Design of 1-D and 2-D IIR Digital Filters using IRLS Technique

by

Shaik Mahaboob Basha

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
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This thesis, written by **Shaik Mahaboob Basha** *under the direction of his Thesis Advisor, and approved by his Thesis committee, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirements for the degree of*
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COLLEGE OF GRADUATE STUDIES

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Dedicated to

MY PARENTS, BROTHER & SISTER

whose patience and perseverance

led to this accomplishment.

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Abstract

Name: Basha, Shaik Mahaboob

Title: Design of 1-D and 2-D IIR Digital Filters

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In this thesis a design technique called iteratively reweighted least-squares(IRLS) is used to design 1-D and 2-D IIR digital filters in the L_p norm sense. In this technique, by selecting the weights in each iteration based on the error in previous iterations, a solution to the L_p approximation problem is found by solving a weighted least-squares problem. In order to successfully design the IIR digital filters using the IRLS technique a convergence control parameter is determined in a manner suitable for the IIR filter design problem. A comparison of various 1-D and 2-D IIR filters designed using the IRLS technique and the Davidon Fletcher Powell (DFP) unconstrained optimization technique is also made, and the relative computational complexity of the two techniques is discussed. Comparisons with some other recent techniques both for 1-D and 2-D IIR digital filters are also presented.

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خلاصة الرسالة

اسم الطالب : شيخ محبوب باشا .

عنوان الرسالة : تصميم المنقيات المتقطعة من نوع IIR ذات الابعاد الاحادية الثنائية باستخدام طريقة IRLS.

مجال الدراسة : الهندسة الكهربائية

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في هذه الرسالة تستخدم طريقة تصميم يسمى iteratively reweighted least-squares (IRLS) وذلك لتصميم المنقيات المتقطعة من نوع IIR ذات البعد الاحادي والثنائي من وجهة نظر Lp norm في هذه الطريقة يتم عن طريق اختيارالاوزان في كل مرحلة تبعاً لمقدار الخطأ في المرحلة السابقة ، ايجاد حل لمشكلة التقريب من وجهة نظر LP وذلك بحل مشكلة من نوع least-squares حتى يتم تصميم المنقيات المتقطعة من نوع IIR باستخدام طريقة IRLS ، يستخدم معامل تحكم في درجة التقارب مناسب لمشكلة تصميم المنقيات من نوع IIR . سوف يتم مقارنة المنقيات من نوع IIR ذات الابعاد الاحادية والثنائية المصممه باستخدام طريقة IRLS وتلك الناتجة عن استخدام طريقة التصميم المثالي غير المشروط (Davidon Fletcher Powell)DFP سوف يتم ايضاً مقارنة المنقيات من نوع IIR ذات الابعاد الاحادية والثنائية الناتجة عن استخدام طريقة IRLS وتلك الناتجة عن استخدام طريقة أخرى .

درجة الماجستير في العلوم

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Chapter 1

Introduction

1.1 General

The field of Digital Signal Processing is concerned with processing of signals which can be represented as sequence of numbers. This representation of signals in digital form permits the processing with digital hardware and it permits signal processing operations to be specified as algorithms or procedures. These digital methods are powerful, flexible and can be designed to be adaptive. Signals in digital form can be stored indefinitely without error. Digital techniques have become more and more cheaper with the advancement in the Integrated Circuit technology.

Digital filters are computational algorithms that transform a given input sequence of numbers into output sequence of numbers according to prespecified rules, hence yielding some desired modification to the characteristics of input sequences. These digital filters can be used for the processing of both analog or digital signals. In most applications digital filters are used for processing the continuous-time signals. Such digital processing of continuous-time signals is commonplace in communication systems, radar and sonar systems. speech and video coding and enhancement. and biomedical engineering.

Multidimensional digital signal processing is concerned with the processing of signals which can be represented as multidimensional arrays, such as sampled images or sampled time waveforms which are received simultaneously from several sensors. Two dimensional digital signal processing, a special case of multidimensional digital signal processing has wide applications in the field of image processing. Two-dimensional signal processing techniques are needed in such areas as video coding, medical imaging, enhancement and analysis of area photographs and analysis of satellite weather photos.

Conceptually there is much similarity between the processing of a one-dimensional signal and multidimensional signals. For example many operations such as sampling, filtering and transform computation, that we might perform on multidimensional signals are conceptually the same as those performed on one dimensional filters. However there exist some differences due to the facts that:(1) as dimensionality increases more amount of data is involved:(2) the mathematics used for multidimensional filters is much more restrictive than for one dimensional case. The reason for this is that a multivariable polynomial cannot be factorized into lower order multivariable polynomials as can be done in the 1-D case. As we move on from 1-D to 2-D dimensions the amount of data involved increases. The Computer Industry, by making computers smaller and cheaper, has helped to solve the data volume problem and allowed sophisticated signal processing algorithms to be implemented in real time at a substantially reduced cost.

Digital filters can be classified into recursive and non-recursive filters. These recursive and non-recursive filters are also known as IIR and FIR filters respectively. In the case of FIR filters the output sample value depends upon the past and present input samples only. On the other hand the output sample value of an IIR filter not only depends on the past and present input samples but also on the previous output samples. While the FIR filters are always stable, the IIR filters are not. One of the major drawbacks of the FIR filter is that in general it requires more computations than an IIR filter to perform a similar filtering operation.

Two dimensional recursive and nonrecursive filters possess some characteristics similar to their one-dimensional counterparts. For example design methods for 2-D FIR digital filters are simpler than 2-D IIR digital filters as is the case with one dimensional FIR and IIR digital filters. We know that in general to meet the same specifications an FIR filter requires, in general, significantly more arithmetic operations per output sample than an IIR filter and for a 2-D FIR filter this requirement is further increased. Hence in general 2-D IIR filters possess the advantage that they require less number of arithmetic operations than 2-D FIR filters.

1-D and 2-D IIR digital filters can be designed in spatial (time) domain, where an error criterion in the spatial domain is minimized. They can also be designed in the frequency domain by using an error criterion in the frequency domain. After selecting an error criterion or performance measure, it is minimized by analytical methods or by iterative techniques using some standard minimization techniques.

1.2 IRLS Technique

In this thesis a new method of designing IIR digital filters in L_p norm sense using a technique called iteratively reweighted least-squares (IRLS) is formulated. This IRLS technique was developed by Burrus *et al.* [1] for the design of optimal L_p-approximation FIR digital filters. The IRLS technique is an iterative technique in which a weighted least-squares problem is solved in every iteration. In this technique, by selecting the weights in each iteration based on the error in previous iteration, a solution to the L_p-approximation problem is found by solving a weighted least-squares problem, which is quite simple to implement and can be written in short computer code. This is one of the major advantages of the IRLS technique.

In this thesis, the IRLS technique is developed for the 1-D and 2-D IIR digital filter design in the L_p norm sense. Chapter 2 presents a literature review of the 1-D and 2-D Frequency-domain IIR digital filter design techniques. In Chapter 3, the design of 1-D IIR digital filter using the IRLS technique is presented. In order to improve the convergence of the IRLS algorithm, it is proposed that a convergence control parameter be selected in a different manner from that used for FIR filters, and suitable for the IIR filter design problem. Several design examples demonstrating the achieved improvement in the convergence, are presented. Comparison of several lowpass and bandpass IIR filter designs using the IRLS technique and the Davidon Fletcher Powell (DFP) unconstrained optimization technique are also presented. The relative computational complexity is also discussed next. Comparison of filter designs using the proposed technique with that of some recent IIR digital filter design techniques are also presented.

In Chapter 4, the 1-D IIR digital filter design using the IRLS technique is extended for the design of 2-D IIR IIR digital filter design. The formulation of the problem for the 2-D case is presented first. Design of 2-D IIR digital filters in the cascade form is presented next. Later several design examples, which demonstrate the effectiveness of the proposed method are presented. In these design examples comparison of the proposed 2-D IIR filter design technique with filters designed using the Davidon Fletcher Powell (DFP) unconstrained optimization technique is presented. The relative computational complexity of the two methods is also presented next. Finally a comparison with some other design techniques is also presented.

Chapter 5 presents the summary and conclusions drawn during the course of this thesis and concludes with a section on recommendation for the future work.

Chapter 2

Literature review of frequency domain design techniques for 1-D and 2-D IIR filters

2.1 Introduction

An infinite impulse response (IIR) digital filter has an impulse response that is infinite in extent. As a result, an IIR filter differs in some major respects from an FIR filter. The input and output signals of an IIR filter obey a constant-coefficient difference equation from which the value of an output sample can be computed using the input sample and previously computed output samples. Because the values of output samples are fed back the IIR filter can be unstable. On the other hand. an

IIR filter typically requires a significantly smaller number of coefficients to meet a particular magnitude specification than does an FIR filter meeting the same specifications.

An IIR filter can be designed in either time domain or frequency domain. In the time domain the filters are designed by using an error criterion in the time domain. In the frequency domain design approach, filters are designed by using an error criterion in the frequency domain. Frequency-domain design methods are popular because of several reasons. Firstly, the filter frequency response function is easily written in closed form as a function of the filter numerator and denominator coefficients. Secondly, often one may be interested in only approximating a certain magnitude response without specifying a particular phase response. Such a partial specification is easier to impose in the frequency domain than in the spatial or time domain. Moreover by a suitable choice of some positive weighting function the relative importance of different frequency components can be taken into account, which is again not possible in the spatial domain.

In the following sections a review of the 1-D and 2-D frequency domain design techniques is presented.

2.2 Frequency domain design techniques: 1-D IIR digital filters

The transfer function $H(z)$ of the 1-D recursive digital filter has the following form:

$$H(z) = \frac{\sum_{n=0}^{M} b_n z^{-n}}{1 + \sum_{n=1}^{N} a_n z^{-n}} \tag{2.1}$$

A 1-D IIR digital filter can be designed by designing an appropriate analog filter and then transforming it into a discrete-time filter. This approach is reasonable when we can take the advantage of continuous time designs that have closed-form formulas such as the Butterworth, Chebyshev, or Elliptic filters. Analytical formulas do not exist for matching an arbitrary frequency response specification in general and the above methods cannot be applied. Hence in such general cases, direct filter design procedures to solve sets of linear or nonlinear equations are needed. First the type of realization of the rational transfer function $H(z)$ of the 1-D IIR digital filter is assumed and the orders of numerator and denominator are fixed. The type of realization could be either direct form, cascade form or parallel form. In the direct form the filter transfer function $H(z)$ is realized as given in (2.1). In the cascade form the filter transfer function is realized as a product of lower order filter sections, usually of second order. In the case of parallel form the filter transfer function is realized as a sum of lower order filter sections. After choosing the type of realization a desired ideal frequency response and approximation error criterion are chosen. By a suitable optimization technique, the filter numerator and denominator coefficients, are varied in a systematic way to minimize the approximation error according to the assumed error criterion. In this section several 1-D IIR frequency-domain design techniques will be reviewed.

2.2.1 Minimum mean-squared error design

Steiglitz [5] proposed a method for designing recursive digital filters by using the optimization algorithm of Fletcher and Powell [38] to minimize a square-error criterion in the frequency domain. The IIR digital filter's Z-transform of the form

$$H(Z) = A\prod_{k=1}^{K}\frac{(1+a_k z^{-1}+b_k z^{-2})}{(1+c_k z^{-1}+d_k z^{-2})} \tag{2.2}$$

was considered, i.e the filter is realized as a cascade of second order sections. Let $\omega_i, i = 1, 2, ...M$, where M is the total number of frequency samples, be the discrete set of frequencies at which the error between the actual and desired responses is evaluated. The squared error in frequency (as a function of filter parameters) can be expressed as

$$Q(\underline{\theta}) = \sum_{i=1}^{M}[|H(e^{j\omega_i})| - |H_d(e^{j\omega_i})|]^2 \tag{2.3}$$

where $H_d(e^{j\omega_i})$ is the desired frequency response value (at the discrete frequency points) and $\underline{\theta}$ is the vector of unknown coefficients given by

$$\underline{\theta} = (a_1, b_1, c_1, d_1, ..., a_k, b_k, c_k, d_k, A)$$

To minimize the squared error, (2.3) implies finding the optimum value of $\underline{\theta}$ say $\underline{\theta}^*$, such that

$$Q(\underline{\theta}^*) \leq Q(\underline{\theta}) \; for \; \underline{\theta} \neq \underline{\theta}^*$$

Steiglitz used Fletcher-Powell nonlinear optimization algorithm to minimize the squared error (2.3). One recent technique based on the squared error criterion was proposed by Shaw [6]. This technique will be used for the comparison purposes in chapter 3, where further details of this technique are provided.

2.2.2 Minimum L_p error design

Dezcky [7] considered minimization of the L_p error criterion using the Fletcher-Powell algorithm [38] in order to meet the given recursive filter specifications. In Dezcky's method, the system function of the 1-D IIR filter is represented in terms of its poles and zeros.

$$H(Z) = k_0 \prod_{k=1}^{N_s} \frac{(1 - z_K z^{-1})(1 - \overline{z}_K z^{-1})}{(1 - p_K z^{-1})(1 - \overline{p}_K z^{-1})} \tag{2.4}$$

The method permits the minimization of approximation error for both the frequency-response magnitude and the group delay. The approximation error is the sum of errors at a discrete set of frequencies. The L_p error function defined in terms of the filter parameter vector $\underline{\theta}$ and desired frequency response H_d is given by

$$L_p(\underline{\theta}) = \sum_{i=1}^{M} W(\omega_i)[|H(e^{j\omega_i})| - |H_d(e^{j\omega_i})|]^p \tag{2.5}$$

When $p = 2$ and $W(\omega_i) = 1$, for all i, the L_p approximation is identical to the squared error criterion. The L_p approximation error is a nonlinear function of the independent variables (filter parameters) and finding the minimum of such a function involves the solution of as many nonlinear equations as the number of filter parameters. Dezcky [7] used the iterative algorithm of Fletcher and Powell [38] for finding a local minimum of the approximation error. Bandler and Bardakjian [9] and [10] also designed recursive digital filters based on the minimum L_p error criterion. In the limit as $p \to \infty$ the L_p approximation tends to be the L_∞ approximation, which is also known as the minimax or Chebyshev approximation. Charalambous [15] applied non-linear minimax optimization techniques to the problem of designing recursive digital filters in the minimax sense, to meet arbitrary magnitude specifications. The unconstrained optimization algorithm due to Fletcher [16] was used in conjunction with the minimax algorithm. Some other IIR filter design methods based on the minimax approach are given in [17] and [18].

2.2.3 Linear Programming design

Another 1-D frequency domain design technique is based on the application of the Linear Programming methods to approximate a prescribed magnitude-squared error characteristic. Rabiner *et al* [11] used Linear Programming technique to obtain an equiripple approximation to an IIR filter with a prescribed magnitude-squared characteristic. The magnitude squared function of the filter $H(z)$ given by,

$$H(z) = \frac{\sum_{i=0}^{m} b_i z^{-i}}{\sum_{i=0}^{n} a_i z^{-i}} \tag{2.6}$$

can be expressed as

$$H(z)H(z^{-1}) = \frac{(\sum_{i=0}^{m} b_i z^{-i})(\sum_{j=0}^{m} b_j z^{j})}{(\sum_{i=0}^{n} a_i z^{-i})(\sum_{j=0}^{n} a_j z^{j})} = \frac{\sum_{i=-m}^{m} c_i z^{-i}}{\sum_{i=-n}^{n} d_i z^{-i}} \tag{2.7}$$

where $c_i = c_{-i}$ and $d_i = d_{-i}$ The magnitude-squared function of the filter is therefore a ratio of trigonometric polynomials i.e.

$$|H(e^{j\omega})|^2 = \frac{\hat{N}(\omega)}{\hat{D}(\omega)} = \frac{c_0 + \sum_{i=1}^{m} 2c_i cos(\omega_i)}{d_0 + \sum_{i=1}^{n} 2d_i cos(\omega_i)} \tag{2.8}$$

Both $\hat{N}(\omega)$ and $\hat{D}(\omega)$ are linear in the coefficients c_i and d_i. Hence a Linear Programming technique can be used to determine c_i's and d_i's such that the peak approximation error is minimized.

If $F(\omega)$ is the desired magnitude-squared characteristic, then the approximation problem consists of finding the filter coefficients, such that

$$-\epsilon(\omega) \leq \frac{\hat{N}(\omega)}{\hat{D}(\omega)} - F(\omega) \leq \epsilon(\omega) \tag{2.9}$$

where $\epsilon(\omega)$ is a tolerance function on the error function. Since $F(\omega)$ and $\epsilon(\omega)$ are generally known function of the frequency, (2.9) can be expressed as a set of linear inequalities in the c_i's and d_i's by writing it in the form

$$\hat{N}(\omega) - \hat{D}(\omega)[F(\omega) + \epsilon(\omega)] \leq 0$$

$$-\hat{N}(\omega) + \hat{D}(\omega)[F(\omega) + \epsilon(\omega)] \leq 0$$

Since the magnitude squared function must always be positive the additional linear inequalities are

$$-\hat{N}(\omega) \leq 0$$

$$-\hat{D}(\omega) \leq 0$$

The above inequalities completely define the approximation problem. The filter coefficients are obtained as the output of a Linear Programming routine. Other papers using the Linear Programming techniques for IIR filter design are [12], [14]

2.3 Frequency domain design techniques: 2-D IIR filters

A 2-D IIR filter with real and stable impulse response $h(n_1, n_2)$ has the z-transform $H(z_1, z_2)$, of the following rational function form:

$$H(z_1, z_2) = \frac{\sum\sum_{k_1,k_2 \epsilon R_b} b(k_1, k_2) z_1^{-k_1} z_2^{-k_2}}{\sum\sum_{k_1,k_2 \epsilon R_a} a(k_1, k_2) z_1^{-k_1} z_2^{-k_2}} \tag{2.10}$$

where R_a is the *region of support* (non-zero area of the 2-D sequence) of $a(k_1, k_2)$ and R_b is the region of support of $b(k_1, k_2)$.

A 2-D IIR filter with an impulse response $h(n_1, n_2)$ is termed QP(quarter-plane) if $h(n_1, n_2)$ has support only in the region $\{n_1 \geq 0, n_2 \geq 0\}$ or its rotations $\{n_1 \geq 0, n_2 \leq 0\}$ etc. It is called a NSHP(nonsymmetric half-plane) filter if $h(n_1, n_2)$ is non-zero in the region $\{n_1 \geq 0, n_2 \geq 0\} \cup \{n_1 < 0, n_2 > 0\}$ or their rotations $\{n_1 \geq 0, n_2 \geq 0\} \cup \{n_1 > 0, n_2 < 0\}$ etc. The transfer function $H(\omega_1, \omega_2)$ of the 2-D recursive digital filter has the following form:

$$H(e^{j\omega_1}, e^{j\omega_2}) = \frac{\sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} b_{n_1 n_2} e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}}{1 + \sum_{n_1=1}^{N_1} \sum_{n_2=1}^{N_2} a_{n_1 n_2} e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}} \tag{2.11}$$

In the frequency domain 2-D IIR filter design the coefficient arrays $a(n_1, n_2)$ and $b(n_1, n_2)$ are to be determined such that the resulting frequency response $H(\omega_1, \omega_2)$

is as close as possible to some specified desired frequency response $D(\omega_1, \omega_2)$. The objective error function is expressed as a difference between the resulting (actual) frequency response and the desired frequency response. The mean-squared error is the same in both domains (Parseval's theorem), that is

$$\sum_{n_1} \sum_{n_2} [h(n_1, n_2) - d(n_1, n_2)]^2 = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |H(\omega_1, \omega_2) - D(\omega_1, \omega_2)|^2 \, d\omega_1 d\omega_2 \tag{2.12}$$

Apart from the mean squared error criteria other error criteria are also possible. The L∞ or Minimax error in the frequency domain is given by

$$E_\infty = max |H(\omega_1, \omega_2) - D(\omega_1, \omega_2)| \tag{2.13}$$

and the Lp error norm is given by

$$E_p = [\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |H(\omega_1, \omega_2) - D(\omega_1, \omega_2)|^p \, d\omega_1 d\omega_2)]^{1/p} \tag{2.14}$$

For large p values, say 20 or more, Lp norm becomes a good approximation to the L∞ norm.

