efghlmo	aefghjklno	befghjlnop	abefghklmop	cefghop
acefghjkmnop	bcefghjmno	abcefghko	defghj	adefghkmn
bdefghmnp	abdefghjkp	cdefghjlmp	acdefghklnp	bcdefghln
abcdefghjklm				

Table D.1: Fractions of the experimental design of the multistage model.

Appendix E

Experimental Design and Results of the Multistage Model

E.1 Experimental Design

	Input parameters														
Run	P	Q	R	σ_1	θ_1	λ_1	C_l^1	C_u^1	C_R^1	σ_2	θ_2	λ_2	C_l^2	C_u^2	C_R^2
1	3	100	110	.49	.1	.5	.1	.1	100	.663	-.2	.4	.15	.15	100
2	5	100	110	.49	.1	.5	.1	.1	150	.8	-.2	.5	.3	.15	100
3	3	150	110	.49	.1	.5	.1	.1	150	.663	-.2	.5	.3	.15	150
4	5	150	110	.49	.1	.5	.1	.1	100	.8	-.2	.4	.15	.15	150
5	3	100	200	.49	.1	.5	.1	.1	100	.663	-.1	.5	.15	.15	150
6	5	100	200	.49	.1	.5	.1	.1	150	.8	-.1	.4	.3	.15	150
7	3	150	200	.49	.1	.5	.1	.1	150	.663	-.1	.4	.3	.15	100
8	5	150	200	.49	.1	.5	.1	.1	100	.8	-.1	.5	.15	.15	100
9	3	100	110	.6	.1	.5	.1	.1	150	.663	-.1	.5	.15	.3	100
10	5	100	110	.6	.1	.5	.1	.1	100	.8	-.1	.4	.3	.3	100
11	3	150	110	.6	.1	.5	.1	.1	100	.663	-.1	.4	.3	.3	150
12	5	150	110	.6	.1	.5	.1	.1	150	.8	-.1	.5	.15	.3	150
13	3	100	200	.6	.1	.5	.1	.1	150	.663	-.2	.4	.15	.3	150
14	5	100	200	.6	.1	.5	.1	.1	100	.8	-.2	.5	.3	.3	150
15	3	150	200	.6	.1	.5	.1	.1	100	.663	-.2	.5	.3	.3	100

16	5	150	200	.6	.1	.5	.1	.1	150	.8	-.2	.4	.15	.3	100
17	3	100	110	.49	.2	.5	.1	.1	100	.8	-.2	.5	.3	.3	100
18	5	100	110	.49	.2	.5	.1	.1	150	.663	-.2	.4	.15	.3	100
19	3	150	110	.49	.2	.5	.1	.1	150	.8	-.2	.4	.15	.3	150
20	5	150	110	.49	.2	.5	.1	.1	100	.663	-.2	.5	.3	.3	150
21	3	100	200	.49	.2	.5	.1	.1	100	.8	-.1	.4	.3	.3	150
22	5	100	200	.49	.2	.5	.1	.1	150	.663	-.1	.5	.15	.3	150
23	3	150	200	.49	.2	.5	.1	.1	150	.8	-.1	.5	.15	.3	100
24	5	150	200	.49	.2	.5	.1	.1	100	.663	-.1	.4	.3	.3	100
25	3	100	110	.6	.2	.5	.1	.1	150	.8	-.1	.4	.3	.15	100
26	5	100	110	.6	.2	.5	.1	.1	100	.663	-.1	.5	.15	.15	100
27	3	150	110	.6	.2	.5	.1	.1	100	.8	-.1	.5	.15	.15	150
28	5	150	110	.6	.2	.5	.1	.1	150	.663	-.1	.4	.3	.15	150
29	3	100	200	.6	.2	.5	.1	.1	150	.8	-.2	.5	.3	.15	150
30	5	100	200	.6	.2	.5	.1	.1	100	.663	-.2	.4	.15	.15	150
31	3	150	200	.6	.2	.5	.1	.1	100	.8	-.2	.4	.15	.15	100
32	5	150	200	.6	.2	.5	.1	.1	150	.663	-.2	.5	.3	.15	100
33	3	100	110	.49	.1	.6	.1	.1	100	.8	-.1	.5	.15	.3	150
34	5	100	110	.49	.1	.6	.1	.1	150	.663	-.1	.4	.3	.3	150
35	3	150	110	.49	.1	.6	.1	.1	150	.8	-.1	.4	.3	.3	100
36	5	150	110	.49	.1	.6	.1	.1	100	.663	-.1	.5	.15	.3	100
37	3	100	200	.49	.1	.6	.1	.1	100	.8	-.2	.4	.15	.3	100
38	5	100	200	.49	.1	.6	.1	.1	150	.663	-.2	.5	.3	.3	100
39	3	150	200	.49	.1	.6	.1	.1	150	.8	-.2	.5	.3	.3	150
40	5	150	200	.49	.1	.6	.1	.1	100	.663	-.2	.4	.15	.3	150
41	3	100	110	.6	.1	.6	.1	.1	150	.8	-.2	.4	.15	.15	150
42	5	100	110	.6	.1	.6	.1	.1	100	.663	-.2	.5	.3	.15	150
43	3	150	110	.6	.1	.6	.1	.1	100	.8	-.2	.5	.3	.15	100
44	5	150	110	.6	.1	.6	.1	.1	150	.663	-.2	.4	.15	.15	100
45	3	100	200	.6	.1	.6	.1	.1	150	.8	-.1	.5	.15	.15	100
46	5	100	200	.6	.1	.6	.1	.1	100	.663	-.1	.4	.3	.15	100
47	3	150	200	.6	.1	.6	.1	.1	100	.8	-.1	.4	.3	.15	150
48	5	150	200	.6	.1	.6	.1	.1	150	.663	-.1	.5	.15	.15	150
49	3	100	110	.49	.2	.6	.1	.1	100	.663	-.1	.4	.3	.15	150
50	5	100	110	.49	.2	.6	.1	.1	150	.8	-.1	.5	.15	.15	150
51	3	150	110	.49	.2	.6	.1	.1	150	.663	-.1	.5	.15	.15	100
52	5	150	110	.49	.2	.6	.1	.1	100	.8	-.1	.4	.3	.15	100
53	3	100	200	.49	.2	.6	.1	.1	100	.663	-.2	.5	.3	.15	100
54	5	100	200	.49	.2	.6	.1	.1	150	.8	-.2	.4	.15	.15	100
55	3	150	200	.49	.2	.6	.1	.1	150	.663	-.2	.4	.15	.15	150
56	5	150	200	.49	.2	.6	.1	.1	100	.8	-.2	.5	.3	.15	150

57	3	100	110	.6	.2	.6	.1	.1	150	.663	-.2	.5	.3	.3	150
58	5	100	110	.6	.2	.6	.1	.1	100	.8	-.2	.4	.15	.3	150
59	3	150	110	.6	.2	.6	.1	.1	100	.663	-.2	.4	.15	.3	100
60	5	150	110	.6	.2	.6	.1	.1	150	.8	-.2	.5	.3	.3	100
61	3	100	200	.6	.2	.6	.1	.1	150	.663	-.1	.4	.3	.3	100
62	5	100	200	.6	.2	.6	.1	.1	100	.8	-.1	.5	.15	.3	100
63	3	150	200	.6	.2	.6	.1	.1	100	.663	-.1	.5	.15	.3	150
64	5	150	200	.6	.2	.6	.1	.1	150	.8	-.1	.4	.3	.3	150
65	3	100	110	.49	.1	.5	.2	.1	150	.8	-.1	.5	.3	.3	150
66	5	100	110	.49	.1	.5	.2	.1	100	.663	-.1	.4	.15	.3	150
67	3	150	110	.49	.1	.5	.2	.1	100	.8	-.1	.4	.15	.3	100
68	5	150	110	.49	.1	.5	.2	.1	150	.663	-.1	.5	.3	.3	100
69	3	100	200	.49	.1	.5	.2	.1	150	.8	-.2	.4	.3	.3	100
70	5	100	200	.49	.1	.5	.2	.1	100	.663	-.2	.5	.15	.3	100
71	3	150	200	.49	.1	.5	.2	.1	100	.8	-.2	.5	.15	.3	150
72	5	150	200	.49	.1	.5	.2	.1	150	.663	-.2	.4	.3	.3	150
73	3	100	110	.6	.1	.5	.2	.1	100	.8	-.2	.4	.3	.15	150
74	5	100	110	.6	.1	.5	.2	.1	150	.663	-.2	.5	.15	.15	150
75	3	150	110	.6	.1	.5	.2	.1	150	.8	-.2	.5	.15	.15	100
76	5	150	110	.6	.1	.5	.2	.1	100	.663	-.2	.4	.3	.15	100
77	3	100	200	.6	.1	.5	.2	.1	100	.8	-.1	.5	.3	.15	100
78	5	100	200	.6	.1	.5	.2	.1	150	.663	-.1	.4	.15	.15	100
79	3	150	200	.6	.1	.5	.2	.1	150	.8	-.1	.4	.15	.15	150
80	5	150	200	.6	.1	.5	.2	.1	100	.663	-.1	.5	.3	.15	150
81	3	100	110	.49	.2	.5	.2	.1	150	.663	-.1	.4	.15	.15	150
82	5	100	110	.49	.2	.5	.2	.1	100	.8	-.1	.5	.3	.15	150
83	3	150	110	.49	.2	.5	.2	.1	100	.663	-.1	.5	.3	.15	100
84	5	150	110	.49	.2	.5	.2	.1	150	.8	-.1	.4	.15	.15	100
85	3	100	200	.49	.2	.5	.2	.1	150	.663	-.2	.5	.15	.15	100
86	5	100	200	.49	.2	.5	.2	.1	100	.8	-.2	.4	.3	.15	100
87	3	150	200	.49	.2	.5	.2	.1	100	.663	-.2	.4	.3	.15	150
88	5	150	200	.49	.2	.5	.2	.1	150	.8	-.2	.5	.15	.15	150
89	3	100	110	.6	.2	.5	.2	.1	100	.663	-.2	.5	.15	.3	150
90	5	100	110	.6	.2	.5	.2	.1	150	.8	-.2	.4	.3	.3	150
91	3	150	110	.6	.2	.5	.2	.1	150	.663	-.2	.4	.3	.3	100
92	5	150	110	.6	.2	.5	.2	.1	100	.8	-.2	.5	.15	.3	100
93	3	100	200	.6	.2	.5	.2	.1	100	.663	-.1	.4	.15	.3	100
94	5	100	200	.6	.2	.5	.2	.1	150	.8	-.1	.5	.3	.3	100
95	3	150	200	.6	.2	.5	.2	.1	150	.663	-.1	.5	.3	.3	150
96	5	150	200	.6	.2	.5	.2	.1	100	.8	-.1	.4	.15	.3	150

