Flavor Conserving Flavor Changing Radiative Decays of Vector Mesons in Quark Model

by

El-Aaoud El-Hassan

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

PHYSICS

July, 1991

Order Number 1354107

Flavor conserving, flavor changing radiative decays of vector mesons in quark model

El-Hassan, El-Aaoud, M.S.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1991

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

FLAVOR CONSERVING FLAVOR CHANGING RADIATIVE DECAYS OF VECTOR MESONS IN QUARK MODEL

BY

EL-AAOUD EL-HASSAN

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

In

PHYSICS

JULY 1991

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

COLLEGE OF GRADUATE STUDIES

This thesis, written by **EL AAOUD EL HASSAN** *under the direction of his Thesis Advisor, and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of* ***MASTER OF SCIENCE IN PHYSICS.***

Thesis Committee

Riazuddin

Chairman (Professor Riazuddin)

Co-Chairman (Dr. A. Al-Haidari)

Robert Lynch

Member (Professor R.F. Lynch)

Harry A. Mavromatis

Member (Professor H.A. Mavromatis)

Department Chairman

Dean College of Graduate Studies

Date :

ACKNOWLEDGEMENT

I am gratefully indebted to professor Riazuddin for his supervision and help to accomplish this research. I also thank him for his advices and guidance during my studies for the last two years. My thanks are also due to my thesis' committee members : Dr. A. Al-Haidari, Professor R. F. Lynch and Professor H. A. Mavromatis, for their cooperation and their constructive comments.

My acknowledgment is also due to K.F.U.P.M., represented by the chairman and faculty of the Physics department for their assistances.

Finally I wish to thank all my colleagues and friends for their help and support all along may stay at K.F.U.P.M. .

TABLE OF CONTENTS

LIST OF TABLES

FIGURES CAPTIONS

THESIS ABSTRACT

NAME OF STUDENT : EL AAOUD EL HASSAN

TITLE OF STUDY : FLAVOR CONSERVING AND FLAVOR CHANGING RADIATIVE DECAYS OF VECTOR MESONS IN QUARK MODEL.

MAJOR FIELD : PHYSICS

DATE OF DEGREE : JULY, 1991

A relativised quark model appropriate to a universal one-gluon exchange plus a screening confining potential motivated by lattice gauge theory was used in studying flavor changing radiative decays of B meson: $B \rightarrow K^* + \gamma$. Recoil as well as relativistic corrections were included in evaluating the form factor. Compared to other predictions, numerical computations showed that the ratio of exclusive ($B \rightarrow K^* + \gamma$) to inclusive ($b \rightarrow s + \gamma$) processes is reduced by introducing screening potential.The above model was extended to flavor conserving radiative decay of ground states mesons $V(1^-) \rightarrow P(0^-) + \gamma$. Here the relativistic corrections are more important than the recoil effects while in B decay opposite is the case. Fitting the predicted decay rates with experimental results provides information about the strong interaction at large distances as one can fix the screening length in fitting the data for these decays. It was found that the screening length increases as we move to light sector.

MASTER OF SCIENCE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran , Saudi Arabia

July 1991

خلاصة الرسالة

اسم الطالب الكامل : العود الحسن

عنوان الاطروحة : ثبوت وتغير النكهة بالتفكك الإشعاعي لمتجهة الميزونات في أنموذج الكوارك

التخصص : فيزياء

تاريخ الشهادة : يوليو ١٩٩١

تم إستعمال أنموذج كوارك نسبي ، لجهد وحيد القليون مع جزء احتوائي حاجب مشتق من نظرية قيج للرصف ، لدراسة تغيّر النكهة بسبب التفكك الإشعاعي للميزون B وفق $B \dashrightarrow K^{\times} + \gamma$. وقد تم حساب معامل الهيئة مع الآخذ بعين الاعتبار الارتداد والتصحيحات النسبية . وبمقارنة حساباتنا مع النتائج الأخرى تبين أن نسبة إعتبار فقط التحول ($B \dashrightarrow K^{*} + \gamma$) الى اعتبار التحولات ($b \dashrightarrow s + \gamma$) قد إنخفضت بإدخال الجهد الحاجب . وقد طبق هذا الأنموذج على حالة التفكك الإشعاعي المنحفظة النكهة للميزونات وهي في أدني مستويات طاقاتها $V(1^-) \dashrightarrow P(0^-) + \gamma$. التصحيحات النسبية في هذه الحاله أهم من مفعول الارتداد بينما في حالة تفكك B يصبح مفعول الإرتداد اهم من التصحيحات النسبية . الموالفة بين نسب التفكك النظرية مع نتائج التجارب أعطت معلومات حول التفاعل الشديد للمسافات البعيدة وتحديد معامل الحجب بواسطة الموالفة . وقد وجد أن معامل الحجب يزداد بانتقالنا فى الميزونات الأثقل الى الميزونات الأخف .

درجة الماجستير في العلوم

جامعة الملك فهد للبترول و المعادن

الظهران ، المملكة العربية السعودية

يوليو ١٩٩١

CHAPTER ONE

INTRODUCTION

1. QUARK MODEL: INTERNAL QUANTUM NUMBERS OF QUARKS.

Atoms and nuclei are built up from small number of elementary constituents, namely from electrons and nucleons. This concept was extended to elementary particle physics. It was revealed that the subnuclear particles called hadrons (among them are protons, neutrons and the mesons) are composite rather than elementary. In 1964 Gell- Mann and Zweig independently attempted to account for the bewildering variety of hadrons by suggesting they are made up of the simpler constituents called quarks[1-3]. At first quarks were interpreted as fictitious particles, or quasi-particles to be used in phenomenological calculations of masses and widths, but they are now regarded as fundamental particles. All known hadrons (more than one hundred) are attributed to combinations of these fundamental entities.

The fundamental assumption of the quark model for hadrons is that baryons are three quarks bound states, whereas mesons are bound states of one quark and one antiquark[4]. In a schematic notation where spin and spatial degrees of freedom are suppressed:

$$| \text{Baryon} \rangle = | q_i q_j q_k \rangle \quad (1.1)$$

$$| \text{Meson} \rangle = | q_i \bar{q}_j \rangle \quad (1.2)$$

In the above equation the indices i,j,k refer to the quark flavor.

Since baryon number is additive we assign B = 1/3 to all quarks. Quarks were supposed to be spin 1/2 particles and existing in different varieties termed flavor. Five of them have been identified - the up (u), down (d), charm (c), strange (s) and bottom (b) quarks - and a sixth flavor, the top quark (t), is believed to exist*, [5] . They also carry fractional electric charge (Q) and fractional baryon number (B), the intrinsic quantum numbers are given in Table (1.1).

For the low-lying baryon states the total wavefunction is

---

* The top quark mass has not been found yet. Experiments at the TRISTAN ring in Japan gives a lower bound for the t-mass $m_t > 26$ Gev (Takasaki 1988)

symmetric under interchange of any two quarks. In fact in the ground state the orbital angular momentum of the three quarks is zero ($l = 0$) and hence the spatial wavefunction must be symmetric. Let's consider the Δ^{++} resonance particle ($J = 3/2$). It is made of three u quarks (flavor symmetric), so the spins wave function must be symmetric, with all three quarks spin aligned up to give $S = J = 3/2$. and the flavors of the three quarks are equal. The combined spin, flavor and space wave function:

$$\psi(\Delta^{++}, J = 3/2) = (U\uparrow U\uparrow U\uparrow)\,\varphi(x_1, x_2, x_2) \qquad (1.3)$$

where $\varphi(x_1, x_2, x_3)$ is totally symmetric under any permutation of x_i. Therefore we have totally symmetric wavefunction for particles with half integral spin, but this contradicts the Pauli exclusion principle. The solution to this puzzle is to suppose that the quark carry a further degree of freedom called color[6]. According to this each quark can occur in three different versions or colors; red (r), blue (b) and green (g):

$$q \rightarrow q_a \qquad a = r, b, g$$

Including this additional degree of freedom it is possible to construct a three quarks wavefunction which is antisymmetric in color. Hence the Δ^{++} wavefunction becomes:

Quarks	Q	B	S	C	t	b	Y
u	2/3	1/3	0	0	0	0	1/3
d	- 1/3	1/3	0	0	0	0	1/3
c	2/3	1/3	0	1	0	0	0
s	- 1/3	1/3	- 1	0	0	0	- 2/3
t	2/3	1/3	0	0	1	0	0
b	- 1/3	1/3	0	0	0	- 1	0

Table 1.1 Intrinsic quantum numbers of the six quarks u,d,s,c,t,b (Q is unit of electron charge)

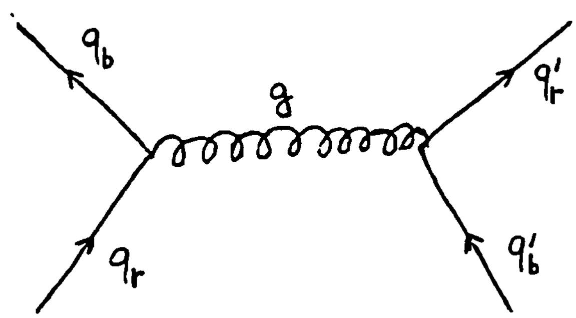


Fig 1.1 Strong scattering via gluon exchange.

$$\psi (\Delta^{++}, J = 3/2) = \frac{1}{\sqrt{6}} \varepsilon_{abc} (U_a\uparrow U_b\uparrow U_c\uparrow) \varphi (x_1, x_2, x_2) \qquad (1.4)$$

where ε_{abc} = 1, - 1 for even, odd permutation of abc respectively.

In general equation (1.1) is replaced by the totally antisymmetric baryon wavefunction which satisfies the Pauli principle (color singlet):

$$|\,\mathrm{Baryon}\,\rangle = \frac{1}{\sqrt{6}}\varepsilon_{abc}\,|\,q_i^a\,q_j^b\,q_k^c\,\rangle \qquad (1.5)$$

Correspondingly, the meson wavefunction in equation (1.2) is replaced by :

$$|\,\mathrm{Meson}\,\rangle = \frac{1}{\sqrt{3}}\sum_a |\,q_i^a\,\bar{q}_j^a\,\rangle \qquad (1.6)$$

The states in equations (1.5) and (1.6) are invariant under rotation in color space[7]. To get agreement with experiment it was necessary to assume that all hadrons are described by quark states that are invariant under color SU(3) transformation. Indeed all known hadrons are color singlets, i.e hadrons are colorless. In order to explain the nonexistence of unobserved states like free quark (q), diquarks (qq), four-quarks (qqqq) states , ... etc., one can make the confinement postulate, which states: " all hadrons and all physical observables (currents, energy-momentum tensor, ... etc.) are color singlets"[3]

2. QUANTUM CHROMODYNAMICS

As far as quarks are concerned, each one exists in three color varieties which act as strong charges and generate the strong forces. The strong quark interaction proceeds by the exchange of force quanta called gluons, which have spin 1 and are believed to be massless. The gluons carry color back and forth between the quarks as shown in the basic exchange process (fig. 1.1).

The quantum theory of interaction of quarks and gluons is called QuantumChromodynamics[3,4,7] (QCD) just as the quantum theory of interaction of charged particles with photons is called Quantum Electrodynamics (QED).

Two types of tests which support QCD are stated below :

1)Hadron spectroscopy:

As already seen the color was introduced in order to satisfy Pauli principle. At short distances the one gluon exchange between two quarks inside hadrons fig.(1.2) give rise to hyperfine splitting between the singlet and triplet spin states of quark and antiquark (cf.2.1.3) This is the source of $\rho - \pi$, K - K^* and N - Δ mass differences.

- In fact $m(\rho) > m(\pi)$ is found in nature. This supports the theory that the gluons carry spin 1.

- And $m(\Delta) > m(N)$ is found in nature. This supports the theory that the vector gluons carry color.

2) Deep inelastic lepton - hadron scattering.

At very short distances or high momentum transfer (Q) quarks behave nearly as free particles within hadrons, and this implies that the strong coupling constant decreases as $Q^2 \longrightarrow \infty$ (asymptotic freedom). The essence of asymptotic freedom is contained in the following equation :

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n)\,Ln(Q^2/\Lambda^2)} \longrightarrow 0 \quad \text{for } n < 16 \qquad (1.7)$$

n denotes the number of active quark flavors and Λ is a free dimensional parameter of the theory to be determined by experiment. A typical hadronic reaction which supports the asymptotic freedom is :

$$e + p \longrightarrow e + \text{hadrons}$$

At the quark level it is depicted in figure (1.3).

Indeed the cross section for the above process remains large, is expected from the fact that hadron constituents are point-like and interact weakly. There are two regimes:

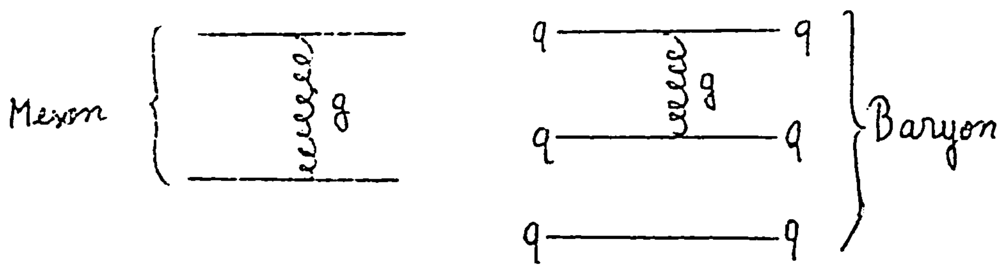


Fig. 1.2 One gluon exchange between two quark inside hadrons

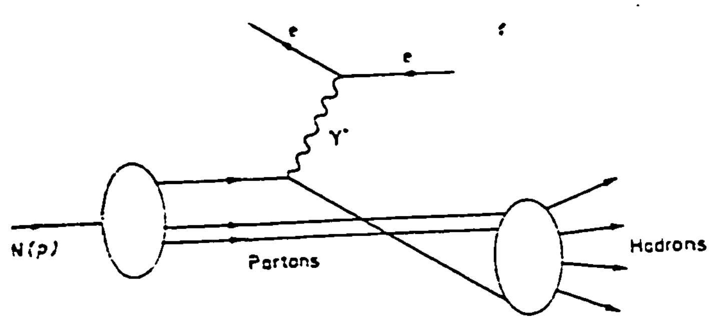


Fig. 1.3 Diagram for electron-proton scattering

- Short distance (high momentum transfer) the running coupling constant α_s becomes small, and hadrons appears to consist of weakly interacting quarks. One can apply perturbation theory calculations.

- Long distances regime (small momentum transfer): perturbation theory is not applicable in this case, the hadrons interact strongly. Quarks and gluons are absent from the physical spectrum (quark confinement).

In fact the quark model has led to important theoretical developments such as (QCD), the theory of strong interactions, and has played a part in electroweak theory (Glashow, 1961, Weinberg 1967, Salam 1968) as well as playing a part in many interesting experimental discoveries involving new flavors of quarks.

Supplemented with ideas from QCD the nonrelativistic quark model has proved to be quite successful in describing the spectroscopy, static properties, and decay amplitudes of hadrons. The success of the quark model in its various guises (the nonrelativistic quark model, the bag model, ...) is quite impressive.

3. WEAK DECAYS

The strong and electromagnetic interactions do not alter the quark flavor . In the weak interaction , on the other hand, a quark can change its flavor (but not its color). The quark picture allows us to write down a simple interaction which describes a variety of weak hadronic decays.

Weak decays have always been an important source of information about the form and symmetry of basic interactions as well as the structure of the constituents of matter. Actually weak decays are essential for testing the standard model and determining its fundamental parameters (quark mass, quark mixing parameters, ...). They also give valuable information about the inner structure of hadrons not yet predictable from strong interaction theory (QCD).

In recent years great effort has gone into theoretical calculation of weak decays of particles containing a heavy quark c or b. Despite this, it still has not been possible to get results from the first principles. However, calculations based on various plausible assumptions have led to a reasonable understanding of charmed and beauty decays[4].

In order to link theory with experiment one must eventually find ways to calculate the hadronic matrix elements of the effective hamiltonian. This proves not too difficult for inclusive* decays of hadrons which contain a sufficiently heavy quark Q. In fact many exclusive decay channels have been calculated in a sophisticated version of the spectator model (cf.

---

* A reaction in which all the final-state particles are identified is called exclusive, whereas a reaction in which only one of the final-state particle is identified is called inclusive.

4.2.1) using model wavefunctions for the hadrons[8]. In such model the light quark constituents act merely as passive spectators.