2.3.1 Design based on transformation:

Some simple frequency-domain transformations can map both 1-D and 2-D IIR filters into other 2-D IIR filters. These transformations can be useful for designing lowpass, highpass, and bandpass filters as well as multiple passband filters. The objective of the frequency transformation is to map a stable, rational system function into another stable, rational system function, while preserving some of the characteristics of the prototype filter, such as the location and number of passbands. Designing 1-D IIR digital filters by frequency transformations of 1-D analog prototype filters or 1-D digital filters is quite simple. In 2-D, however, design by frequency transformation of 1-D or 2-D analog prototype filters to 1-D or 2-D digital prototype filters is not straightforward.

Pendergrass *et al.*[19], Chakrabarti and Mitra [20], Shenoi and Mishra [21] and Mastorakis and Nikos [22] designed two-dimensional digital filters based on spectral transformations. Some other designs based on transformation are considered in [23, 24].

2.3.2 General minimization procedures

The Standard descent algorithms can be applied to the frequency-domain error minimization problem. Usually the error is summed over a finite number of samples in the frequency domain rather than being integrated over the square $-\pi \leq \omega_1, \omega_2 < \pi$. For mean-squared error case. the problem becomes one of minimizing the approximation error functional

$$J_a = \sum_K W(\omega_{1K}, \omega_{2K}) \left[\frac{A(\omega_{1k}, \omega_{2k})}{B(\omega_{1k}, \omega_{2k})} - D(\omega_{1k}, \omega_{2k})\right]^2 \qquad (2.15)$$

where $W(\omega_{1k}, \omega_{2k})$ is a weighting function, $(\omega_{1k}, \omega_{2k})$ are the frequency samples selected for minimization. General optimization algorithms, such as Fletcher-Powell [38] and Levenberg-Marquardt [39], or linearization techniques can be applied to find the filter coefficients $a(n_1, n_2), b(n_1, n_2)$ that minimizes J_a.

Maria and Fahmy [40] had studied the design of 2-D IIR digital filters in the L_p norm sense by using the Fletcher-Powell algorithm as the optimization technique. They assumed a cascade form realization for the transfer function $H(\omega_1, \omega_2) = \frac{A(\omega_1, \omega_2)}{B(\omega_1, \omega_2)}$. The cascade formulation has several important advantages for practical applications. Firstly the frequency response of a 2-D IIR filter will be less sensitive to coefficient perturbations in a cascade structure than in direct form structure. Secondly, a stability check can be incorporated into the design procedure since checking the stability of lower-order subfilters in the cascade is relatively easy. Other design methods using the cascade implementation are considered in [31, 32, 33, 34, 35].

Aly and Fahmy [36] exploited a general class of symmetry [37] (that could possibly exist in 2-D frequency responses) to simplify the design and implementation complexities by reducing the number of approximation parameters.

Antoniou and Lu [25] , and Deng and Kawamata [25] designed 2-D digital filters by decomposing the desired 2-D frequency specifications into a set of 1-D ones and then applying the 1-D filter design techniques.

2.3.3 Zero-phase IIR frequency domain design methods

Often in applications such as image processing, one may wish to filter a signal with a filter whose impulse response is symmetric. Such filters will have a real-valued , or zero-phase, frequency response.

The frequency response of a zero-phase 2-D IIR filter can be expressed as:

$$H(\omega_1, \omega_2) = \frac{\sum_{n_1} \sum_{n_2} a'(n_1, n_2) cos(\omega_1 n_1 + \omega_2 n_2)}{\sum_{m_1} \sum_{m_2} b'(m_1, m_2) cos(\omega_1 m_1 + \omega_2 m_2)} \tag{2.16}$$

where $a'(n_1, n_2)$ and $b'(m_1, m_2)$ are related to actual 2-D filter coefficients $a(n_1, n_2)$ and $b(m_1, m_2)$ given in (2.11), by [27]

$$a'(0,0) = a(0,0)$$

$$a'(n_1, n_2) = 2a(n_1, n_2) \; for \quad (n_1, n_2) \neq (0,0)$$

$$b'(0,0) = b(0,0)$$

$$b'(m_1, m_2) = 2b(m_1, m_2) \; for \quad (m_1, m_2) \neq (0,0)$$

Using (2.16) a mean-squared error functional can be formulated and minimized by the techniques described earlier to obtain zero-phase filter coefficients $a'(n_1, n_2)$, $b'(n_1, n_2)$ and hence $a(n_1, n_2)$,$b(n_1, n_2)$. Alternatively an $L\infty$ error functional of the following form:

$$E = \left\| D(\omega_1, \omega_2) - \frac{A(\omega_1, \omega_2)}{B(\omega_1, \omega_2)} \right\| \tag{2.17}$$

$$= max \left| D(\omega_1, \omega_2) - \frac{A(\omega_1, \omega_2)}{B(\omega_1, \omega_2)} \right| \tag{2.18}$$

may be minimized. Dudgeon [27] used an iterative technique called differential correction [28, 29, 30] which can minimize E by solving a Linear Programming problem at each iteration. Here A, B, and D are real valued functions.

2.3.4 Separable denominator design

The frequency response of a 2-D separable denominator filter is given by

$$H(\omega_1, \omega_2) = \frac{A(\omega_1, \omega_2)}{B_1(\omega_1)B_2(\omega_2)} \tag{2.19}$$

Separable filters have a number of advantages, both in design and implementation, since they are an outer product of two 1-D filters. Because of their simplicity they offer a limited range of possible impulse and frequency responses. In [41, 42] a nonseparable numerator polynomial with a separable denominator polynomial was considered.

A separable numerator and denominator 2-D filter is an outer product of two 1-D filters. The class of filter specifications it can realize is more restrictive than that which can be realized by a non-separable filter. The separable denominator designs retain much of this flexibility of non-separable designs and yet offer the implementation advantage of separable IIR filters.

2.3.5 Stability of 2-D IIR digital filters:

A system is stable if its output is well behaved for all reasonable inputs. The most extensively studied stability criterion is the bounded-input bounded-output (BIBO) stability criterion. A system is stable in BIBO sense if every bounded input sequence produces bounded output sequence.

The earliest stability theorem, which is due to Shanks [43],[44] is based on examining the zero set of the denominator polynomial of 2-D IIR filter transfer function. Huang [45] proved theorems that is equivalent to Shanks theorem and simplified its stability conditions. The Huang's stability theorem is stated as follows:

Theorem: Let $H(z_1, z_2) = \frac{1}{B(z_1,z_2)}$ be a first quadrant recursive filter. This filter is stable if and only if $B(z_1, z_2)$ satisfies the following two conditions:

1. $B(z_1, z_2) \neq 0, for|z_1| \geq 1, \; |z_2| = 1$

2. $B(a, z_2) \neq 0, |z_2| \geq 1,$ for any a such that $|a| \geq 1$.

Decarlo et al [46] and Strintzis [47] independently showed that Huang's test could also be simplified. The condition that $|a| \geq 1$ is simplified to $|a| = 1$. Note that the second condition of Huang's theorem is a 1-D stability condition, the first condition is 2-D but z_2 is confined to the unit circle. Since it is much easier to check the 1-D stability condition, it must be done first. Since a filter can occasionally be found to be unstable, checking the necessary 1-D stability conditions may save computations needed for checking the 2-D conditions.

In order to test the stability of the 2-D IIR filters, numerical algorithms (tests) are required to check the conditions of the stability theorems. Most stability conditions are based on the Huang's theorem. Maria and Fahmy [48] tested the first condition of the Huang's theorem by considering

$$B(z_1, z_2) = \sum_{n=0}^{M} a_n(z_1) z_2^n$$

where

$$a_n(z_1) = \sum_{i=0}^{M} b(i, n) z_1^i$$

(and M is the order of filter) and then employing the Marden-Jury table. This table performs basically the same function for determining a complex polynomial's root distribution with respect to the unit circle as the Routh array does for the left-half plane. Anderson and Jury [49] tested the same condition by considering $B(z_1, z_2)$ as a one-dimensional polynomial with polynomial coefficients $a_n(z_1)$ and then employing the Schur-Cohn test. Bose [50] extended the Schussler's [51] stability theorem for one-dimensional polynomials with real coefficients, to implement a new stability test for two-dimensional filters.

Chapter 3

Design of 1-D IIR digital filter by IRLS Technique

3.1 Introduction

In this thesis the Iteratively Reweighted Least-Squares Technique (IRLS) developed in [1] for design of FIR digital filter, is applied for the design of IIR digital filter design problem in the L_p norm sense. In IRLS technique the L_p approximation problem is solved by solving a weighted least-squares problem in every single iteration, in which the weights are based on the previous iteration error. Hence it is necessary to formulate a weighted least-squares based IIR filter design problem. Lim *et al.* [2] had addressed this problem of designing IIR digital filters in weighted least-squares sense. In this technique the filter coefficients are obtained simply by solving a set of normal equations. This method has been applied to solve the weighted least squares problem occurring in every single iteration of the IRLS technique. The design of IIR digital filter in the L_p norm sense is a nonlinear problem with respect to the filter coefficients. Nonlinear optimization techniques such as the Fletcher-Powell technique had been applied in the past [7] for the minimization of the L_p error function. Such techniques are computationally intensive. Since the coefficients of the IIR digital filters are obtained by solving simple linear equations, in the IRLS technique there is no need for any nonlinear solution techniques.

The following section covers the IRLS design of IIR digital filters.

3.2 IRLS design of 1-D IIR digital filter

In this section, the Iteratively Reweighted Least Squares design method developed in [1] for FIR filter design is extended for designing IIR digital filter. Let $\frac{P(z)}{1+Q(z)}$ be the z-transfer function of an IIR filter. i.e

$$H(z) = \frac{P(z)}{1+Q(z)} = \frac{\sum_{n=0}^{N} b_n z^{-n}}{1+\sum_{n=1}^{N} a_n z^{-n}} \tag{3.1}$$

Let $D(\omega)$,which, in general, is complex, be the desired frequency response of the IIR filter and let $H(e^{j\omega})$ be the actual frequency response. It is required to find filter coefficients b_n, n=0,1,..,N and a_n. n=1,2,..,N, where N is the order of the filter, such that the L_p error norm

$$E_p = [\sum_{k=1}^{r} |H(e^{j\omega_k}) - D(\omega_k)|^p]^{\frac{1}{p}},\ p \text{ is an even integer.} \tag{3.2}$$

is minimized. Here ω_k is the kth frequency sample, where $k = 1, 2, .., r$. These frequency samples cover the frequency bands of interest (that is, passbands and stopbands). There are no analytical methods which can be used to solve this approximation problem. Therefore an iterative method such as the IRLS method, which by appropriately choosing the weights allows us to find solution to the L_p-approximation problem as the limit to a sucessive approximation of a weighted least-squares problem.

By setting the weights to be

$$w_k = |H(e^{j\omega_k}) - D(\omega_k)|^{p-2} \tag{3.3}$$

the problem of minimizing the L_p norm is converted to that of solving a weighted least-squares problem as follows:

$$E_p^p = \sum_{k=1}^{r} w_k |H(e^{j\omega_k}) - D(\omega_k)|^2 \tag{3.4}$$

By choosing the weights as given by (3.3) and minimizing 3.4, we would minimize the L_p error norm given by 3.2. This cannot be done in one step because we need to find the weights. Hence an iterative algorithm in which the weights are updated from the error in previous iteration, is used to solve the L_p-approximation problem. By solving the above weighted least-squares problem in each iteration of the IRLS algorithm we obtain the required filter coefficient parameters $\{b_n, a_n\}$ for the 1-D IIR digital filter. The weighted least-squares design of IIR digital filter, which is to be carried out in every iteration of the IRLS algorithm is described in the next section.

The L_p error function given by 3.2 is a nonlinear function of the filter coefficients. Hence, the solution obtained cannot be guarenteed to be a global optimum. It corresponds to a local optimum which could be global one.

3.3 Solution of the weighted least-squares problem for 1-D IIR filters

The solution of weighted least-squares problem was carried out in a similar manner as discussed in [2], for the design of IIR digital filters in the weighted least-squares sense.

Let the difference between $D(\omega)$ and $H(e^{j\omega}) = \frac{P(e^{j\omega})}{1+Q(e^{j\omega})}$ be $\zeta(e^{j\omega})$, i.e

$$\zeta(e^{j\omega}) = \frac{P(e^{j\omega})}{1+Q(e^{j\omega})} - D(\omega) \tag{3.5}$$

Multiplying (3.5) by $[1 + Q(e^{j\omega})]$ we get

$$P(e^{j\omega}) - Q(e^{j\omega})D(\omega) = D(\omega) + [1 + Q(e^{j\omega})]\zeta(e^{j\omega}) \tag{3.6}$$

Let

$$V(e^{j\omega}) = [1 + Q(e^{j\omega})]\zeta(e^{j\omega})$$

Equation (3.6) can be written as

$$P(e^{j\omega}) - Q(e^{j\omega})D(\omega) = D(\omega) + V(e^{j\omega}) \tag{3.7}$$

By putting $\omega = \omega_k, k = 1, 2, .., r$ in (3.7), we obtain the following system of equations,

$$P(e^{j\omega_k}) - Q(e^{j\omega_k})D(\omega_k) = D(\omega_k) + V(e^{j\omega_k}), k = 1, 2, .., r. \tag{3.8}$$

Defining

$$E = [V(e^{j\omega_1}), V(e^{j\omega_2}), .., V(e^{j\omega_k}), .., V(e^{j\omega_r})]^T \tag{3.9}$$

$$
\mathbf{U} = \begin{bmatrix}
1, e^{-j\omega_1}, e^{j2\omega_1}, ..., e^{-jN\omega_1}, -e^{-j\omega_1}D_1, -e^{j2\omega_1}D_1, ..., -e^{-jN\omega_1}D_1 \\
1, e^{-j\omega_2}, e^{j2\omega_2}, ..., e^{-jN\omega_2}, -e^{-j\omega_2}D_2, -e^{j2\omega_2}D_2, ..., -e^{-jN\omega_2}D_2 \\
.. \quad\quad .. \quad\quad .. \\
1, e^{-j\omega_r}, e^{j2\omega_r}, ..., e^{-jN\omega_r}, -e^{-j\omega_r}D_r, -e^{j2\omega_r}D_r, ..., -e^{-jN\omega_r}D_r \\
.. \quad\quad .. \quad\quad .. \\
1, e^{-j\omega_k}, e^{j2\omega_k}, ..., e^{-jN\omega_k}, -e^{-j\omega_k}D_k, -e^{j2\omega_k}D_k, ..., -e^{-jN\omega_k}D_k
\end{bmatrix} \tag{3.10}
$$

where $D_k = D(\omega_k)$.

$$
\mathbf{a} \;=\; [b_0, b_1, b_2, ..., b_N, a_1, a_2, ..., a_N]^T \tag{3.11}
$$

$$
H \;=\; [D_1, D_2, .., D_k, ..., D_r]^T \tag{3.12}
$$

The following vector equations can be formed:

$$
E = \mathbf{U}\mathbf{a} - H \tag{3.13}
$$

The weighted error can be expressed in the matrix notation as :

$$
e \;=\; \sum_{k=1}^{k=r} w_k^2 |V(e^{j\omega_k})|^2 \;=\; [WE]^T W\overline{E} \tag{3.14}
$$

where W is a diagonal weighting matrix with elements w_k and E is given by (3.13). Next expressing E in terms of the real and imaginary parts of the matrix $\mathbf{U}$ and H, we can rewrite (3.14) as

$$e = [W(\mathbf{U}\mathbf{a} - H)]^T[W(\overline{\mathbf{U}}\mathbf{a} - \overline{H})] \tag{3.15}$$

$$= [\mathbf{a}^T(\mathbf{U}r^TW^T + j\mathbf{U}i^TW^T) - (Hr^TW^T + jHi^TW^T)]$$

$$\cdot[(W\mathbf{U}r - jW\mathbf{U}i)\mathbf{a} - (WHr - jWHi)] \tag{3.16}$$

where the matrices $\mathbf{U}r$ and $\mathbf{U}i$ represent the real and imaginary parts of the matrix $\mathbf{U}$, and Hr and Hi represent the real and imaginary parts of the matrix H, respectively. Simplifying e, differentiating with respect to the vector $\mathbf{a}$ and equating it to zero, we obtain

$$\frac{\partial e}{\partial \mathbf{a}} = 2\mathbf{U}r^TW^TW\mathbf{U}r\mathbf{a} - 2\mathbf{U}r^TW^TWHr - 2\mathbf{U}i^TW^TW\mathbf{U}i\mathbf{a} - 2\mathbf{U}i^TW^TWHi = 0 \tag{3.17}$$

which gives us,

$$[\mathbf{U}r^TW^TW\mathbf{U}r + \mathbf{U}i^TW^TW\mathbf{U}i]\mathbf{a} = \mathbf{U}r^TW^TWHr + \mathbf{U}i^TW^TWHi \tag{3.18}$$

Thus minimizing $\sum_{k=1}^{k=r} w_k^2|V(e^{j\omega_k})|^2$ leads to the solution

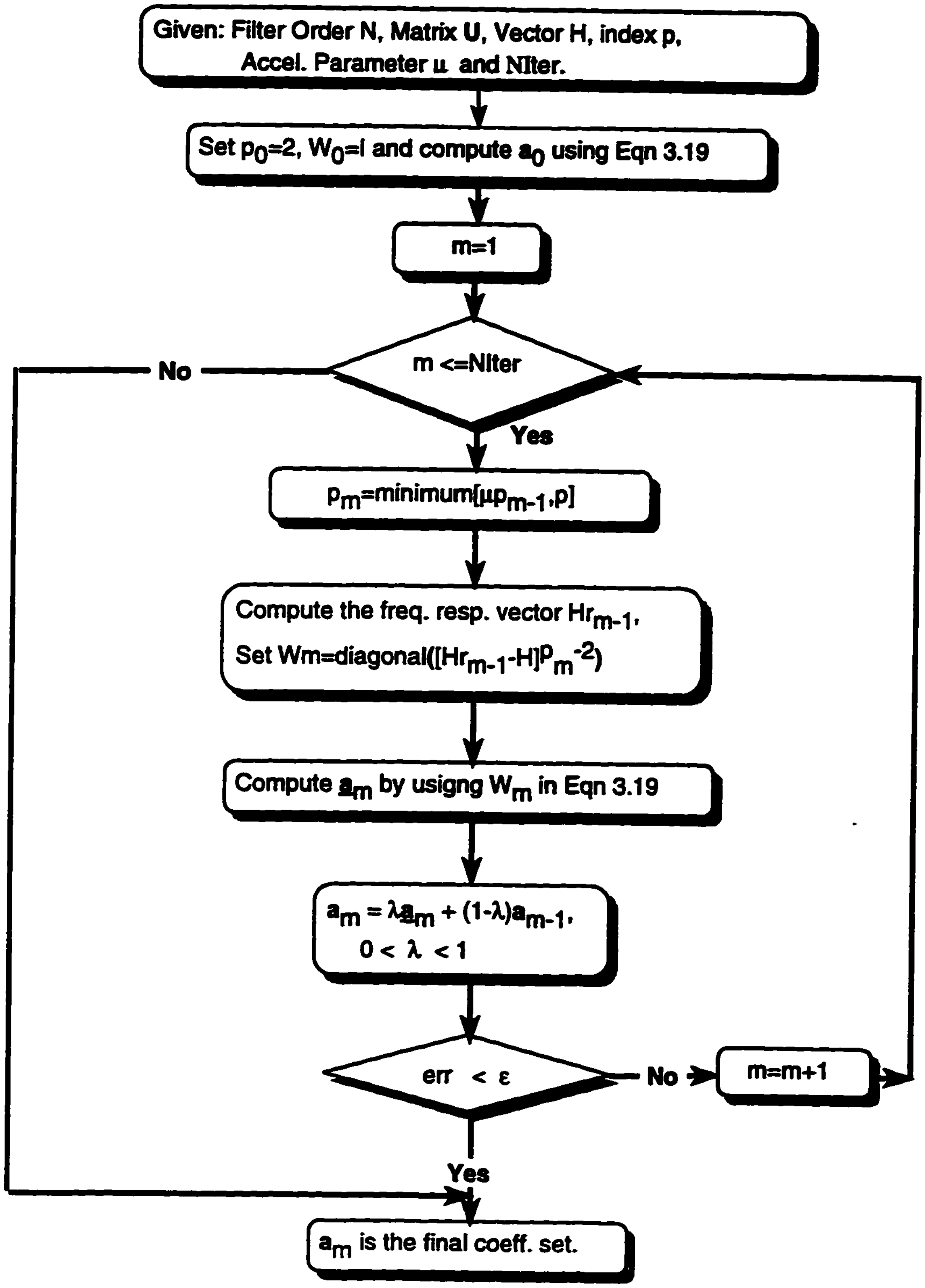
$$\mathbf{a} = [Re(\mathbf{U}^T)\mathbf{R}Re(\mathbf{U}) + Im(\mathbf{U}^T)\mathbf{R}Im(\mathbf{U})]^{-1}$$

$$\cdot\left[\, Re(\mathbf{U}^T)\mathbf{R}Re(H) \;+\; Im(\mathbf{U}^T)\mathbf{R}Im(H)\right] \tag{3.19}$$

where $\mathbf{R}=W^TW$ is a diagonal weighting matrix whose nth diagonal element is w_n^2, and $Re(.)$, $Im(.)$ means the real and imaginary part of (.) respectively.