97	3	100	110	.49	.1	.6	.2	.1	150	.663	-.2	.4	.3	.15	100
98	5	100	110	.49	.1	.6	.2	.1	100	.8	-.2	.5	.15	.15	100
99	3	150	110	.49	.1	.6	.2	.1	100	.663	-.2	.5	.15	.15	150
100	5	150	110	.49	.1	.6	.2	.1	150	.8	-.2	.4	.3	.15	150
101	3	100	200	.49	.1	.6	.2	.1	150	.663	-.1	.5	.3	.15	150
102	5	100	200	.49	.1	.6	.2	.1	100	.8	-.1	.4	.15	.15	150
103	3	150	200	.49	.1	.6	.2	.1	100	.663	-.1	.4	.15	.15	100
104	5	150	200	.49	.1	.6	.2	.1	150	.8	-.1	.5	.3	.15	100
105	3	100	110	.6	.1	.6	.2	.1	100	.663	-.1	.5	.3	.3	100
106	5	100	110	.6	.1	.6	.2	.1	150	.8	-.1	.4	.15	.3	100
107	3	150	110	.6	.1	.6	.2	.1	150	.663	-.1	.4	.15	.3	150
108	5	150	110	.6	.1	.6	.2	.1	100	.8	-.1	.5	.3	.3	150
109	3	100	200	.6	.1	.6	.2	.1	100	.663	-.2	.4	.3	.3	150
110	5	100	200	.6	.1	.6	.2	.1	150	.8	-.2	.5	.15	.3	150
111	3	150	200	.6	.1	.6	.2	.1	150	.663	-.2	.5	.15	.3	100
112	5	150	200	.6	.1	.6	.2	.1	100	.8	-.2	.4	.3	.3	100
113	3	100	110	.49	.2	.6	.2	.1	150	.8	-.2	.5	.15	.3	100
114	5	100	110	.49	.2	.6	.2	.1	100	.663	-.2	.4	.3	.3	100
115	3	150	110	.49	.2	.6	.2	.1	100	.8	-.2	.4	.3	.3	150
116	5	150	110	.49	.2	.6	.2	.1	150	.663	-.2	.5	.15	.3	150
117	3	100	200	.49	.2	.6	.2	.1	150	.8	-.1	.4	.15	.3	150
118	5	100	200	.49	.2	.6	.2	.1	100	.663	-.1	.5	.3	.3	150
119	3	150	200	.49	.2	.6	.2	.1	100	.8	-.1	.5	.3	.3	100
120	5	150	200	.49	.2	.6	.2	.1	150	.663	-.1	.4	.15	.3	100
121	3	100	110	.6	.2	.6	.2	.1	100	.8	-.1	.4	.15	.15	100
122	5	100	110	.6	.2	.6	.2	.1	150	.663	-.1	.5	.3	.15	100
123	3	150	110	.6	.2	.6	.2	.1	150	.8	-.1	.5	.3	.15	150
124	5	150	110	.6	.2	.6	.2	.1	100	.663	-.1	.4	.15	.15	150
125	3	100	200	.6	.2	.6	.2	.1	100	.8	-.2	.5	.15	.15	150
126	5	100	200	.6	.2	.6	.2	.1	150	.663	-.2	.4	.3	.15	150
127	3	150	200	.6	.2	.6	.2	.1	150	.8	-.2	.4	.3	.15	100
128	5	150	200	.6	.2	.6	.2	.1	100	.663	-.2	.5	.15	.15	100
129	3	100	110	.49	.1	.5	.1	.2	150	.8	-.1	.4	.15	.15	100
130	5	100	110	.49	.1	.5	.1	.2	100	.663	-.1	.5	.3	.15	100
131	3	150	110	.49	.1	.5	.1	.2	100	.8	-.1	.5	.3	.15	150
132	5	150	110	.49	.1	.5	.1	.2	150	.663	-.1	.4	.15	.15	150
133	3	100	200	.49	.1	.5	.1	.2	150	.8	-.2	.5	.15	.15	150
134	5	100	200	.49	.1	.5	.1	.2	100	.663	-.2	.4	.3	.15	150
135	3	150	200	.49	.1	.5	.1	.2	100	.8	-.2	.4	.3	.15	100
136	5	150	200	.49	.1	.5	.1	.2	150	.663	-.2	.5	.15	.15	100

137	3	100	110	.6	.1	.5	.1	.2	100	.8	-.2	.5	.15	.3	100
138	5	100	110	.6	.1	.5	.1	.2	150	.663	-.2	.4	.3	.3	100
139	3	150	110	.6	.1	.5	.1	.2	150	.8	-.2	.4	.3	.3	150
140	5	150	110	.6	.1	.5	.1	.2	100	.663	-.2	.5	.15	.3	150
141	3	100	200	.6	.1	.5	.1	.2	100	.8	-.1	.4	.15	.3	150
142	5	100	200	.6	.1	.5	.1	.2	150	.663	-.1	.5	.3	.3	150
143	3	150	200	.6	.1	.5	.1	.2	150	.8	-.1	.5	.3	.3	100
144	5	150	200	.6	.1	.5	.1	.2	100	.663	-.1	.4	.15	.3	100
145	3	100	110	.49	.2	.5	.1	.2	150	.663	-.1	.5	.3	.3	100
146	5	100	110	.49	.2	.5	.1	.2	100	.8	-.1	.4	.15	.3	100
147	3	150	110	.49	.2	.5	.1	.2	100	.663	-.1	.4	.15	.3	150
148	5	150	110	.49	.2	.5	.1	.2	150	.8	-.1	.5	.3	.3	150
149	3	100	200	.49	.2	.5	.1	.2	150	.663	-.2	.4	.3	.3	150
150	5	100	200	.49	.2	.5	.1	.2	100	.8	-.2	.5	.15	.3	150
151	3	150	200	.49	.2	.5	.1	.2	100	.663	-.2	.5	.15	.3	100
152	5	150	200	.49	.2	.5	.1	.2	150	.8	-.2	.4	.3	.3	100
153	3	100	110	.6	.2	.5	.1	.2	100	.663	-.2	.4	.3	.15	100
154	5	100	110	.6	.2	.5	.1	.2	150	.8	-.2	.5	.15	.15	100
155	3	150	110	.6	.2	.5	.1	.2	150	.663	-.2	.5	.15	.15	150
156	5	150	110	.6	.2	.5	.1	.2	100	.8	-.2	.4	.3	.15	150
157	3	100	200	.6	.2	.5	.1	.2	100	.663	-.1	.5	.3	.15	150
158	5	100	200	.6	.2	.5	.1	.2	150	.8	-.1	.4	.15	.15	150
159	3	150	200	.6	.2	.5	.1	.2	150	.663	-.1	.4	.15	.15	100
160	5	150	200	.6	.2	.5	.1	.2	100	.8	-.1	.5	.3	.15	100
161	3	100	110	.49	.1	.6	.1	.2	150	.663	-.2	.5	.15	.3	150
162	5	100	110	.49	.1	.6	.1	.2	100	.8	-.2	.4	.3	.3	150
163	3	150	110	.49	.1	.6	.1	.2	100	.663	-.2	.4	.3	.3	100
164	5	150	110	.49	.1	.6	.1	.2	150	.8	-.2	.5	.15	.3	100
165	3	100	200	.49	.1	.6	.1	.2	150	.663	-.1	.4	.15	.3	100
166	5	100	200	.49	.1	.6	.1	.2	100	.8	-.1	.5	.3	.3	100
167	3	150	200	.49	.1	.6	.1	.2	100	.663	-.1	.5	.3	.3	150
168	5	150	200	.49	.1	.6	.1	.2	150	.8	-.1	.4	.15	.3	150
169	3	100	110	.6	.1	.6	.1	.2	100	.663	-.1	.4	.15	.15	150
170	5	100	110	.6	.1	.6	.1	.2	150	.8	-.1	.5	.3	.15	150
171	3	150	110	.6	.1	.6	.1	.2	150	.663	-.1	.5	.3	.15	100
172	5	150	110	.6	.1	.6	.1	.2	100	.8	-.1	.4	.15	.15	100
173	3	100	200	.6	.1	.6	.1	.2	100	.663	-.2	.5	.15	.15	100
174	5	100	200	.6	.1	.6	.1	.2	150	.8	-.2	.4	.3	.15	100
175	3	150	200	.6	.1	.6	.1	.2	150	.663	-.2	.4	.3	.15	150
176	5	150	200	.6	.1	.6	.1	.2	100	.8	-.2	.5	.15	.15	150

177	3	100	110	.49	.2	.6	.1	.2	150	.8	-.2	.4	.3	.15	150
178	5	100	110	.49	.2	.6	.1	.2	100	.663	-.2	.5	.15	.15	150
179	3	150	110	.49	.2	.6	.1	.2	100	.8	-.2	.5	.15	.15	100
180	5	150	110	.49	.2	.6	.1	.2	150	.663	-.2	.4	.3	.15	100
181	3	100	200	.49	.2	.6	.1	.2	150	.8	-.1	.5	.3	.15	100
182	5	100	200	.49	.2	.6	.1	.2	100	.663	-.1	.4	.15	.15	100
183	3	150	200	.49	.2	.6	.1	.2	100	.8	-.1	.4	.15	.15	150
184	5	150	200	.49	.2	.6	.1	.2	150	.663	-.1	.5	.3	.15	150
185	3	100	110	.6	.2	.6	.1	.2	100	.8	-.1	.5	.3	.3	150
186	5	100	110	.6	.2	.6	.1	.2	150	.663	-.1	.4	.15	.3	150
187	3	150	110	.6	.2	.6	.1	.2	150	.8	-.1	.4	.15	.3	100
188	5	150	110	.6	.2	.6	.1	.2	100	.663	-.1	.5	.3	.3	100
189	3	100	200	.6	.2	.6	.1	.2	100	.8	-.2	.4	.3	.3	100
190	5	100	200	.6	.2	.6	.1	.2	150	.663	-.2	.5	.15	.3	100
191	3	150	200	.6	.2	.6	.1	.2	150	.8	-.2	.5	.15	.3	150
192	5	150	200	.6	.2	.6	.1	.2	100	.663	-.2	.4	.3	.3	150
193	3	100	110	.49	.1	.5	.2	.2	100	.663	-.2	.5	.3	.3	150
194	5	100	110	.49	.1	.5	.2	.2	150	.8	-.2	.4	.15	.3	150
195	3	150	110	.49	.1	.5	.2	.2	150	.663	-.2	.4	.15	.3	100
196	5	150	110	.49	.1	.5	.2	.2	100	.8	-.2	.5	.3	.3	100
197	3	100	200	.49	.1	.5	.2	.2	100	.663	-.1	.4	.3	.3	100
198	5	100	200	.49	.1	.5	.2	.2	150	.8	-.1	.5	.15	.3	100
199	3	150	200	.49	.1	.5	.2	.2	150	.663	-.1	.5	.15	.3	150
200	5	150	200	.49	.1	.5	.2	.2	100	.8	-.1	.4	.3	.3	150
201	3	100	110	.6	.1	.5	.2	.2	150	.663	-.1	.4	.3	.15	150
202	5	100	110	.6	.1	.5	.2	.2	100	.8	-.1	.5	.15	.15	150
203	3	150	110	.6	.1	.5	.2	.2	100	.663	-.1	.5	.15	.15	100
204	5	150	110	.6	.1	.5	.2	.2	150	.8	-.1	.4	.3	.15	100
205	3	100	200	.6	.1	.5	.2	.2	150	.663	-.2	.5	.3	.15	100
206	5	100	200	.6	.1	.5	.2	.2	100	.8	-.2	.4	.15	.15	100
207	3	150	200	.6	.1	.5	.2	.2	100	.663	-.2	.4	.15	.15	150
208	5	150	200	.6	.1	.5	.2	.2	150	.8	-.2	.5	.3	.15	150
209	3	100	110	.49	.2	.5	.2	.2	100	.8	-.2	.4	.15	.15	150
210	5	100	110	.49	.2	.5	.2	.2	150	.663	-.2	.5	.3	.15	150
211	3	150	110	.49	.2	.5	.2	.2	150	.8	-.2	.5	.3	.15	100
212	5	150	110	.49	.2	.5	.2	.2	100	.663	-.2	.4	.15	.15	100
213	3	100	200	.49	.2	.5	.2	.2	100	.8	-.1	.5	.15	.15	100
214	5	100	200	.49	.2	.5	.2	.2	150	.663	-.1	.4	.3	.15	100
215	3	150	200	.49	.2	.5	.2	.2	150	.8	-.1	.4	.3	.15	150
216	5	150	200	.49	.2	.5	.2	.2	100	.663	-.1	.5	.15	.15	150
217	3	100	110	.6	.2	.5	.2	.2	150	.8	-.1	.5	.15	.3	150