The flavor-changing one-loop processes proved to be successful in predicting various physical quantities. Particularly successful was the Gaillard and Lee[9] prediction of the mass of the charmed quark in the following processes :

$$K \rightarrow \mu + \bar{\mu} \quad , \quad K \rightarrow \pi + e^{+} + e^{-}$$

3.1 RARE B DECAY

Similarly the rare decay of B meson proceeding through flavour changing neutral current transitions b -----> s γ, which are allowed at the one-loop level have been identified as a valuable source of information on the standard model and its extensions, in particular with respect to:

1) Top quark mass and Kobayashi-Maskawa mixing angles.

2) Perturbative QCD corrections which enhance the branching ratio for transition b ----> s γ.

In order to illustrate the above points consider the process: B ------> K^{*} γ involving the b -----> s γ transition, which is at the hadronic level is described by an effective interaction hamiltonian, H_{eff}, described in terms of operators involving quarks and gluon fields[10,11]:

$$H_{eff} = C\, m_b\, \bar{s}\, \sigma_{\mu\nu} q^{\nu}\, b_r\, \varepsilon^{\mu} \qquad (m_s << m_b) \tag{1.8}$$

$b_r = \frac{1}{2} (1 + \gamma_5)\, b$, (where s and b are quark field.

q^{ν} is the photon momentum, and ε^{μ} its polarization.)

3.1.a QCD CORRECTIONS

The constant C is calculated in the model of interest, it contains the dependance on the Cabibbo-Kobayashi-Maskawa angles and the charm and the top quark masses. Without QCD corrections, the decay b ----> s γ proceeds at the one-loop level as shown in figure 1.4. In lowest order the constant C can be written as follow:

$$C = \frac{G_f}{\sqrt{2}} \frac{e}{4\pi^2} V_{tb} V_{ts}^{+} F_2(m_t) \text{, where}$$

G_f : Fermi coupling constant.

V_{tb} and V_{ts} are the Kobayashi-Maskawa matrix elements.

The function F_2 can be found in Inami and Lim[12]. For x << 1, F(x) ~ O(x) because of hard Glashow-Iliopoulos-Maiani (GIM) suppression, and only the t quark is important.

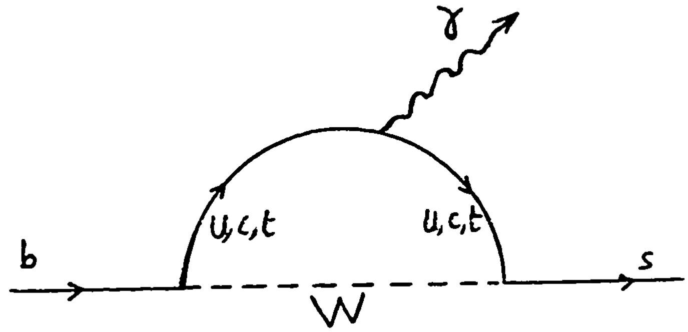


Fig 1.4 : Feynman diagram contributing to the bsγ vertex function at the one-loop level.

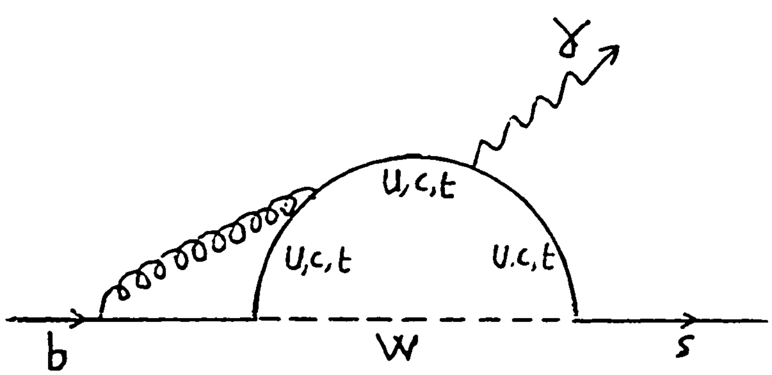


Fig.1.5 Diagram responsible for the QCD corrections for the transition b -----> s the curly line indicate gluon exchange.

The effective hamiltonian H_{eff} eq.(1.8) is treated as an effective - short distance operator. Therefore the constant C should include all short - distance perturbative QCD corrections (ref.13, 34) . In fact there are large QCD corrections to the constant C , because there is an accidental cancelation of $Ln(m_t/m_c)$ in the lowest order evaluation of C. Taking gluon exchange, between external and internal fermion lines, into account (fig 1.5) removes the Glashow - Iliopoulos - Maiani suppression and turns it into a logarithmic suppression. The computation of QCD correction to first order in α_s modifies the constant C as follow[13].

$$C = \frac{G_f}{\sqrt{2}} \frac{e}{4\pi^2} V_{tb} V^{+}_{ts} \left(F_2(m_t) + \frac{4\alpha_s}{3\pi} Ln\left(\frac{m_t^2}{m^2} \right) \right) \qquad (1.9)$$

where $m = m_b$ is a typical hadronic scale of the process. Numerical calculation[13] show that the QCD correction factor $(4\alpha_s/3\pi)Ln(m_t/m_c)$ dominates over $F_2(x_t)$ for $m_t < M_W$

In figure 1.6 we give the plot of the branching ratio[49] for b -----> s γ as a function of m_t with and without QCD corrections (QCD corrections here include the summation of leading - logarithmic contributions). It is clear that the corrected branching ratio is in the experimentally accessible range of order 10^{-5}. We conclude also that from figure 1.6 that the upper bound of the inclusive decay b ---> s γ, would imply a severe constraint on the top quark mass.

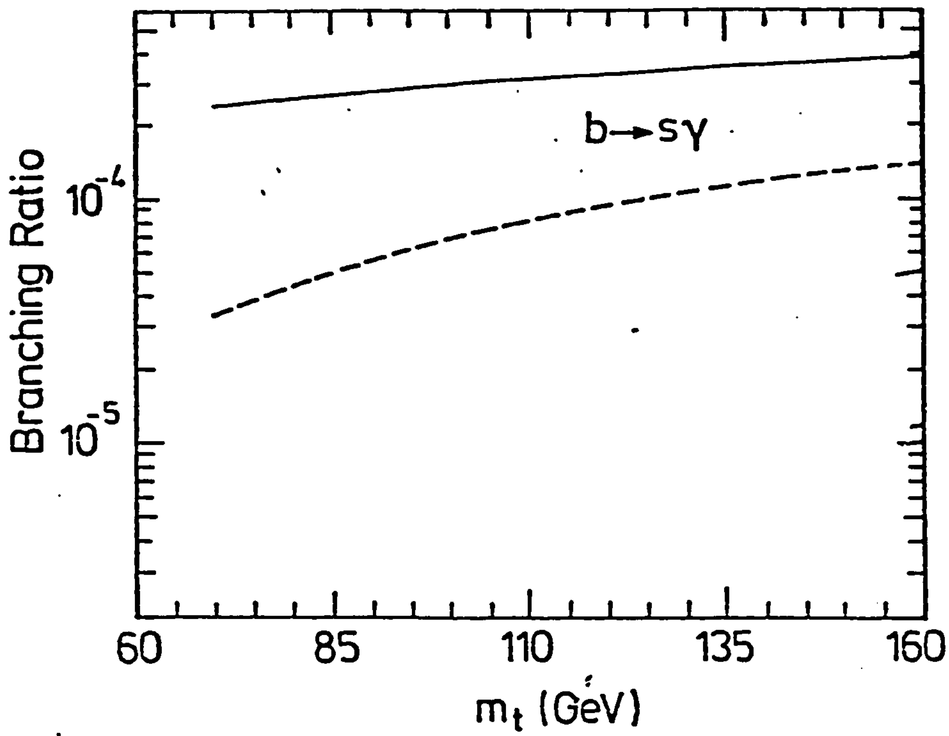


Fig. 1.6 Branching ratios for b -----> s γ as a function of m_t , with (solid line) or without (dashed line) the inclusion of QCD corrections (Ref.49).

10^4 BR	b ——> s γ		B ——> K* γ	
mt (Gev)	No QCD	QCD	No QCD	QCD
40	0.08	2.18	0.01	0.13
65	0.34	2.73	0.02	0.16
80	0.48	3.05	0.03	0.18
100	0.74	3.46	0.04	0.21
120	1.02	3.83	0.06	0.23
150	1.43	4.33	0.09	0.26
200	2.02	5.00	0.21	0.30

Table 1.2 The branching ratios , with and without QCD corrections for b -----> s γ, and B -----> K γ as a function of top-quark mass (Ref.29)

Numerical evaluation of the QCD corrected and QCD uncorrected branching ratios (BR) for B ----> K (892) γ are given in table 1.2 as a function of m_t (Ref.29). We see that for m_t = 40 Gev the ratio $BR_{corr}/BR_{uncorr} \sim 27$ and for m_t = 120 Gev is about 4. This illustrates how the above decays can provide important tests of higher order QCD corrections and top quark mass.

3.1.b HADRONIZATION MODEL

However, despite the above mentioned importance of B decays in testing the standard model, there is a difficulty, which hinders explicit calculation of the relevant matrix element, for the above mentioned test. In fact the estimation of the exclusive hadronic radiative mode is rather difficult and depends on the choice of the hadronization model. Two frameworks have been used in the literature for this purpose. One is based on QCD sum rules embedded in a general Vector Meson Dominance[14,15].

An other framework which have been used extensively is the constituent quark model (CQM). In the present work the CQM based on the Schrodinger equation appropriate to the Coulombic plus screening potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \frac{1 - e^{-\mu r}}{\mu r} \qquad (1.10)$$

will be used to determine the spatial part of the final and initial meson wave functions. To avoid extensive numerical calculations, we will use variational solutions of the Schrodinger equation based on harmonic-oscillator wavefunctions:

$$\phi(r) = \left(\frac{\beta^2}{\pi}\right)^{\frac{3}{4}} e^{-\frac{\beta^2 r^2}{2}}$$

in which the β's are employed as variational parameters.

The hadronic matrix elements involve those of a current expressible in term of the relativistic quark operators. Each matrix elements is expressed in terms of form factors.

In chapter two, we give a detailed account of quark antiquark interactions at short distance, and a phenomenological screening confining potential at large distances will be mentioned. In chapter three we briefly describe the meson states in the weak binding limit. Chapter four is devoted to flavor changing radiative decays of B meson, while in chapter five we extend the same procedure to flavor conserving radiative decays of light vector meson, and finally a general conclusion will be given in Chapter six.

CHAPTER TWO

QUARK-ANTIQUARK POTENTIAL

1. INTRODUCTION

In the quark model hadrons are built up from quarks and antiquarks. Due to the fact that quarks must obey Fermi statistics a new degree of freedom was introduced in mid 1960's known as color charge or color (cf. Chap.1, Sec.1).

The interaction which bind quarks and antiquarks to form mesons is mediated by eight massless gauge bosons[16], the gluons. The quark and antiquark carry the colour charge which is analogous to the electric charge in QED, with a very important difference : whereas the photon has no electric charge in QED, gluons carry color, i.e, the field quanta themselves are a field source; gluons couple both to each other and to quarks. The theory of interaction of gluons and quarks is known as quantum chromodynamics (QCD).

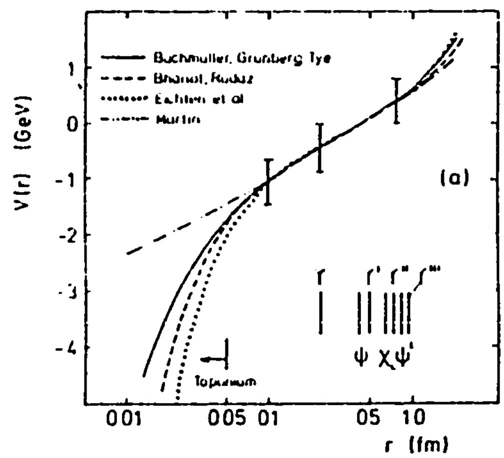


Fig. 2.1 The radial dependence of some typical $Q\bar{Q}$ potential (Ref.21)

It is believed that QCD is the correct theory of strong interactions. One popular approach to deal with this interaction in many cases is the use of potential model supplemented with ideas from QCD to motivate the asymptotic behavior of the potential in two limits[17]:

- For short inter-quark distance (r ---> 0), the coupling constant α_s becomes small (gluon exchange are very weak) and one gluon exchange gives rise to Coulomb-like potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} \qquad (2.1)$$

is expected to be a good approximation

- For large separation of quark and antiquark, the intermediate gluon fields are thought to form a linear tube so that the potential is expected to increases linearly with distance.

There are many phenomenological potential models[18,19,20] which accommodate these two features appropriately. But all approaches lead (numerically) to similar potentials[21] in the region of distances from about 0.1 to 1.0 fm (fig.2.1) all the proposed models are essentially identical in the medium range.

2. COULOMB - LIKE POTENTIAL.

2.1 Quark anti quark scattering.

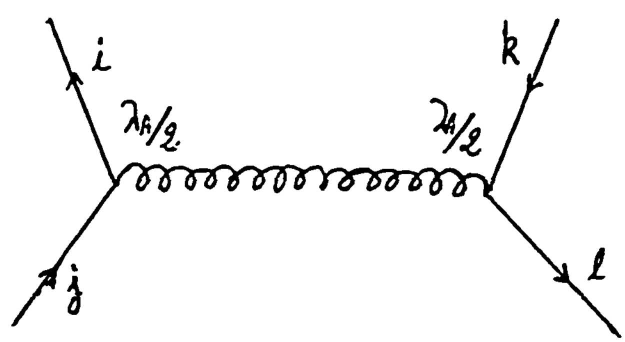


Fig. 2.2 Lowest order Feynman diagrams involving gluon exchange. The color indices are i,j,k for the quark and A for gluon.

The - 4/3 factor which appears in the short range potential (eq. 2.1) comes from the color $SU(3)_{colour}$ group. It may be justified as follow: each quark can have three color degrees of freedom and couple with eight color gluon states: averaging over quark color we get a factor 8/3 and we have to divide by two because the coupling constant is defined as twice the square of the strong color charge. However a rigorous derivation of such factor can be done.

To form color singlet we have to combine final quark and antiquark into a symmetric combination and similarly for the initial quark antiquark. Thus summing over the color indices i,j,k and over the eight colors of gluons, we get:

$$-\left(\frac{\lambda_a}{2}\right)_{ij}\left(\frac{\lambda_a}{2}\right)_{kl}\frac{d_{0jk}}{\sqrt{2}}\frac{d_{0il}}{\sqrt{2}} = -\left(\frac{\lambda_a}{2}\right)_{ij}\left(\frac{\lambda_a}{2}\right)_{kl}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\sqrt{\frac{2}{3}}\sqrt{\frac{2}{3}}\,\delta_{jk}\,\delta_{il}$$

$$= -\frac{1}{12}(\lambda_a)_{ij}(\lambda_a)_{jl} = -\frac{1}{12}\mathrm{Tr}\,(\lambda_a\lambda_a) = -\frac{8}{12}\,2 = -\frac{4}{3}$$

The minus sign come from the fact that in dealing with antiquarks $\bar{v}\,v = -\,\bar{u}\,u$ (u, v are Dirac spinors for particle and antiparticle).

2.2 Matrix elements

Perturbative QCD can be used to calculate the quark - quark scattering amplitude:

$$q_i\,(p_i) + q_j(p_j) \quad \text{------------>} \quad q'_i\,(p'_i) + q'_j(p'_j)$$

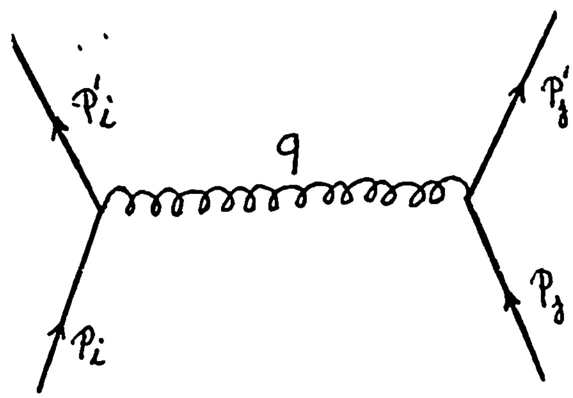


Fig 2.3 Strong scattering via gluon exchange.