The coefficient vector a is obtained as the solution to a weighted least-squares problem. This weighted least-squares solution has to be carried out in each iteration of the IRLS algorithm as will be shown next.

3.4 IRLS Algorithm



Algorithm description

1. Initially the error vector is a null vector since there is no error due to the previous itaration. Hence W_0 is set equal to the identity matrix for the first iteration.

2. Rather than starting the iterations of the IRLS algorithm with the desired value of p, the p value in the first iteration is set to be $p_0 = 2$ and increased every iteration by a factor of μ, $\mu > 1$, as $p_m = \mu p_{m-1}$, until the value of p_m reaches the desired p value. By selecting a μ value greater than one the algorithm starts with an L_2 design and accelerates towards an L_p design. One should be careful not to use too high a value for μ, otherwise convergence problems would occur. In the examples used in this work, values of 1.1 and 1.2 were used.

3. The temporary filter coefficient set $\underline{\mathbf{a}}_m$ obtained by solving (3.19), and the past filter coefficient set $\mathbf{a}_{m-1}$ are used to compute the new filter coefficient set $\mathbf{a}_m$ by the relation $\mathbf{a}_m = \lambda\underline{\mathbf{a}}_m + (1-\lambda)\mathbf{a}_{m-1}$. For the design of 1-D FIR digital filter Burrus and Barreto [1] used $\lambda = \frac{1}{p_m - 1}$ in the IRLS algorithm and good convergence was obtained.

3.5 Results for 1-D IIR digital filter using $\lambda = \frac{1}{p_m - 1}$ in the IRLS algorithm

In this section several lowpass filters with different passband and stopband frequencies were designed using $\lambda = \frac{1}{p_m - 1}$ in the IRLS algorithm. This is to demonstrate that this choice of λ results in poor convergence for the IIR filter design problem.

Example 1. In this example we design an eighth order lowpass filter to approximate in the L_p sense, the ideal specifications given by

passband edge frequency $\omega_p = 0.48\ \pi$

stopband edge frequency $\omega_s = 0.51\ \pi$.

The parameters of IRLS algorithm chosen were p=10, $\mu = 1.2$, NIter=30.

Fig. 3.1 shows the frequency response of the designed filter and Fig. 3.2 illustrates the convergence of the L_p error.

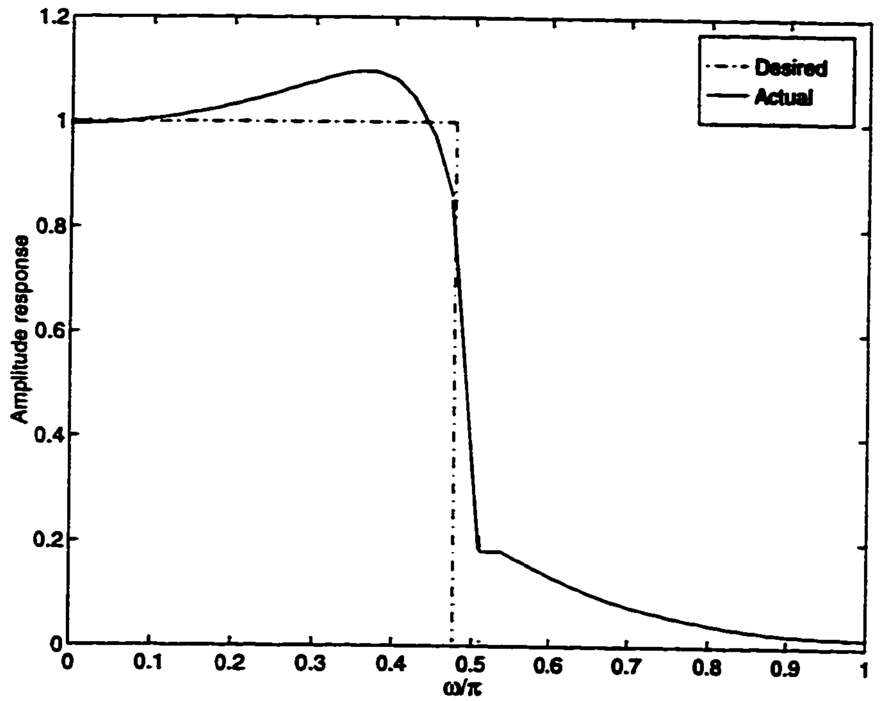


Figure 3.1: Frequency response for the filter in Example 1, corresponding to a minimum L_p error value of 0.2052, which occurs at iteration number five of Fig. 3.2.

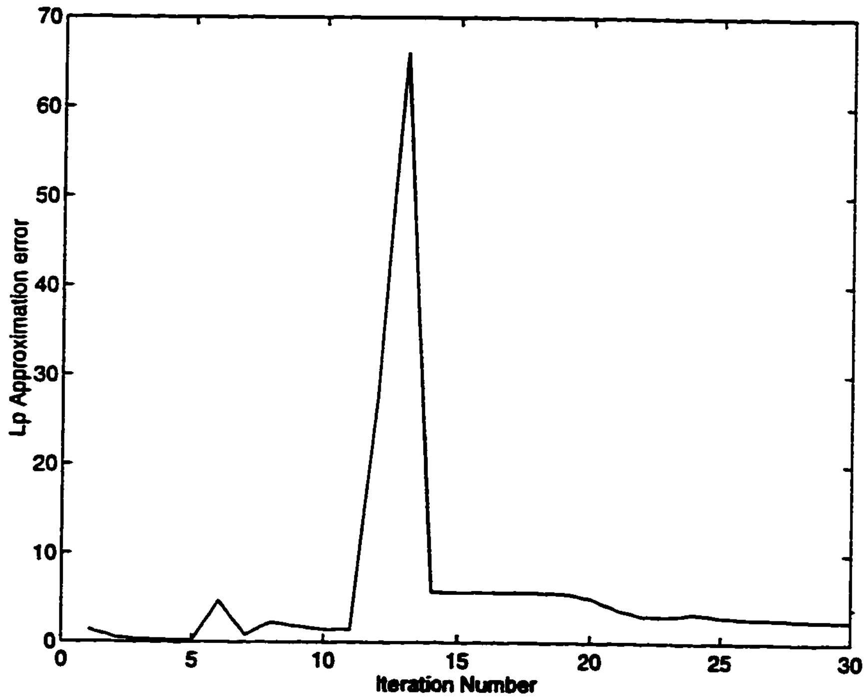


Figure 3.2: Lp Approximation error for the filter in Example 1.

Example 2. In this example we design a tenth order lowpass filter to approximate in the L_p sense, the ideal specifications given by

passband edge frequency $\omega_p = 0.280\ \pi$

stopband edge frequency $\omega_s = 0.301\ \pi$.

The parameters of IRLS algorithm chosen were p=10 , $\mu = 1.2$, NIter=55.

Fig. 3.3 shows frequency response of the designed filter and Fig. 3.4 illustrates the convergence of the L_p error. By looking at the plot of L_p error function it was

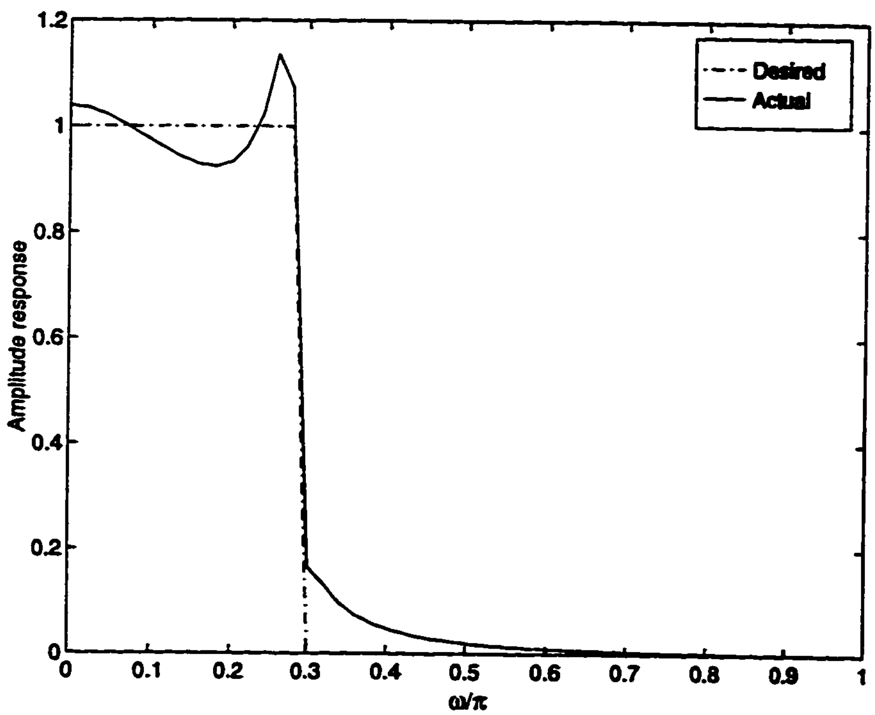


Figure 3.3: Frequency response for the filter in Example 2, corresponding to a minimum L_p error value of 0.1775, which occurs at iteration number seven of Fig. 3.4.

observed that although the error was minimized in subsequent iterations it was not monotonically decreasing. Fig. 3.2 and Fig 3.4 show a big jump in the L_p error at

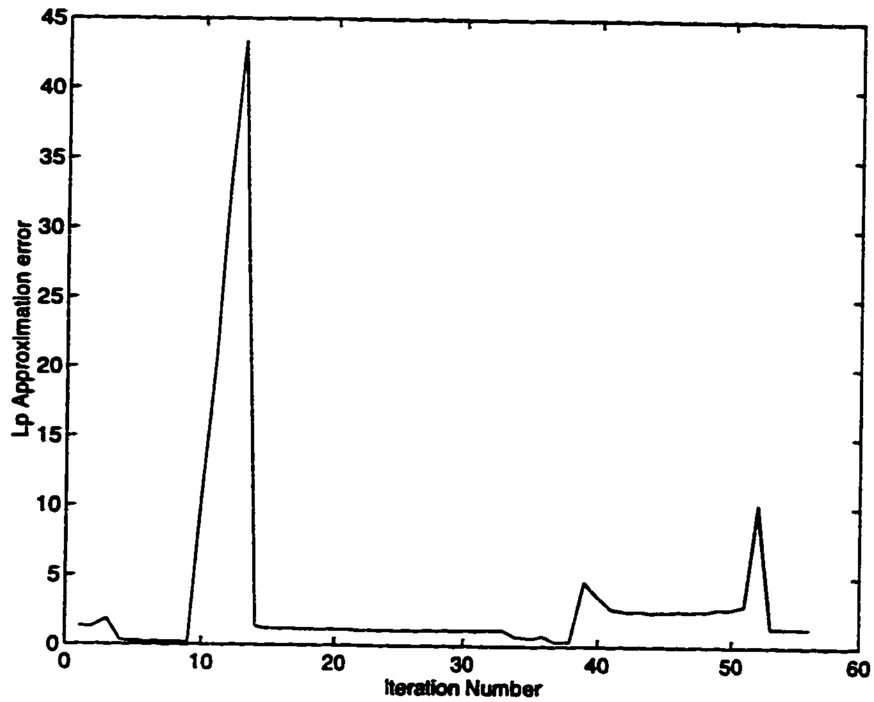


Figure 3.4: Lp Approximation error for the filter in Example 2.

the 13*th* and 12*th* respectively. The reason for this behavior is that the choice of $\lambda = \frac{1}{p_m - 1}$, which was used for FIR filter design, is not appropriate for the design of IIR filters. Since this choice of λ results in a very poor convergence for the IIR filter design problem in the L_p norm sense, in order to improve the convergence an appropriate choice of λ must be considered.

3.6 Determination of the optimal Convergence Parameter λ analytically

It has been shown above that the value of the parameter λ has to be determined in manner which will be appropriate for the IIR filter design problem. For a given previous and temporary filter coefficient set, an optimal λ value would be the one that will give the minimum L_p error. Hence in order to determine the optimal value of λ analytically, the L_p error criterion has to be differentiated with respect to λ and made equal to zero. The pth powered L_p error norm is given by

$$E_p^p = \sum_{k=1}^{r}[H(e^{j\omega_k}) - D(\omega_k)]^p \tag{3.20}$$

where $H(e^{j\omega_k})$ expressed in terms of the IIR filter coefficients $\{b_n, n = 0, 1, .., N,$ $a_n, n = 1, .., N\}$, is given by

$$H(e^{j\omega_k}) = \frac{b_0 + b_1 e^{-j\omega_k} + b_2 e^{-2j\omega_k} + .. + b_N e^{-Nj\omega_k}}{1 + a_1 e^{-j\omega_k} + a_2 e^{-2j\omega_k} + .. + a_N e^{-Nj\omega_k}} \tag{3.21}$$

Let us denote the $(m-1)$th filter coefficient as

$$\mathbf{a}_{m-1}^T = [b_0, b_1, b_2, ..., b_N, a_1, a_2, ..., a_N] \tag{3.22}$$

and the difference between mth temporary filter coefficient set $\underline{\mathbf{a}}_m$ and $(m-1)$th coefficient given by 3.22, as

$$(\underline{\mathbf{a}}_m - \mathbf{a}_{m-1})^T = [db_0, db_1, db_2, ..., db_N, da_1, da_2, ..., da_N] \tag{3.23}$$

where

$$db_i = \mathbf{a}_m(i) - \mathbf{a}_{m-1}(i)$$

. Now the expression for E_p^p, for the case N=2, in terms of λ is given by

$$E_p^p = \sum_{k=1}^{r}[\frac{(b_0+\lambda db_0)+(b_1+\lambda db_1)e^{-j\omega_k}+(b_2+\lambda db_2)e^{-2j\omega_k}}{1+(a_1+\lambda da_1)e^{-j\omega_k}+(a_2+\lambda da_2)e^{-2j\omega_k}} - D(\omega_k)]^p \tag{3.24}$$

Differentiating (3.24) with respect to λ and equating the resulting expression to zero yields the the following expression:

$$\sum_{k=1}^{r} p(\frac{db_0+db_1e^{-j\omega_k}+db_2e^{-2j\omega_k}}{1+(a_1+da_1\lambda)e^{-j\omega_k}+(a_2+da_2\lambda)e^{-2j\omega_k}}$$

$$-\frac{(da_1e^{-j\omega_k}+da_2e^{-2j\omega_k})(b_0+db_0\lambda+(b_1+db_1\lambda)e^{-j\omega_k}+(b_2+db_2\lambda)e^{-2j\omega_k})^2}{(1+(a_1+da_1\lambda)e^{-j\omega_k}+(a_2+da_2\lambda)e^{-2j\omega_k})^2}$$

$$(\frac{b_0+db_0\lambda+(b_1+db_1\lambda)e^{-j\omega_k}+(b_2+db_2\lambda)e^{-2j\omega_k}}{1+(a_1+da_1\lambda)e^{-j\omega_k}+(a_2+da_2\lambda)e^{-2j\omega_k}} - D(\omega_k))^{(p-1)} = 0 \tag{3.25}$$

It can be seen that this is a highly nonlinear expression with respect to λ and it is very difficult to obtain an analytical solution for the optimal value of λ. This means that we have to resort to numerical solution techniques to solve for λ using (3.25). Such techniques are iterative in nature and computationally expensive to be incorporated in every single iteration of the IRLS algorithm. Since this optimal search for λ would affect the computational complexity of the IRLS algorithm we resort to an alternative approach of λ selection.

The performance of the original IRLS algorithm is enhanced when the value of the parameter λ is chosen as follows. The value of λ is varied from 0 to 1 in steps of 0.1 and the λ value which results in the minimum L_p error is selected as the current value of λ. Let λ_M denote the λ value which gives the minimum L_p error value as described above. Using this method of selecting λ the previous two designs were carried out and resulted in rapid convergence of the L_p error function, as demonstrated in the next section.

3.7 Results for 1-D IIR digital filter using $\lambda = \lambda_M$ in the IRLS algorithm

It has been shown that the choice of $\lambda = \frac{1}{p_m - 1}$ resulted in poor convergence and hence a more appropriate choice of λ for the IIR filter design problem must be considered. In this section the previous two examples of filter design are solved using $\lambda = \lambda_M$ as defined earlier. Fig. 3.5 and Fig. 3.6 show the frequency response of the designed filters. Equation (3.26) and (3.27) give the transfer functions of the two filter designs of Examples 1 and 2, respectively in the cascade form of second-order sections.

$$
\begin{aligned}
H(Z) &= 0.0535\frac{(1+2.2938z^{-1}+1.6860z^{-2})(1+0.9310z^{-1}+1.5300z^{-2})}{(1-0.1208z^{-1}+0.1905z^{-2})(1-0.0399z^{-1}+0.7557z^{-2})} \\
&\quad \times \frac{(1+0.2892z^{-1}+1.1341z^{-2})(1+0.0809z^{-1}+1.0037z^{-2})}{(1-0.0429z^{-1}+0.9407z^{-2})(1+0.0757z^{-1}+0.2753z^{-2})} \qquad (3.26)
\end{aligned}
$$

$$
\begin{aligned}
H(Z) &= 0.0042\frac{(1-2.0617z^{-1}+3.8428z^{-2})(1-0.0587z^{-1}+2.5888z^{-2})}{(1-1.0220z^{-1}+0.4652z^{-2})(1-1.1280z^{-1}+0.8277z^{-2})} \\
&\quad \times \frac{(1-1.0456z^{-1}+1.0161z^{-2})(1-0.2500z^{-1}+0.4567z^{-2})}{(1-1.0580z^{-1}+0.6275z^{-2})(1-0.9145z^{-1}+0.2070z^{-2})} \\
&\quad \times \frac{(1-1.1769z^{-1}+1.0060z^{-2})}{(1-1.2175z^{-1}+0.9787z^{-2})} \qquad (3.27)
\end{aligned}
$$

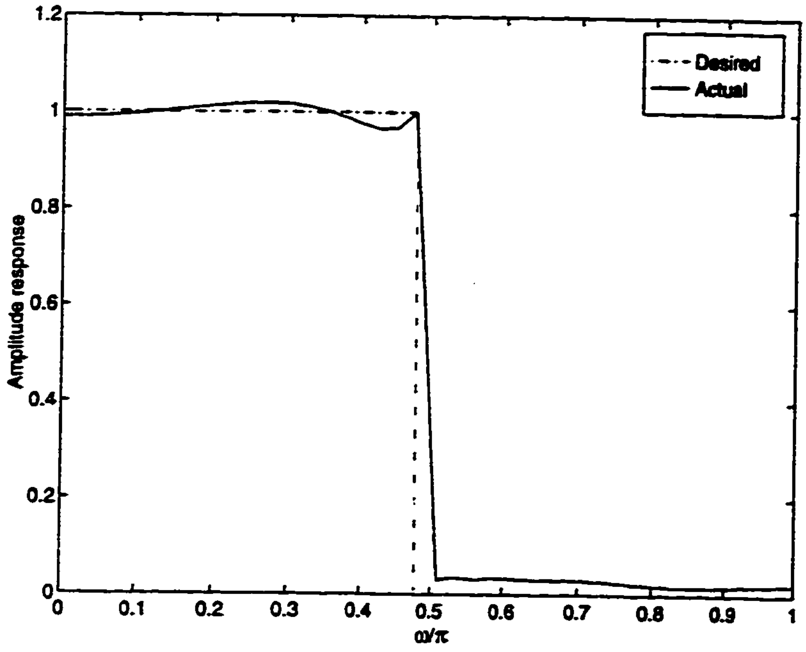


Figure 3.5: Frequency response for the filter in Example 1 by using $\lambda = \lambda_M$.