218	5	100	110	.6	.2	.5	.2	.2	100	.663	-.1	.4	.3	.3	150
219	3	150	110	.6	.2	.5	.2	.2	100	.8	-.1	.4	.3	.3	100
220	5	150	110	.6	.2	.5	.2	.2	150	.663	-.1	.5	.15	.3	100
221	3	100	200	.6	.2	.5	.2	.2	150	.8	-.2	.4	.15	.3	100
222	5	100	200	.6	.2	.5	.2	.2	100	.663	-.2	.5	.3	.3	100
223	3	150	200	.6	.2	.5	.2	.2	100	.8	-.2	.5	.3	.3	150
224	5	150	200	.6	.2	.5	.2	.2	150	.663	-.2	.4	.15	.3	150
225	3	100	110	.49	.1	.6	.2	.2	100	.8	-.1	.4	.3	.15	100
226	5	100	110	.49	.1	.6	.2	.2	150	.663	-.1	.5	.15	.15	100
227	3	150	110	.49	.1	.6	.2	.2	150	.8	-.1	.5	.15	.15	150
228	5	150	110	.49	.1	.6	.2	.2	100	.663	-.1	.4	.3	.15	150
229	3	100	200	.49	.1	.6	.2	.2	100	.8	-.2	.5	.3	.15	150
230	5	100	200	.49	.1	.6	.2	.2	150	.663	-.2	.4	.15	.15	150
231	3	150	200	.49	.1	.6	.2	.2	150	.8	-.2	.4	.15	.15	100
232	5	150	200	.49	.1	.6	.2	.2	100	.663	-.2	.5	.3	.15	100
233	3	100	110	.6	.1	.6	.2	.2	150	.8	-.2	.5	.3	.3	100
234	5	100	110	.6	.1	.6	.2	.2	100	.663	-.2	.4	.15	.3	100
235	3	150	110	.6	.1	.6	.2	.2	100	.8	-.2	.4	.15	.3	150
236	5	150	110	.6	.1	.6	.2	.2	150	.663	-.2	.5	.3	.3	150
237	3	100	200	.6	.1	.6	.2	.2	150	.8	-.1	.4	.3	.3	150
238	5	100	200	.6	.1	.6	.2	.2	100	.663	-.1	.5	.15	.3	150
239	3	150	200	.6	.1	.6	.2	.2	100	.8	-.1	.5	.15	.3	100
240	5	150	200	.6	.1	.6	.2	.2	150	.663	-.1	.4	.3	.3	100
241	3	100	110	.49	.2	.6	.2	.2	100	.663	-.1	.5	.15	.3	100
242	5	100	110	.49	.2	.6	.2	.2	150	.8	-.1	.4	.3	.3	100
243	3	150	110	.49	.2	.6	.2	.2	150	.663	-.1	.4	.3	.3	150
244	5	150	110	.49	.2	.6	.2	.2	100	.8	-.1	.5	.15	.3	150
245	3	100	200	.49	.2	.6	.2	.2	100	.663	-.2	.4	.15	.3	150
246	5	100	200	.49	.2	.6	.2	.2	150	.8	-.2	.5	.3	.3	150
247	3	150	200	.49	.2	.6	.2	.2	150	.663	-.2	.5	.3	.3	100
248	5	150	200	.49	.2	.6	.2	.2	100	.8	-.2	.4	.15	.3	100
249	3	100	110	.6	.2	.6	.2	.2	150	.663	-.2	.4	.15	.15	100
250	5	100	110	.6	.2	.6	.2	.2	100	.8	-.2	.5	.3	.15	100
251	3	150	110	.6	.2	.6	.2	.2	100	.663	-.2	.5	.3	.15	150
252	5	150	110	.6	.2	.6	.2	.2	150	.8	-.2	.4	.15	.15	150
253	3	100	200	.6	.2	.6	.2	.2	150	.663	-.1	.5	.15	.15	150
254	5	100	200	.6	.2	.6	.2	.2	100	.8	-.1	.4	.3	.15	150
255	3	150	200	.6	.2	.6	.2	.2	100	.663	-.1	.4	.3	.15	100
256	5	150	200	.6	.2	.6	.2	.2	150	.8	-.1	.5	.15	.15	100

Table E.1: Experimental design of the multistage model.

E.2 Experimental Results

Run	Output				
	μ_1^*	T_1^*	μ_2^*	T_2^*	ETC^*
1	10.89081	12.67044	16.09991	10.89081	89.26627
2	11.85704	12.30849	6.571999	5.525211	136.9653
3	12.01389	12.47201	12.20331	7.83292	232.3918
4	11.58309	12.33549	15.83106	6.247237	376.9231
5	12.07903	12.36307	10.76574	10.76574	33.55952
6	11.61182	12.59132	16.68729	15.74694	43.75159
7	12.18233	12.23533	10.51584	8.984275	39.77925
8	11.1004	12.24581	8.054111	9.803755	36.19281
9	11.58482	12.36104	7.021291	8.606115	58.398
10	12.13626	12.13562	9.897673	6.625797	115.4578
11	11.59706	12.24827	12.91871	8.402944	207.8558
12	11.84054	12.25595	8.744054	8.277237	363.3722
13	11.95855	12.56399	13.57508	9.447792	49.21775
14	11.4265	12.5316	18.4906	8.541109	57.65366
15	11.77999	12.6358	11.77999	10.20448	60.99925
16	11.35239	12.19743	7.036497	5.468983	61.45547
17	10.99829	12.81253	8.631838	12.56322	136.4087
18	10.83045	12.63927	12.42185	10.53978	139.8798
19	10.98114	12.67022	10.84122	11.71636	280.1645
20	11.19675	12.45332	7.531262	6.748674	356.2789
21	11.2389	12.26553	16.38521	10.0757	50.91274
22	11.08995	12.33239	11.55574	9.726874	43.41487
23	11.15576	12.44643	12.02801	11.84774	42.43959
24	10.86138	12.39411	14.64915	11.66999	37.29153
25	11.31004	12.31004	9.508966	9.543064	88.82634
26	11.39411	12.4265	8.523011	9.638278	65.13117

27	11.14986	12.25495	11.23479	8.882532	229.0691
28	11.27678	12.22356	9.519818	8.572819	332.2199
29	10.63211	12.69815	11.06604	10.46829	57.58823
30	11.01223	12.47973	11.97962	8.987768	39.7048
31	10.92811	12.58309	11.58309	9.681186	81.56789
32	11.03194	12.52254	9.372594	8.49966	48.51494
33	11.28802	12.39479	7.955918	11.58428	85.05163
34	12.11006	12.28002	10.17955	9.131377	80.41449
35	11.62378	12.11871	10.86617	6.193422	238.2143
36	12.03262	12.2761	9.552661	7.902604	304.6013
37	11.44748	12.44666	14.73491	8.15841	44.25266
38	11.98583	12.44666	8.520951	7.802621	55.90907
39	11.15062	12.47905	9.020611	7.988222	87.99222
40	12.22986	12.54451	11.6496	9.270398	46.63042
41	11.00883	12.28304	10.16134	5.433892	104.8045
42	11.85524	12.38193	8.417675	6.185379	96.30713
43	11.23533	12.76806	12.25549	11.23533	267.3203
44	11.94254	12.50035	8.792304	8.922732	365.3247
45	11.43465	12.27565	10.2641	9.040996	38.81905
46	11.13798	12.37889	19.80445	11.74445	29.93422
47	11.67445	12.36528	11.70396	11	44.2967
48	11.43375	12.58701	18.12064	15.95413	31.06193
49	11.1947	12.26973	10.92945	9.065098	58.08896
50	11.25883	12.33289	9.96761	10	112.254
51	11.35404	12.35404	9.776217	10.93887	208.7786
52	11.34932	12.26649	6.067958	5.985132	353.1212
53	10.83352	12.76148	11.37999	11.38043	44.29414
54	10.90707	12.73194	12.97967	11.90227	41.25734
55	10.94243	12.51144	12.66573	9.259752	42.80985
56	10.97346	12.73386	7.440273	10.86768	144.7795
57	11.47905	12.47905	9.470463	7.955238	93.48947
58	10.9819	12.39343	12.7178	7.177102	147.9032
59	11.24699	12.43083	10.69964	7.888814	226.4169
60	11.55707	12.55707	8.868743	8.487225	410.7522
61	11.09829	12.28925	11.93387	9.064868	43.23843
62	11.53219	12.25495	10.24371	9.232475	40.75719
63	11.3317	12.4526	9.056522	9.805375	40.67837
64	10.99955	12.30872	11.47928	9.256626	57.98631
65	12.19801	12.30872	9.483395	10.04487	89.52457
66	11.30917	12.22356	19.27882	8.142965	66.56713
67	11.95855	12.2231	11.51423	5.783617	219.6995

68	12.02691	12.40659	7.9407	9.34729	322.4011
69	11.46978	12.37303	10.22867	7.046649	60.35502
70	11.44861	12.63932	10.15832	9.808607	43.16794
71	11.65028	12.51212	11.5316	8.297169	73.1038
72	12.20933	12.38908	9.979022	8.338531	59.709
73	11.52297	12.52297	10	8.545464	107.1844
74	11.35379	12.55455	10	8.399971	116.4859
75	12.24574	12.64732	7.900428	10.7428	269.2322
76	11.3995	12.21456	12.61838	4.6648	325.2947
77	11.85175	12.34866	10.03199	10.50309	39.98743
78	11.61334	12.26568	11.55802	8.566924	34.14052
79	11.82086	12.38643	10.1051	11.38643	42.93401
80	11.55822	12.54908	15.33349	13.79356	35.3976
81	11.28066	12.25549	8.915062	8.703511	61.94627
82	10.81594	12.02792	12.11177	4.35641	113.7886
83	11.28834	12.30917	8.862061	8.520951	196.8795
84	10.947	12.37395	8.378493	7.326398	360.3304
85	10.788	12.51144	14.41851	8.27701	36.81832
86	10.97573	12.55707	11.55707	9.148261	46.65448
87	11.04227	12.43706	11.85201	8.168149	46.68529
88	11.32247	12.62809	11.26636	10.34218	126.5106
89	11.47973	12.45889	8.435334	7.862288	84.10345
90	10.96648	12.63715	12.63669	11.53205	196.3382
91	11.13656	12.47767	11.15155	8.84845	241.1254
92	11.25663	12.44666	10.03239	7.244168	389.198
93	11.04957	12.26649	11.43214	8.89916	35.1189
94	10.96238	12.46717	13.08004	12.51992	49.34707
95	10.87454	12.38643	14.35554	11.38643	46.37093
96	11.2231	12.28788	10.18857	9.914383	46.92656
97	11.07593	12.26339	21.08819	5.881343	61.29157
98	11.41209	12.70228	10.76137	10.83028	162.0032
99	11.65783	12.29717	11.87432	5.861551	218.1255
100	11.7137	12.45889	15.74311	8.329558	398.1641
101	11.46741	12.34342	10.11112	8.38785	45.5336
102	11.04983	12.7152	22.78121	17.69692	30.87432
103	11.22698	12.41184	19.66465	11.68963	24.49497
104	12.10431	12.43284	11.17258	13.20082	43.34376
105	11.69493	12.30081	11.30081	8.390692	46.48292
106	11.72299	12.23533	11.05907	5.918049	98.72126
107	11.46216	12.44148	11.8933	12.82104	216.6373
108	11.7139	12.30475	12.4288	9.997494	362.3459