The Feynman rules (Appendix B) for QCD gives the matrix element of the above process.

$$S = (2\pi)^4\, \delta^4(p_i+p_j - p_i'+p_j')(-i)^2 \frac{1}{(2\pi)^3}\sqrt{\frac{m_j m_j}{E_j E_j'}}\, \overline{U}\,(p_j')(-g\,\gamma_\mu)U(p_j)$$
$$(-i\frac{\delta_{\mu\nu}}{q^2})\, \frac{1}{(2\pi)^3}\sqrt{\frac{m_i m_i}{E_i E_i'}}\, \overline{U}\,(p_i')(-ig\,\gamma_\nu)U(p_i)$$

$$S = -i\,\widetilde{H}\,(2\pi)^4\, \delta^4(p_i+p_j - p_i'+p_j')$$

where,

$$\widetilde{H} = \frac{g^2}{(2\pi)^6 q^2}\sqrt{\frac{m_j m_j}{E_j E_j'}}\, \overline{U}\,(p_j')\,\gamma_\mu U(p_j)\sqrt{\frac{m_i m_i}{E_i E_i'}}\, \overline{U}\,(p_i')\,\gamma_\nu U(p_i)\,\delta_{\mu\nu}$$

$$= \frac{g^2}{(2\pi)^2} M$$

The matrix element M is given by

$$M = \frac{1}{q^2}\sqrt{\frac{m_j m_j}{E_j E_j'}}\, \overline{U}(p_j')\, \gamma_\mu U(p_j) \sqrt{\frac{m_i m_i}{E_i E_i'}}\, \overline{U}(p_i')\, \gamma_\mu U(p_i) \qquad (2.2)$$

It is well known that it is possible to represent the wave functions of free Dirac particle by two components instead of four. This could be done by performing a canonical transformation of the form e^{iS}, where S is hermitian. The transforming function is given by:

$$\exp(iS(p)) = \frac{E(p)+m + i\,\vec{\gamma}.\vec{p}}{\sqrt{2E(E+M)}} \qquad (2.3)$$

The above transformation known as Foldy Wouthuysen[22,23] transformation and has the effect of removing all odd operators so only even operators contribute. Using:

$$\sqrt{\frac{m}{E}}\ U(p) = e^{-iS(p)}\chi \quad , \quad \sqrt{\frac{m}{E}}\ U^{+}(p)\,\gamma_4 = \overline{\chi}\, e^{+iS(p)}$$

$$\text{Where } \chi \text{ are Pauli spinors: } \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

then the evaluation of the matrix element involves the calculation of $\bar{u}(p')\,\Gamma_a u(p)$, i.e,

$\bar{\chi}\, e^{+iS(p)}\,\Gamma_a\, e^{-iS(p)}\,\chi$, where $\Gamma_a = \gamma_4, \gamma$ stand for Dirac matrices. The Pauli spinors $\bar{\chi}$ and χ on left and right respectively can be omitted, and only even operators* will contribute.

2.3 NON RELATIVISTIC LIMIT.

Consider the non - relativistic limit to order ($|p|^2/m^2$) of

$$\frac{m}{\sqrt{E\,E'}}\,\bar{U}(p')\Gamma_a U(p) \qquad (2.3.1)$$

This is given in appendix A. Using the results of this appendix, the matrix element M has the form:

$$\begin{aligned} M = \frac{1}{\vec{q}^{\,2}} \Big(1 &- \frac{\vec{q}^{\,2}}{8}\left(\frac{1}{m_i^2} + \frac{1}{m_j^2}\right) - \frac{i}{4m_j^2}\vec{\sigma}_j \cdot (\vec{q}\times\vec{p}_j) + \frac{i}{4m_i^2}\vec{\sigma}_i \cdot (\vec{q}\times\vec{p}_i) \\ &+ \frac{1}{4m_im_j}\left(-4\,\vec{p}_i\cdot\vec{p}_j + 4\,(\vec{p}_j\cdot\vec{q})\,(\vec{p}_i\cdot\vec{q})\right) + \\ &\frac{i}{4m_im_j}\,2\,\vec{p}_i\cdot(\vec{\sigma}_j\times\vec{q}) + \frac{i}{4m_im_j}\,2\,(\vec{\sigma}_i\times\vec{q})\cdot\vec{p}_j \\ &- \frac{1}{4m_im_j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)\,\vec{q}^{\,2} + \frac{1}{4m_im_j}(\vec{\sigma}_i\cdot\vec{q})(\vec{\sigma}_j\cdot\vec{q}) \Big) \end{aligned} \qquad (2.4)$$

---

*Operators which couple the large and the small components of Dirac wavefunction are called odd, whereas even operators do not couple large and small components.

where:

$$\vec{q} = \vec{p}_i' - \vec{p}_i \quad = \vec{p}_j - \vec{p}_j'$$

The form of the potential in r - space is found by taking the Fourier transform of the matrix element M:

$$V(r) = -\frac{4}{3}\frac{g^2}{(2\pi)^3}\int_0^\infty e^{i\vec{q}.\vec{r}} M(\vec{q})\, d^3\vec{q}\ , \qquad g^2 = 4\pi\,\alpha_s$$

$$= -\frac{4}{3}\frac{\alpha_s}{2\pi^2}\int_0^\infty e^{i\vec{q}.\vec{r}} M(\vec{q})\, d^3\vec{q} \tag{2.5}$$

Evaluation of the integral gives the following one gluon exchange potential.

$$V(r) = -\frac{4}{3}\alpha_s\Big(\frac{1}{r} - \frac{1}{2m_im_j}\Big(\frac{\vec{p}_j \cdot \vec{p}_i}{r} + \frac{\vec{r}\,(\vec{r}\,.\,\vec{p}_i\,).\,\vec{p}_i}{r^3}\Big) - \frac{\pi}{2}\,\delta^3(\vec{r})\,\Big(\frac{1}{m_i^2} + \frac{1}{m_j^2}$$

$$+ \frac{16}{3m_im_j}\vec{S}_i.\vec{S}_j\Big) - \frac{1}{r^3}\Big(\frac{1}{m_i^2}(\vec{r}\times\vec{p}_i).\vec{S}_i - \frac{1}{m_j^2}(\vec{r}\times\vec{p}_j).\vec{S}_j\Big) - \frac{2}{m_im_j}\Big(\,(\,\vec{r}\times\vec{p}_i\,).\,\vec{S}_j$$

$$- (\,\vec{r}\times\vec{p}_j\,)\,.\,\vec{S}_i + 3\,(\,(\,\vec{S}_i.\vec{r}\,)\,(\,\vec{S}_j.\vec{r}\,) - \frac{1}{3}\,\vec{S}_i\,.\,\vec{S}_i\,)\,)\Big) \tag{2.6.a}$$

.

For l = 0, the S state of quark antiquark, the angular momentum and tensor force part vanish. Thus the S - wave interaction between quark and antiquarks is given by:

$$V(r) = -\frac{4}{3}\alpha_s\left(\frac{1}{r} - \frac{1}{2m_im_j}\left(\frac{\vec{p}_j \cdot \vec{p}_i}{r} + \frac{\vec{r}\,(\vec{r}\cdot\vec{p}_i).\vec{p}_i}{r^3}\right) - \frac{\pi}{2}\delta^3(\vec{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16}{3m_im_j}\vec{S}_i.\vec{S}_j\right)\right) \qquad (2.6.b)$$

The term:

$$-\frac{\pi}{2}\delta^3(\vec{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16}{3m_im_j}\vec{S}_i.\vec{S}_j\right)$$

gives the hyperfine splitting between $^3S_1$ and $^1S_0$ states.

However at short distances in the leading non - relativistic limit the Coulombic term can be considered as the dominant one.

$$V(r) = -\frac{4}{3}\alpha_s\frac{1}{r} \qquad (2.7)$$

3. LONG RANGE POTENTIAL

3.1 STRING PICTURE

Free quark have never been observed, thus Coulomb-like potential is not sufficient to confine quarks inside hadrons. Although, one can knock an electron out of the hydrogen atom (which is also bound by coulomb - like potential), one cannot knock out a quark out of hadrons by hitting it with sufficient energy. Hence the one gluon exchange potential dominate at short distances for qq system. At long range a confining potential is needed.

As the distance between qq increases, the interaction (eq. 2. 7) is modified in QCD at large distances of order one Fermi or more. The chromoelectric flux lines bunch together into a tube and the force lines tend to be parallel[3,17] (fig 2.4). This leads to a constant force between quark-antiquark i.e the potential has the form

$$V(r) = \sigma \cdot r \qquad (2.8)$$

An infinitely rising potential such as (eq 2.8) permanently confines quarks since an infinite amount of energy would be needed to produce quarks as separate entities. The force between quark and antiquark is transmitted by a color flux with a constant energy per unit length corresponding to σ. As the interquark distances increase, one expects that the system will breaks up as soon as there is enough energy available to create a new quark-antiquark pair (fig 2.4), which combine with the initial pair forming two color singlets. These considerations form a link to the string picture of hadrons.

In the string picture quark antiquark are linked by a string (fig 2.5). Simple calculations based on such a picture, supplemented with experimental data, permit rough estimation of the string constant.

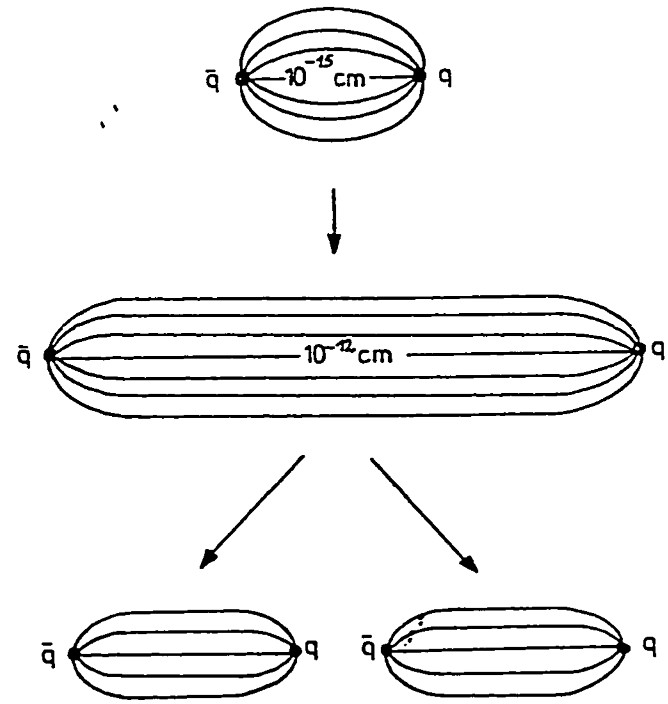


Fig. 2.4 Hypothetical collimation of the flux lines in QCD. If energy exceeds the threshold for quark pair creation the string breaks.

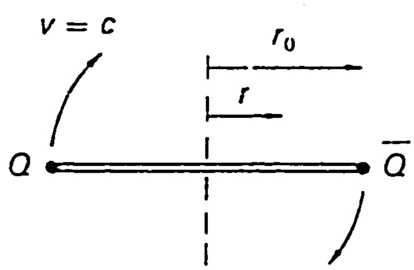


Fig 2.5 String model used in calculating the relation between angular momentum and mass of hadron.

Assume now that the string rotates about its center and the quark , antiquark located at the ends of the string are massless so that the maximum velocity is equal to the velocity of light . The angular momentum of the gluon tube is (fig. 2. 5)

$$J = 2\sigma \int_0^{r_0} \frac{r\, v(r)\, dr}{\sqrt{1 - v^2(r)}}, \quad \text{where we have } v(r) = \frac{r}{r_0} \text{ (velocity of light } c = 1 \text{)}$$

Thus $J = \dfrac{\sigma r_0^2 \pi}{2}$, r_0 being half of the string length, while

the total mass $E = M = 2\sigma \displaystyle\int_0^{r_0} \frac{dr}{\sqrt{1 - v^2(r)}} = \sigma r_0 \pi$

$$\text{Thus} \quad J = \frac{1}{2\pi\sigma} E^2 \tag{2.9}$$

Equation (2. 9) is known as the Regge slope. It holds for cases of constant energy density of the string, i.e, for a linear potential. The linear dependance of the angular momentum, on the square of the mass (eq. 2. 9) is in a good agreement with data on the rotational excitation of different hadrons. This fact is illustrated in fig. 2. 6. a & 2. 6. b

3. 2 SCREENING EFFECT

The basic understanding of the confinement of quarks is not yet complete and the increasing of linear potential V(r) ~ r seems too fast. There are now many phenomenological potential models which accommodate the end points[21] (r -----> 0 and r ------> ∞) to account for the experimental data of heavy quarkonuim which are mostly in the region of short to intermediate distances. It is uncertain whether the confining potential has a linear form. In QCD the interaction among gluons leads to a diminution of the effective strong interaction charge at short distances. Qualitatively, at large distances, the potential would be strongly weakened by the screening effects due to the quark pair creation, and the confining interaction potentials must fall below the linear potential[19]. In fact many people proposed different potential weaker than the linear one[17,19].

Recently F. Langhamer et al[24] used lattice gauge theory to investigate the potential between quarks. They found indications that at large distances the potential deviates from a linear potential . This is expected from the fact that spontaneous quark pair creation in the field stretched between the static quark screens the charge of the color sources at a scale of order one fermi and this turns the linearly rising potential at large distances into a short range one.

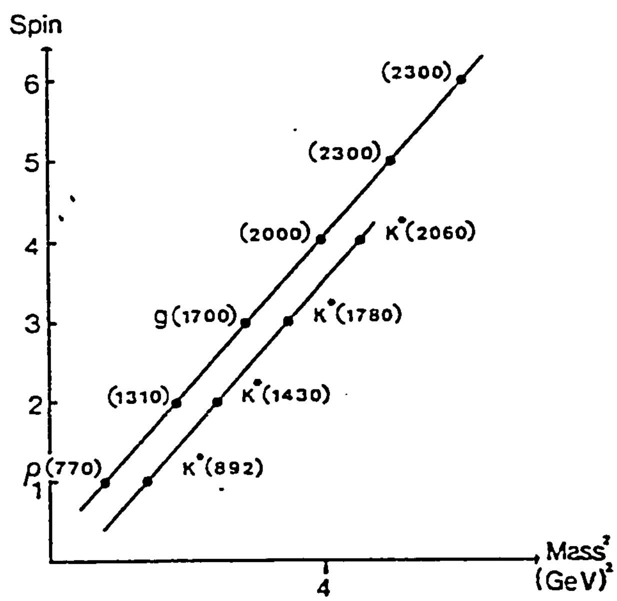


Fig.2.6.a Plot of ρ and K* Regge trajectories (ref.3)

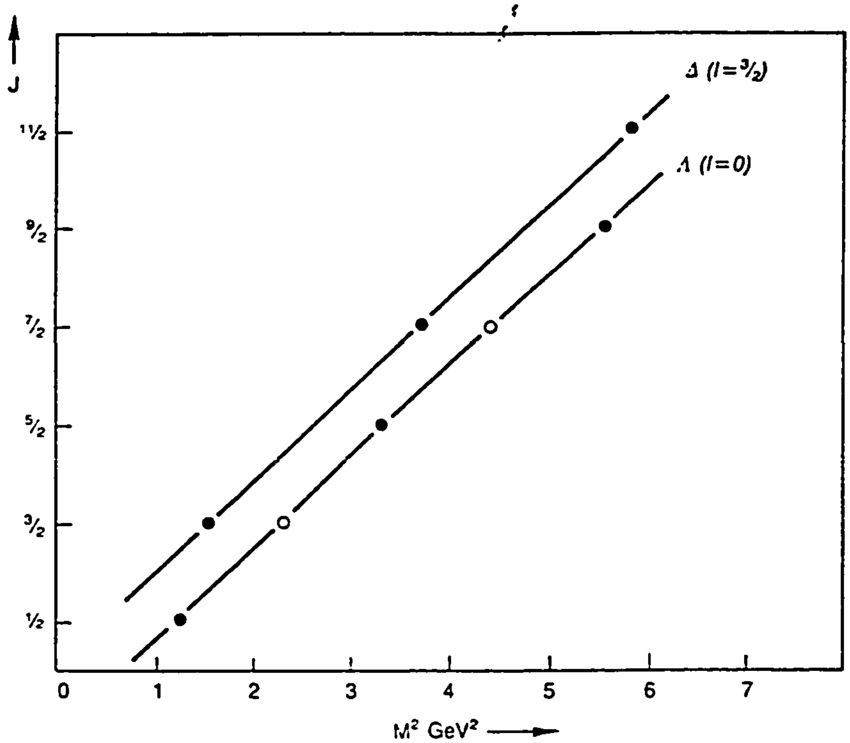


Fig 2.6.b Plots of spin J against mass, squared, for baryon resonances of the Δ family (I = 3/2) and the Λ family (I = 0). Positive and negative parity states are shown as full and open circles (ref. 51)

For a system of a heavy quark and a heavy antiquark F. Langhamer et al get a potential curve presented in figure 2.7 . Such a curve could be fitted by superposition of a Coulombic and a linear confining term. For the case where a light quark is incorporated into the system their calculation gives the curve shown in figure 2.8.