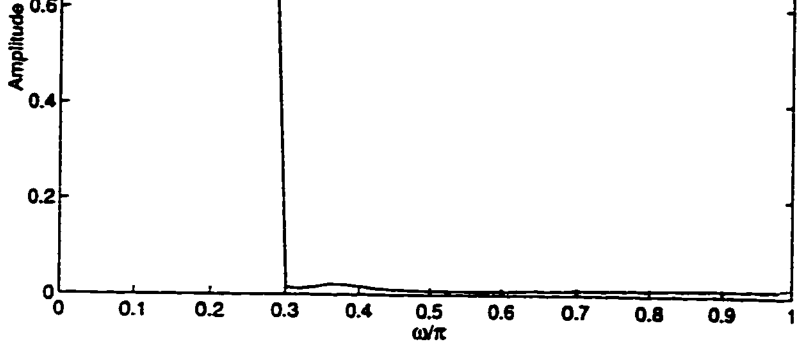


Figure 3.6: Frequency response for the filter in Example 2 by using $\lambda = \lambda_M$.

Fig. 3.7 shows a comparison of the Lp error by using $\lambda = \frac{1}{(p_m - 1)}$ as given in [1] for FIR filters and $\lambda = \lambda_M$, for example 1. For the case of $\lambda = \lambda_M$, the L_p error and peak deviations were 0.0466 and 0.0334 respectively. Whereas for the case of $\lambda = \frac{1}{(p_m - 1)}$, the L_p error and peak deviations were 0.2052 and 0.1505 respectively.

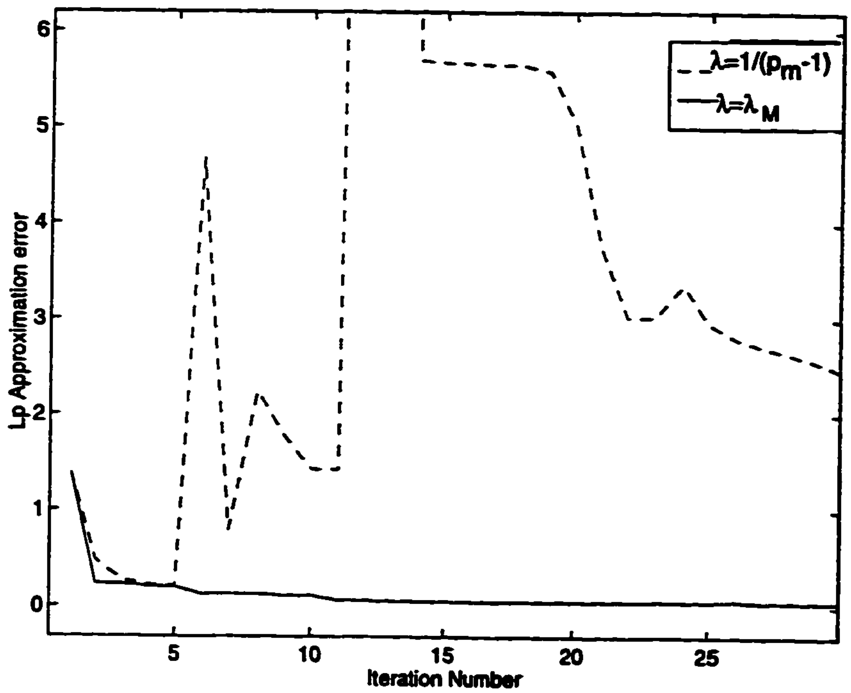


Figure 3.7: Comparison of Lp error by using $\lambda = \frac{1}{p_m - 1}$ and $\lambda = \lambda_M$ in the IRLS algorithm, for Example 1.

Fig. 3.8 shows a comparison of the Lp error for the two cases, for example 2. For the case of $\lambda = \lambda_M$, the L_p error and peak deviations were 0.0272 and 0.0236 respectively. Whereas for the case of $\lambda = \frac{1}{(p_m-1)}$, the L_p error and peak deviations were 0.1775 and 0.0758 respectively.

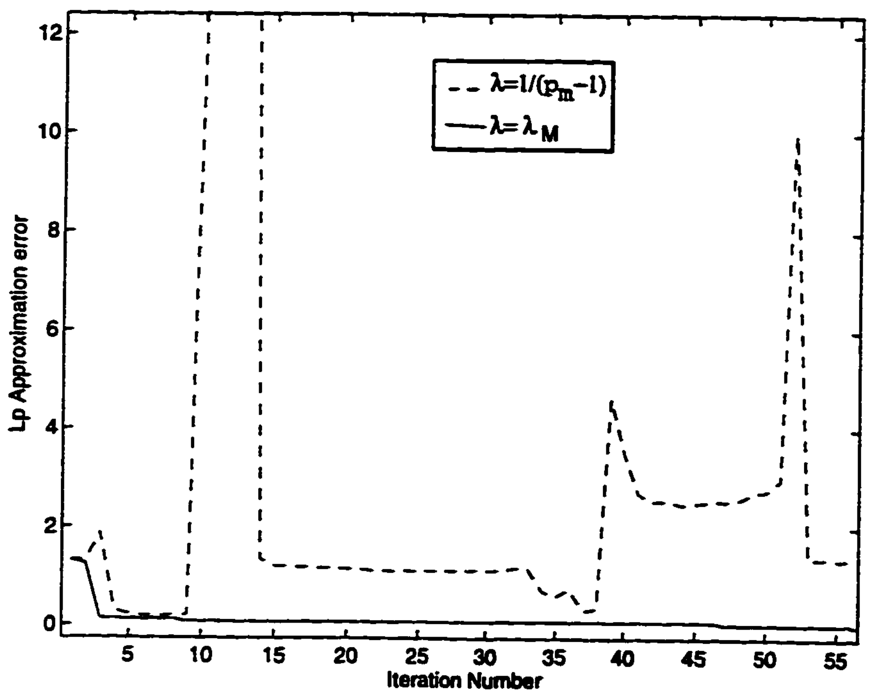


Figure 3.8: Comparison of Lp error by using $\lambda = \frac{1}{p_m-1}$ and $\lambda = \lambda_M$ in the IRLS algorithm, for Example 2.

From the results obtained it is seen that the choice of $\lambda = \lambda_M$ in the IRLS algorithm results in rapid convergence of the L_p error function and the resulting filters have smaller peak deviations.

Example 3. In this example an eighth order lowpass filter is designed to approximate both magnitude and phase specifications, with passband edge frequency $\omega_p = 0.40\ \pi$, stopband edge frequency $\omega_s = 0.46\ \pi$ and group delay= 10 samples. The parameters of IRLS algorithm chosen were p=16 , $\mu = 1.1$, NIter=50. Fig. 3.9 shows the frequency response of the designed filter and Fig. 3.10 shows the resulting passband group delay.

The transfer function of the designed filter in the cascade form of second order sections is given by (3.28).

$$\begin{aligned} H(Z) &= 0.0036\frac{(1-2.6984z^{-1}+3.0634z^{-2})(1+0.9181z^{-1}+1.6486z^{-2})}{(1-0.5049z^{-1}+0.9942z^{-2})(1-0.7535z^{-1}+0.7008z^{-2})} \\ &\times \frac{(1-0.2453z^{-1}+1.0800z^{-2})(1-4.8692z^{-1}+5.7078z^{-2})}{(1-1.1801z^{-1}+0.5983z^{-2})(1-1.4632z^{-1}+0.5668z^{-2})} \end{aligned} \tag{3.28}$$

The resulting L_p error was 0.1563.

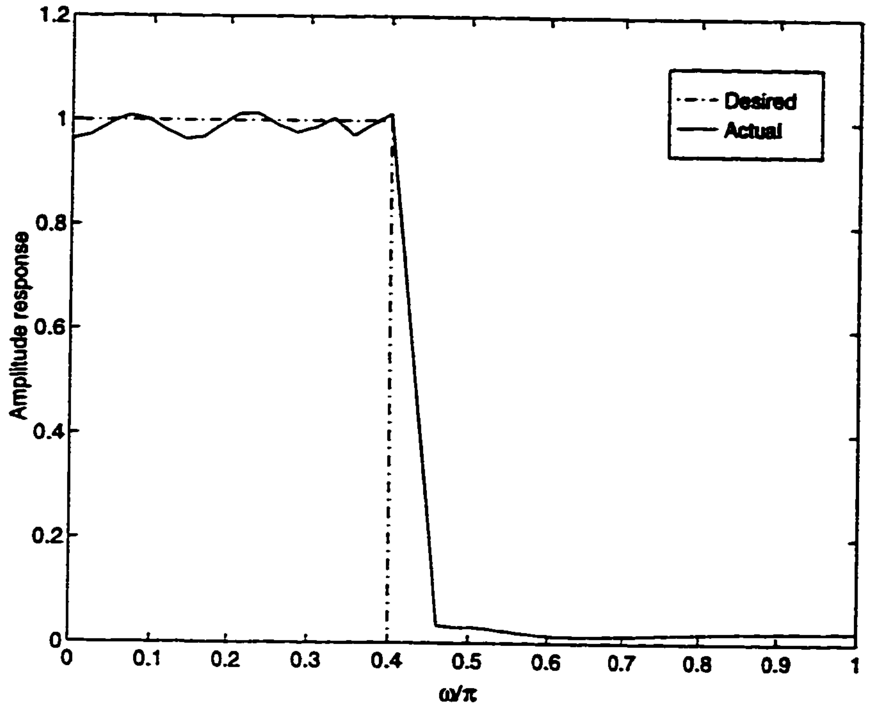


Figure 3.9: Frequency response for the filter in Example 3.

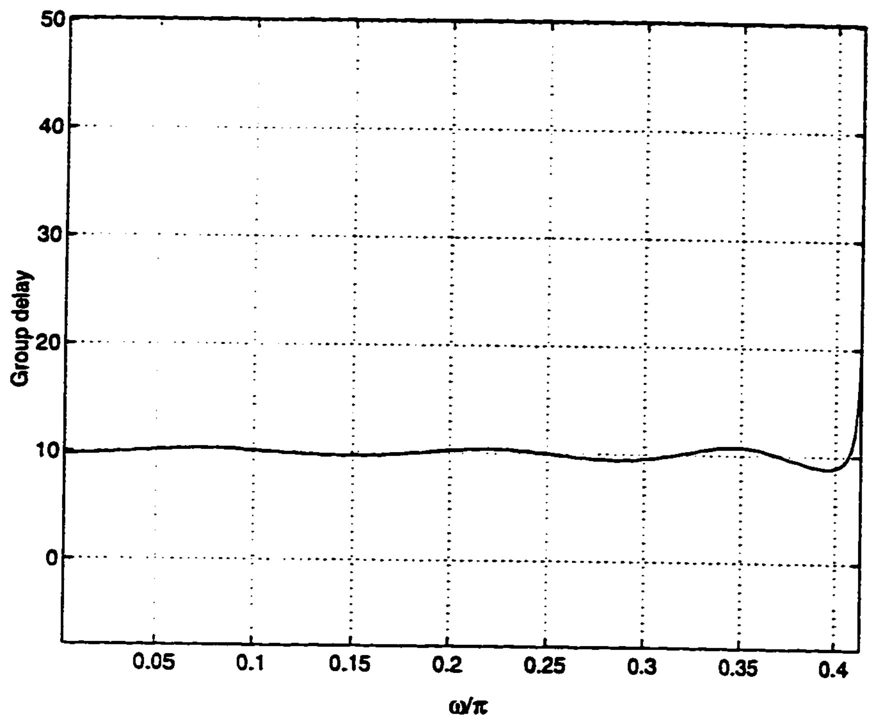


Figure 3.10: Passband group delay for the filter in Example 3.

3.8 Performance comparison of IRLS technique with Davidon Fletcher Powell (DFP) unconstrained optimization technique.

In this section a comparison between the designs by IRLS technique and the DFP unconstrained optimization technique is presented.

Lowpass filter design: Using the IRLS and DFP techniques five lowpass filters of order five, with following specifications: same passband edge $=0.3\pi$ radians stopband edge varying from 0.31π radians to 0.39π radians in steps of 0.02π radians p=4. The DFP optimization technique was implemented using the fminu routine available in the MATLAB's Optimization toolbox. Fig. 3.11-3.15 shows the comparison of log-magnitude response for the two techniques. Table 3.1 shows a comparison of the L_p error, and the number of Flops (the floating point operations) needed to achieve the value of L_p error, for the designed filters using the two techniques. These Flops are computed by using the 'Flops' function available in MATLAB.

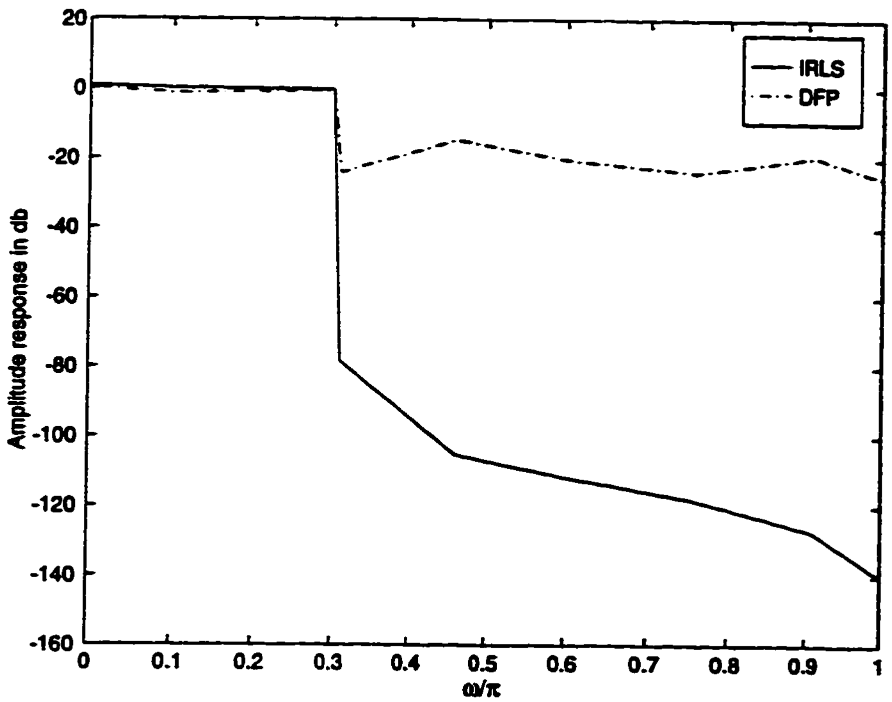


Figure 3.11: Comparison of log-magnitude response for the IRLS and DFP techniques, $\omega_s = 0.31\pi$ radians .

ω_p (radians)	ω_s (radians)	IRLS		DFP	
		L_p error	Flops	L_p error	Flops
0.30 π	0.31 π	0.1101	995,130	0.9058	2,675,848
0.30 π	0.33 π	0.0797	994,769	0.9680	1,889,665
0.30 π	0.35 π	0.0377	994,840	1.3349	2,456,706
0.30 π	0.37 π	0.0293	995,066	0.4329	2,690,601
0.30 π	0.39 π	0.0183	997,716	0.3644	2,492,747

Table 3.1: Comparison of L_p error for IRLS and DFP techniques for p=4, N=5.

It can be observed from the obtained results that the IRLS technique performs much better in terms of the L_p Approximation error as well as in terms of stopband attenuation, as compared to the DFP optimization techique. It is also seen that the

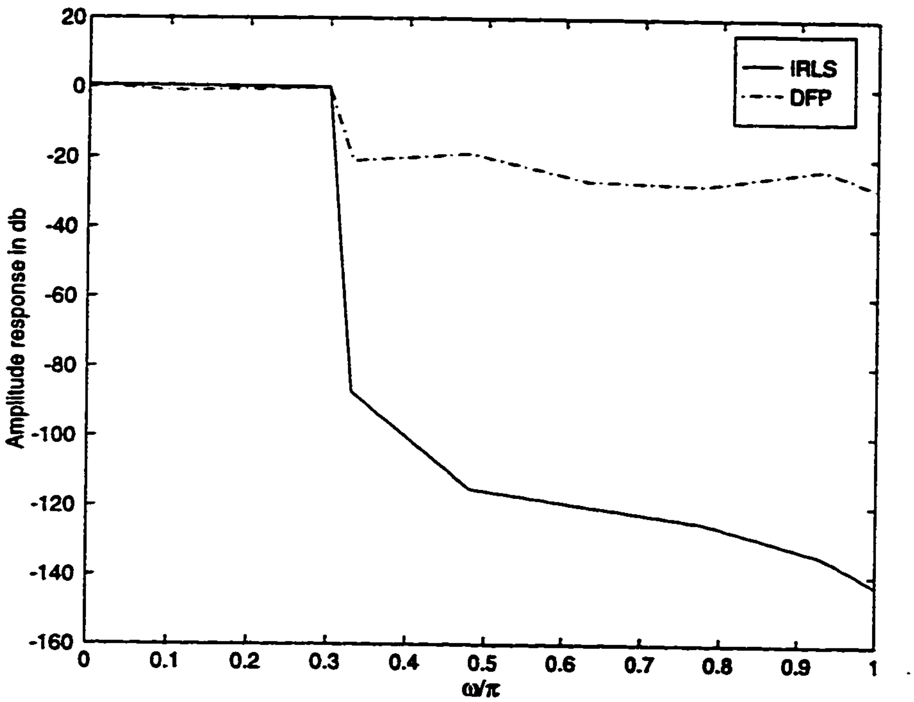


Figure 3.12: Comparison of log-magnitude response for the IRLS and DFP techniques, $\omega_s = 0.33\pi$ radians .