109	11.73522	12.54383	9.264552	9.199773	55.95147
110	11.14813	12.78076	10.5818	11.57676	58.60349
111	10.98544	12.21954	8.252989	5.444686	53.03167
112	11.43916	12.49617	11.75132	8.174886	87.82361
113	10.9389	12.43082	9.248796	6.787405	102.1846
114	11.65893	12.61576	9.198035	11.56449	157.1835
115	11.19896	12.60965	8.203507	10.54803	275.2184
116	10.97916	12.5316	11.37418	8.520725	370.487
117	11.25495	12.25495	10.22378	8.93576	52.65712
118	10.84396	12.4265	14.40146	12.54634	42.13876
119	10.58134	12.2881	11.46837	8.822301	47.04989
120	11.10998	12.34717	12.6231	10.11064	37.44312
121	11.04102	12.22644	11.66522	7.372071	79.87563
122	11.0119	12.20757	10.49498	7.29672	64.34575
123	11.21456	12.29739	10.16565	9.366712	237.6365
124	11.14556	12.26605	10.73119	9.313969	326.7577
125	11.0168	12.73522	13.59226	11.47973	42.45042
126	11.04995	12.44406	12.5207	8.438753	50.82069
127	11.0404	12.50922	8.374301	8.091516	80.78128
128	10.94243	12.52968	13.32891	8.470323	34.28825
129	11.4464	12.22283	17.17893	5.74571	66.71556
130	11.43308	12.21108	11.4884	6.808604	50.40187
131	12.17019	12.2904	9.487727	9.844407	230.3862
132	11.7026	12.28066	15.33252	9.472276	320.5888
133	11.01431	12.50983	9.214658	8.897967	51.1917
134	10.86373	12.55024	26.83868	8.893874	42.25324
135	12.20732	12.32172	8.352797	6.243731	51.18425
136	11.86358	12.59648	12.67738	9.300142	37.28172
137	11.80838	12.45889	12.25595	7.744054	96.24483
138	12.0111	12.32027	9.691506	6.065793	92.74742
139	11.61343	12.37719	8.845523	6.504673	261.6398
140	11.20654	12.36788	12.50304	5.966872	344.1869
141	11.47205	12.51212	12.89855	13.86045	42.5169
142	11.38643	12.38643	8.581178	9.318826	51.61532
143	10.9114	12.34759	22.49253	11.29224	43.99329
144	11.58048	12.30872	14.1123	9.865954	32.21358
145	11.15189	12.39272	8.899599	11.25841	66.43254
146	11.11747	12.25495	11.14986	8.745047	112.5892
147	11.15837	12.32057	10.31196	9.838442	208.9086
148	11.23087	12.16302	9.236628	6.528717	365.4767
149	10.96358	12.42582	12.9162	8.179528	56.48113

150	11.41197	12.73444	8.588029	11.41197	55.74189
151	10.89058	12.3973	12.51531	6.932481	40.48581
152	11.5472	12.58191	8.948002	10.59731	143.2932
153	11.21478	12.54565	9.783951	9.51663	87.57615
154	11.52617	12.54499	6.044053	9.115209	161.6009
155	11.40103	12.45145	8.588326	7.268399	233.6258
156	10.99932	12.67022	11.39411	10.72322	426.5754
157	11.07772	12.45852	12.68913	11.72277	38.04531
158	11.24487	12.19702	8.996477	9.104412	47.75069
159	10.97148	12.41882	12.91611	11.86616	32.2023
160	11.19405	12.27966	12.47493	10.08562	39.25432
161	11.41162	12.41162	18.31969	7.083512	75.08131
162	11.37776	12.32244	14.29296	5.65172	134.4721
163	11.08494	12.17055	17.10795	4.297535	210.3803
164	11.66321	12.30105	12.90333	5.337681	375.1904
165	11.1453	12.39544	22.9405	12.87719	31.61119
166	11.05209	12.39343	10.37441	11.39343	46.59208
167	11.93859	12.46765	14.83931	12.61006	41.32218
168	10.90944	12.36104	11.50369	10.14265	49.88392
169	11.61021	12.30081	11.24827	9.896144	54.66778
170	11.61767	12.37395	11.4265	11.1504	116.3421
171	11.61721	12.32865	9.756059	8.435334	201.2677
172	11.69461	12.27966	11.31228	6.126613	343.243
173	12.1679	12.46893	9.210545	8.994058	36.72821
174	11.59117	12.61239	12.71395	9.420263	49.83699
175	12.03473	12.49213	9.535146	9.273607	53.83346
176	11.92311	12.54078	10.24745	8.391285	83.51237
177	11.08494	12.55105	12.72138	9.80974	117.8899
178	10.94682	12.46239	13.16853	7.224313	102.3289
179	11.39343	12.39343	11.04462	7.192073	242.8449
180	10.87185	12.42238	11.92964	7.922773	357.1706
181	11.22707	12.36073	10.08217	11.20757	44.84527
182	11.15528	12.2881	11.703	10	28.95753
183	11.13283	12.26649	12.13134	10.2476	37.86164
184	11.71735	12.27966	8.755239	8.176875	48.19855
185	10.95254	12.41435	9.783513	11.49696	91.05724
186	11.28415	12.26007	9.577232	9.305441	83.66021
187	11.28599	12.24699	8.835177	8.663172	238.9841
188	11.17698	12.3427	11.28762	8.387134	317.0972
189	10.83885	12.49729	11.85878	8.915062	55.45986
190	10.89597	12.46216	12.26347	7.672546	45.13843

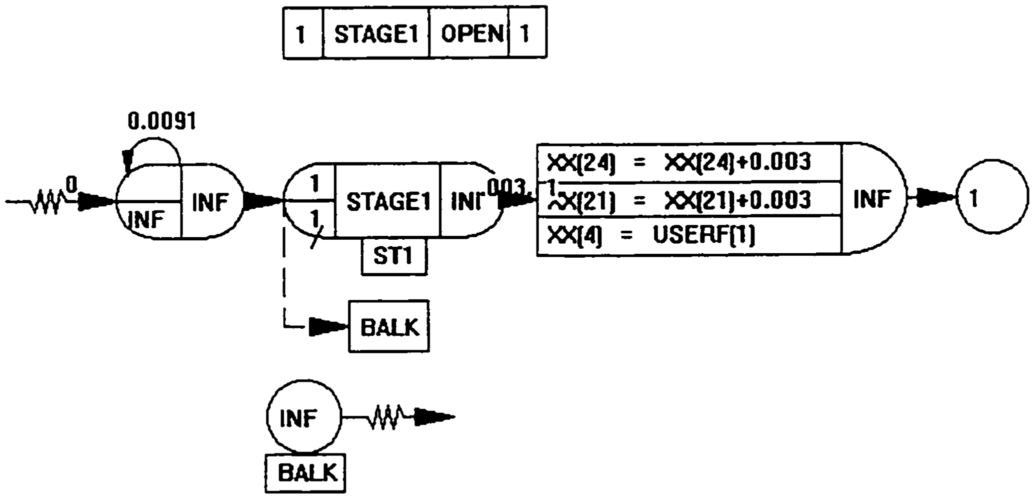
191	11.63737	12.41359	6.126613	7.38256	82.42761
192	11.28786	12.63409	7.951012	10.92876	96.65872
193	12.33874	12.4239	7.109055	7.214831	84.31284
194	11.37776	12.17627	16.02846	4.758423	130.8324
195	11.38243	12.33437	18.59375	6.710997	215.7583
196	11.56467	12.28788	11.04524	5.548649	377.2412
197	11.906	12.34111	12.40612	11.06501	37.98789
198	12.25572	12.32865	9.638732	12.06433	45.54959
199	11.36943	12.36943	11.5497	8.720701	43.32991
200	12.08308	12.26928	12.99598	10	51.96934
201	11.78329	12.31235	10.14545	9.854546	63.01568
202	11.70283	12.32165	10.80115	10.25595	105.9523
203	11.61699	12.44666	11.44666	9.915289	196.0347
204	11.43531	12.2187	19.38179	5.758254	346.1244
205	12.45296	11.75013	9.249873	5.388718	60.32357
206	11.79266	12.63263	12.77567	10.96276	37.0681
207	11.55802	12.6131	12.24976	9.473744	38.9918
208	11.27932	12.46767	11.02056	7.751193	87.93939
209	11.0586	12.37035	9.264514	6.873091	98.39539
210	11.10705	12.469	6.81744	7.648611	119.3352
211	11.2231	12.39411	9.829445	6.969444	250.4017
212	11.43817	12.41045	6.707415	8.117863	353.8159
213	11.15433	12.44679	10.32827	12.57077	32.44274
214	10.93409	12.25617	11.51212	8.999773	39.08069
215	11.23579	12.3812	10.05255	11.25595	50.68834
216	11.03194	12.26842	9.256626	7.975289	38.16048
217	11.14217	12.34294	11.34294	9.764413	88.15556
218	11.21488	12.27966	8.613568	8.581178	80.68227
219	11.17002	12.2761	7.626813	6.318319	231.5855
220	11.5497	12.36943	8.540433	8.450299	320.7902
221	11.20362	12.47905	11.45957	8.776897	51.65005
222	11.26583	12.43149	10.72567	7.677847	48.36628
223	11.6472	12.41162	7.009378	6.538203	75.22038
224	11.00014	12.39343	11.56399	7.711524	52.35546
225	11.6849	12.11912	9.008079	6.204056	81.81329
226	12.26795	12.31205	7.347063	8.179388	63.89655
227	11.20139	12.1964	18.53026	7.70388	224.1273
228	11.32014	12.22059	17.30637	8.025473	316.9955
229	11.54315	12.62877	12.57896	9.390918	51.30979

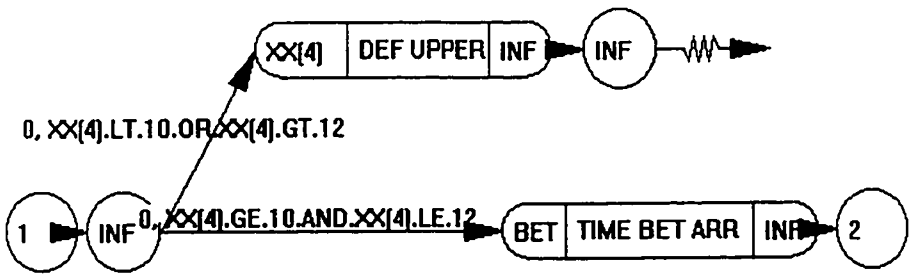
230	11.11755	12.58505	12.64983	9.500113	42.68493
231	11.35266	12.34088	13.40521	6.094903	42.80683
232	11.64732	12.44266	11.8947	7.695993	40.32382
233	11.75856	12.48911	12.75527	7.445977	103.7625
234	11.70228	12.52297	8.207582	8.386895	108.6078
235	11.75597	12.44666	10.47891	8.468544	256.0371
236	11.85658	12.63142	13.04066	10.3227	394.9999
237	11.77531	12.2231	12.03216	8.948132	57.5102
238	11.13732	12.52636	19.50979	14.42435	34.42195
239	11.04754	12.25568	24.50147	10.87774	35.49051
240	11.72277	12.40425	15.51789	12.07236	40.3588
241	11.2447	12.24592	9.585099	7.308131	45.08387
242	11.21488	12.34444	8.785116	6.614722	112.5484
243	11.08038	12.24603	11.24103	8.970456	215.1414
244	11.0809	12.21456	10.36796	7.938836	357.2584
245	10.54778	12.58093	12.23547	9.723983	48.10978
246	10.96214	12.59117	11.21037	8.609605	66.0455
247	11.4106	12.68351	9.72623	11.31105	88.76967
248	11.59706	12.20204	8.574179	5.647791	51.55777
249	11.03865	12.48149	8.680073	7.794401	81.19608
250	11.12679	12.55745	9.716345	8.589943	150.7416
251	11.44666	12.39434	8.608262	6.800108	229.3061
252	10.91461	12.43828	11.76761	8.041223	403.0957
253	11.13817	12.41427	12.99887	9.244167	39.12162
254	11.35895	12.41875	10.09744	12.62679	45.59526
255	11.24727	12.31205	11.00014	9.999407	35.15463
256	11.10428	12.34027	11.83894	10.60562	37.12536

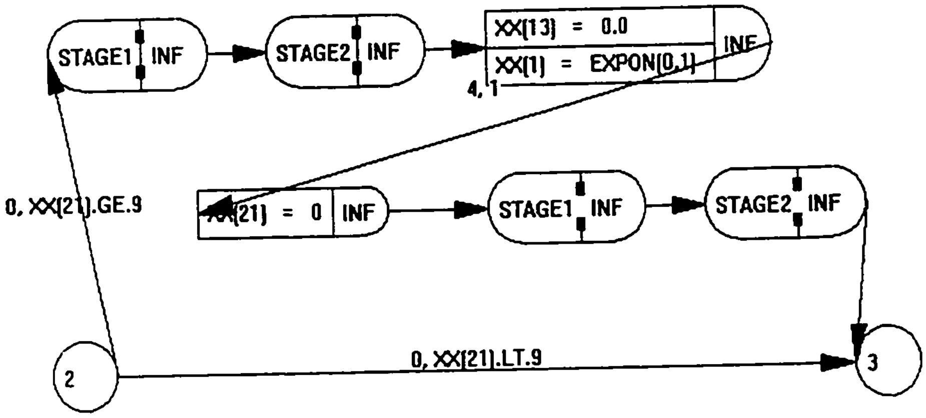
Table E.2: Experimental results of the multistage model.