It is clear from fig.2.8 that for large distances the potential is weaker than a linear one. F. Langhamer et al parametrize the potential by an ansatz that corresponds to a cut - off confinement form with screening length μ^{-1}.

Such parametrization maintains the Coulombic plus linear behavior for small and medium range , but for large distances the linear part is damped.

While the Coulombic term is flavor dependant through the running coupling constant α_s, only the linear term which is flavor independent will be damped, therefore, in the presence of a light quark the potential has the form:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \frac{1 - e^{-\mu r}}{\mu r} \qquad (2.10)$$

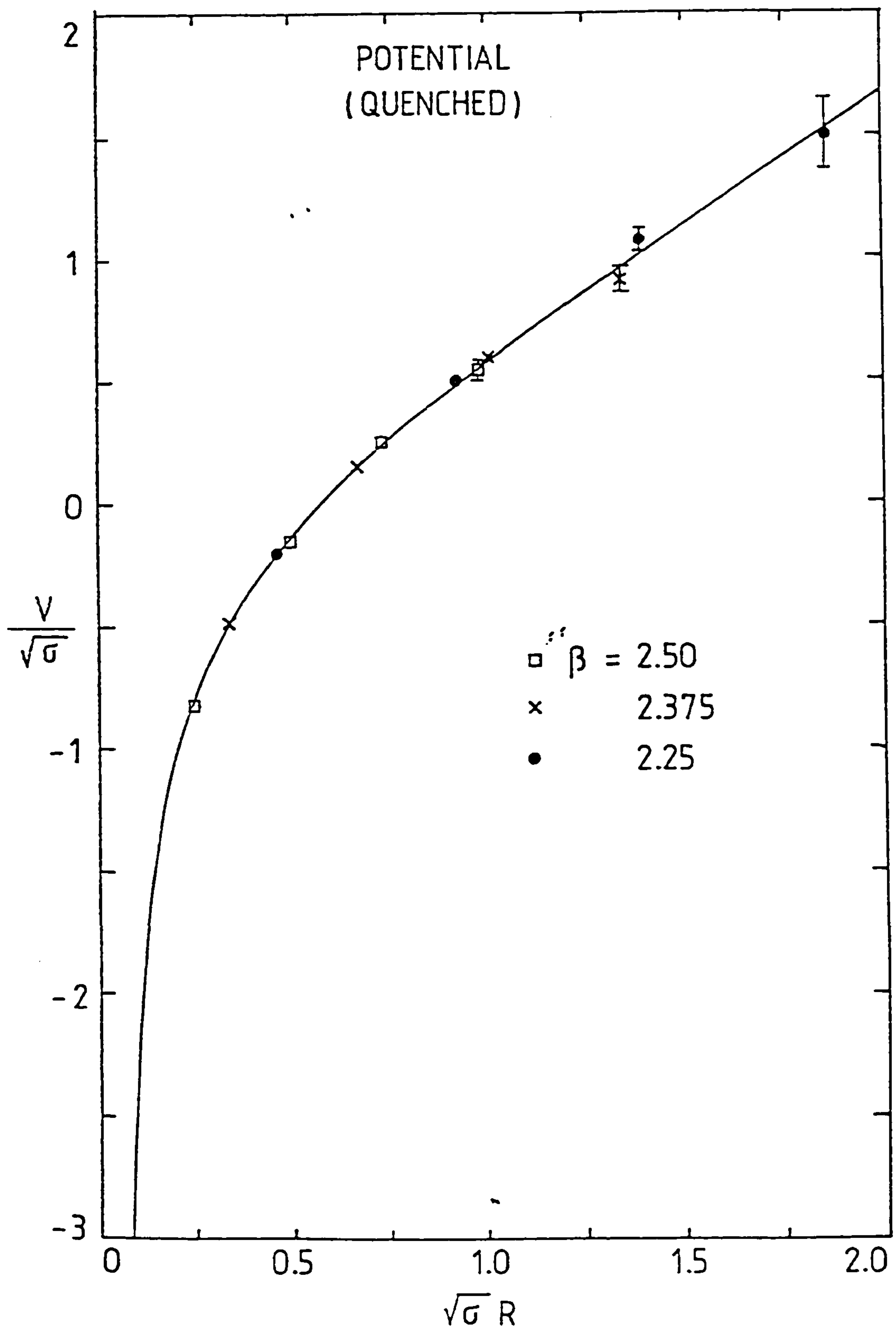


Fig. 2.7 Radial variation of Potential between two heavy quarks (ref.24)

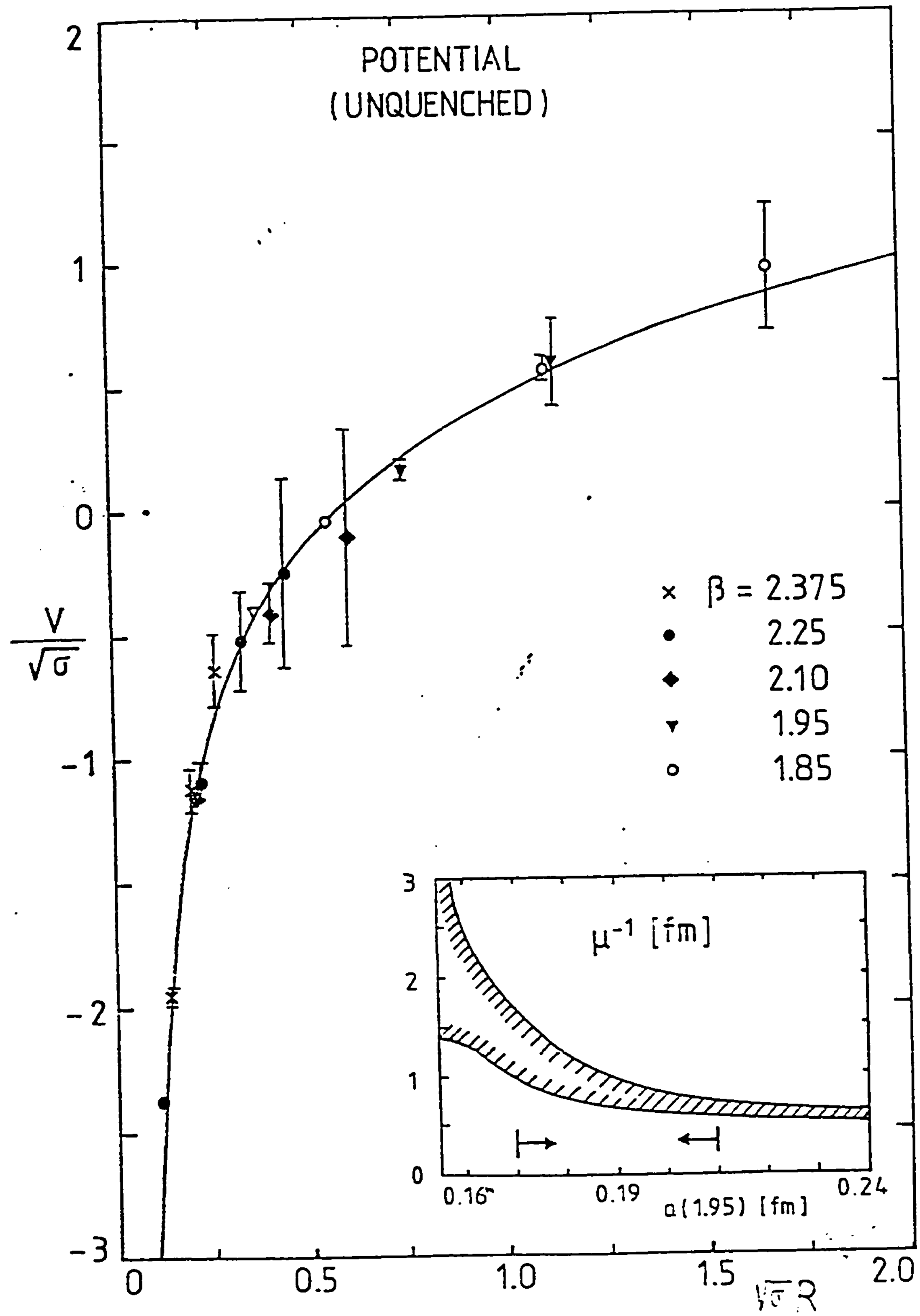


Fig. 2.8 Potential between a heavy and light quark (ref.24)

CHAPTER THREE

MESON STATES

1. MESON WAVEFUNCTIONS.

The meson wavefunctions may be explicitly constructed as a product of the usual three-dimensional momentum wavefuctions and the internal space wavefunctions. In the weak binding limit, the construction of the meson state[25-28] starts with the construction of a free quark-antiquark momentum shell in its center of mass frame with definite J^P and definite relative momentum **p**. The general quark-model states are superpositions of such states weighted with a wavefunction that depends on **p**. Then noting that:

$$P_1 = \frac{m_1}{M} P - p \quad \text{and} \quad P_2 = \frac{m_2}{M} P + p$$

where P_1 is momentum of antiquark with $m_1 = m_{\bar{q}}$ and P_2 is that of the quark with $m_2 = m_q$ and $\mathbf{P}$ is the center of mass momentum. The ground state meson wavefunction (since the orbital angular momentum is zero, thus there is no spin angular momentum coupling) takes the following form:

$$|\mathcal{M}(\mathbf{P},S,S_3)\rangle = \int d^3\mathbf{p}\ \phi_M(\mathbf{p})\ \chi(S,S_3)\phi_f\phi_c \times$$
$$b^{+}\left(\frac{m_q}{M}\mathbf{P}+\mathbf{p}\right) d^{+}\left(\frac{m_{\bar{q}}}{M}\mathbf{P}-\mathbf{p}\right) |\,0\,\rangle \qquad (3.1)$$

$$M = m_q + m_{\bar{q}} = m_1 + m_2$$

where ϕ_M , $\chi(S,S_3)$, ϕ_f and ϕ_c are momentum, spin, flavor and color wavefunctions of the physical meson $\mathcal{M}$.

The operators b^+, d^+ create a quark and antiquark respectively, while $|\ 0\ \rangle$ is the vacuum state (which should not be confused with the null vector). The vacuum state is normalized so that $\langle 0 \mid 0 \rangle = 1$.

The function $\phi_M(\mathbf{p})$ is the Fourier transform of the ordinary Schrodinger wavefunction $\psi(\mathbf{x})$:

$$\psi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\ e^{i\,\mathbf{p}.\mathbf{x}}\ \phi_M(\mathbf{p}) \qquad (3.2)$$

note that: $\int d^3\mathbf{p} |\ \phi_M(\mathbf{p})\ |^2 = 1$

It is convenient to normalize the meson states to a momentum conserving δ-function i.e:

$$< M\,(P',S',S'_3)\mid M\,(P,S,S_3) > = \delta_{ss'}\,\delta_{s_3 s'_3}\,\delta^3(\,P' - P\,) \qquad (\,3.3\,)$$

SPIN PART:

From a spin 1/2 quark and spin 1/2 antiquark one can only construct total spin zero (singlet), or spin one (triplet) states. Denoting the spin projections $S_3 = \pm\,1/2$ by an arrow pointing up or down respectively, the resulting spin wavefunction $\chi\,(\,S,\,S_3)$ takes the form :

- Symmetric (spin 1)

$$\begin{aligned} S=1\,,\quad & S_3=1 & \chi(1,1) &= \mid\uparrow\uparrow\,> \\ & S_3=0 & \chi(1,0) &= \frac{1}{\sqrt{2}}[\,\mid\uparrow\downarrow>+\mid\downarrow\uparrow\,>] \\ & S_3=-1 & \chi(1,-1) &= \mid\downarrow\downarrow\,> \end{aligned} \qquad (\,3.4\,)$$

- Antisymmetric (spin 0)

$$S=0\,,\quad S_3=0 \qquad \chi(0,0) = \frac{1}{\sqrt{2}}[\,\mid\uparrow\downarrow>-\mid\downarrow\uparrow\,>]$$

COLOR WAVEFUNCTION:

The hadrons have no net color (hence they are "colorless"), i.e. they are color singlets. Thus a meson consists of quark and antiquark having equal and opposite color. The color singlet wavefunction is given by

$$\psi_c = \frac{1}{\sqrt{3}} (R\,\overline{R} + B\,\overline{B} + G\,\overline{G}) \ , \qquad \text{symmetric}$$

FLAVOR WAVEFUNCTION:

The following table gives the flavor wavefunctions for the different mesons to be studied.

mesons	flavor wavefunction
B^+	$u\bar{b}$
ω^0	$(1/2)^{1/2}(u\bar{u} + d\bar{d})$
ρ^+ π^+	$u\bar{d}$
ρ^0 π^0	$(1/2)^{1/2}(u\bar{u} - d\bar{d})$
ρ^- π^-	$d\bar{u}$
K^{0*} K^0	$d\bar{s}$
K^{+*} K^+	$u\bar{s}$

2. EXPLICIT SPACE WAVEFUNCTION.

In the nonrelativistic limit the hamiltonian of the quark and antiquark system is:

$$H = p^2/2m + V(r)$$

where m stands for the the reduced mass of the quark - antiquark system. The potential is given by[24]:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \frac{1 - e^{-\mu r}}{\mu r} \qquad (3.5)$$

The average value of the energy is : $E = \langle \psi | H | \psi \rangle$, where the ψ states are normalized to unity. To proceed further we take a Gaussian wavefunction:

$$\psi(r, \beta) = \frac{\beta^{3/2}}{\pi^{3/4}} e^{-\frac{\beta^2 r^2}{2}} \qquad (3.6)$$

as a trial function with β as the variational parameter. Hence the expectation value of the hamiltonian is:

$$E(\beta) = \int \psi^*(r, \beta) \left(\frac{\nabla^2}{2m} + V(r) \right) \psi(r, \beta)\, d^3r \qquad (3.7)$$

which on using equations (3.5) and (3.6) for the potential and the wavefunction respectively, yields the following result:

$$E(\beta)=\frac{3}{4m}\beta^2-\frac{8\,\alpha_s}{3\sqrt{\pi}}\beta+\frac{\sigma}{\mu}+\frac{\sigma}{\sqrt{\pi}\,\beta}-\left(\frac{\sigma\mu}{2\beta^2}+\frac{\sigma}{\mu}\right)e^{\frac{\mu^2}{4\beta^2}}\left(1-\frac{2}{\sqrt{\pi}}\int_0^{\frac{\mu}{2\beta}} e^{-t^2}dt\right) \qquad (3.8)$$

REMARK:

As μ goes to zero (the screening length $1/\mu$ becomes large) the potential V(r) behave like a coulombic plus linear potential. In fact one can calculate the limit of E (β) as μ approaches zero. Then the expectation value is the same as the one obtained by Deshpande et al[29], in the case of a linear confining potential.

The values of β will be needed for the numerical estimation of the different form factors involved in the hadronic matrix elements (in Chap.4 and 5). Thus these values are those for which the expectation value E (β), is minimum.

We take the same numerical values as Deshpande et al for the different parameters in the equation $E(\beta)$ which are:
α_S = 0.50, σ = 0.18 Gev^2, $m_u = m_d$ = 0.33 Gev, m_s = 0.55 Gev, m_b = 5.12 Gev.

While, for the screening length we used the numerical value (ref. 24): $\mu^{-1} = 1.24$ fm.

To determine the desired values of β which minimize $E(\beta)$, one has to solve:

$$\frac{\partial E(\beta)}{\partial \beta} = 0 \quad \text{and} \quad \frac{\partial^2 E(\beta)}{\partial^2 \beta} > 0 \qquad (3.10)$$

This can be done by use of the Mathematica (ref. 50) software as illustrated in the next section. The minimum of $E(\beta)$ occurs for $\beta = 0.32$ Gev, and $\beta = 0.24$ Gev, for B and K meson respectively.