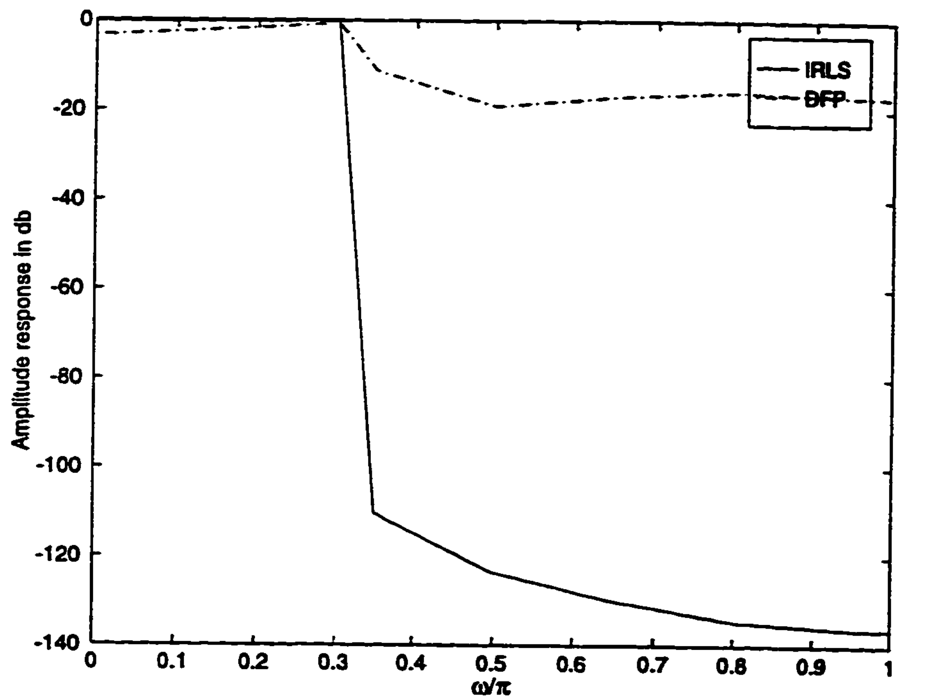


Figure 3.13: Comparison of log-magnitude response for the IRLS and DFP techniques, $\omega_s = 0.35\pi$ radians .

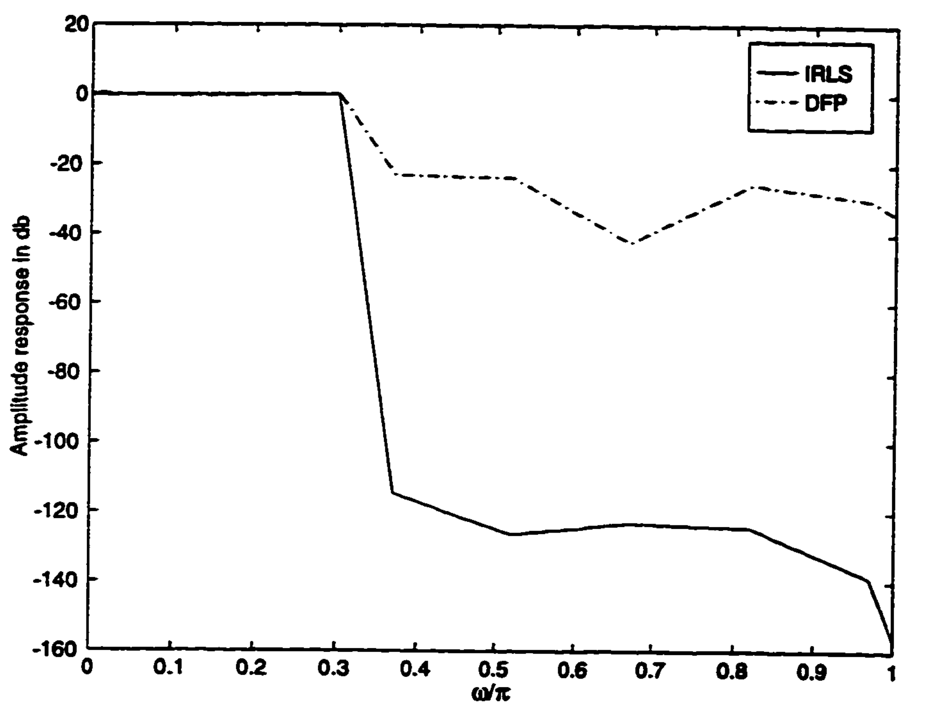


Figure 3.14: Comparison of log-magnitude response for the IRLS and DFP techniques, $\omega_s = 0.37\pi$ radians.

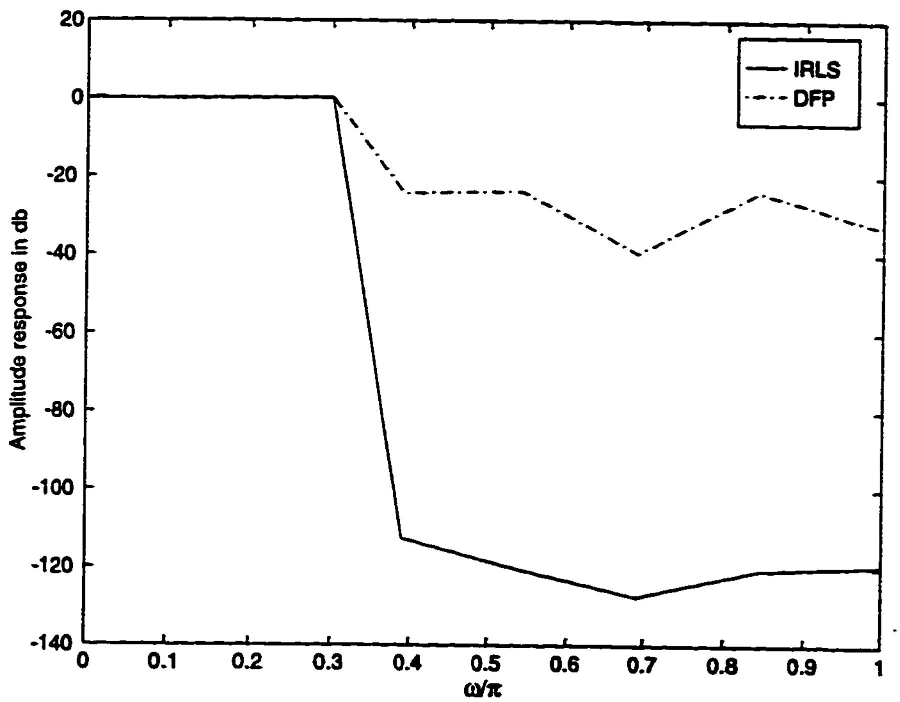


Figure 3.15: Comparison of log-magnitude response for the IRLS and DFP techniques, $\omega_s = 0.39\pi$ radians .

IRLS technique needs much smaller number of computations as compared to the DFP optimization technique. The details of the computational complexity for the two techniques is discussed in the section 3.8.1.

Table 3.2 and 3.3 give the transfer function of the five filters designed using the IRLS and DFP techniques in the cascade form of second order sections. The coefficients indicated in the tables are those of (3.29), namely

$$H(Z) = A \prod_{k=1}^{K} \frac{(1 + a_k z^{-1} + b_k z^{-2})}{(1 + c_k z^{-1} + d_k z^{-2})} \tag{3.29}$$

$\omega_s(rad)$	A	a_1	b_1	c_1	d_1	a_2	b_2	c_2	d_2	a_3	c_3
0.31 π	0.00125	-1.6205	1.9281	-1.1756	1.0000	-0.5259	-1.7136	-1.9021	1.0000	-1.5982	-1.0010
0.33 π	0.00746	-1.4892	5.7505	-1.1756	1.0000	-0.6160	1.7620	-1.9019	0.9998	0.9137	-1.002
0.35 π	0.00427	-0.9695	1.3746	-1.1756	1.0000	0.6851	0.1859	-1.9021	1.0000	-4.2580	-1.0000
0.37 π	0.00610	-0.8591	1.0689	-1.1756	1.0000	0.3468	1.0007	-1.9017	0.9996	0.9877	-1.0004
0.39 π	0.00228	-0.8281	1.1245	-1.1756	1.0000	0.7375	0.8817	-1.9018	0.9997	-3.1952	-0.9997

Table 3.2: Filter coefficients for the cascade form transfer function, for the five lowpass filters designed using the IRLS technique.

$\omega_s(rad)$	A	a_1	b_1	c_1	d_1	a_2	b_2	c_2	d_2	a_3	c_3
0.31 π	0.00891	-1.1075	1.0007	-1.1916	0.9880	0.8802	0.4936	-0.1779	-0.3096	1.0686	2.1072
0.33 π	0.00521	-0.9599	1.0074	-1.1528	0.9266	1.0165	0.7590	-0.2225	-0.2486	1.0741	2.5024
0.35 π	0.00283	-0.8032	1.0061	-1.1969	0.9120	1.0524	1.0912	-0.2767	-0.1569	0.9108	3.5565
0.37 π	0.00307	1.1148	1.1087	-1.0314	0.8273	-0.6939	1.0010	-0.2190	-0.1553	1.0167	3.3383
0.39 π	0.0065	-0.5505	1.0067	0.1003	-0.2862	1.2519	1.1184	0.1003	-0.2862	0.9409	1.7020

Table 3.3: Filter coefficients for the cascade form transfer function, for the five lowpass filters designed using the DFP technique.

Bandpass filter design: A sixteenth order band-pass filter with two passbands and three stopbands was designed using the IRLS and DFP optimization techniques. The passband and stopband frequencies are, $\omega_{s_1} = 0.17\pi$, $\omega_{p_1} = 0.21\pi$, $\omega_{p_2} = 0.37\pi$, $\omega_{s_2} = 0.42\pi$, $\omega_{s_3} = 0.58\pi$, $\omega_{p_3} = 0.63\pi$, $\omega_{p_4} = 0.79\pi$, $\omega_{s_4} = 0.83\pi$. Fig.3.16 shows the desired and actual Magnitude responses for the design using IRLS technique. The resulting L_p error for p=8 was 0.0893 and peak magnitude deviation was 0.1210.

Fig.3.17 shows the desired and actual Magnitude responses for the design using DFP optimization technique. The resulting L_p error for p=8 was 0.2171 and the peak magnitude deviation was 0.2215.

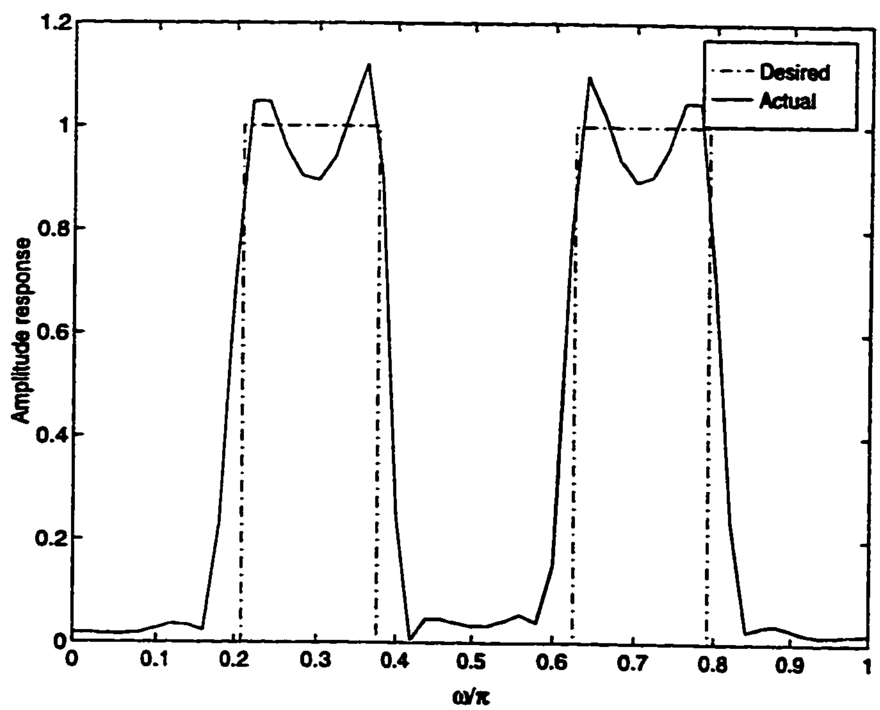


Figure 3.16: Bandpass filter design: Comparison of desired and actual magnitude response for the IRLS technique.

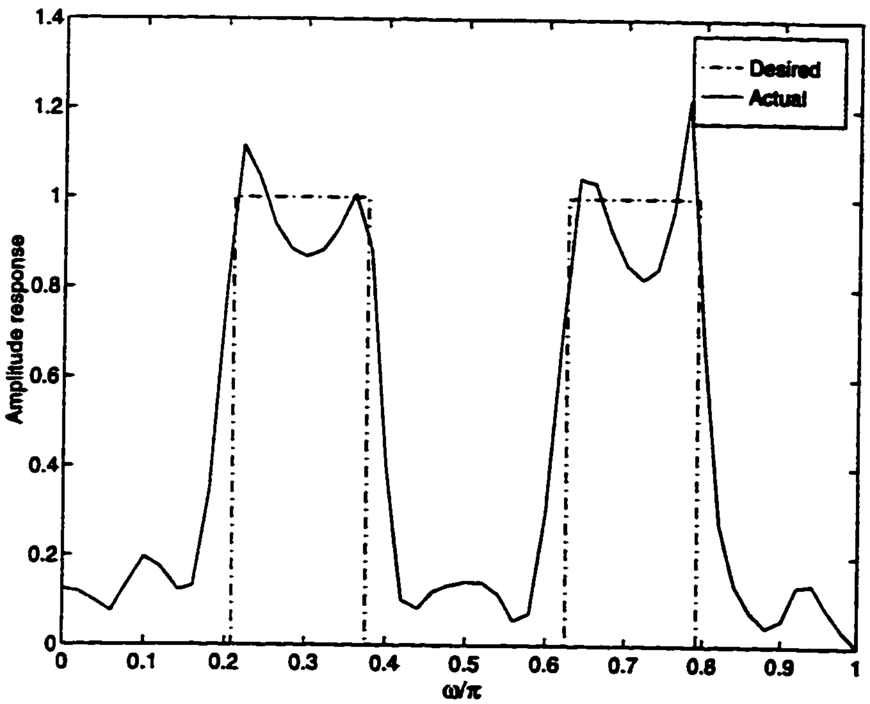


Figure 3.17: Bandpass filter design: Comparison of desired and actual magnitude response for the DFP optimization technique.

Table 3.4 and 3.5 give the transfer function of the bandpass filter designed using the IRLS and DFP techniques in the cascade form of second order sections. The coefficients indicated in the tables are those of (3.30)), namely

$$H(Z) = A\prod_{k=1}^{K}\frac{(1+a_k z^{-1}+b_k z^{-2})}{(1+c_k z^{-1}+d_k z^{-2})} \tag{3.30}$$

k	A	a_k	b_k	c_k	d_k
1	0.1294	1.7782	1.0131	1.4219	0.7703
2		1.7717	0.8196	1.4234	0.7915
3		-1.7795	1.0144	0.6777	0.8700
4		-1.7560	0.8046	0.6760	0.7693
5		0.5490	1.0152	-1.4222	0.7712
6		-0.5026	0.9964	-1.4226	0.7924
7		0.0874	0.7844	-0.6628	0.8776
8		-0.1311	0.5981	-0.6901	0.7622

Table 3.4: Filter coefficients for the cascade form transfer function, for the bandpass filter designed using the IRLS technique.

k	A	a_k	b_k	c_k	d_k
1	0.2544	-1.8708	0.9098	1.4876	0.9034
2		-1.6883	0.9084	1.8180	0.8730
3		-0.4378	0.9481	-1.7588	0.8304
4		0.4420	1.0161	-1.4549	0.8460
5		-0.1017	0.7616	0.7336	0.8257
6		1.4699	0.7140	-0.6729	0.8512
7		1.7983	0.9159	-0.1174	0.6713
8		0.9718	-0.0289	0.3087	0.1983

Table 3.5: Filter coefficients for the cascade form transfer function, for the bandpass filter designed using the DFP technique.

3.8.1 Comparison of the Computations required by the two Algorithms

IRLS Algorithm.

The number of Computations needed for a single iteration of the IRLS algorithm are as follows:

To find the filter coefficient set **a** the following equation has to be solved.

$$\mathbf{a} = [Re(\mathbf{U}^T)\mathbf{R}Re(\mathbf{U}) + Im(\mathbf{U}^T)\mathbf{R}Im(\mathbf{U})]^{-1}$$
$$.[Re(\mathbf{U}^T)\mathbf{R}Re(H) + Im(\mathbf{U}^T)\mathbf{R}Im(H)] \tag{3.31}$$

(1.) Let N be the filter order. If we assume the number of frequency samples to be $5 \times N$ and since there are $2N+1$ filter coefficients to be determined, the dimensions of the matrix **U** will be $5N \times (2N+1) \doteq 5N \times 2N$. The dimensions for the matrices **R** and H will be $5N \times 5N$ and $5N \times 1$ respectively. The computations needed for obtaining each of the matrix products in (3.31) are shown in tabular form in Table 3.6. The computations required for finding the inverse of the resulting 2N×2N matrix in (3.31) is negligible compared to the those needed for above matrix operations. Hence for solving (3.19) to obtain the filter coefficient set **a**, we need about $40N^3 + 40N^2$ multiplications, where N is the filter order.

(2.) For computing the frequency response vector H, we need about about $5N \times N$

Matrix operation	Matrix dimensions	No. of Multiplications
$Re(\mathbf{U}^T) \times \mathbf{R}$	(2N x 5N) x (5N x 5N)	(2N x 5N)=$10N^2$ **R** being diagonal
$Re(\mathbf{U}^T)\mathbf{R} \times Re(\mathbf{U})$	(2N×5N)×(5N x 2N)	2N×5N×2N=$20N^3$
$Im(\mathbf{U}^T) \times \mathbf{R}$	(2N×5N)×(5N×5N)	(2N×5N)=$10N^2$ **R** being diagonal
$Im(\mathbf{U}^T)\mathbf{R} \times Im(\mathbf{U})$	(2N×5N)×(5N×2N)	2N×5N×2N=$20N^3$
$Re(\mathbf{U}^T)\mathbf{R} \times Re(H)$	(2N×5N)×(5N×1)	(2N×5N)=$10N^2$
$Im(\mathbf{U}^T)\mathbf{R} \times Im(H)$	(2N×5N)×(5N×1)	(2N×5N)=$10N^2$
		Total= $40N^3 + 40N^2$

Table 3.6: Computations needed for obtaining each of the matrix products.

multiplications for the numerator and another $5N \times N$ multiplications for the denominator. Hence a total of $10N^2$ multiplications are needed.

(3.) For the selection of best λ we need to evaluate the frequency response vector H at ten different λ values. Hence we need about $10 \times 10N^2 = 100N^2$ multiplications. So we need about $40N^3 + 150N^2$ multiplications per single iteration of the IRLS algorithm.

DFP unconstrained optimization technique.

The number of Computations needed for a single iteration of the DFP unconstrained optimization technique are as follows:

(1.) For the calculations of the gradients the objective function needs to be evaluated as many times as the number of variables. The number of multiplications needed for one function evaluation is about $10N^2$. Since there are $2N$ variables $20N^3$ multiplications are needed to compute the gradients per iteration.

(2.) For the computation (updating) of the inverse hessian matrix (required for the Line search) $8N^3 + 20N^2$ multiplications are needed.

Thus about $28N^3 + 20N^2$ multiplications, per single iteration are needed for the DFP unconstrained optimization technique.

From the above discussion it is clear that the Computational complexity, per single iteration, of both algorithms is almost same. Hence the algorithm which takes more number of iterations to achieve a certain minimum error level would be less efficient in terms of computations. From Table 3.1 showing the comparison of L_p Approximation error and the number of computations needed, for the two algorithms, it can be observed that the DFP technique, despite taking more number of computations does not perform well in terms of the L_p error. The IRLS technique gives smaller L_p approximation error and requires less number of computations. Despite the similar computational complexity per single iteration of the two techniques the IRLS algorithm is more efficient computationally as it requires smaller number of total computations and hence less design time.

3.9 Comparison with some recent IIR digital filter design techniques.

3.9.1 LCK Technique [10]

Lu, Cui, and Kirlin in [10] proposed a technique which we denote here by LCK technique. In this technique, they derived closed form formulas for the evaluation of the gradient vector and the Hessian matrix of an IIR transfer function. These formulas were then used in a modified Newton optimization algorithm for the optimal design of IIR digital filters in L_p norm sense. The design example considered was as follows: passband edge frequency=0.45 π radians and stopband edge frequency=0.55 π radians. The method was applied to minimize the following objective function.

$$E(x) = \sum_i |H(\omega_i) \ - H_d(\omega_i)|^p \tag{3.32}$$

The filter order considered was eight and p=4. Table 3.7 gives a comparison of the results for the above design example using the technique of [10] and the IRLS technique.