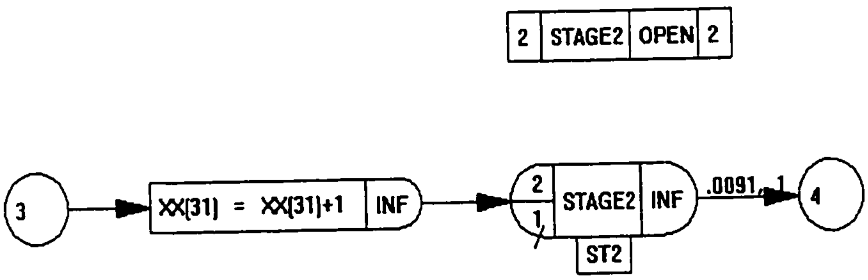
Appendix F

SLAM II Model for Two-Stage Lines without a Buffer


1
STAGE1
OPEN
1
0.0091
0
INF
INF
1
1
STAGE1
INF
ST1
.003, 1
XX[24] = XX[24]+0.003
XX[21] = XX[21]+0.003
XX[4] = USERF[1]
INF
1
BALK
INF
BALK


XX[4]
DEF UPPER
INF
INF
0, XX[4].LT.10.OR.XX[4].GT.12
1
INF
0, XX[4].GE.10.AND.XX[4].LE.12
BET
TIME BET ARR
INF
2


STAGE1 INF
STAGE2 INF
XX[13] = 0.0
XX[1] = EXPON[0,1]
INF
4, 1
0, XX[21].GE.9
XX[21] = 0
INF
STAGE1 INF
STAGE2 INF
2
0, XX[21].LT.9
3


2
STAGE2
OPEN
2
3
XX[31] = XX[31]+1
INF
2
1
STAGE2
INF
ST2
.0091, 1
4

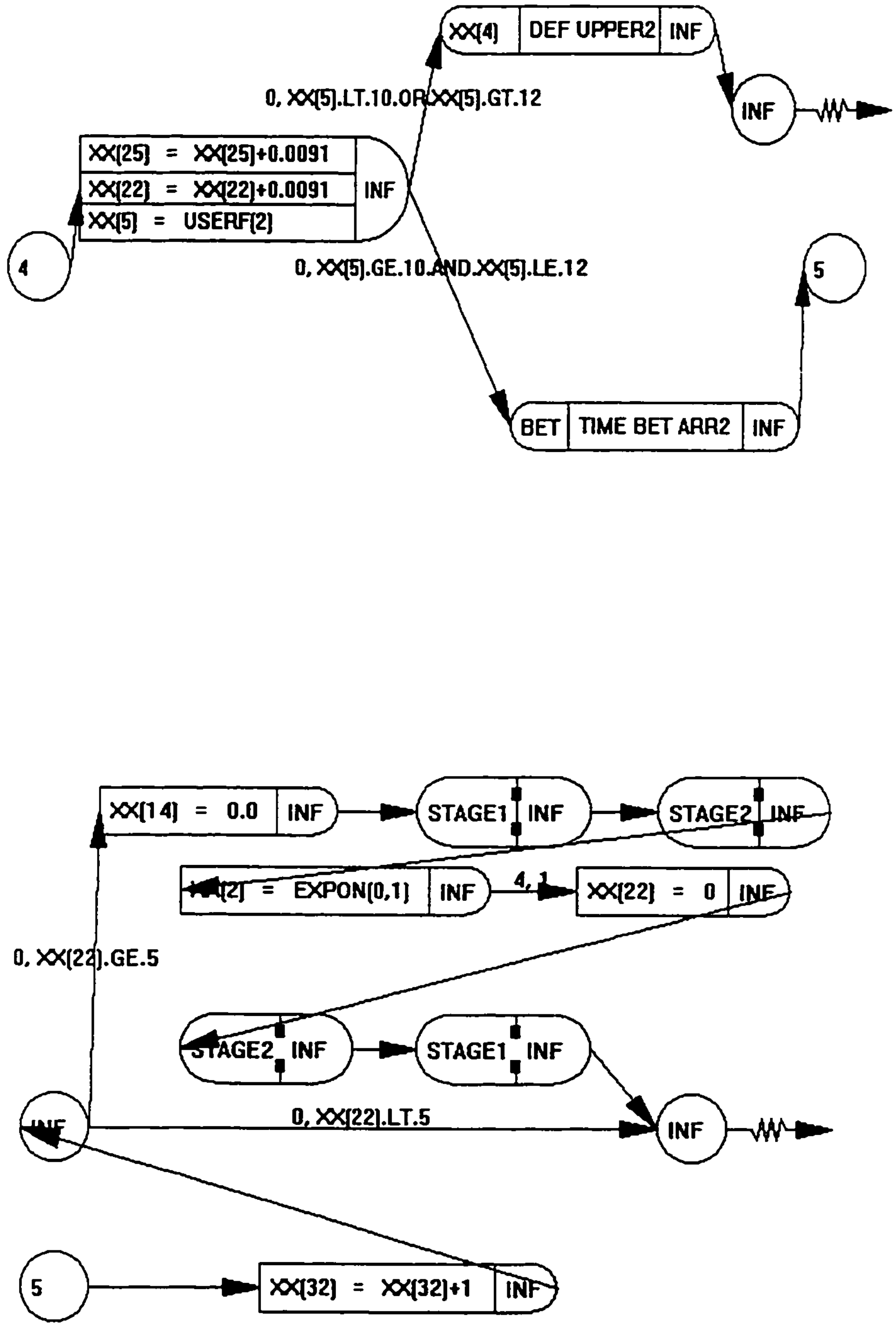
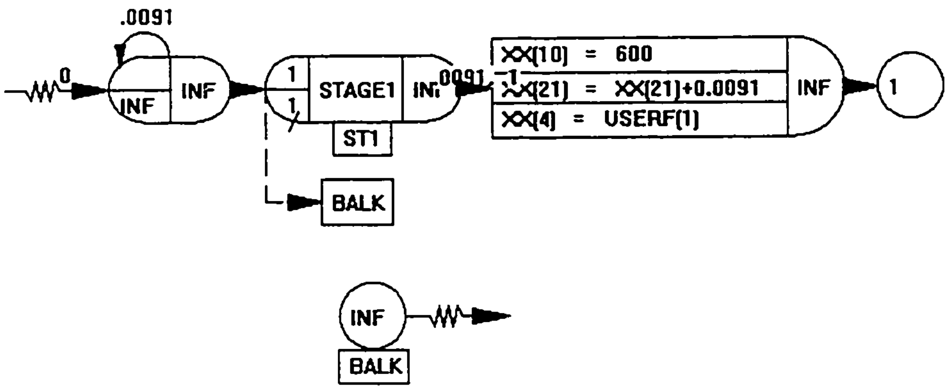


Figure F.1: *SLAM II* Model for Two-Stage Lines without a Buffer.