3. NUMERICAL COMPUTATION

In what follows we show the output of the Mathematica program; b, e[b] and k[b] stand for the variational parameter (beta), the average energy for B and K mesons respectivly. The numerical values of beta which minimize e[b] and k[b] will be determined as follow:(The unit used is Gev)

```
m1 = 5.12; m2 = 0.33; σ = 0.18  ; αs = 0.50
μ = 0.244687; mB = m1 m2 /(m1 + m2);
```

■ **The average energy for B meson is :**

```
e[b_] := 3 b^2/(4 mB) - 8 αs b/(3 Sqrt[Pi])+ σ/μ+
σ/(Sqrt[Pi] b) - (σ μ/(2 b^2) +σ/μ) Exp[μ^2/(4 b^2)] *
(1-(2/Sqrt[Pi]) Integrate[Exp[-t^2], {t,0,μ/(2 b)}])
```

```
m1 = 0.33; m2 = 0.55; mk = m1 m2 /( m1 + m2 );
```

■ **The average energy for K meson is :**

```
k[b_] := 3 b^2/(4 mk) - 8 αs b/(3 Sqrt[Pi])+ σ/μ+
σ/(Sqrt[Pi] b) - (σ μ/(2 b^2) +σ/μ) Exp[μ^2/(4 b^2)] *
(1-(2/Sqrt[Pi]) Integrate[Exp[-t^2], {t,0,μ/(2 b)}])
```

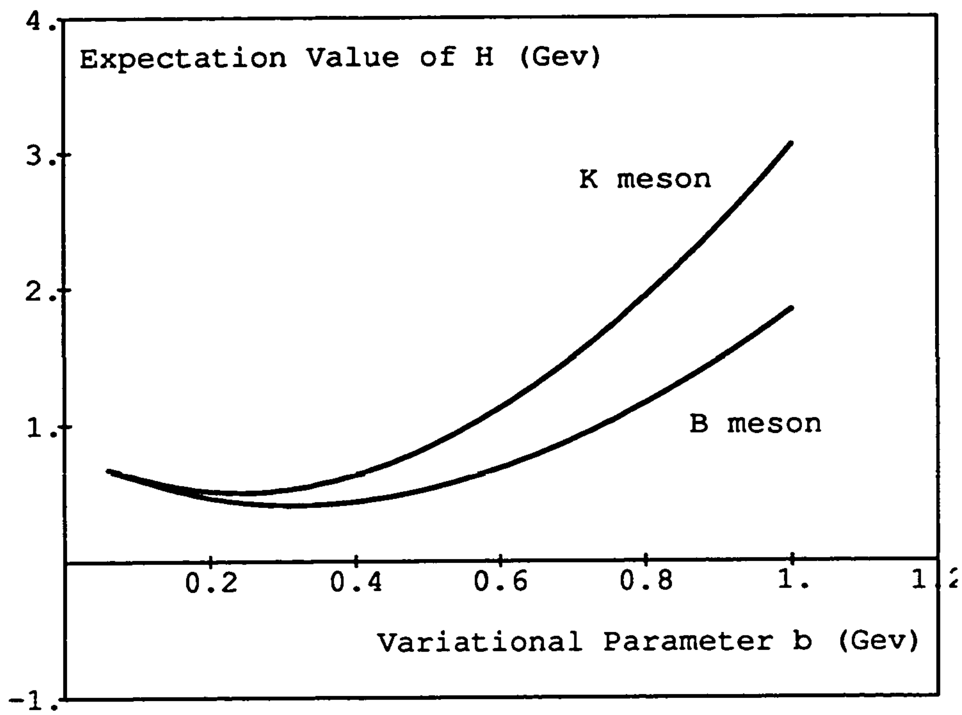
■ Plot of e(b) and k(b) as functions of b.

```
text = {Text["Variational Parameter b (Gev)",
 {0.8, -0.6}],
 Text["Expectation Value of H (Gev)", {0.38, 3.7}],
 Text["B meson", {0.95, 1}],
 Text["K meson", {0.8, 2.8}]};

 plot = Plot[{e[b],k[b]}, {b, 0.06, 1},
        Ticks -> {Range[0, 1.2, 0.2], Automatic},
        PlotRange -> {{0, 1.2}, {-1, 4}},
        Framed -> True]

-Graphics-

Show[Graphics[text], plot,
 AspectRatio -> .8]
```



■ Determination of minima

The above graph shows that e(b) and k(b) have minima for b around 0.3 and 0.2 respectively so the required values are given below.

```
FindMinimum[e[b], {b, 0.3}]
{0.412149, {b -> 0.315975216539}}
```

```
FindMinimum[k[b], {b, 0.2}]
{0.504904, {b -> 0.242289}}
```

CHAPTER FOUR

FLAVOR CHANGING

1. INTRODUCTION

The lightest mesons containing one b or $\bar{b}$ quark will be stable with respect to the strong and electromagnetic interactions. They can decay by weak interaction, since emission of charged boson W^- takes one from the lower to the upper components (u, c or t) of the weak isospin doublets.

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L , \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

In this chapter we will use a relativised constituent quark model to evaluate the different form factors and estimate the recoil effects for the flavor-changing radiative B-decay. In particular we will focus on the standard model prediction for the rate of B-meson decay to a hard photon and a strange hadronic final states: B -----> $K^*(892)\ \gamma$. The rate for this inclusive process is likely to be dominated by short-distance physics which give rise to a local b ----> s γ single quark transition. The short-distance dynamics is described by an effective hamiltonian H_{eff}, which can be calculated[10-13] using the short-distance techniques of QCD.

If QCD corrections are not taken into consideration, the contribution from one-loop diagrams, due to W-exchange, are more or less suppressed due to the GIM mechanism. The QCD corrections for the process B -------> $K^*\ \gamma$ change the GIM suppression in the amplitude from being a power law, $(m_t^2 - m_c^2)/M_w^2$, to the softer form of a logarithm, $\ln(m_t^2/m_c^2)$, thus enhancing the amplitude of the indicated process[13].

The long-distance effects of the strong interactions, on the other hand, are too weak to affect the quark decay, and they are taken into consideration by taking the matrix element of the concerned operator (H_{eff}) between the hadron states.(For a recent study of long distances effects see ref. 30).

The B-meson has several advantages; the emitted photon in B ----> $K^*\ \gamma$ is monochromatic[13] and the branching ratio of few times 10^{-5} is accessible to presently planned experiments. The presence of a heavy b-quark permits the use of the spectator approximation[32,33] . This reduces the problem to the free decay of a b quark, and neglects contributions from u and d quarks.

2. MATRIX ELEMENTS

2.1 SPECTATOR MODEL

To proceed with the hadronic matrix element calculations it is advantageous to make certain approximations. Firstly the partner of the b quark in B meson is treated as a spectator. Figure 4.1 shows a diagram contributing to the process B --->K^* γ . The calculations of the matrix element of the effective weak hamiltonian are carried out at the quark level[31] . In fact, since the typical momentum transfer in the inclusive decay of hadrons

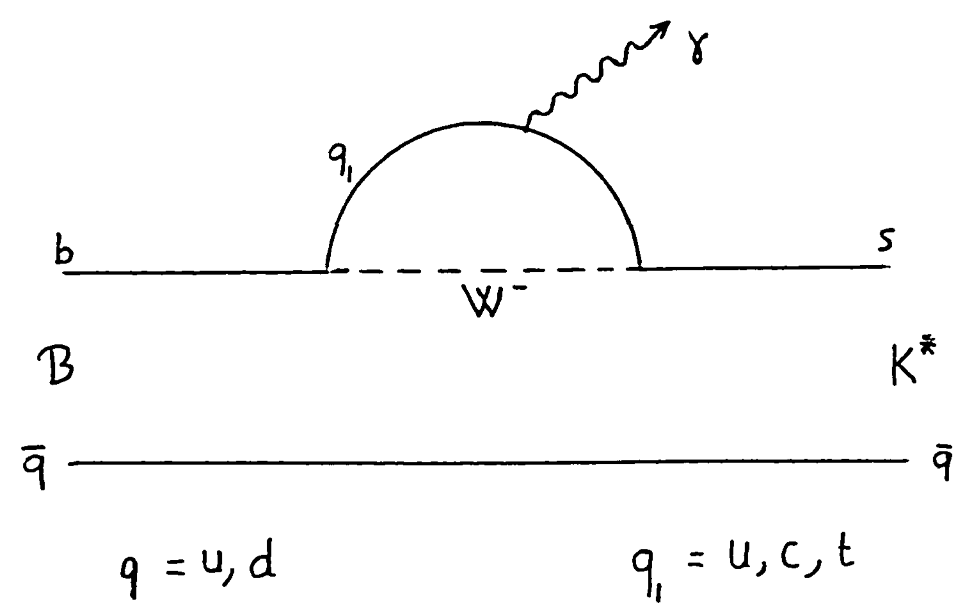


Fig 4.1 Quark level diagram depicting an amplitude contributing to B ---> $K^*\gamma$

which contain a sufficiently heavy quark Q, is of order its mass m_Q , one can neglect all other effects such as bound-state effects, soft hadronic interaction,..., on the decaying hadron, since the heavy-quark mass is much larger than the ordinary hadronic scales represented by light constituent mass, confinement radius,.. etc. . Such a model is called the spectator model[32].

In the spectator model one directly carries over the quark level calculation as the hadronic results. Furthermore we assume that the spectator quark and gluon arrange themselves to form final bound state particles together with the quark coming from the decaying (heavy) quark with no benefit in the overall rate.

2.2 FORM FACTOR.

It is straightforward to calculate the inclusive process B -----> X γ (where X contains no charm) by equating it with b -----> s γ. The matrix element, for b -----> s γ is eq.(1.8). Without the approximation $m_s << m_b$ it is given by

$$M(b \longrightarrow s\gamma) = C[m_b \bar{s}\, \sigma_{\mu\nu} q^{\nu}\ b_R + m_s \bar{s}\, \sigma_{\mu\nu} q^{\nu}\ b_L]\ \varepsilon^{\mu}(q) \qquad (4.1)$$

$$b_L = \frac{1}{2}(1 + \gamma_5)b \ , \qquad b_R = \frac{1}{2}(1 - \gamma_5)b$$

and the rate [29]:

$$\Gamma(b \longrightarrow s\,\gamma) = \frac{m_b^5\, C^2}{16\,\pi}\left(1-\frac{m_s^2}{m_b^2}\right)^3\left(1+\frac{m_s^2}{m_b^2}\right) \qquad (4.1\text{-a})$$

The small correction caused by the second term of eq. (4.1) is given by m_s^2/m_b^2 . s and b in eq. 4.1 stand for the quarks field which is decomposed as follows

$$S(x) = \int d^3 p_s \left(\frac{m_s}{E_s}\right)^{\frac{1}{2}} \sum_{n,c} \left[u_n^c(p_s)\, b_{n,c}^s(p_s) e^{i p_s . x} + v_n^c(p_s)\, d_{n,c}^{s\,+}(p_s) e^{-i p_s . x} \right] \qquad (4.2)$$

here $\bar{s} = s^+ \gamma_4$, n,c are spin and color indices. $b_{n,c}^f(p_f)$, $d_{n,c}^{f\,+}(p_f)$ represents the annihilation operators of a quark and the creation-operator of an antiquark, with spin-n, flavor-f, color-c, and momentum p_f respectively. u and v are free Dirac spinors defined by

$$u_f^c(p_f) = \sqrt{\frac{E_f + m_f}{2m_f}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma} . p_f}{E_f + m_f} \end{pmatrix} \chi_f^c \qquad (4.3)$$

$$v_f^c(p_f) = \sqrt{\frac{E_f + m_f}{2m_f}} \begin{pmatrix} \frac{\vec{\sigma} . p_f}{E_f + m_f} \\ 1 \end{pmatrix} \bar{\chi}_f^c \qquad (4.4)$$

where $\bar{\chi}_n^c = i\, \sigma_2 \chi_n^c$ and the spinors are normalized to one, that is:

$\bar{u}(p)\, u(p) = 1$

2.2-a LORENTZ STRUCTURE OF MATRIX ELEMENTS

Considering Lorentz invariance the matrix element can be expressed in terms of a few real Lorentz scalar functions (form factors) which allow one to parametrize the internal structure of the particle in a very condensed form. This is achieved by making use of some general properties of the matrix element, such as its behavior under Lorentz transformations, current conservation, hermiticity, parity conservation, etc.

As the operator $\bar{s}\,\sigma_{\mu\nu} q^{\nu}\, b_R$ contain a vector and an axial vector , its matrix element between initial and final meson states must be a linear combination of vector and axial vectors. For the construction of such a vector and the axial vector we have at our disposal the vectors (momentum of B and K^*) P, K and the polarization vector $\varepsilon(K)$. Therefore the most general Lorentz invariant decomposition of the above operator between the vector and pseudoscalar mesons state K^* and B is

$$M_\mu \equiv i\sqrt{2K_0\, 2P_0}\; \left< K^*(K) \right| \bar{s}\,\sigma_{\mu\nu} q^{\nu}\, b_R \left| B(P) \right> \qquad (4.5)$$

$$= i\varepsilon_{\mu\nu\lambda\sigma}\, \varepsilon^{\nu}(K) P^{\lambda} q^{\sigma} f_1(q^2) + (m_B^2 - m_{K^*}^2)\, \varepsilon_\mu(K)\, f_2(q^2)$$
$$+ (q.\, \varepsilon(K))\,(P_\mu + K_\mu)\, f_3(q^2) + (q.\, \varepsilon(K))\,(P_\mu - K_\mu)\, f_4(q^2) \qquad (4.6)$$

where $q = P - K$, is the four momentum transfer, and $f_i(q^2)$ are the real form factors - functions of the square momentum transfer q^2.

The states we are using satisfy the non-covarient orthogonality condition equation (3.3), and this is the origin of the square root factor on the right-hand side of the above eqs.(4,5).

The different functions $f_i(q^2)$ must be Lorentz scalars and thus can only depend on Lorentz scalar quantities such as P^2, K^2 and q^2 . Since, the final and initial meson state are on-mass-shell states of a given momentum and spin, then $P^2 \sim m_B{}^2$, $K^2 \sim m_K{}^2$ are constants, so the only variation must be in the scalar invariant variable q^2.

2.2-b GAUGE CONDITION

The decay B -----> $K^* \gamma$, with a real photon, is described by the interaction $M = M_\mu \varepsilon^\mu(q)$. Due to the gauge condition $q_\mu \varepsilon^\mu(q) = 0$, the fourth term in eq. (4.6), which is proportional to $f_4(q^2)$, does not contribute.

2.2-c CURRENT CONSERVATION

The current conservation condition $q^\mu M_\mu = 0$, gives

$$(m_B^2 - m_{K*}^2)\, \varepsilon_\mu(K)\, q^\mu f_2(q^2) + (q.\, \varepsilon(K))\, (P_\mu + K_\mu)\, q^\mu f_3(q^2) = 0$$

Since $(P_\mu + K_\mu)\, q^\mu = P^2 - K^2 = m_B^2 - m_{K*}^2$, it follows that $f_2 + f_3 = 0$. We are thus left with two form factor instead of four. Eq. (4.6) expressed in term of the independent form factors f_1 and f_2 becomes:

$$\begin{aligned} M_\mu \;=\; & i\varepsilon_{\mu\nu\lambda\sigma}\, \varepsilon^\nu(K)\, P^\lambda q^\sigma f_1(q^2) + \\ & \left[(m_B^2 - m_{K*}^2)\, \varepsilon_\mu(K) - (q.\, \varepsilon(K))\, (P_\mu + K_\mu) \right] f_2(q^2) \qquad (4.7) \end{aligned}$$

As far as Lorentz invariance is concerned we cannot go further. In the rest frame of B meson, $\mathbf{p}_B = 0$, $P_0 = m_B$ and we take the fourth component of the polarization vector of the K^* meson equal zero, $\varepsilon_0(K) = 0$. The equation (4.7) then reduces to :

$$\begin{aligned} M \;=\; & - i\, m_B (\vec{q} \times \varepsilon(q)) \,.\, \vec{\varepsilon}(K)\, f_1(q^2) + \\ & .\left[- (m_B^2 - m_{K*}^2)\, \vec{\varepsilon}(q) . \vec{\varepsilon}(K) + 2\, m_B \varepsilon^0(q)\, \vec{q} \,.\, \vec{\varepsilon}(K) \right] f_2(q^2) \qquad (4.8) \end{aligned}$$

3. MATRIX ELEMENT IN QUARK MODEL

For B at rest equation (4.5) becomes

$$M = \sqrt{\frac{1}{2}(m_B^2 + m_{K^*}^2)} \left[-i(\vec{q} \times \vec{\varepsilon}(q) - q^0 \vec{\varepsilon}(q) + \varepsilon^0(q)\vec{q} \right] .$$

$$. < K^*(K) \left| \bar{s}\vec{\sigma}(1+\gamma_5)b \right| B(P)> \qquad (4.9)$$

In the absence of rigorous methods to calculate the matrix element from first principles, we adopt the quark model as a phenomenological model valid in the non-perturbative regime of QCD where | M(P)> is the quark model state vector in the weak binding limit eq. (3.1).