	Ref. [10]	IRLS tech.
$E(x)$	1.375×10^{-5}	4.395×10^{-6}
Max. passband dev.	0.0305	0.0085
Max. stopband dev.	0.0215	0.0106

Table 3.7: Comparison of deviations for IRLS technique and the technique of [10] for p=4 and filter order = 8.

3.9.2 Shaw's Technique [6]

Shaw in [6] proposed a frequency-domain approach for optimal estimation of the IIR filter coefficients to match a given frequency specification in the least-squares sense. The method decouples the numerator and denominator estimation problem into two subproblems. One for the estimation of the numerator coefficients and the other for the estimation of the denominator coefficients. After decoupling, the denominator is estimated by an iterative minimization algorithm. The numerator is found only once by using the least-squares technique, based on the estimated denominator. Since this technique performs iterative minimization with respect to the denominator coefficients only and does not consider the numerator coefficients, the numerator computed from the estimated denominator may not be optimal. Two design examples considered in the paper are compared with the IRLS design method.

Lowpass filter design

A lowpass filter design was considered in [6] where the desired magnitude specifications were given in the dB scale and are shown by solid line in Fig. 3.18. By using a sixth order filter Shaw [6] obtained a minimum stopband attenuation of about 35 dB. By using a same filter order for the IRLS technique the maximum stopband attenuation of about 30 dB was achieved. The the resulting response is shown in dot-dashed line, in the Fig. 3.18. Hence the performance of the IRLS technique is close to that of [6].

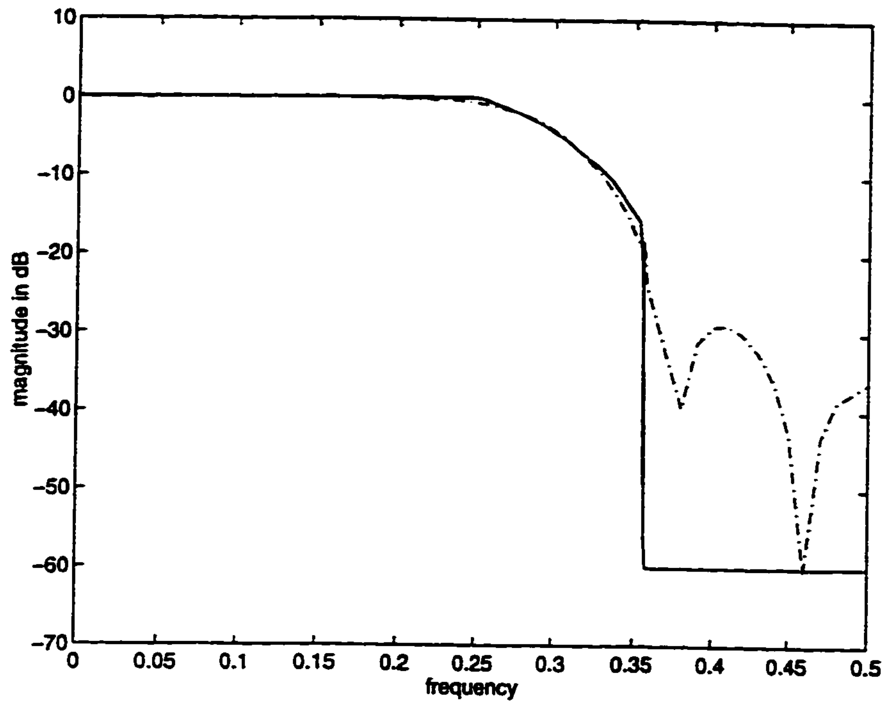


Figure 3.18: Comparison of desired (solid line) and actual (dot-dashed) frequency response for lowpass filter design using the IRLS technique.

Notch filter design

A notch filter design was considered in [6] using a tenth order filter. The desired magnitude specification are shown by solid line in Fig. 3.19. The desired attenuation at the notch frequency was 20 dB. In [6] the value of attenuation achieved at the notch frequency was about 18 dB. The resulting response using the IRLS technique is shown in dot-dashed line, in Fig. 3.19. The attenuation at the notch frequency obtained was 19.3 dB. Hence for this design example also the IRLS technique performs favorably.

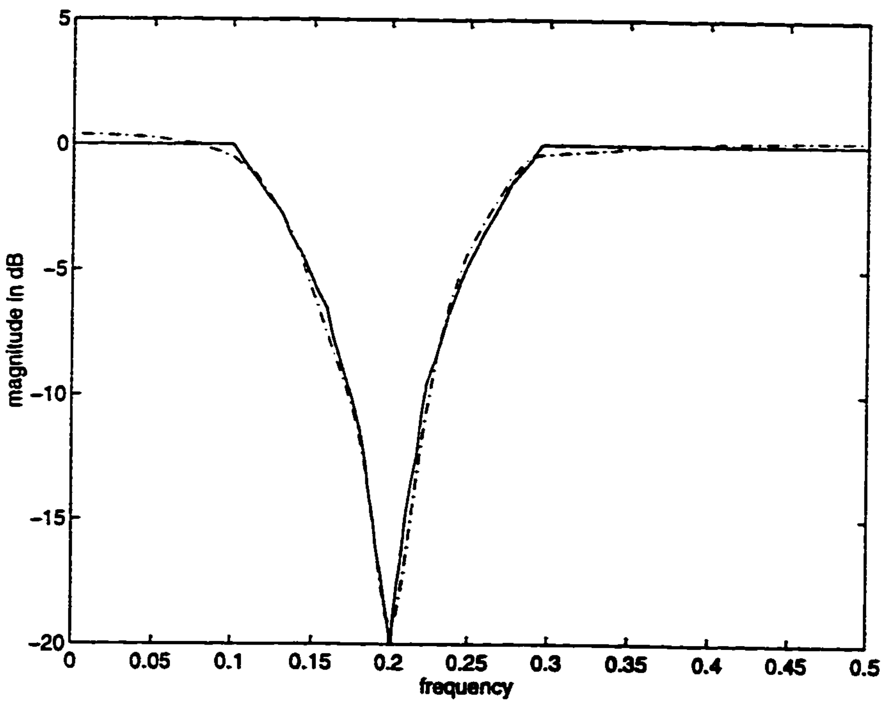


Figure 3.19: Comparison of desired (solid line) and actual (dot-dashed) frequency response for notch filter design using the IRLS technique.

3.10 Conclusions

One of the major contributions of this Thesis is the formulation of the IIR digital filter design problem in the L_p norm sense. By transforming the problem of minimization of the L_p norm of the filter error function to that of solving an equivalent weighted least-squares problem, the IIR filter coefficients are obtained without the need of using any nonlinear optimization techniques.

The choice of the convergence parameter λ as used by Burrus and Barreto [1] in the IRLS algorithm for the FIR problem was found to be inappropriate for the IIR filter design problem. An optimal value of the parameter λ would be the one which will give the minimum L_p error in the subsequent iterations. An attempt to find such an optimal value resulted in a highly nonlinear expression with respect to λ, the solution of which was very difficult. Although numerical solution techniques to solve such an equation exist, incorporating them into every iteration of the IRLS algorithm would be computationally too expensive. Hence a simple method of selection of λ (denoted by λ_M) is proposed. As a result of this method of selection of λ rapid convergence of the L_p error was obtained. Several design examples, demonstrating the effectiveness of the proposed IRLS technique, are presented. A comparison of several lowpass and bandpass IIR filters designed using the modified IRLS technique and the Davidon Fletcher Powell (DFP) unconstrained optimization technique is also presented. It is seen that the IRLS technique compares favorably in terms of the

L_p error as well as the Computational complexity. The results of comparison with some recent IIR filter design techniques, demonstrate the usefulness of the proposed design approach.

Chapter 4

Design of 2-D IIR digital filter

4.1 Introduction

In this chapter the design of 2-D IIR digital filters using the IRLS technique is presented. The IIR filter design technique developed for the 1-D case in the previous chapter is extended for the design of 2-D IIR digital filters. The first two sections present the formulation of the 2-D IIR filter design problem. The next section presents the design of 2-D IIR digital filters in the cascade form. In the following sections various design examples and comparisons with the other 2-D IIR filter design techniques are presented.

4.2 IRLS design of 2-D IIR digital filter

Let $H(z_1, z_2)$ be the system function of a 2-D recursive digital filter described as

$$H(z_1, z_2) = \frac{P(z_1, z_2)}{1 + Q(z_1, z_2)} = \frac{\sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} b_{n_1 n_2} z_1^{-n_1} z_2^{-n_2}}{1 + \sum_{n_1=1}^{N_1} \sum_{n_2=1}^{N_2} a_{n_1 n_2} z_1^{-n_1} z_2^{-n_2}} \tag{4.1}$$

Let $D(u, v)$ be the desired frequency response of the 2-D IIR filter and let $H(e^{ju}, e^{jv})$ be the actual frequency response. The filter coefficients $b_{n_1 n_2}$ and $a_{n_1 n_2}$, $n_1 = 0, 1, .., N_1, n_2 = 0, 1, .., N_2$, are to be found such that the following p^{th} power error norm

$$E_p^p \quad = \quad \sum_{k_1} \sum_{k_2} |H(e^{ju_{k_1}}, e^{jv_{k_2}}) \quad - \quad D(u_{k_1}, v_{k_2})|^p \tag{4.2}$$

is minimized. By setting the weights to be

$$w_{k_1 k_2} = |H(e^{ju_{k_1}}, e^{jv_{k_2}}) \quad - \quad D(u_{k_1}, v_{k_2})|^{p-2} \tag{4.3}$$

the p^{th} power error norm can be defined as weighted least-squares as follows

$$E_p^p \quad = \quad \sum_{k_1} \sum_{k_2} w_{k_1 k_2} |H(e^{ju_{k_1}}, e^{jv_{k_2}}) \quad - \quad D(u_{k_1}, v_{k_2})|^2 \tag{4.4}$$

By solving the above set of equations as a weighted least-squares problem we obtain the required filter coefficient parameters $\{b_{n_1 n_2}, a_{n_1 n_2}\}$ for the 2-D IIR digital filter.

4.3 Solution of the weighted least-squares problem

The design equations for 2-D IIR digital filter in weighted least-squares sense are quite similar to those of 1-D case except for an increase in the dimensionality.

Let the difference between $D(u,v)$ and $H(e^{ju},e^{jv})$ be $\zeta(e^{ju},e^{jv})$, i.e

$$\frac{P(e^{ju},e^{jv})}{1+Q(e^{ju},e^{jv})} - D(u,v) = \zeta(e^{ju},e^{jv}) \tag{4.5}$$

multiplying (4.5) by $[1+Q(e^{ju},e^{jv})]$ we get

$$P(e^{ju},e^{jv}) - Q(e^{ju},e^{jv})D(u,v) = D(u,v) + V(e^{ju},e^{jv}) \tag{4.6}$$

where

$$V(e^{ju},e^{jv}) = [1+Q(e^{ju},e^{jv})]\zeta(e^{ju},e^{jv}) \tag{4.7}$$

By putting $u = u_{k_1}, k_1 = 1,2,..,r_1$, and $v = v_{k_2}, k_2 = 1,2,..,r_2$ in 4.6 the following system of equations can be formed.

Defining

$$E = [V(e^{ju_1},e^{jv_1}), V(e^{ju_1},e^{jv_2}), .., V(e^{ju_1},e^{jv_{r_2}}),$$

$$V(e^{ju_2},e^{jv_1}), V(e^{ju_2},e^{jv_2}), .., V(e^{ju_2},e^{jv_{r_2}}),$$

$$..............., V(e^{ju_{r_1}}, e^{jv_1}), .., V(e^{ju_{r_1}}, e^{jv_{r_2}})]^T \tag{4.8}$$

$$\mathbf{U} = \begin{bmatrix} \tilde{u}(u_1, v_1) \\ \tilde{u}(u_1, v_2) \\ : \\ \tilde{u}(u_1, v_{r_2}) \\ \tilde{u}(u_2, v_1) \\ : \\ \tilde{u}(u_2, v_{r_2}) \\ : \\ : \\ \tilde{u}(u_{r_1}, v_{r_2-1}) \\ \tilde{u}(u_{r_1}, v_{r_2}) \end{bmatrix} \tag{4.9}$$

where

$$\tilde{u}(u_i, v_j) = [1, e^{-jv_j}, e^{-j2v_j}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, e^{-jN_2v_j},$$
$$e^{-ju_i}, e^{-j(u_i+v_j)}, \ldots\ldots\ldots\ldots\ldots\ldots, e^{-j(u_i+N_2v_j)}$$
$$, \ldots\ldots\ldots\ldots\ldots\ldots, e^{-jN_1u_i}, e^{-j(N_1u_i+N_2v_j)},$$
$$D(u_i, v_j)e^{-jv_j}, D(u_i, v_j)e^{-j2v_j}, .., D(u_i, v_j)e^{-jN_2v_j},$$
$$D(u_i, v_j)e^{-ju_i}, D(u_i, v_j)e^{-j(u_i+v_j)}, .., D(u_i, v_j)e^{-j(u_i+N_2v_j)}$$
$$, \ldots\ldots\ldots, D(u_i, v_j)e^{-jN_1u_i}, D(u_i, v_j)e^{-j(N_1u_i+N_2v_j)}] \tag{4.10}$$

Let

$$\mathbf{a} = [b_{00}, b_{01}, b_{02}, .., b_{0N_2}, b_{10}, b_{11}, .., b_{1N_2}, ..., b_{N_1N_2},$$
$$a_{01}, a_{02}, a_{03}, .., a_{0N_2}, a_{10}, a_{11}, .., a_{1N_2}, ..., a_{N_1N_2}]^T \tag{4.11}$$

$$H = [D(u_1, v_1), D(u_1, v_2), \ldots\ldots\ldots, D(u_1, v_{r_2}),$$
$$D(u_2, v_1), D(u_2, v_2), \ldots\ldots\ldots, D(u_2, v_{r_2}),$$
$$\ldots\ldots, D(u_{r_1}, v_1), D(u_{r_1}, v_2), ..., D(u_{r_1}, v_{r_2})]^T \tag{4.12}$$

The following vector equations can be formed:

$$E = \mathbf{U}\mathbf{a} - H \tag{4.13}$$

Minimizing $\sum_{i=1}^{i=k}\sum_{j=1}^{j=k} w(u_i,v_j)^2|V(e^{ju_i},e^{jv_j})|^2$ leads to the solution

$$\begin{aligned}\mathbf{a} \;=\; & [Re(\mathbf{U}^T)\mathbf{R}Re(\mathbf{U}) \;+\; Im(\mathbf{U}^T)\mathbf{R}Im(\mathbf{U})]^{-1} \\ & .[\; Re(\mathbf{U}^T)\mathbf{R}Re(H) \;+\; Im(\mathbf{U}^T)\mathbf{R}Im(H)]\end{aligned} \tag{4.14}$$

The derivation for (4.14) is similar to that for the 1-D case

where $\mathbf{R}$ is a diagonal weighting matrix with the weights $w(u_i,v_j)^2$, $Re(.)$ means the real part of (.) and $Im(.)$ means the imaginary part of (.)

The coefficient vector $\mathbf{a}$ given by (4.11), is obtained as the solution to a weighted least-squares problem. This weighted least-square solution has to be carried out in every single iteration of the IRLS algorithm as discussed in chapter 3. The stability of the resulting filter is tested using the Maria and Fahmy stability test [48].

4.4 Cascade design of 2-D IIR filters

In order to approximate a given 2-D IIR digital filter specification a cascade form design of 2nd or first order filter sections is best suited because of several reasons. First the frequency response of cascade form is less sensitive to coefficient quantization. Secondly, lower order 2-D difference equations are easier to implement than the higher order direct forms. Also stability check of lower order filters sections is easier than that of the overall filter. In order to perform the design of 2-D IIR filters in cascade form using the IRLS technique the following procedure is adopted. At each iteration, the coefficients of all but one of the filter sections are held constant while the coefficients of the remaining filter section are found such that the overall error is reduced. In the next iteration the coefficients of another filter section are varied. In this algorithm one cycle consists of N iteration for N cascaded sections.

Cascade design of 2-D IIR digital filters

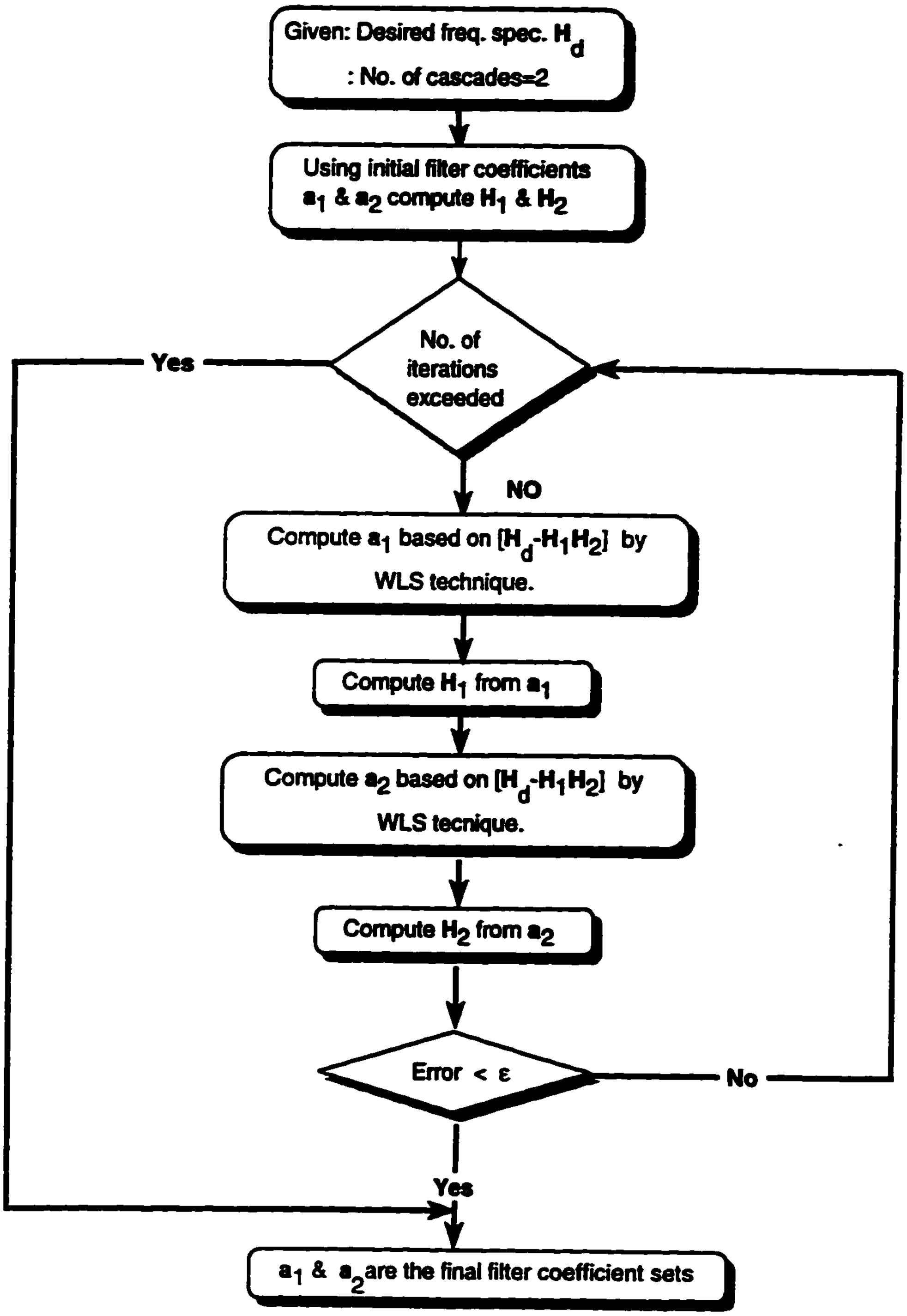


Figure 4.1: Flowchart describing the cascade design of 2-D IIR filters, using a cascade of two second order sections.