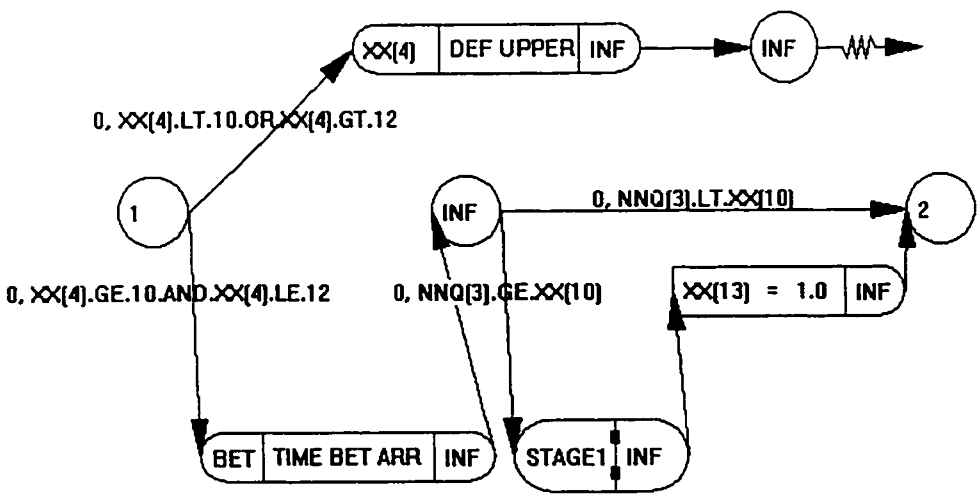
Appendix G

SLAM II Model for Two-Stage Lines with a Buffer

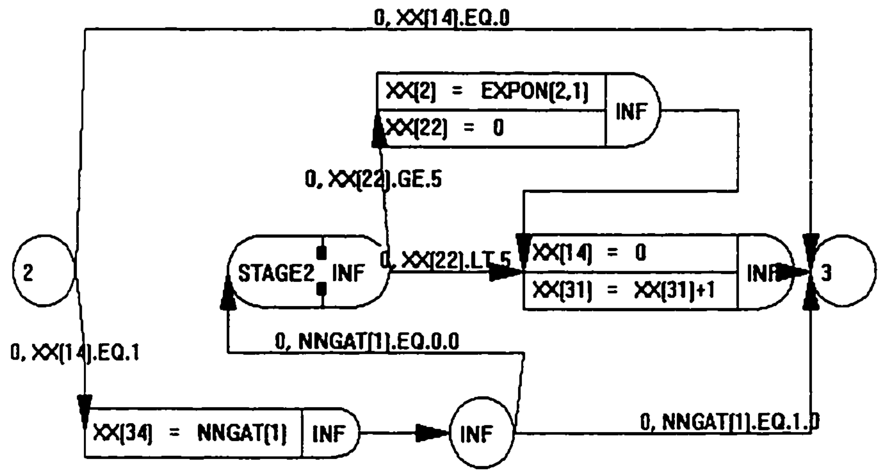
1
STAGE1
OPEN
1

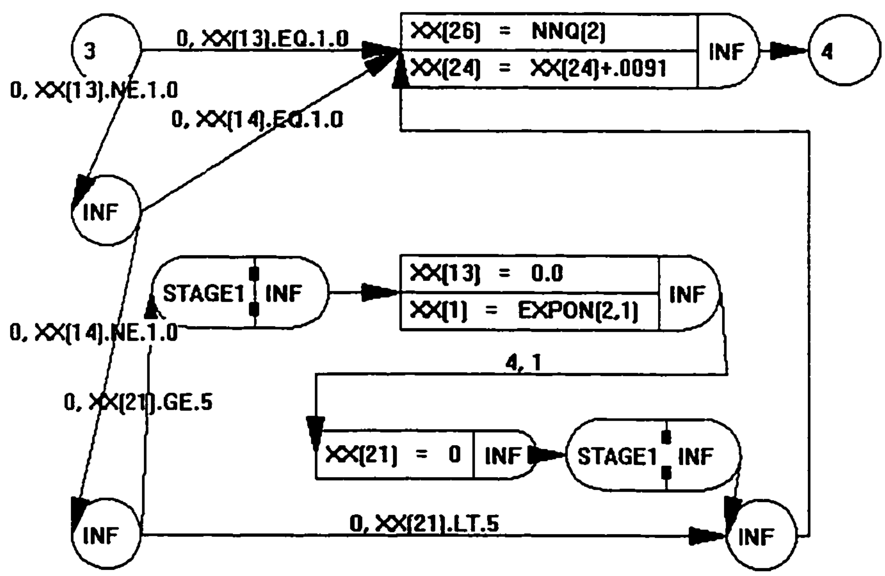


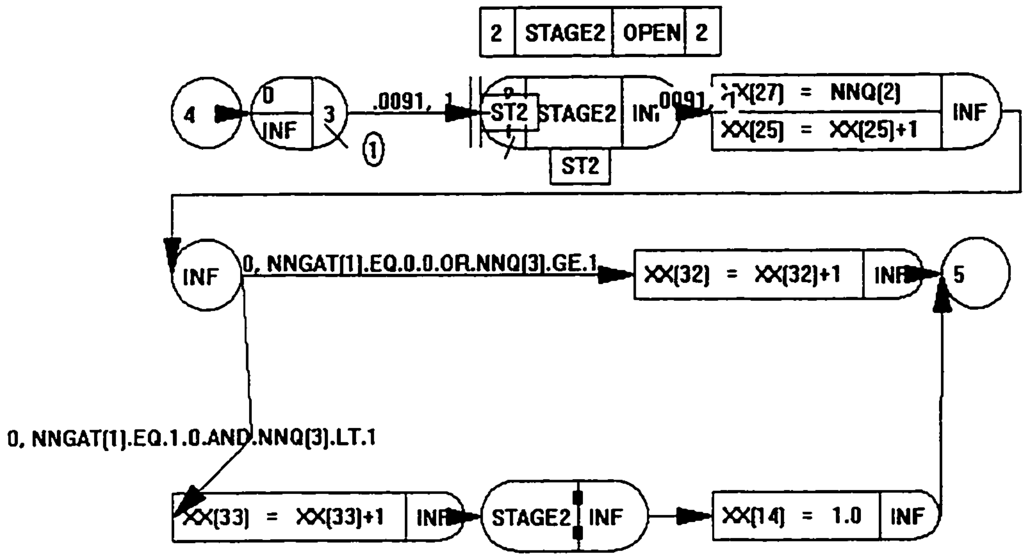
.0091
0
INF
INF
1
STAGE1
INF
1
ST1
.0091
1
XX(10) = 600
XX(21) = XX(21)+0.0091
XX(4) = USERF(1)
INF
1
BALK
INF
BALK



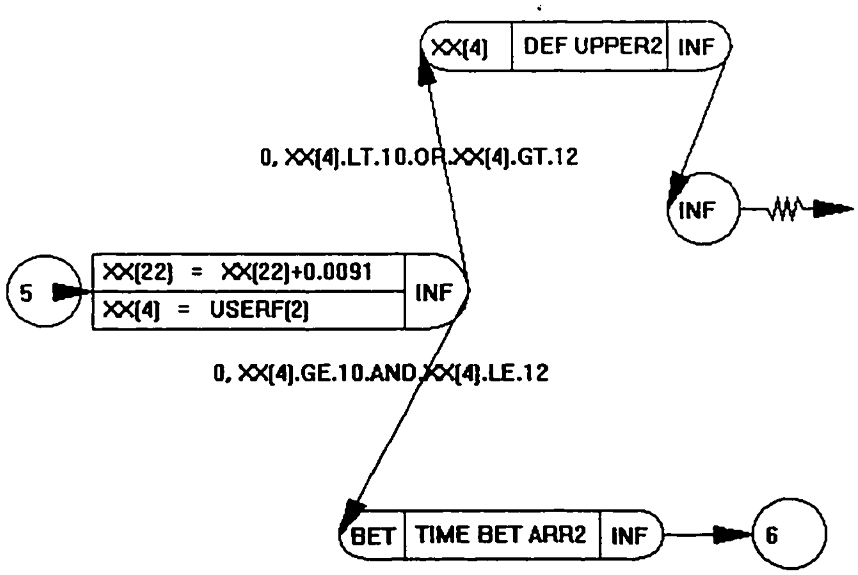
XX(4)
DEF UPPER
INF
INF
0, XX(4).LT.10.OR.XX(4).GT.12
1
INF
0, NNQ(3).LT.XX(10)
2
0, XX(4).GE.10.AND.XX(4).LE.12
0, NNQ(3).GE.XX(10)
XX(13) = 1.0
INF
BET
TIME BET ARR
INF
STAGE1
INF


0, XX[14].EQ.0
XX[2] = EXPON[2,1]
XX[22] = 0
INF
0, XX[22].GE.5
2
STAGE2 INF
0, XX[22].LT.5
XX[14] = 0
XX[31] = XX[31]+1
INF
3
0, XX[14].EQ.1
0, NNGAT[1].EQ.0.0
XX[34] = NNGAT[1]
INF
INF
0, NNGAT[1].EQ.1.0


3
0, XX[13].EQ.1.0
XX[26] = NNQ[2]
XX[24] = XX[24]+.0091
INF
4
0, XX[13].NE.1.0
0, XX[14].EQ.1.0
INF
STAGE1 INF
XX[13] = 0.0
XX[1] = EXPON[2,1]
INF
0, XX[14].NE.1.0
4, 1
0, XX[21].GE.5
XX[21] = 0
INF
STAGE1 INF
INF
0, XX[21].LT.5
INF



2
STAGE2
OPEN
2
4
0
INF
3
1
.0091, 1
ST2
STAGE2
INF
ST2
.0091, 1
XX[27] = NNQ[2]
XX[25] = XX[25]+1
INF
INF
0, NNGAT[1].EQ.0.0.OR.NNQ[3].GE.1
XX[32] = XX[32]+1
INF
5
0, NNGAT[1].EQ.1.0.AND.NNQ[3].LT.1
XX[33] = XX[33]+1
INF
STAGE2
INF
XX[14] = 1.0
INF



XX[4]
DEF UPPER2
INF
0, XX[4].LT.10.0.OR.XX[4].GT.12
INF
5
XX[22] = XX[22]+0.0091
XX[4] = USERF[2]
INF
0, XX[4].GE.10.AND.XX[4].LE.12
BET
TIME BET ARR2
INF
6

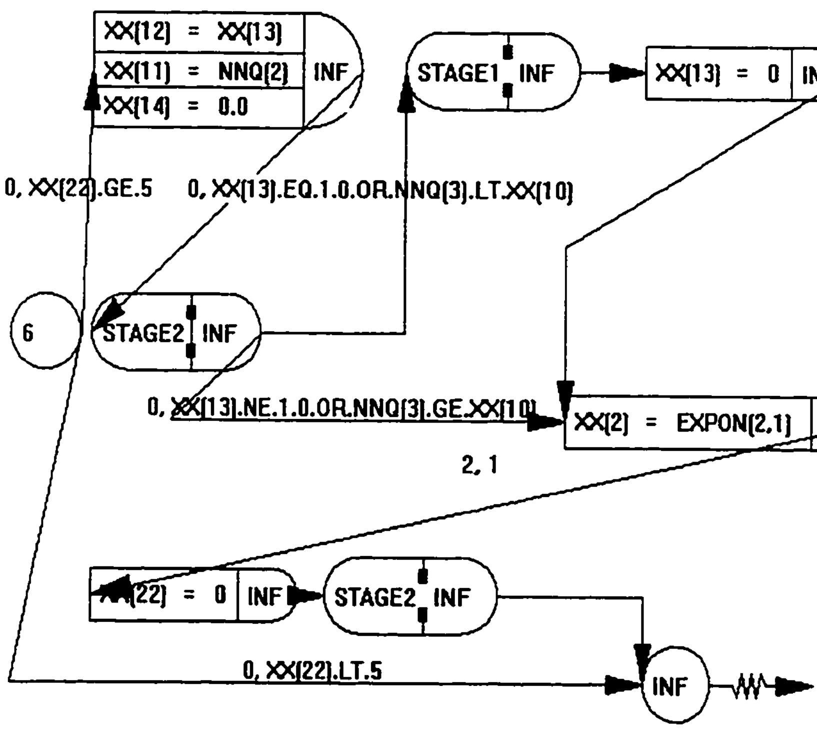
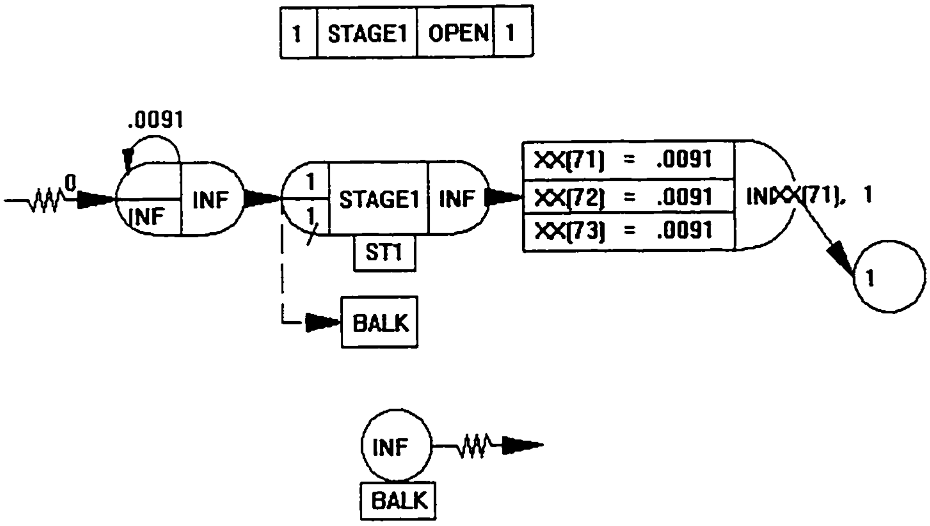
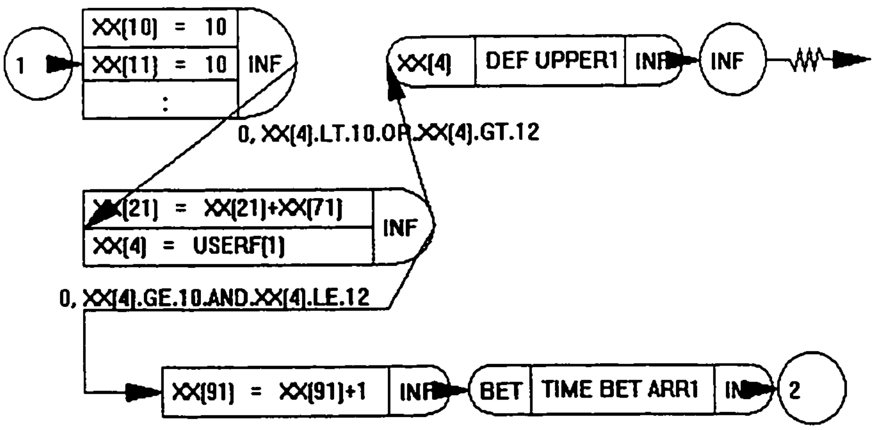


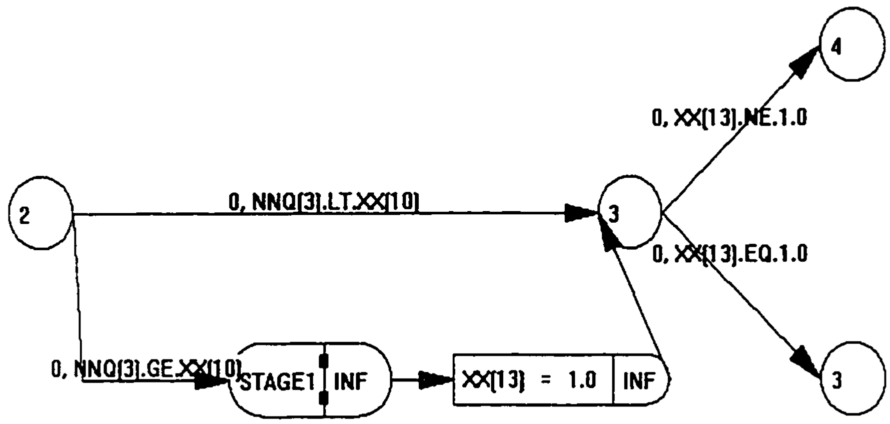
Figure G.1: *SLAM II* Model for Two-Stage Lines with a Buffer.