Because B and K^* mesons have the same parity, the term containing γ_5 makes zero contribution to the process B ----> K^*. Calculated in any quark model the matrix element has the form

$$< K^*(K) \left| \bar{s}\vec{\sigma} b \right| B(P)> = \vec{\varepsilon}(K)\, F(q^2) \qquad (4.10)$$

Using eq.(4.10) and by identifying eq.(4.9) and (4.8) one finds that

$$f_1(q^2) = \sqrt{\frac{1}{2}(1+\frac{m_{K^*}^2}{m_B^2})}\; F(q^2), \quad f_1(q^2) = \frac{1}{2} f_2(q^2) \qquad (4.11)$$

where F (q^2) is the wave function overlap to be evaluated in the constituent quark model. The partial width for B -----> K* γ is[29]

$$\Gamma(B \rightarrow K^* \gamma) = \frac{C^2}{32\pi} m_b^2 m_B^3 \left(1 - \frac{m_{K^*}^2}{m_B^2}\right)^3 . (f_1^2 + 4 f_2^2) \qquad (4.12)$$

Therefore the ratios of exclusive (B -----> K* γ) to inclusive (b -----> s γ) processes is

$$\eta = \frac{\Gamma(B \rightarrow K^* \gamma)}{\Gamma(b \rightarrow s\ \gamma)} = \left[\frac{m_b (m_B^2 - m_{K^*}^2)}{m_B (m_b^2 - m_s^2)} \right]^3 \left(1 + \frac{m_s^2}{m_b^2}\right)^{-1} \frac{1}{2}(f_1^2 + 4 f_2^2) \qquad (4.13)$$

3.1 CALCULATION OF F(q^2)

The effective weak hamiltonian involves the product of the s and b quark field at the same point, and this gives rise to a divergence. However this problem can be avoided by introducing the normal-ordered product or normal product of quark field denoted by a pair of colons as shown in equation (4.14) below. "In the normal product, all creation operators are made to operate as they stood to the left of annihilation operators, and for every exchange of Fermi operators a factor -1 is introduced corresponding to their anticommutation rules"[4].

Because of translational invariance we may take the field operator at $x = 0$. Using eq. (4.2) for s and b quark fields, we get

$$:\bar{s}(0)\,\vec{\sigma}\,b(0): = \int d^3 p_s d^3 p_b \left(\frac{m_s}{E_s}\frac{m_b}{E_b}\right)^{\frac{1}{2}}$$

$$\sum_{n,m}\left[\overline{u}^{s}_{r,m}(p_s)\,\vec{\sigma}\,u^{b}_{c,n}(p_b)\,:b^{s+}_{r,m}(p_s)\,b^{b}_{c,n}(p_s): + \ldots\right] \qquad (4.14)$$

The other terms of eq. (4.14) do not contribute to the relevant matrix element. Here n and m are spin indices and summation over color indices r, c is understood.

Using eq.(3.1), K^* and B meson states take the following form called "mock-meson states[29]"

$$|B(P,S_3{=}0)> = \frac{1}{\sqrt{6}}\int d^3p\,\phi_B(p)\;\left[d^{d+}_{i,\uparrow}\left(\frac{m_d}{M_B}P + p\right) b^{b+}_{i,\downarrow}\left(\frac{m_b}{M_B}P - p\right)\right.$$

$$\left. - d^{d+}_{i,\downarrow}\left(\frac{m_d}{M_B}P + p\right) b^{b+}_{i,\uparrow}\left(\frac{m_b}{M_B}P - p\right)\right]|0> \qquad (4.15)$$

$$|K^*(K,S_3{=}1)> = \frac{1}{\sqrt{3}}\int d^3k\,\phi_{K^*}(k)$$

$$\cdot\left[d^{d+}_{j,\uparrow}\left(\frac{m_d}{M_K}K + k\right) b^{s+}_{j,\uparrow}\left(\frac{m_s}{M_K}K - k\right)\right]|0> \qquad (4.16)$$

Substituting equations (4.14), (4.15) and (4.16) into (4.10) we obtain the following

$$<K^*(K,S_3=1) \mid :\bar{s}(0)\, \vec{\sigma}\, b(0): \mid B(P,S_3=0) > =$$

$$\frac{1}{\sqrt{18}}\int d^3p\, d^3k\, d^3 p_b\, d^3 p_s\, \phi^*_{K^*}(\mathbf{k})\phi_B(\mathbf{p}) \left(\frac{(E_s+ m_s)(E_s+ m_b)}{4E_sE_b}\right)^{\frac{1}{2}}$$

$$\sum_{n,m} \sum_{i,j,r,c} \bar{u}^s_{n,r}(p_s)\, \vec{\sigma}\, u^b_{m,c}(p_b) \times <0 \mid\ b^s_{j,\uparrow}(\frac{m_s}{M_K} K - k) d^d_{j,\uparrow}(\frac{m_d}{M_K} K + k)$$

$$:b^{s+}_{r,m}(p_s)\, b^b_{c,n}(p_b): \Big[d^{d+}_{i,\uparrow}(\frac{m_d}{M_B} P + p)\, b^{b+}_{i,\downarrow} (\frac{m_b}{M_B} P - p)$$

$$- d^{d+}_{i,\downarrow}(\frac{m_d}{M_B} P + p)\, b^{b+}_{i,\uparrow} (\frac{m_b}{M_B} P - p) \Big] \mid 0 > \qquad (4.17)$$

After contraction, as indicated (the contraction with the second term vanish), and integration over $d^3\mathbf{p}_s$, $d^3\mathbf{p}_b$ $d^3\mathbf{k}$ and for B at rest ($\mathbf{P}$ = o) we have for eq. (4.15) the expression

$$\frac{1}{\sqrt{2}}\, \delta_{m,\uparrow}\delta_{n,\downarrow} \int d^3p\, \phi^*_{K^*}(p - \frac{m_d}{M_K} K)\, \phi_B(p)\, \alpha(K - p, p)$$

$$\chi^+_m \left[\vec{\sigma} - \frac{(\vec{\sigma}(K - p))\vec{\sigma}\, (\vec{\sigma}\, p)}{(E_s+ m_s)(E_s+ m_b)} \right] \chi_n \qquad (4.18)$$

The form factor $F(q^2)$ is computed from eq.(4.18) and is found to be

$$F(q^2) = \int d^3p\, \phi_{K^*}\left(\mathbf{p} - \frac{m_d}{M_K}\mathbf{K}\right) \phi_B(\mathbf{p})\, \alpha(\mathbf{K} - \mathbf{p}, \mathbf{p}) \left[1 - \frac{\mathbf{p}\cdot(\mathbf{K}-\mathbf{p})}{3\,(E_s + m_s)(E_s + m_b)} \right] \tag{4.19}$$

where $M_K = m_s + m_d$, $M_B = m_b + m_d$ and

$$E_s = \sqrt{m_s^2 + (\mathbf{K} - \mathbf{p})^2}\ , \quad E_b = \sqrt{m_b^2 + p^2} \tag{4.20}$$

$$\alpha(\mathbf{K} - \mathbf{p}, \mathbf{p}) = \sqrt{\frac{(E_s + m_s)(E_s + m_b)}{4E_sE_b}} \tag{4.21}$$

3.2 RELATIVISTIC CORRECTIONS

Equations (4.19), (4,20) and (4,21) show that our calculation includes both recoil as well as the relativistic corrections[29,34].

3.2-a RECOIL CORRECTIONS

Recoil corrections are caused by the motion of the hadron as a whole. The momentum mismatch $k' = m_d/M_K\, K$ term appearing in $F(q^2)$ arise since the B meson rest frame is not the center-of-mass frame of K meson because $m_B >> m_{K^*}$. This term is in fact very large consequently its effects are very important. It gives rise to the damping exponential factor, as will be shown in the next section, which describes the main recoil corrections.

3.2-b RELATIVISTIC CORRECTIONS

There are two sources of relativistic corrections in equation (4.19):

- Those in the function $\alpha(K - p, p)$ contribute a factor $1/\sqrt{2}$, because the s quark is quite energetic, and very relativistic so that Es >> ms while the b quark is extremely non-relativistic, Eb ~ mb .

- Corrections caused by the motion of the quarks inside the potential mix with ones from the recoil. They are given by the term $\frac{p \cdot (K - p)}{6E_s m_b}$.

4. NUMERICAL COMPUTATION OF F(q^2)

The overlap integrals (4.19) are evaluated with the use of Gaussian wave functions [c.f. eq. (3.6)] which were used in the solution of the Schrodinger equation utilizing the variational method (chap. 3). The momentum wave function is determined by the Fourier transform of eq. (3.6) is :

$$\phi_M(\mathbf{p}) = (\pi \beta_M^2)^{-3/4} \exp(-\mathbf{p}^2/2\beta_M^2), \quad M = K^*, B \qquad (4.22)$$

With these wave functions and the approximations $E_s \sim |\mathbf{K}|$, $E_b \sim m_b$ and $|\mathbf{K}| = m_b/2$ we can perform analytically the integral (4.19). The analytical expression for $F(q^2)$ is given by

$$F(q^2) = \frac{1}{\sqrt{2}} \left(\frac{2\beta_B \beta_K}{\beta_B^2 + \beta_K^2} \right)^{\frac{3}{2}} \exp\left(\frac{-m_d^2 K^2}{2 M_K^2 (\beta_B^2 + \beta_K^2)} \right).$$

$$.\left[1 + \frac{\beta_B^2 \beta_K^2}{m_b^2 (\beta_B^2 + \beta_K^2)} \left(1 - \frac{-m_d K^2}{3 M_K \beta_B^2} \left(1 - \frac{-m_d \beta_B^2}{M_K (\beta_B^2 + \beta_K^2)} \right) \right) \right] \qquad (4.23)$$

One can show that $K^2 = M_{K^*}(t_m - q^2)/M_B$ where t_m is the square of the momentum transfer corresponding to the zero recoil of the K^* meson ie $K = 0$, hence $t_m = (m_B - m_{K^*})^2$. Thus

$$F(q^2) = \frac{1}{\sqrt{2}} I(K = 0) \,.\, e^{-D(q^2)} \,.\, R(q^2, K \neq 0),$$

where $I(K = 0) = \left(\frac{2\beta_B \beta_K}{\beta_B^2 + \beta_K^2} \right)^{\frac{3}{2}}$, $D(q^2) = \frac{m_d^2 (t_m - q^2)}{2 M_K M_B (\beta_B^2 + \beta_K^2)}$

$$R(q^2, K \neq 0) = 1 + \frac{\beta_B^2 \beta_K^2}{m_b^2(\beta_K^2 + \beta_K^2)} \left[1 - \frac{m_d (t_m - q^2)}{3 M_B \beta_{K*}^2} \left(1 - \frac{m_d \beta_B^2}{M_K(\beta_B^2 + \beta_K^2)}\right)\right]$$

Note that we have to evaluate the form factors at the physical point $q^2 = 0$ which is far from the zero-recoil of the K^* meson, i.e $K = 0$. With the numerical values from chapter 3 for β and values of the different masses, the following table 4.1 summarizes the numerical calculation for both linear and screening potential.

table 4.1 numerical results for linear and screening potential.

	linear[29]	screening
β_{K^*}	0.34 Gev	0.24 Gev
β_B	0.41 Gev	0.32 Gev
$I(K = 0)$	0.974	0.941
$R(0, K \neq 0)$	0.996	0.994
$D(0)$	0.767	1.365
$\exp(-D(0))$	0.464	0.255
$F(0)$	0.319	0.169
$f_1(0)$	0.228	00121
$f_2(0)$	0.114	0.0605
η	0.06	0.015

The calculations (cf. table 4.1) show that the corrections to the form factors is due to , mainly, the sharp exponential damping factor ,exp(-D(0)), because of recoil and the relativistic correction factor $(1/2)^{1/2}$. In fact neglecting these two corrections factors one can see that F(0) is nearly equal to one (0.94). From our results it is clear that the Kaon's momentum K plays a very important role in the decay B-----> K^{*} γ.

The process B-----> K^{*} γ is suppressed by a sharply exponential damping factor. A realistic calculation of $f_1(0)$ requires an understanding of nonperturbative effects of QCD responsible for binding the quarks into hadrons; since this is lacking the approach followed here is the best one can do at the moment.

Comparison of our results with those of Deshpande et al., obtained in case of linear confining potential, shows that the ratio η is reduced, by introducing the screening potential. The main correction comes from the recoil effect which is more important than in the case of linear potential. Future experimental results would hopefully test this, as the present calculation shows the flavor changing radiative decays of heavy B meson is sensitive to the screening effect.

The estimation of the branching ratio depends on the particular choice of hadronization model. Depending on the choice of the hadronic wavefunction, the predictions of η in the different attempts made in the framework of the Constituent Quark model span the range (5 -- 40)% (ref.15, 31, 34).

CHAPTER FIVE

FLAVOR CONSERVING

1. INTRODUCTION

Since the time of proposal of the non-relativistic quark model (NRQM) many attempts have been made to apply it to various processes involving elementary particles (hadrons decays, hadrons spectroscopy, ...). We note that in application to hadrons containing heavy quarks, the (NRQM) works well. However, attempts[35] to calculate the radiative decays of the ground-state mesons ($V \to P+\gamma$), where V (P) are vector (pseudoscalar) mesons, gives unsatisfactory results. Even using symmetry breaking through quark mass differences the experimentally known results could not be reproduced[36].

Many improvements[37] taking into account different possible corrections (recoil, relativistic, ...) have been made to reduce the discrepancies between theory and experiment. Singh et al[38] used the concept of effective mass (this was suggested by Sogami and Oh' Yamaguchi[39] to explain the observed characteristics of baryon magnetic moments) to study radiative decays of vector mesons. They got improved results for the decay rates which are close to experimental ones. In radiative decays a final meson (especially a final π-meson) generally moves relativistically because of large mass difference between initial and final mesons, hence a relativistic treatment is necessary[40].

In fact a nonrelativistic treatment of hadrons containing as constituents light quarks u, d, and s is not justified in the weak-binding limit. The relative velocity of light quarks is so large that relativistic corrections cannot be neglected. Calculations show that the role of relativistic corrections increases as the quark mass decreases. So its effect increases and substantially changes nonrelativistic predictions for decay widths especially for mesons in the light sector.

As mentioned above several authors have attempted to improve different theoretical approaches to fill the gap between theory and experimental results. S. Godfrey and N. Isgure[27] proposed a unified quark model, using a universal one - gluon exchange plus a linear confining potential motivated by quantum chromodynamics, to study both heavy and light mesons. They have attempted to identify all possible types of relativistic effects, including smearing, nonlocality, and momentum-dependent effective potentials. Another approach which includes relativistic corrections was followed by R. N. Faustov and V. O. Galkin[40,41]. They used a relativistic quark model based on a quasi-potential formalism and found that relativistic corrections play an important role in meson decays and strongly depend on the Lorentz properties and the shape of the confining potential.

The relativised constituent quark model, with the screening confining potential eq. (3.5), used for the study of flavor changing radiative decays of (heavy) B meson (cf. Chap. 4) will be extended to investigate the radiative decay of light vector mesons to pseudoscalar one. Calculations will include estimation of relativistic as well as recoil corrections.

2. MATRIX ELEMENTS

2.1 MAGNETIC DIPOLE M_1 TRANSITIONS

The process $V(1^-) \to P(0^-) + \gamma$ is a magnetic dipole transition (M_1), since the parity of the initial and final state does not change. (For a review of M_1 with regard to relativistic M_1 transitions in atomic and particle physics see ref. 42. See also ref. 40, for recent studies of M_1 in the framework of a relativistic quark model). In the quark model the M_1 transitions take place in analogy with ordinary M_1 transitions in hydrogenelike atoms for decays $^3S_1 \rightarrow {}^1S_0 + \gamma$.

A common feature of nearly all quark models[43] is that $V \rightarrow P+\gamma$ is triggered by a single quark $q \rightarrow q+\gamma$ and the full amplitude is obtained by summing over all the constituent quarks in the vector meson. This assumes that the spectator model is approximately valid[27]. The mesons decay by the electromagnetic interaction. Therefore, the quark flavor will not be altered, hence, the name "flavor conserving". At the quark level the electromagnetic current[44] is given by

$$J_\mu^{em} = \sum_f e_f \overline{\Psi}_f \gamma_\mu \psi_f \quad , \qquad (5.1)$$

where the summation is over all flavors forming the meson state, e_f is the flavor electric charge, ψ_f is the quark field (eq. 4.2), and the γ_μ are Dirac matrices.