The flow diagram given in Fig.4.1 describes the optimization of cascaded filter sections in stages. Suppose we wish to approximate a given 2-D frequency specification H_d by two second order filter sections. Let H_1 and H_2 be the frequency responses of the two filter sections for the initial iteration. In the next iteration the filter coefficients for the first filter section H_1 are found by the IRLS technique by setting the weights based on the error which is a function of $(H_d - H_1.H_2)$. From the filter coefficient set of the first section we find the frequency response of H_1 to be used for error calculation in the subsequent iteration. In the next iteration the filter coefficients for the second filter section are found by the IRLS technique by setting the weights based on the error which is a function of $(H_d - H_1.H_2)$. From the filter coefficient set of the second section we find the frequency response of H_2 to be used for error calculation in the next cycle. The above procedure is continued in order to reduce the overall error.

As a result of this method of optimization in stages the IRLS technique requires significantly less number of computations as compared to any other nonlinear optimization technique, (such as the DFP unconstrained optimization technique), which considers optimizing the coefficients of all the cascade sections simultaneously. The details of which are given later.

4.5 Results for 2-D IIR digital filter

Example 1. In this example a two-dimensional circularly symmetric lowpass filter with following specifications is considered.

$$H(u,v)\begin{cases} = 1 & [u^2 + v^2]^{1/2} \leq 0.08 \\ = 0.5 & 0.08 < [u^2 + v^2]^{1/2} < 0.12 \\ = 0 & [u^2 + v^2]^{1/2} \geq 0.12 \end{cases} \tag{4.15}$$

A cascade of three second order sections and p=10 was used to approximate the specifications. The frequency response of the resulting filter is shown in Fig. 4.3. The frequency specification is shown in Fig. 4.2. An L_p error of 0.3762 was obtained after 5,730,013 flops (floating point operations). The peak magnitude error was 0.2093.

Davidon Fletcher Powell (DFP) unconstrained optimization technique was used to approximate the same filter specifications, using the same initial filter. The resulting response of the designed filter is shown in the Fig. 4.4 and the L_p error after 18,994,665 flops was 0.4048. The peak magnitude error for this case was 0.2382.

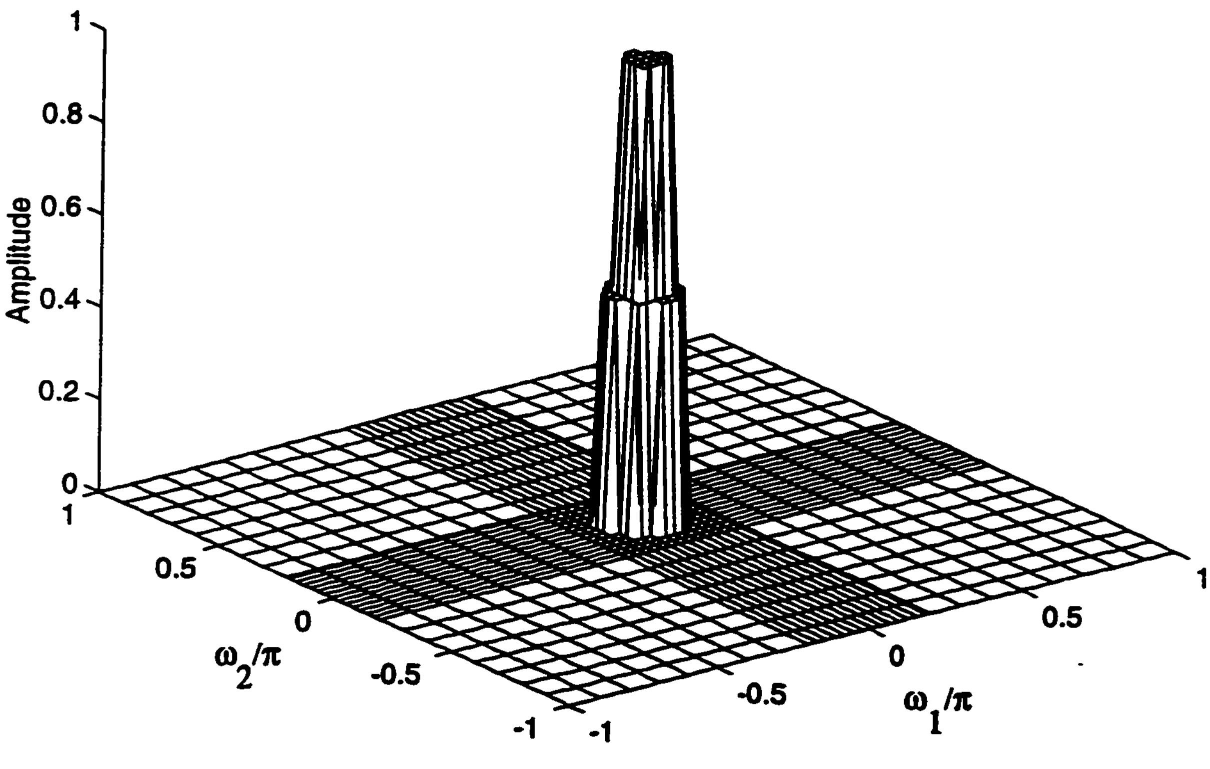


Figure 4.2: Frequency specification of the 2-D low-pass filter for Example 1.

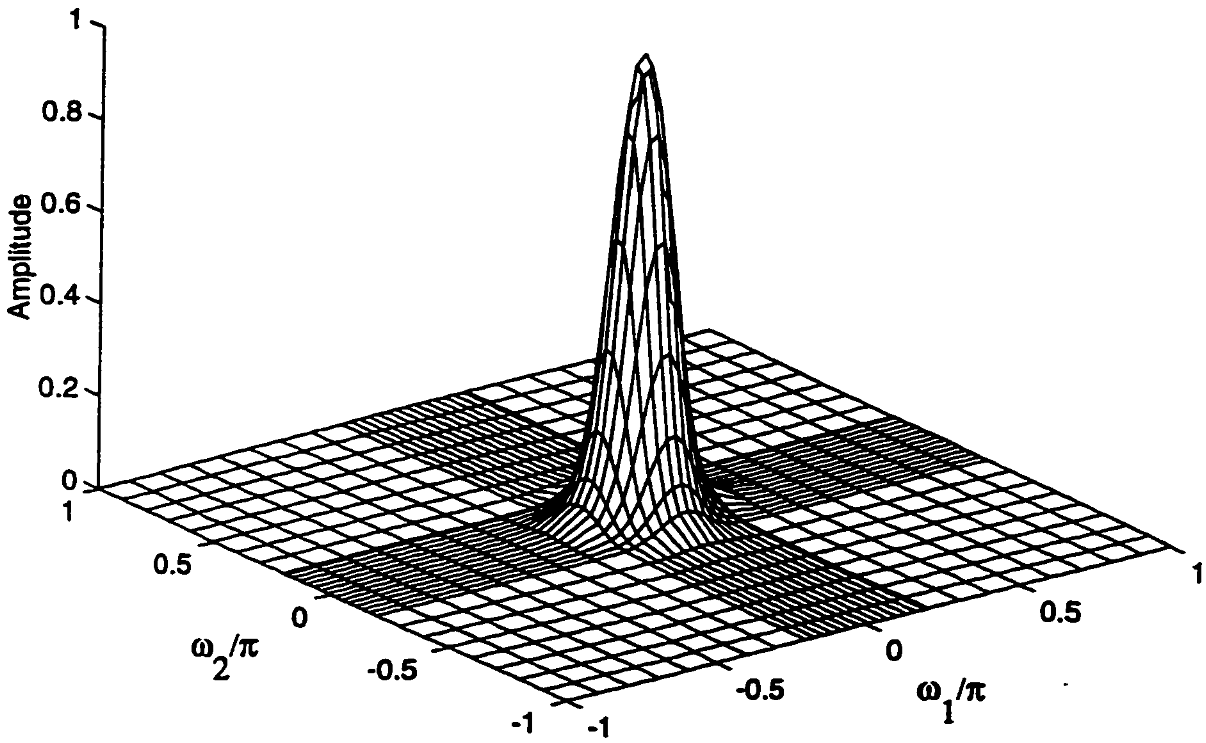


Figure 4.3: Frequency response of the 2-D low-pass filter designed using the IRLS technique for Example 1.

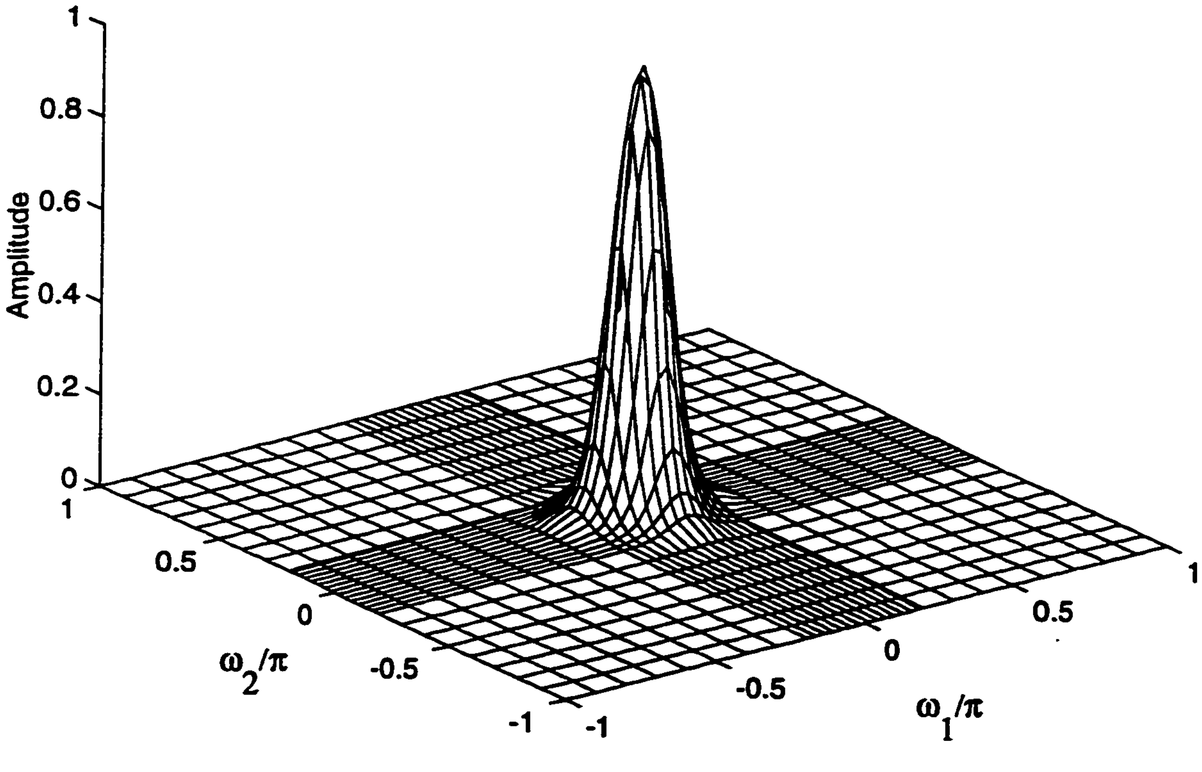


Figure 4.4: Frequency response of the 2-D low-pass filter designed using DFP technique for Example 1.

The resulting 2-D IIR filter using the IRLS technique was

$$H(z_1^{-1}, z_2^{-1}) = \frac{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 0.001227 & -0.000081 & 0.000870 \\ -0.000081 & 0.021075 & -0.003137 \\ 0.000870 & -0.003137 & 0.001111 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 1.000000 & -1.388637 & 0.511270 \\ -1.388637 & 1.953046 & -0.730068 \\ 0.511270 & -0.730068 & 0.277841 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}$$

$$\times \frac{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 0.001229 & -0.000085 & 0.000872 \\ -0.000085 & 0.021083 & -0.003140 \\ 0.000872 & -0.003140 & 0.001113 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 1.000000 & -1.388638 & 0.511271 \\ -1.388638 & 1.953048 & -0.730069 \\ 0.511271 & -0.730069 & 0.277842 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}$$

$$\times \frac{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 0.001198 & -0.000028 & 0.000842 \\ -0.000028 & 0.020976 & -0.003084 \\ 0.000842 & -0.003084 & 0.001083 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 1.000000 & -1.388792 & 0.511324 \\ -1.388792 & 1.953572 & -0.730249 \\ 0.511324 & -0.730249 & 0.277868 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}$$

The resulting 2-D IIR filter using the DFP technique was

$$H(z_1^{-1}, z_2^{-1}) = \frac{\begin{bmatrix}1 & z_1^{-1} & z_1^{-2}\end{bmatrix}\begin{bmatrix}-0.000210 & 0.001002 & -0.000934 \\ 0.001002 & 0.018151 & -0.001912 \\ -0.000934 & -0.001912 & 0.000010\end{bmatrix}\begin{bmatrix}1 \\ z_2^{-1} \\ z_2^{-2}\end{bmatrix}}{\begin{bmatrix}1 & z_1^{-1} & z_1^{-2}\end{bmatrix}\begin{bmatrix}1.0000 & -1.356032 & 0.480520 \\ -1.356032 & 1.825470 & -0.648516 \\ 0.480520 & -0.648516 & 0.235639\end{bmatrix}\begin{bmatrix}1 \\ z_2^{-1} \\ z_2^{-2}\end{bmatrix}}$$

$$\times \frac{\begin{bmatrix}1 & z_1^{-1} & z_1^{-2}\end{bmatrix}\begin{bmatrix}-0.000131 & 0.001214 & -0.000702 \\ 0.001214 & 0.018213 & -0.001710 \\ -0.000702 & -0.001710 & 0.000011\end{bmatrix}\begin{bmatrix}1 \\ z_2^{-1} \\ z_2^{-2}\end{bmatrix}}{\begin{bmatrix}1 & z_1^{-1} & z_1^{-2}\end{bmatrix}\begin{bmatrix}1.000000 & -1.355811 & 0.480715 \\ -1.355811 & 1.825613 & -0.648120 \\ 0.480715 & -0.648120 & 0.235700\end{bmatrix}\begin{bmatrix}1 \\ z_2^{-1} \\ z_2^{-2}\end{bmatrix}}$$

$$\times \frac{\begin{bmatrix}1 & z_1^{-1} & z_1^{-2}\end{bmatrix}\begin{bmatrix}0.000110 & 0.001511 & -0.000401 \\ 0.001511 & 0.018321 & -0.001540 \\ -0.000401 & -0.001540 & 0.000203\end{bmatrix}\begin{bmatrix}1 \\ z_2^{-1} \\ z_2^{-2}\end{bmatrix}}{\begin{bmatrix}1 & z_1^{-1} & z_1^{-2}\end{bmatrix}\begin{bmatrix}1.000000 & -1.355942 & 0.480561 \\ -1.355942 & 1.825431 & -0.648503 \\ 0.480561 & -0.648503 & 0.235650\end{bmatrix}\begin{bmatrix}1 \\ z_2^{-1} \\ z_2^{-2}\end{bmatrix}}$$

Example 2. In this example we will consider a high-pass filter with the following specification:

$$H(u,v) = [u^2 + v^2]^{1/2}$$
$$u = 0, \frac{1}{14}, ..., 1.0$$
$$v = 0, \frac{1}{14}, ..., 1.0$$

The filter was approximated by a second order section. First the IRLS design technique was used to approximate the desired filter specifications. Fig. 4.5 shows the desired amplitude specification. The amplitude response of the resulting filter is shown in the Fig. 4.6 , the resulting L_p error in 9 iterations was 0.2090 for $p = 12$ and $\mu = 1.11$. The resulting filter was

$$H(z_1^{-1}, z_2^{-1}) = \frac{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 0.4979 & 0.0204 & 0.0372 \\ 0.0204 & -0.3399 & 0.0043 \\ 0.0372 & 0.0043 & 0.0481 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 1.0000 & 0.5671 & 0.0575 \\ 0.5671 & 0.1686 & -0.0066 \\ 0.0575 & -0.0066 & 0.0184 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}.$$

Davidon Fletcher Powell (DFP) unconstrained optimization technique was used to approximate the same filter specifications for the purpose of comparison, using the same initial filter. The resulting response of the designed filter is shown in the Fig. 4.7 and the L_p error after ten iterations was 0.2187. The resulting filter was

$$H(z_1^{-1}, z_2^{-1}) = \frac{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 0.4617 & -0.0363 & 0.0086 \\ -0.0363 & -0.4090 & -0.0081 \\ 0.0086 & -0.0081 & 0.0735 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}{\begin{bmatrix} 1 & z_1^{-1} & z_1^{-2} \end{bmatrix} \begin{bmatrix} 1.0000 & 0.6297 & 0.1032 \\ 0.6297 & 0.1814 & -0.0084 \\ 0.1032 & -0.0084 & 0.0141 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix}}$$

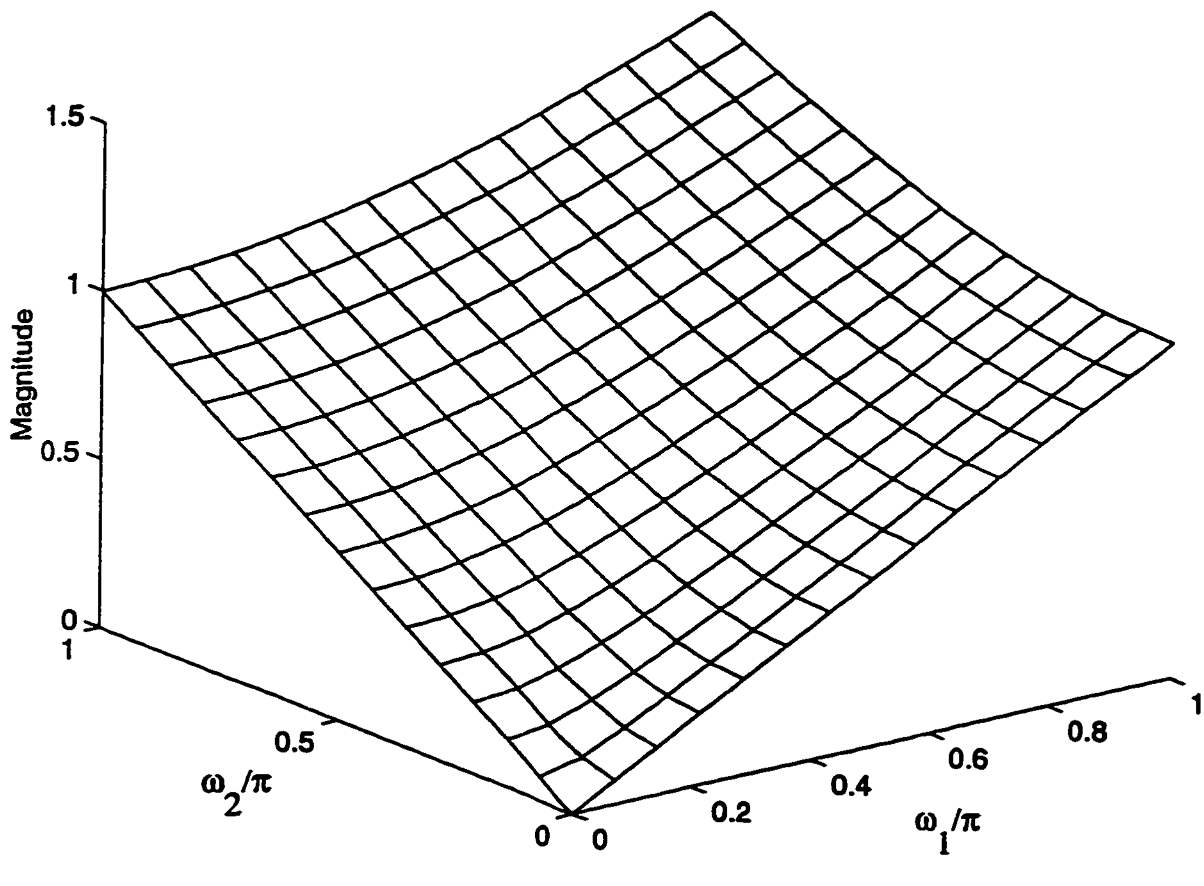


Figure 4.5: Amplitude specification of the 2-D high-pass filter for Example 2.