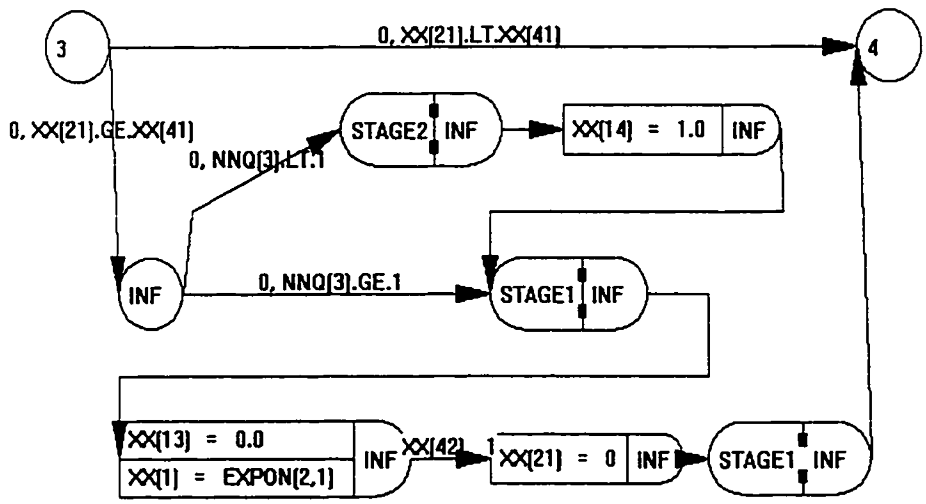
Appendix H

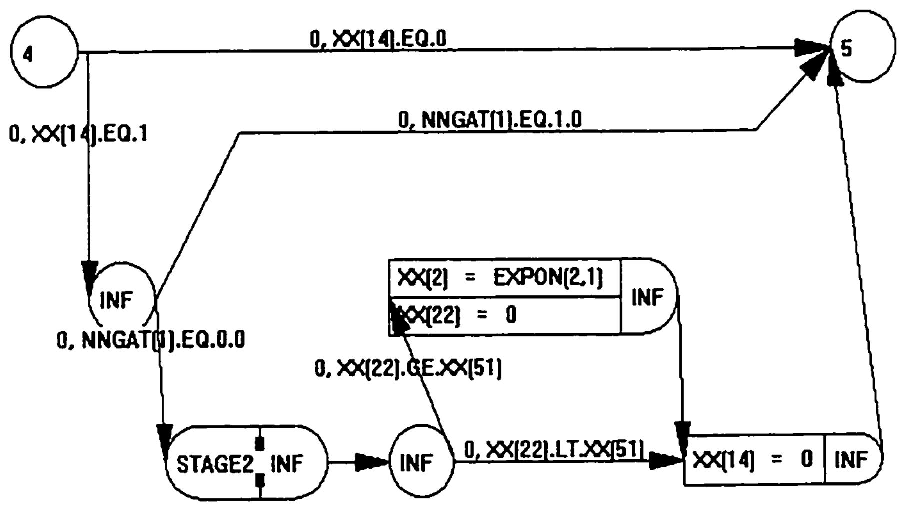
SLAM II Model for Multistage Lines with Buffers

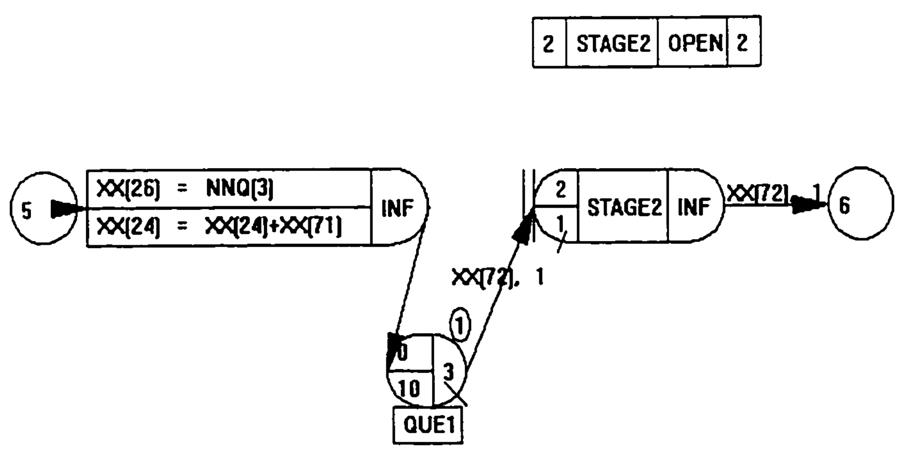

1
STAGE1
OPEN
1
.0091
0
INF
INF
1
1
STAGE1
INF
ST1
BALK
XX[71] = .0091
XX[72] = .0091
XX[73] = .0091
INF
XX[71], 1
1
INF
BALK

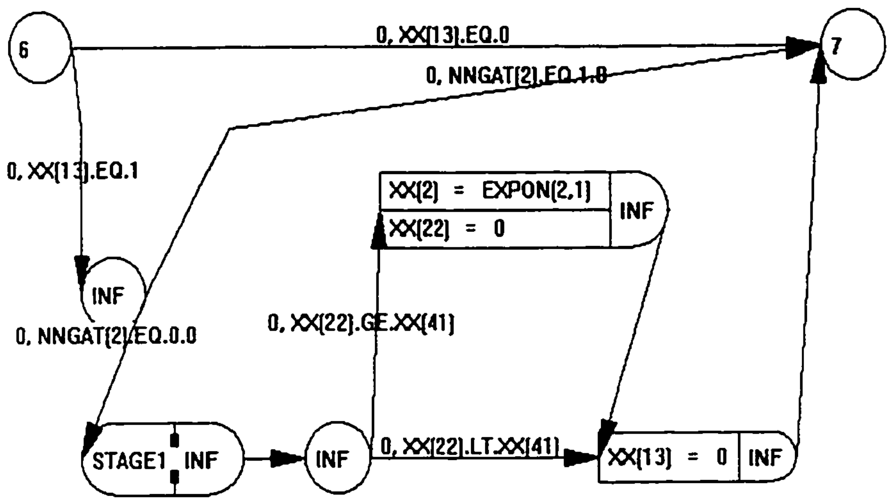

1
XX[10] = 10
XX[11] = 10
:
INF
0, XX[4].LT.10.OR.XX[4].GT.12
XX[21] = XX[21]+XX[71]
XX[4] = USERF[1]
INF
0, XX[4].GE.10.AND.XX[4].LE.12
XX[4]
DEF UPPER1
INF
XX[91] = XX[91]+1
BET
TIME BET ARR1
2

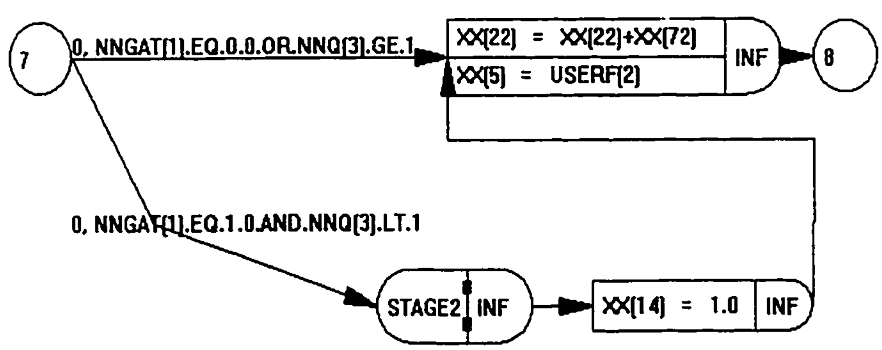

2
0, NNQ[3].LT.XX[10]
3
0, XX[13].NE.1.0
4
0, XX[13].EQ.1.0
3
0, NNQ[3].GE.XX[10]
STAGE1
INF
XX[13] = 1.0
INF

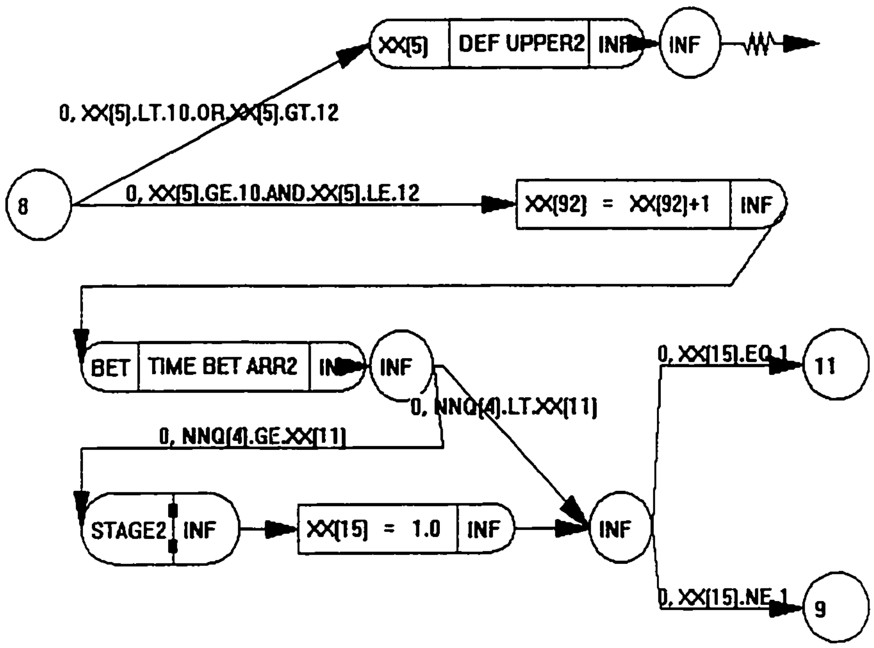

3
0, XX[21].LT.XX[41]
4
0, XX[21].GE.XX[41]
0, NNQ[3].LT.1
STAGE2
INF
XX[14] = 1.0
INF
INF
0, NNQ[3].GE.1
STAGE1
INF
XX[13] = 0.0
XX[1] = EXPON[2,1]
INF
XX[42]
XX[21] = 0
INF
STAGE1
INF