The Lorentz - invariant hadronic matrix element operative in $V \rightarrow P+\gamma$ may be written in general as.

$$M_\mu \equiv < P(K) \,|\, : J_\mu^{e.m}(0) : |\, V(P,e) > \propto \mu_{PV} \varepsilon_{\mu\nu\rho\sigma} e^\mu K^\rho P^\sigma \qquad (5.2)$$

where μ_{PV} is the matrix element of the magnetic transition from vector to pseudoscalar meson state, and e^μ denotes the polarization vector of the vector meson. We shall derive in some detail the expression of μ_{PV} for the particular decay $\omega^0 \rightarrow \pi^0 + \gamma$.

(The procedure is same for all other decays). The meson states are

$$| \pi^0(K,S_3=0) > = \int d^3 p\, \phi_\pi(p) \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}) \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \times$$

$$\frac{1}{\sqrt{3}} \sum_i d_i^+ (\frac{m_q}{M} K + p)\, b_i^+ (\frac{m_q}{M} K - p)] \,| 0 > \qquad (5.3)$$

$$| \omega^0(P,S_3=1) > = \int d^3 p'\, \phi_\omega(p') \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) (\uparrow\uparrow) \times$$

$$\frac{1}{\sqrt{3}} \sum_j d_j^+ (\frac{m_q}{M} P + p')\, b_j^+ (\frac{m_q}{M} P - p')] \,| 0 > \qquad (5.4)$$

Due to translational invariance the electromagnetic current is evaluated at x = 0. Introducing the normal product, to avoid divergences as in chapter four, we have

$$: J_\mu(0) : = \sum_f : \bar{\psi}_f(0) \gamma_\mu \psi_f(0) : = \sum_f \sum_{n,mc,c'} e_f \int d^3q\, d^3 q' \left(\frac{m_f}{E_f} \frac{m_f}{E'_f} \right)^{\frac{1}{2}} \times$$

$$[\bar{u}^f_{c',m}(q') \gamma_\mu u^f_{c,n}(q) : b^{f+}_{c',m}(q')\, b^f_{c,n}(q) : +$$

$$\bar{v}^f_{c',m}(q') \gamma_\mu v^f_{c,n}(q) : d^f_{c',m}(q')\, d^{f+}_{c,n}(q) : + ...] \qquad (5.5)$$

In equations (5.3), (5.4) and (5.5) the indices i, j, c and c' refer to color while n and m refer to spin, and f stands for the flavor. From equation (5.5) we see that two terms contribute to the matrix elements of flavor conserving processes while only one contributes in the case of flavor changing processes. Substituting equations (5.5), (5. 4), (5.3) in the left hand side of equation (5.2), one obtains in the rest frame of the ω meson (P = 0) the following result

$$M_\mu . \varepsilon^\mu \equiv - \frac{1}{6}\left(\frac{I_d}{m_d} + \frac{2I_u}{m_u} \right) \vec{e} \, . \, (\vec{\varepsilon} \times \vec{K}) \qquad (5.6)$$

where ε is the photon polarization. Therefore the matrix element of the magnetic transition is

$$\mu_{\omega\pi} = - \frac{1}{6}\left(\frac{I_d}{m_d} + \frac{2I_u}{m_u} \right)$$

where I_d, I_u are overlap integrals to be evaluated. They are equal to unity in the non-relativistic limit.

2.2 OVERLAP INTEGRAL

Calculations show that the overlap integral I_f has the general form given below

$$I_f = \int d^3 p' \, \phi_p(p' - \frac{m_f}{M} K) \; \phi_v(p')\left(\frac{2m_f}{M}\right) m_f \sqrt{\frac{E + m_f}{E \, E'(E' + m_f)}} \qquad (5.7)$$

where $M = m_f + m_{sp}$, m_f and m_{sp} are the mass of the decaying and spectator quark which form the meson and;

$$E = \sqrt{\vec{p}'^2 + m_f^2} \ , \quad E' = \sqrt{\left(\frac{2m_f}{M} K - \vec{p}'\right)^2 + m_f^2} \qquad (5.8)$$

ϕ_p and ϕ_v are pseudoscalar and vector meson momentum wavefunctions given by (eq.(4.22))

$$\phi_p(\mathbf{p}) = \phi_v(\mathbf{p}) = (\pi\beta^2)^{-3/4} \exp(-\mathbf{p}^2/2\beta^2) \qquad (5.9)$$

Following the same procedure, as with ω decays, one can get the same overlap integrals for the process $\rho^0 \to \pi^0 + \gamma$. However, calculations show that the overlap integrals are somewhat what different for the other decays of interest. They have the form

$$I_f' = \int d^3 p' \, \phi_p(p' - \frac{m_{sp}}{M} K) \; \phi_v(p') m_f \sqrt{\frac{E + m_f}{E E'(E' + m_f)}} \qquad (5.10)$$

$$E = \sqrt{\vec{p}'^2 + m_f^2} \ , \quad E' = \sqrt{(K - \vec{p}')^2 + m_f^2} \qquad (5.11)$$

The transition moments, μ_{PV} expressed in unit of electron charge, for different radiative decay are grouped in table 5.1. Comparison with the results of ref.27 shows that our results are more or less in agreement with theirs. However, this reference uses an ad hoc smearing factor of the type $(m/E)^f$ where the exponent f is chosen to fit $\rho \to \pi + \gamma$, without any theoretical justification. On the other hand our calculation has theoretical basis in the sense of using a screening confining potential where the screening length becomes more important in the light sector.

process	μ_{PV}
$\omega^0 \to \pi^0 \gamma$	$-\frac{1}{6}\left(\frac{I_d}{m_d} + \frac{2I_u}{m_u}\right)$
$\rho^- \to \pi^- \gamma$	$-\frac{1}{6}\left(\frac{2I'_u}{m_u} - \frac{I'_d}{m_d}\right)$
$\rho^- \to \pi^- \gamma$	$-\frac{1}{6}\left(\frac{2I'_u}{m_u} - \frac{I'_d}{m_d}\right)$
$K^{0*} \to K^0 \gamma$	$\frac{1}{6}\left(\frac{I'_s}{m_s} + \frac{I'_d}{m_d}\right)$
$K^{+*} \to K^+ \gamma$	$\frac{1}{6}\left(\frac{I'_s}{m_s} - \frac{2I'_u}{m_u}\right)$

Table 5.1 Magnetic moment of vector mesons

REMARKS:

-If we go to the nonrelativistic limit, i.e, put all the overlap integrals equal to one, we obtain the well known transition magnetic moment for nonrelativistic quark model.

-Assuming the isospin symmetry with equal u- and d-quark masses, I_f and I'_f will have the same shape.

2.3 RELATIVISTIC CORRECTIONS

Equations (5.7) and (5.10) show that both recoil and relativistic corrections are included in our calculations. They are described by the momentum mismatch $m_{sp}K/M$ and the factor $m_f\sqrt{\frac{E+m_f}{E\,E'(\,E'+m_f)}}$ respectively.

Contrary to the case of B-meson radiative decay where recoil is very important, because of the large mass difference between B and K^* mesons, in the ground meson state radiative decay the mass difference $m_v - m_p$ is not quite large to give rise to a high momentum recoil caused by the photon emission. Taking the nonrelativistic limit, i.e, putting $E = E' = m_f$, then only the recoil corrections will be included in the overlap integral I_f and are given by $\exp(-\,(\,m'\,K\,)^2/(2\beta M)^2\,)$, where m' is m_f or m_{sp} depending on the decay in interest. which is of the order of and 0.86 for ω and ρ decays. On the other hand in the Kaon radiative decay the recoil corrections are more important for the (light) u, d quarks, and calculations of recoil effects shows that $I_s \sim 0.96$ while $I_{u,d} \sim 0.87$. These values give strongly overestimated decay rates, hence the necessity of relativistic corrections.

Including relativistic corrections, the different overlap integrals are strongly damped especially for u and d quark. (Both recoil and relativistic corrections are evaluated numerically in section four). It turns out that relativistic corrections are much important in the radiative decay of ground-state mesons and they drastically modify the non- relativistic quark model decay rates .

3. DECAY RATE

Having obtained the transition moment, the calculation of the decay width is straightforward[45-47], and is given by the formula

$$\Gamma = 4\,\alpha\,\mu_{pv}^2\,K^3 \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \text{ for } \begin{bmatrix} V \to P + \gamma \\ P \to V + \gamma \end{bmatrix} , \qquad (5.12)$$

where $\alpha = 1/137$ is the hyperfine constant, μ_{PV} is the transition magnetic moment, and K is the photon momentum which is given by $(m_v^2 - m_p^2)/ 2m_v$ in the rest frame of decaying vector meson of mass m_V, while m_P is the mass of the pseudoscalar meson. An additional factor of 3 appears for the reversed reaction $P \to V + \gamma$ since there is no averaging over initial spins. The evaluation of the decay rates requires the numerical values of the overlap integrals.

Using equations (5.9), (5.8) and (5.11) the overlap integral takes the form

$$I = \frac{2m_f}{\sqrt{\pi}\,\beta^3} e^{-\frac{1}{2}\left(\frac{m_{sp}K}{\beta M}\right)^2} \int_0^\infty dp \int_0^\pi d\theta\, p^2 e^{-p^2/\beta^2} \sqrt{1 + \frac{m_f}{\sqrt{p^2+m_f^2}}}\ \mathrm{Exp}\left(\frac{m_{sp}}{M\beta^2} p\, K \cos\theta\right) \times$$

$$\frac{\sin\theta}{\sqrt{K^2 + p^2 - 2\,p\,K\cos\theta + m_f^2 + m_f\sqrt{K^2+p^2-2pK\cos\theta + m_f^2}}} \qquad (5.13)$$

Note that we have assumed isospin symmetry so that I_f and I'_f are identical. One cannot perform this integration analytically. However, numerical estimation is possible by using the MATHEMATICA software. For the quark mass in this calculation we use typical relativised quark masses[27,48] : $m_u = m_d = 220$ Mev, and $m_s = 419$ Mev. The value of beta in the integration is fixed such that we get the best fit for the predicted decay rate with the experimental results table (5.3) . It is found that required values are $\beta = 0.24$ for ρ and ω meson and $\beta = 0.28$ for K - meson decays.

Once the values of beta are known we can determine the screening length $1/\mu$ such that the energy expectation value $E(\beta)$ equation (3. 8) is minimum at the points where β is equal to the above values. (Recall that β is the variational parameter). It is found that for $\mu = 0.0736585$ Gev and $\mu = 0.0545048$ Gev, and the minima of $E(\beta)$ occur at $\beta = 0.24$ Gev and $\beta = 0.28$ Gev respectively.

Process	Radiative decay width Γ(kev) NRQM (Ref.41)	Present case	Results of (Ref. 41)	Experiment
$\rho^{\pm} \rightarrow \pi^{\pm} \gamma$	122	64	71	67.1 ± 8.8
$\omega^{0} \rightarrow \pi^{0} \gamma$	1210	579	782	717 ± 51
$K^{0*} \rightarrow K^{0} \gamma$	203	122	105	115 ± 12
$K^{+*} \rightarrow K^{+} \gamma$	133	56	47	50 ± 5

Table 5.3 Radiative decay widths of ground state mesons: present calculation and results of (Ref. 41) are presented together with the experimental values.The results of simple quark model (NRQM) are also given.

CONCLUSIONS

The radiative M_1 - decays widths are calculated and summarized in table(5.3), in the framework of a relativised quark model with a screening confining potential. As mentioned before the relativistic corrections are very important in decays of ground-state mesons. The agreement obtained with the experimental data is quite good-within 10 % (except for the ω decay) and is at least as good as that obtained in ref.41, where a very different approach has been used to obtain agreement with the experimental data.

Comparing predicted results with the experimental data we can extract important information on the quark - antiquark interaction at large distances. Compared with heavy mesons our calculations show , as was expected, that the screening length increase by a factor of three for ρ and ω systems, while, it increase by a factor of about 2.2 for K meson .

5. NUMERICAL CALCULATIONS OF THE OVERLAP INTEGRAL.

We present here the method by which the overlap integral equation (5.13) is evaluated. The calculations are performed for ω decay; for the other processes one just changes the input values of the different parameters.In what follow mf, msp are the mass of the decaying and spectator quark. K is the photon energy and b stands for the the variational parameter beta.

```
mf = 0.22
msp = 0.22
mt = mf + msp
K = 0.379
b = 0.23

a = 2/(b^3 Sqrt[Pi]) mf Exp[-(msp K)^2/
    (2 (b mt)^2)] //N


14.5307


f = p^2 +  K^2 - 2 p K Cos[t] + mf^2
s = Sin[t] Exp[msp p K Cos[t]/(b^2 mt)]/
    Sqrt[Sqrt[f] mf + f]
```

```
  3.58223 p Cos[t]
(E                    Sin[t]) /

                          2
     Sqrt[0.192041 + p   - 0.758 p Cos[t] +

                                      2
         0.22 Sqrt[0.192041 + p   - 0.758 p Cos[t]]]
```

■ **Lets first integrate over the angular variable t from zero to π.**

```
y = Integrate[s, {t, 0, Pi}]
```

```
              3.58223 p Cos[t]
Integrate[(E                    Sin[t]) /

                          2
        Sqrt[0.192041 + p   - 0.758 p Cos[t] +

                                      2
            0.22 Sqrt[0.192041 + p   - 0.758 p Cos[t]]]

   {t, 0, Pi}]
```

```
h = Sqrt[p^2 + mf^2]

g = a y p^2 Exp[-(p/b)^2] Sqrt[h + mf]/Sqrt[h]
```

```
                2                 3.58223 p Cos[t]
   (14.5307 p  Integrate[(E                         Sin[t]) /

                                            2
                Sqrt[0.192041 + p  - 0.758 p Cos[t] +

                    0.22

                                                2
                    Sqrt[0.192041 + p  - 0.758 p Cos[

                                                                2
          {t, 0, Pi}] Sqrt[0.22 + Sqrt[0.0484 + p ]]

                                    1
                2                   -
    18.9036 p                   2  4
  (E              (0.0484 + p ) )
```

■ Now we integrate over the variable p from zero to infinity.

```
N[Integrate[g, {p, 0, Infinity}]]

 0.463257
```

CHAPTER SIX

CONCLUSIONS

In studying the flavor changing and flavor conserving radiative decays $P \rightarrow V \gamma$ or $V \rightarrow P \gamma$, a relativised quark model has been used in the following sense:

i) In writing the relevant operator responsible for the above decays in terms of a two components Pauli form, no relativistic approximation has been made.

ii) Recoil effects due to the large mass differences between the initial and final meson states are fully taken into account in calculating the overlap integrals in the evaluation of the relevant operator.

iii) Although Schrodinger equation, appropriate to short range Coulomb type potential arising due to one gluon exchange and QCD inspired long range confining potential is used, but only to the extent in fixing the variational parameter in a trial wave function which is used in calculating the overlap integrals.

iv) The new element in our calculation is the use of a screened long range confining potential instead of a pure linear term. Such a potential is indicated by lattice gauge theory calculations when a light quark is involved in $Q\bar{q}$ or $q\bar{Q}$ system.

For the flavor changing radiative decay of B meson: $B \rightarrow K^{*}(892)\ \gamma$ in the spectator approximation, where in the weak hamiltonian responsible for this decay, the QCD corrections arising from one gluon exchange between the internal and external lines of the decaying quark are taken into account. Numerical evaluation shows that the form factor is damped by an exponential term arising mainly from the recoil effect because of large mass differences between the B and K^{*} meson. The relativistic corrections of the type mentioned in (i) above are found not to be very important but also are not negligible. However, the recoil effects are much more important. Moreover the results are found to be sensitive to the screening parameter introduced previously by lattice gauge theory calculations in study of spectroscopy of a heavy quark (antiquark) and light antiquark (quark) system.

We extended the above model to study radiative decays of ground state mesons which are flavor conserving. It is found that the relativistic corrections are more much pronounced for the light sector. The recoil as well as relativistic corrections are taken into account in evaluating the decay rates. Here the screening length is used as a free parameter to be fixed by making a best fit to decay rates. In fact one obtains a good agreement with the experimental data (except for ω) for mesons involving strange quark for a screening length μ^{-1} to be twice as larger as for B meson , while it is about three times larger in the case of non strange mesons. As one would expect we found that the screening length $1/\mu$ increases as we move from the heavy to the light sector. Thus our calculation give some information on the quark - antiquark interaction at large distances particularly when lighter quarks are involved. Previously the flavor conserving radiative decays were fitted by using an ad hoc factor ($E/m)^f$ where the parameter f was fixed from $\rho \rightarrow \pi \ \gamma$. In our case the screening parameter has a physical meaning in terms of screening of the linear confining potential at large distances.