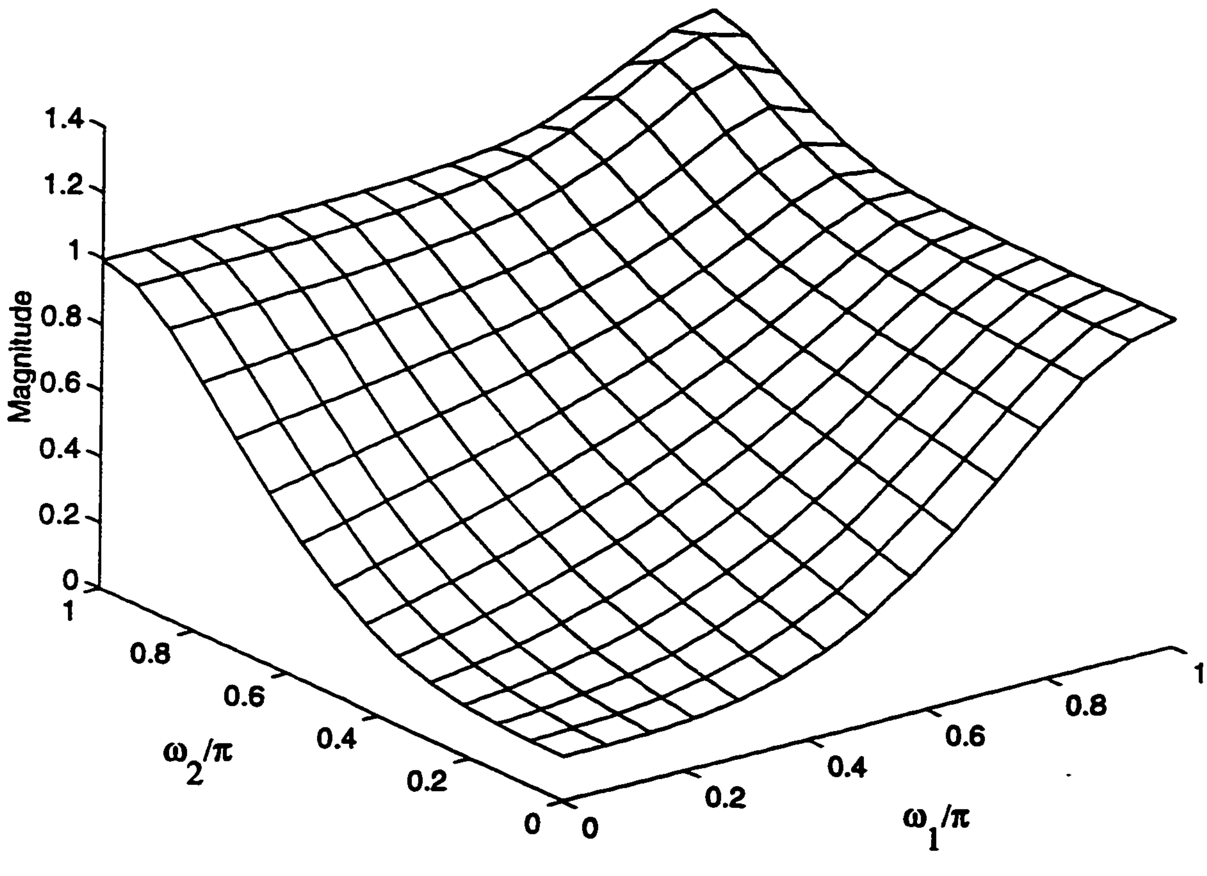


Figure 4.6: Frequency response of the 2-D high-pass filter using IRLS technique for Example 2.

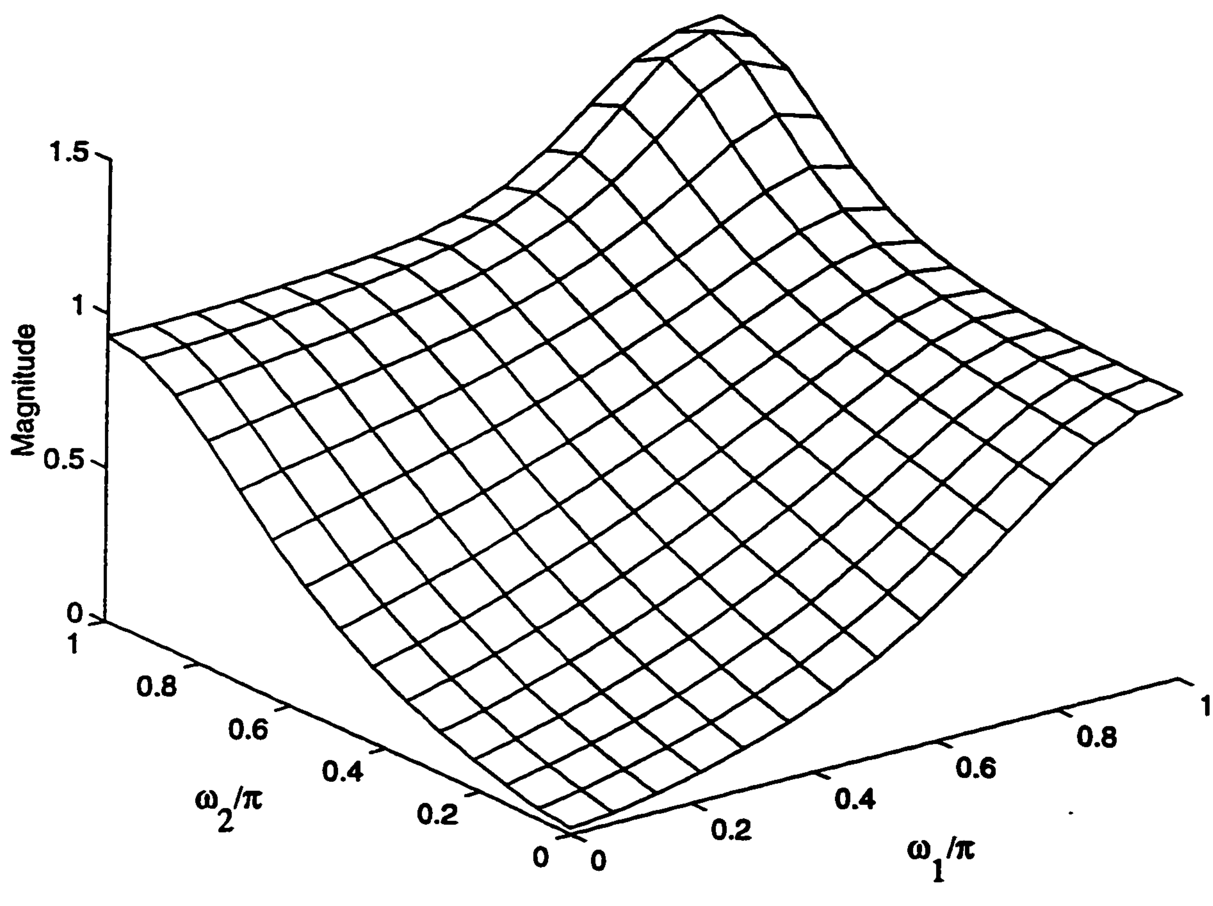


Figure 4.7: Frequency response of the 2-D high-pass filter using DFP Optimization technique for Example 2.

4.5.1 Comparison of the Computations required for IRLS and DFP techniques.

IRLS Algorithm.

The number of Computations needed for a single iteration of the IRLS algorithm are as follows:

(1.) Let N be the order of the 2-D IIR filter. If we assume the number of frequency samples to be $(5N)^2$ and since there are $2(N+1)^2$ filter coefficients to be determined, the dimensions of the matrix U will be $25N^2 \times 2(N+1)^2 \doteq 25N^2 \times 2N^2$. Proceeding in a similar manner as for the 1-D case the required number of computations can be calculated. For solving equation 4.14 to obtain the filter coefficient set a, we need about $200N^6 + 150N^4$ multiplications.

(2.) For computing the frequency response vector H, we need about about $50N^4$ multiplications.

(3.) For the selection of best λ we need to evaluate the frequency response vector H at ten different λ values. Hence we need about $500N^4$ multiplications.

So we need about $200N^6 + 700N^4$ multiplications per single iteration of the IRLS algorithm.

DFP unconstrained optimization technique.

The number of Computations needed for a single iteration of the DFP unconstrained optimization technique are as follows:

(1.) For the calculations of the gradients the objective function needs to be evaluated as many times as the number of variables. The number of multiplications needed for one function evaluation is about $50N^4$. Since there are $2N^2$ variables $100N^6$ multiplications are needed to compute the gradients per iteration.

(2.) For the computation (updating) of the inverse hessian matrix (required for the Line search) $8N^6 + 20N^4$ multiplications are needed.

Thus about $108N^6 + 20N^4$ multiplications, per single iteration are needed for the DFP unconstrained optimization technique for the design of 2-D IIR digital filter.

It can be seen that the order of complexity is same for both the techniques.

Computational complexity for cascade design.

Let there be K cascade sections each of order N. By optimizing each of the K Nth order sections in stages using the IRLS technique would require about $K(200N^6 + 700N^4)$, whereas the DFP optimization technique by optimizing the coefficients of all the cascade sections simultaneously requires about $108(KN)^6 + 20(KN)^4$. Clearly as the number of cascades K, increases, the computational complexity of the DFP optimization technique would increase at a greater rate than for the IRLS technique.

4.6 Comparison with some other 2-D IIR digital filter design techniques.

Deng [57] proposed a new technique of designing 2-D digital filters based on iterative singular value decomposition of a 2-D magnitude specification matrix. By using this decomposition approach the 2-D magnitude specification was decomposed into a pair of 1-D ones and thus 1-D design techniques are used. A circularly symmetric lowpass 2-D magnitude specification given by

$$H_d(\omega_1, \omega_2) = \begin{cases} = 1.0, & R \in [0.0, 0.1] \\ = 0.8, & R \in (0.1, 0.2] \\ = 0.44, & R \in (0.2, 0.3] \\ = 0.14, & R \in (0.3, 0.4] \\ = 0.03, & R \in (0.4, 0.5] \\ = 0.02, & R \in (0.5, 0.6] \\ = 0.001, & R \in (0.6, 1.0] \end{cases} \tag{4.16}$$

where $R = \sqrt{(\omega_1^2 + \omega_2^2)/\pi}$. The results of comparison of the design using the technique of [57] and the IRLS technique are shown in table 4.1. It can be concluded that the design results are comparable to the previous techniques.

Figure 4.8 shows the response of the sixth order filter designed using the IRLS technique in cascade form.

Design Methods	Filter Order	$e_m(\%)$	Maximum Passband Atten.(dB)	Minimum Stopband Atten.(dB)
IRLS	2,2	31.07	2.3875	19.89
[57]	2,2	42.85	6.62	33.46
[35]	2,2	34.21	0.91	22.35
IRLS	4,4	23.29	1.6216	37.72
[57]	4,4	15.58	2.20	58.26
[35]	4,4	24.36	-0.28	37.56
IRLS	6,6	21.20	1.18	54.01
[57]	6,6	14.6	1.91	51.88

Table 4.1: Comparison of the design results

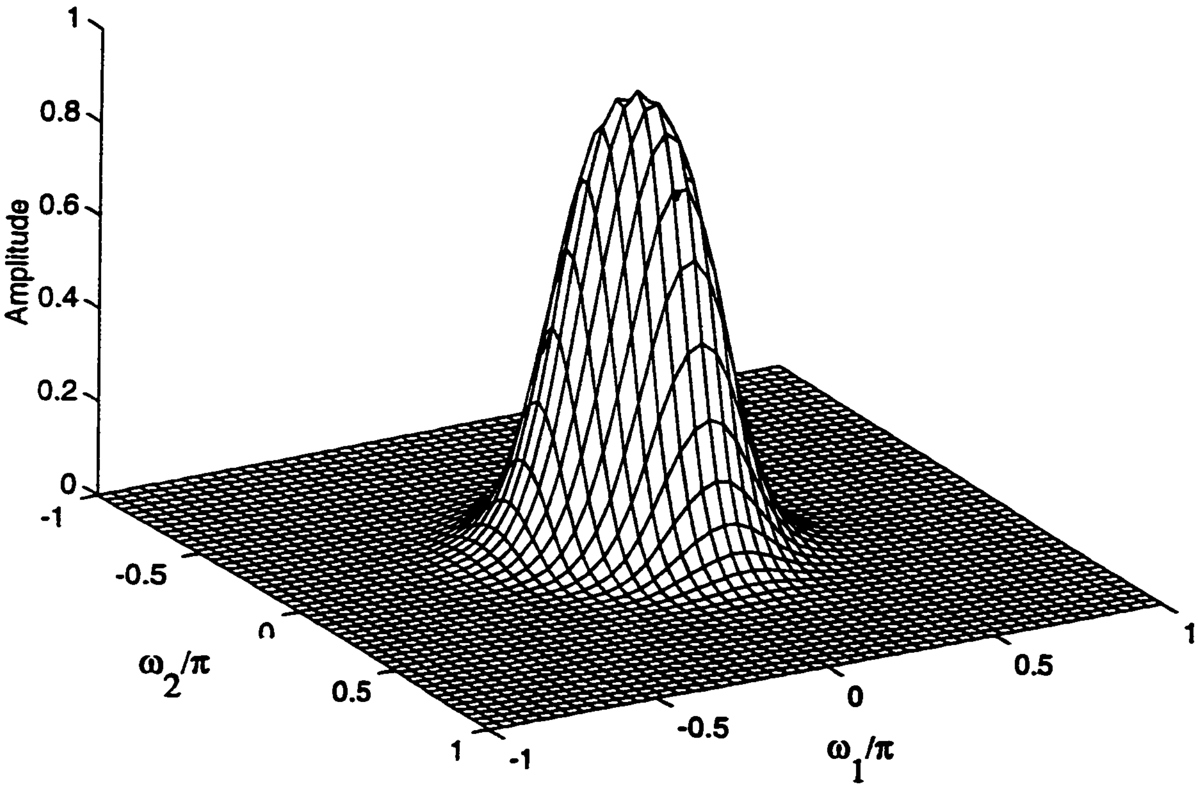


Figure 4.8: Example 3:Frequency response of the 2-D low-pass filter using the IRLS technique.

4.7 Conclusions

In this chapter a formulation for the 2-D IIR filter design, in the L_p norm sense, based on the IRLS technique is presented. The IRLS technique developed for the design of 1-D IIR digital filters is extended to the design of 2-D IIR filters. For the design of 2-D IIR digital filter a cascade form is considered. The reason for choosing the cascade form is its less sensitivity to coefficient quantization and ease of implementation of lower order subfilter sections. This cascade form design method optimizes the coefficients of each second order filter section individually to reduce the overall error. This method of optimization of filter sections in stages is computationally more efficient than any nonlinear optimization technique such as the DFP unconstrained optimization technique, which optimizes the coefficients of all the cascade sections simultaneously. The computational complexity of the DFP technique increases at a greater rate than the cascade design using IRLS technique, as the number of cascade sections increases. A comparison of the filter designs using the cascade form IRLS technique and the DFP optimization technique is also presented. The results show that the IRLS technique compares favorably with the DFP optimization design of 2-D IIR filters. From the results obtained it is also seen that IRLS technique by optimizing each second order section individually, requires much less computations as compared to the DFP unconstrained optimization technique. Finally a comparison with some other well established techniques is also presented.

Chapter 5

Summary and Conclusions

This chapter summarizes the work described in this thesis and presents conclusions drawn during the course of this thesis work.

5.1 Summary and Conclusions

The major contribution of this thesis work is the application of the Iteratively Reweighted Least-Squares technique for the IIR digital filter design problem for both 1-D and 2-D cases in the L_p norm sense. Since the IIR filter design problem is a nonlinear problem with respect to the filter coefficients, nonlinear optimization techniques such as the Fletcher-Powell technique has been applied in the past for the minimization of the frequency domain error function. The proposed IRLS IIR filter design technique solves the L_p approximation problem by solving a weighted least-squares problem (which is linear in the filter coefficients) and can be written in short computer code.

Based on the study of several 1-D IIR digital filter designs it was observed that the L_p error was not monotonically decreasing, the reason for this being the inappropriate choice of λ as $\lambda = \frac{1}{p_m - 1}$. This choice of λ was used by Burrus *et al.* [1] for the design of optimal L_p approximation FIR digital filters. An optimal value of the parameter λ would be the one which will give the minimum L_p error in the subsequent iterations. An attempt to find such an optimal value resulted in a highly nonlinear expression with respect to λ, the solution of which was very difficult. Although numerical solution techniques to solve such an equation exist, incorporating them into every iteration of the IRLS algorithm would be computationally too expensive. Hence in order to improve the convergence and successfully design the IIR digital filters a simple method for the selection of the convergence parameter is proposed. In this method the value of λ is varied from 0 to 1 in steps of 0.1 and the λ value which results in the minimum L_p error is selected as the current value of λ. As a result of this method of selection of λ, rapid convergence of the L_p error was obtained. Several design examples, demonstrating the effectiveness of the proposed method of choosing λ in the IRLS technique, are presented.

A comparison of several lowpass and bandpass IIR filters designed using the IRLS technique and the Davidon Fletcher Powell (DFP) unconstrained optimization technique is also presented. It is seen that the IRLS technique compares favorably with the DFP technique, in terms of the L_p error and the Computational complexity. The IRLS algorithm, requiring about $40N^3 + 150N^2$ multiplications per iteration, and the DFP technique requiring about $28N^3 + 20N^2$ multiplications for the same, are of same computational complexity order. It has been observed from the results obtained that although the IRLS technique requiring slightly more number of computations as compared to the DFP technique, per iteration, shows a faster convergence in less number of iterations. A comparison with some recent filter design techniques is also presented and results show the usefulness of the proposed technique. It is seen that the L_p error for the modified IRLS technique is lower than that of the DFP technique.

The IRLS technique used for the design of 1-D IIR digital filters is extended to design of 2-D IIR filters in the L_p norm sense. In order to design a 2-D IIR digital filter for the given specification a cascade of second order filter sections is considered. The reason for choosing the cascade form of second order sections being its less sensitivity to coefficient quantization, than the direct form, easier implementation of second order sections and easier stability check for the second order sections. This cascade form design method optimizes the coefficients of a single second order filter section in every iteration by keeping the coefficients of the other filter sections fixed such that the overall error is reduced. In the next sub-iteration the coefficients of another second order filter section are optimized to reduce the overall error. Here one primary iteration or one cycle consists of N sub-iterations, where N is the number of cascaded sections. This method of optimization of filter sections in stages, will be computationally more efficient than any nonlinear optimization technique(such as the DFP unconstrained optimization technique), which optimizes the coefficients of all the cascade sections simultaneously. The computational complexity of the DFP technique increases at a greater rate than the cascade design using IRLS technique, as the number of cascade secctions increase. A comparison of the filter designs using the cascade form IRLS technique and the DFP optimization technique is also presented. The results show that the IRLS technique compares reasonably with the DFP optimization design of 2-D IIR filters. It is also seen from the results obtained that IRLS technique by optimizing each second order section individually, requires much less computations as compared to the DFP unconstrained optimization technique. Finally a comparison with some other well established techniques is also presented.

5.2 Recommendations for future work

The IRLS technique used here for the design of IIR digital filters in L_p norm sense, is a frequency domain design technique in which the frequency domain error is minimized in the L_p norm sense. The same IRLS technique can also be considered as a technique for designing the IIR digital filters in the time domain, by minimizing a time domain error criterion in the L_p norm sense. In other words the problem needs to be formulated in the time domain. This could be a possible area for the future work.

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