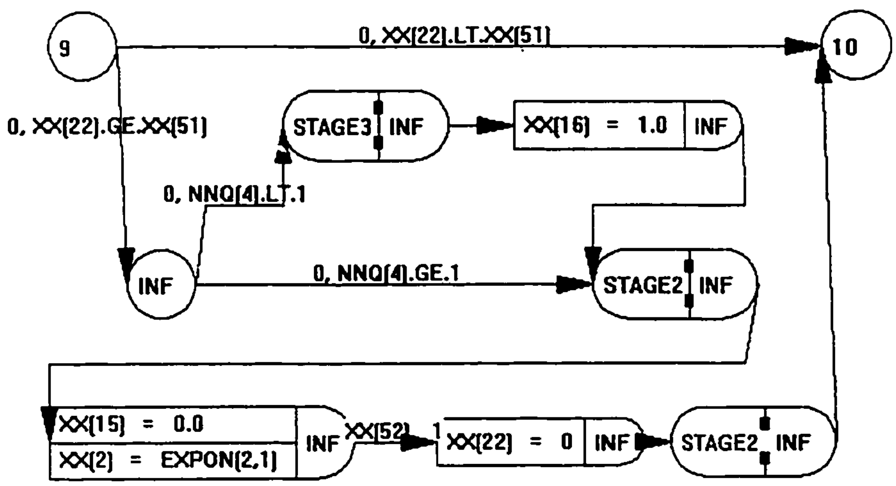

4
0, XX[14].EQ.0
5
0, XX[14].EQ.1
0, NNGAT[1].EQ.1.0
INF
0, NNGAT[1].EQ.0.0
XX[2] = EXPON[2,1]
XX[22] = 0
INF
0, XX[22].GE.XX[51]
STAGE2
INF
INF
0, XX[22].LT.XX[51]
XX[14] = 0
INF

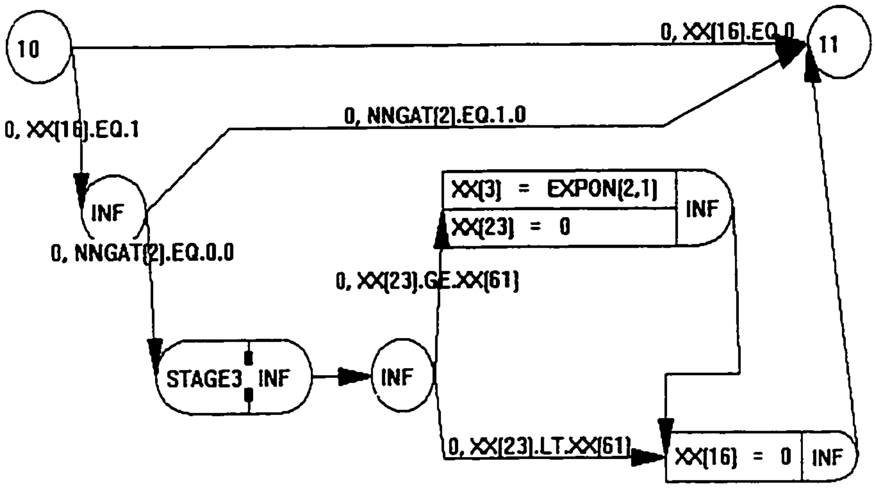

2
STAGE2
OPEN
2
5
XX[26] = NNQ[3]
XX[24] = XX[24]+XX[71]
INF
2
1
STAGE2
INF
XX[72], 1
6
XX[72], 1
1
0
10
3
QUE1

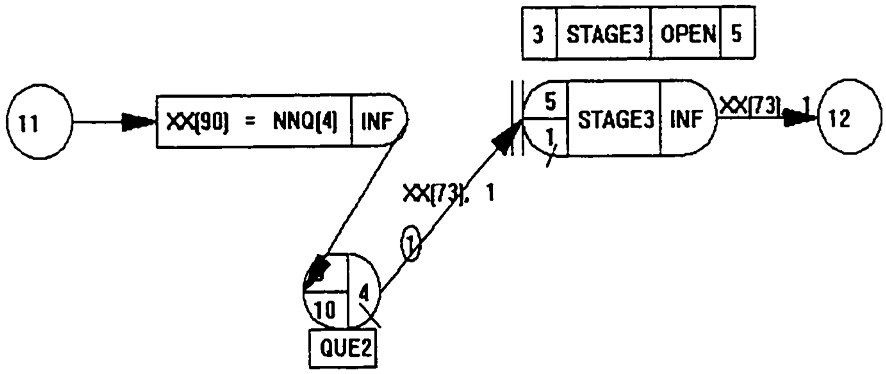

6
0, XX[13].EQ.0
7
0, NNGAT[2].EQ.1.0
0, XX[13].EQ.1
XX[2] = EXPON[2,1]
XX[22] = 0
INF
INF
0, NNGAT[2].EQ.0.0
0, XX[22].GE.XX[41]
STAGE1
INF
INF
0, XX[22].LT.XX[41]
XX[13] = 0
INF

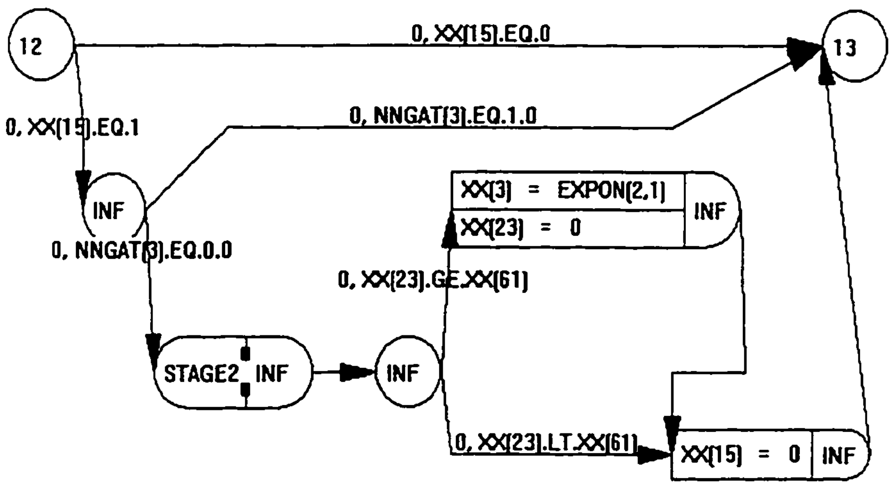

7
0, NNGAT[1].EQ.0.0.OR.NNQ[3].GE.1
XX[22] = XX[22]+XX[72]
XX[5] = USERF[2]
INF
8
0, NNGAT[1].EQ.1.0.AND.NNQ[3].LT.1
STAGE2
INF
XX[14] = 1.0
INF

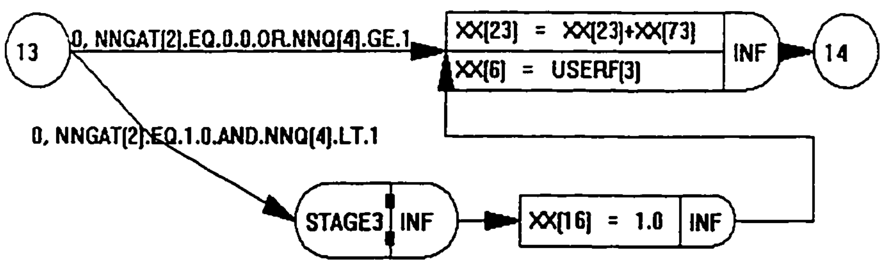

XX[5]
DEF UPPER2
INF
INF
0, XX[5].LT.10.OR.XX[5].GT.12
8
0, XX[5].GE.10.AND.XX[5].LE.12
XX[92] = XX[92]+1
INF
BET
TIME BET ARR2
INF
0, XX[15].EQ.1
11
0, NNQ[4].LT.XX[11]
0, NNQ[4].GE.XX[11]
STAGE2
INF
XX[15] = 1.0
INF
INF
0, XX[15].NE.1
9

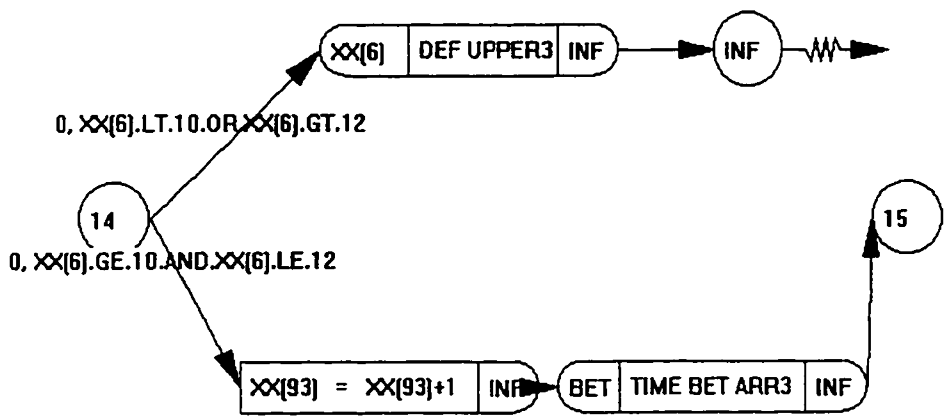

9
0, XX[22].LT.XX[51]
10
0, XX[22].GE.XX[51]
STAGE3
INF
XX[16] = 1.0
INF
0, NNQ[4].LT.1
INF
0, NNQ[4].GE.1
STAGE2
INF
XX[15] = 0.0
XX[2] = EXPON[2,1]
INF
XX[52]
1
XX[22] = 0
INF
STAGE2
INF


10
11
0, XX[16].EQ.0
0, XX[16].EQ.1
0, NNGAT[2].EQ.1.0
INF
0, NNGAT[2].EQ.0.0
XX[3] = EXPON[2,1]
XX[23] = 0
INF
0, XX[23].GE.XX[61]
STAGE3
INF
INF
0, XX[23].LT.XX[61]
XX[16] = 0
INF


3
STAGE3
OPEN
5
11
XX[90] = NNQ[4]
INF
5
1
STAGE3
INF
XX[73], 1
12
XX[73], 1
1
10
4
QUE2


12
0, XX[15].EQ.0
13
0, XX[15].EQ.1
0, NNGAT[3].EQ.1.0
INF
XX[3] = EXPON[2,1]
XX[23] = 0
INF
0, NNGAT[3].EQ.0.0
0, XX[23].GE.XX[61]
STAGE2
INF
INF
0, XX[23].LT.XX[61]
XX[15] = 0
INF


13
0, NNGAT[2].EQ.0.0.OR.NNQ[4].GE.1
XX[23] = XX[23]+XX[73]
XX[6] = USERF[3]
INF
14
0, NNGAT[2].EQ.1.0.AND.NNQ[4].LT.1
STAGE3
INF
XX[16] = 1.0
INF



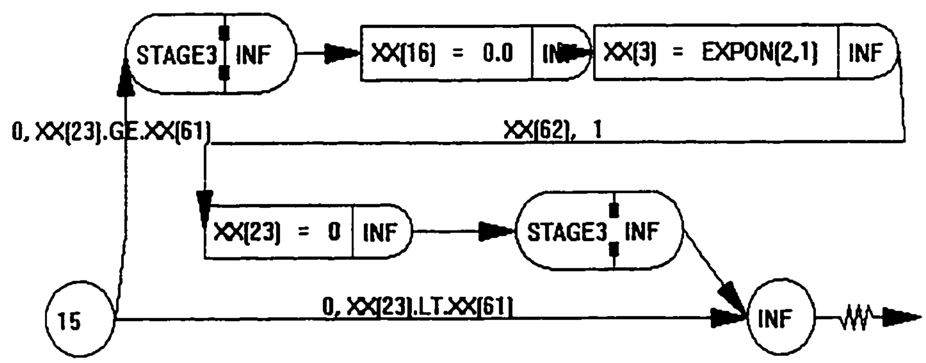


Figure H.1: *SLAM II* Model for Multistage Lines with Buffers.

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