Finally we may mention that the calculations presented here are not fully relativistic, because of using Schrodinger equations although in limited form. One may use the Bethe Salpeter equation or some version of it, for the quark antiquark system for fully relativistic calculations. This, for instance, has been done in ref.40, 41; and as table 4.2 indicates our calculations are not very different from those in ref.41. We note also that more realistic calculations must include contributions coming from gluon exchange with the spectator quark, but such calculations requires an understanding of QCD at large distances which is not well understood at the present time.

Appendix. A

MATRIX ELEMENTS

A detailed calculation of the matrix elements arising from one gluon exchange (cf. Fig. 2.3) approximation between two quarks will be performed.

$\Gamma_a = \gamma_4$, we have:

$$\frac{m}{\sqrt{E\,E'}}\,\overline{U}(p')\gamma_4 U(p) \equiv e^{+iS(p')}\,\gamma_4\,e^{-iS(p)}$$

$$\frac{(E'+m) - i\vec{\gamma}.\vec{p}'}{\sqrt{2E'(E'+m)}}\,\gamma_4\,\frac{(E+m) - i\vec{\gamma}.\vec{p}}{\sqrt{2E(E+m)}} = \frac{(E'+m)(E+m) - \vec{\gamma}.\vec{p}'\gamma_4\vec{\gamma}.\vec{p}}{2\sqrt{EE'(E'+m)(E+m)}}$$

$$= \frac{1}{2}\,\frac{\sqrt{(E'+m)(E+m)}}{\sqrt{EE'}} + \frac{(\vec{\sigma}.\vec{p}')(\vec{\sigma}.\vec{p})}{2\sqrt{EE'(E'+m)(E+m)}}$$

$$(\vec{\sigma}.\vec{p}')(\vec{\sigma}.\vec{p}) = \vec{p}'.\vec{p} + i\,\vec{\sigma}.(\,\vec{p}' \times \vec{p}\,)$$

We made the following approximation:

$$E=\sqrt{m^2+\vec{p}^2}=m(1+\frac{\vec{p}^2}{m^2})^{\frac{1}{2}}\approx m+\frac{\vec{p}^2}{2m^2}\ ,\ E'=\ m+\frac{\vec{p}'^2}{2m^2}$$

$$(E+m)^{\frac{1}{2}}=(2m+\frac{\vec{p}^2}{2m^2})^{\frac{1}{2}}=\sqrt{2m}(1+\frac{\vec{p}^2}{4m^2})^{\frac{1}{2}}\approx\sqrt{2m}\ (\ 1+\frac{\vec{p}^2}{8m^2})$$

$$\frac{1}{E}\approx\frac{1}{m}(1-\frac{\vec{p}^2}{2m^2})\ ,\ \frac{1}{\sqrt{E}}=\frac{1}{\sqrt{m}}(1-\frac{\vec{p}^2}{4m^2})\ ,\ (E+m)^{-\frac{1}{2}}\approx\frac{1}{\sqrt{2m}}(\ 1-\frac{\vec{p}^2}{8m^2})$$

Substituting in the above equation and keep only term of order p^2/m^2 we get the following:

$$\begin{aligned}\gamma_4 &\equiv 1-\frac{\vec{p}^2}{8m^2}-\frac{\vec{p}'^2}{8m^2}-\frac{\vec{p}^2}{4m^2}-\frac{\vec{p}'^2}{4m^2}+\frac{i}{4m^2}\vec{\sigma}.(\ \vec{p}'\times\vec{p}\ )\\ &=1-\frac{(\ \vec{p}'-\vec{p})^2}{8m^2}+\frac{i}{4m^2}\vec{\sigma}.(\ \vec{p}'\times\vec{p}\ )\end{aligned}$$

For $\Gamma_a=\gamma$:

$$e^{-iS(p')}\,\vec{\gamma}\,e^{-iS(p)}=\frac{(E'+m)-i\vec{\gamma}.\vec{p}'}{\sqrt{2E'(E'+m)}}\vec{\gamma}\ \frac{(E+m)-i\vec{\gamma}.\vec{p}}{\sqrt{2E(E+m)}}$$

$$=\frac{-\,i\,(E'+m)\,\vec{\gamma}\,(\,\vec{\gamma}\,.\,\vec{p}\,)-i\,(\,E+m\,)\,(\,\vec{\gamma}\,.\,\vec{p}'\,)\,\vec{\gamma}}{2\sqrt{EE'(E'+m)(E+m)}}$$

$$=-i\frac{\sqrt{E'+m}}{2\sqrt{EE'(E+m)}}\ \vec{\sigma}(\,\vec{\sigma}\,.\,\vec{p}\,)-i\frac{\sqrt{E+m}}{2\sqrt{EE'(E'+m)}}(\,\vec{\sigma}\,.\,\vec{p}'\,)\,\vec{\sigma}$$

$$\approx\ -\frac{i}{2m}(\,\vec{p}-i\,(\,\vec{\sigma}\times\vec{p}\,))-\frac{i}{2m}(\,\vec{p}'-i\,(\,\vec{\sigma}\times\vec{p}'\,))$$

$$e^{-iS(p')}\,\vec{\gamma}\,e^{-iS(p)}\approx-\frac{i}{2m}(\,\vec{p}'+\vec{p}\,)+\frac{1}{2m}\vec{\sigma}\times(\ \vec{p}'-\vec{p}\,)+O\,(\frac{|\vec{p}|^3}{m^3})$$

In the non relativistic limit the matrix element M is given by the approximation:

$$M = \frac{1}{q^2}\left[1 - \frac{(\vec{p_i'} - \vec{p_i})^2}{8m_i^2} + \frac{i}{4m_i^2}\vec{\sigma_i}.(\vec{p_i'} \times \vec{p_i})\right].$$

$$\left[1 - \frac{(\vec{p_j'} - \vec{p_j})^2}{8m_j^2} + \frac{i}{4m_j^2}\vec{\sigma_j}.(\vec{p_j'} \times \vec{p_j})\right] +$$

$$\left[- \frac{i}{2m_i}(\vec{p_i'} + \vec{p_i}) + \frac{1}{2m_i} i\vec{\sigma_i} \times (\vec{p_i'} - \vec{p_i})\right].$$

$$\left[- \frac{i}{2m_j}(\vec{p_j'} + \vec{p_j}) + \frac{1}{2m_j}\vec{\sigma_j} \times (\vec{p_j'} - \vec{p_j})\right]$$

put

$$\vec{q} = \vec{p_i'} - \vec{p_i} \quad = \vec{p_j} - \vec{p_j'}$$

$$M = \frac{1}{q^2}\left[1 - \frac{\vec{q}^2}{8m_i^2} + \frac{i}{4m_i^2}\vec{\sigma_i}(\vec{q} \times \vec{p_i})\right].\left[1 - \frac{\vec{q}^2}{8m_j^2} - \frac{i}{4m_j^2}\vec{\sigma_j}(\vec{q} \times \vec{p_j})\right] +$$

$$\left[-\frac{i}{2m_i^2}(\vec{p_i} + \vec{p_i'}) - \frac{1}{2m_i^2}\vec{\sigma_i} \times (\vec{p_i} - \vec{p_i'})\right].\left[\frac{i}{2m_j^2}(\vec{p_j} + \vec{p_j'}) - \frac{1}{2m_j^2}\vec{\sigma_j} \times (\vec{p_j} - \vec{p_j'})\right]$$

$$M = \frac{1}{q^2}\Big(1 - \frac{\vec{q}^2}{8m_i^2} + \frac{i}{4m_i^2}\vec{\sigma_i}.(\vec{q} \times \vec{p_i}) - \frac{\vec{q}^2}{8m_j^2} - \frac{i}{4m_j^2}\vec{\sigma_j}.(\vec{q} \times \vec{p_j})$$

$$- \frac{1}{4m_im_j}(\vec{q} + 2\vec{p_i})(2\vec{p_j} - \vec{q}) - \frac{i}{4m_im_j}(\vec{q} + 2\vec{p_i}).(\vec{\sigma_j} \times \vec{q})$$

$$+ \frac{i}{4m_im_j}(\vec{\sigma_i} \times \vec{q})(2\vec{p_j} - \vec{q}) - \frac{1}{4m_im_j}(\vec{\sigma_i} \times \vec{q})(\vec{\sigma_j} \times \vec{q})\Big)$$

with:

$$q^2 = \vec{q}^2 - q_0^2 = \vec{q}^2\left(1 - \frac{q_0^2}{\vec{q}^2}\right), \quad \frac{1}{\vec{q}^2}\left(1 + \frac{q_0^2}{\vec{q}^2}\right)$$

$$q_0 = \frac{\vec{p}_i'^{\,2} - \vec{p}_i^{\,2}}{2\,m_i} = \frac{\vec{p}_j^{\,2} - \vec{p}_j'^{\,2}}{2\,m_j}$$

$$\frac{q_0^2}{\vec{q}^{\,2}} = \frac{\vec{q}\cdot\vec{p}_i\;\vec{q}\cdot\vec{p}_j}{2\,m_i\,m_j} + \frac{1}{2\,m_i\,m_j}\left(-\vec{q}\cdot\vec{p}_i + \vec{q}\cdot\vec{p}_j\right) - \frac{\vec{q}^{\,2}}{4\,m_i\,m_j}$$

M has the form given in equation (2.4).

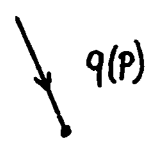
Appendix B

FEYNMAN RULES OF QCD

We present here the Feynman rules necessary to perform the calculations of one gluon exchange matrix element (chap.2) ref.4, 7 and 16.

Particle (spin 1/2) **Factor included**

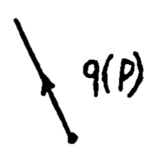
incoming quark line
quark initial state



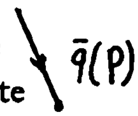
$u(p)$

quark

outgoing quark line
quark in final state

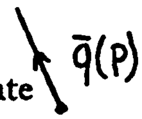


$\bar{u}(p)$

incoming quark line
antiquark in final state 

$v(p)$

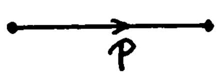
antiquark

outgoing quark line
antiquark in initial state 

$\bar{v}(p)$

Propagators

quark

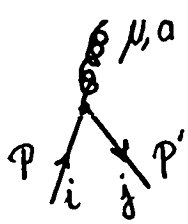


$$\frac{i}{\not{p} - m}$$

gluon

$$i\,\delta^{ab}\left(-\frac{g_{\mu\nu}}{q^2}\right)$$

Vertices



$$-\,i\,g\,\gamma_\mu \frac{1}{2}\lambda_a^{ij}$$

The parameters g and m_j appearing in the above Feynman an rules are the coupling constant and the quark mass respectively.

REFERENCES

1) Kokkedee, J. J. J. , *The quark model* (Benjamin, New York, 1969).

2) A. W Hendry and D. Lichtenberg Rep. Prog. Phys. 41, 1707 (1978).

3) D. Flamm, F. Schoberl, *Introduction to the quark model of elementary particles;* Volume 1: *Quantum Numbers, Gauge Theories and Hadron spectroscopy.* (Gordon and Breach Science publishers, 1982).

4) Otto Nachtman, *Elementary Particle Physics Concepts and Phenomena* (Springer - Verlag, 1990)

5) Takasaki, F., Nuc. Phys. B (Proc. Suppl.)3, 17(1988).

6) Greenberg O. W. and Nelson C. A. , Phys. Rep. 32 C, 69-121 (1977).

7) Aitchison I. J. R., Anathony J. G. H., *Gauge theories in Particle Physics,* 2nd Edition (Adam Hilger, 1989).

8) Bauer, M., B. Stech, and M. Wirbel, Z. Phys. C 34, 103(1987) and references therein.

9) M. K. Gaillard, B. W. Lee: Phys. Rev. D 10, 897(1974).

10) N. G. Deshpande, G. Eilam : Phys. Rev. D 26, 2463 (1982).

11) N. G. Deshpande, M. Nazerimonfared: Nucl. Phys. B 213,

390(1983).

12) T. Inami, C. S. Lim: Prog. Theor. Phys. 65, 297(1981).

13) S. Bertolini. F. Borzumati. A. Masiero : Phys. Rev. Lett. 59, 180(1987).

14) C. A. Dominguez, N. Paver and Riazuddin, Phys. lett. B 214, 459(1988).

15) C. A. Dominguez, N. Paver and Riazuddin, (Preprint) IFN/AE - 89/5.

16) P. Renton, *Electroweak Interaction an Introduction to Physics of Quarks and Leptons* (Cambredge University Press 1990).

17) W. Kwong, J. L. Rosner, C. Quigg Fermilab - Pub - 87/15 -T .

18) A. Martin Comments Nuc. Part. Phys. 16, 149-266(1986).

19) Xiaotong Song, (Preprint) IC/88/223 and references therein.

20) N. Brambulla and G. M. Prosperi IFUM 362/FT, May 1989.

21) Kay Konigsman, AIP conf. Proc. 185, Particles and fields Series 36, page 36-49(1988) ed. SUH-URK CHUNG. and references therein.

22) Silvan S. Schweber, *An Introduction to Relativistic Quantum Field theory* (Hoper and Row, New Work 1961).

23) Bjorken, J. D. Drell, *Relativistic Quantum Mechanics* (Mc, Graw- Hill, New York 1966).

24) E. Laerman, F. Langhammer, I. Schmit, P. M. Zerwas, Phys. Lett. B173, 437(1986); CERN - TH. 4393/86.

25) N. Isgur, Phys. Rev. D 12, 3666(1975).

26) C. Hayne and N. Isgur, Phys. Rev. D 25, 1944(1982).

27) S. Godfrey, N. Isgur, Phys. Rev. D 32, 189(1985).

28) N. Isgur, D. Scora, B. Grinstein and M. B. Wise, Phys. Rev. D 39, 799(1989).

29) N. G. Deshpande, P. Lo, J. Trampotic, Z. Phys. C - Particles and fields 40, 369-377(1988).

30) P. Colangelo, G. Nardulli, N. Paver, Riazuddin: Z. Phys. C-Particles and Fields 45, 575-580(1990).

31) Timothy Altomari, Phys. Rev. D 37, 677(1988).

32) Frederick J. Gilman, SLAC - PUB - 4514 December 1987 (T/E).

33) R. Ruckl, "Elementary Particles" International School of Physics (Enrico Fermi), (1987)43 .

34) N. G. Deshpande, P. Lo and J. Trampetic, Phys. Rev. Lett. 39, 183(1987).

35) P. J. O' Donnell, Rev. Mod. Phys. 53, 673(1981) and references therein.

36) L. P. Singh, Phys. Rev. D 19, 2812(1979).

37) H. Grotch, K. J. Sebastian, Phys. Rev. D25 2944(1982); V. Zambitakis, N. Byers, Phys. Rev. D 28 2908(1983); E. Sucipto, R. L. Thews, Phys. Rev. D 38 (1987).

38) C. P. Singh, S. K. Das, and C.P.S. Verma, Phys. Rev. D 39, 2072(1989).

39) Ikuo S. Sogami, Noboru Oh' yamaguchi, Phys. Rev. Lett. 54,

2295(1985).

40) V. O. Galkin and R. N. Faustove, Sov. J. Nucl. Phys. 44, 1023(1986).

41) V. O. Galkin and R. N. Faustove, Proceeding of the International Seminar " QUARKS' 88" Tbilisi, USSR 17 - 21 May 1988 (page 624).

42) J. Sucher, Rep. Prog. Phys. 41, 1781(1978).

43) E. F. Close, *An Introduction to Quarks and Partons* (Academic London 1979).

44) E. D. Commins, P. H. Bucksbaum, *Weak interactions of leptons and quarks* (Cambridge university press 1983).

45) R.Van Royen, and V. F. Weisskopf, 1967a, Nuvo cimento A 50, 617.

46) C. Becchi and G. Morpurgo. Phys. Rev. 140, 687B (1965).

47) S. Ishida et al. Prog. Theor. Phys. 71, 806(1984).

48) C. Hayne and N. Isgure, Phys. Rev. D 25, 1944(1982).

49) S. Bertolini, (Preprint) Desy 89 - 169 December 1989.

50) Stephen Wolfram, " *Mathematica A System for Doing Mathematics by Computer*" Addison - Wesley Publishing Company 1988.

51) D. H. Perkins, Introduction to High Energy Physics (Third ed. Addison- Wesley, 1987).