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Rigidity Influence of Suspended Cable on Free Vibration Modes

The problem of vibration control of overhead line conductors subjected to laminar transverse wind, inducing stationary vibrations by Karmán effect is of high importance due to consequences upon these structures lifetime and service. We consider the cable model as Euler-Bernoulli beam that include the influence of cable rigidity and that respect the author condition which detaches the suspended cable model of the beam model with viscous, hysteretic or dry friction internal damping hypothesis. The original analytical expression of the free vibration modes and the resonance frequencies equation for the cable with clamped extremities has produced. Some experimental aspects are underlined in the paper.

Keywords: *Euler-Bernoulli beam, suspended cable model, vibration mode, damping, analytical solution*

1. Introduction

We consider the cable model derived from the Euler-Bernoulli beam with viscous, hysteretic or dry friction internal damping [1]-[12]. The analytical expression of the free vibration modes and the resonance frequencies equation for the cable with clamped extremities are produced using our hypothesis of the cable imposed to Euler-Bernoulli beam, essentially for accurate identification of the cable model parameters. The property of any Euler-Bernoulli beam model to be substituted, for sufficient high frequencies, by our cable model, is underlined. Some relative recent studies in our domain of interest appeared [13], [14]. Our experimental research was performed on a specialized stand endowed with the overhead conductor using clamped extremities, alone or with a choice of Stockbridge dampers, mounted on the extreme zones of the span. The resonance frequencies and vibration modes are identified theoretically and also experimentally, on the conductor in the stand. The analytical aspects on the internal damping terms influence versus frequency, in the cable models, are discussed. A possible equivalence between the internal damping coefficients of the cable models, using the described damping hypothesis, is studied, equalizing analytical expressions of corresponding damping energies of cable. The formulas of equivalence between two possible hypotheses of internal damping coefficients of cable are deduced and analyzed.

2 Euler-Bernoulli beam model

The following equation of free vibrations is considered [1]-[7]:

$$m_L \frac{\partial^2 w_i}{\partial t^2} = -(c_i^{H*} + k_i) w_i - \left(c_i^V + \frac{c_i^H}{\omega_i^{VH}} \right) \frac{\partial w_i}{\partial t} + T \frac{\partial^2 w_i}{\partial x^2} - EI \frac{\partial^4 w_i}{\partial x^4} + q \quad (1)$$

The Eq. (1) describes the behavior of the beam, excited by the force $q = q(x,t)$, applied transversal on the beam, acting in the point of abscissa x, at the time t, on viscous damping hypothesis by the constant coefficient c_i^V, on hysteretic damping hypothesis by the constant coefficient of the form c_i^H / ω_i^{VH} and on dry friction (Coulomb) damping hypothesis, expressed by the coefficient c_i^{H*}. The coefficient c_i^{H*} is piecewise constant, as function of time t, and the sign is such that the sign of the damping force $c_i^{H*} w_i(x,t)$ to be opposite to that of the velocity $\dot{w}_i(x,t) = \partial w_i(x,t)/\partial t$ at any time t. Other explicit expression of the dry friction force is $c_i^{H1} |w_i(x,t)| sign(\dot{w}_i(x,t))$, where c_i^{H1} is constant [8]. The first expression of dry friction force is deduced, in our case, taking into account the properties that the functions $w_i(x,t)$ and $\partial w_i(x,t)/\partial t$ are continue and with separable variables. We denote by ω_i^{VH} the circular frequency of order i for damped free vibration, by $f_i^{VH} = \omega_i^{VH} / \pi / 2$ the resonance frequency of order i for damped free vibration, by m_L the mass unit length of the beam, by EI the bending rigidity of the beam, by T the tension in the beam, by k_i the rigidity coefficient of the beam, by $y_i(x,t)$ corresponding vertical displacement of the beam for vibration mode of order i and by L the length of the beam.

Firstly, we search the stabilized free transverse vibrations of the beam without damping and with clamped extremities that are of standing waves form:

$$w_r(x,t) = w_r(x) \sin(\omega_r t + \varphi) \quad (2)$$

In Eq. (2), the notations signify: $\omega_r = 2\pi\, f_r$ is the circular frequency, with f_r the resonance frequency of the cable in free vibrations without damping; and φ is the phase angle between the initial impulse and displacement. Bellow appears, in condition $m_L \omega_r^2 - k_r > 0$, the following dimensionless notations [3]:

$$\alpha^2 = \frac{TL^2}{EI},\ \delta_r^2 = \frac{\alpha^2}{2} + \left(\frac{\alpha^4}{4} + \beta_r^4\right)^{\frac{1}{2}},$$

$$\beta_r^4 = \frac{(m_L \omega_r^2 - k_r)\, L^4}{EI},\ \varepsilon_r^2 = -\frac{\alpha^2}{2} + \left(\frac{\alpha^4}{4} + \beta_r^4\right)^{\frac{1}{2}}.$$

The expressions α, β_r, δ_r, ε_r verify the relationships:

$$\delta_r^2 - \varepsilon_r^2 = \alpha^2,\ \delta_r \varepsilon_r = \beta_r^2,\ \delta_r^4 - \alpha^2 \delta_r^2 - \beta_r^4 = 0,\ \varepsilon_r^4 + \alpha^2 \varepsilon_r^2 - \beta_r^4 = 0. \quad (3)$$

The free vibrations of the beam are defined by the following equation, derived from equation (1), where $w_i = w_i(x)$:

$$EI \frac{d^4 w_i}{d\, x^4} - T\ \frac{d^2 w_i}{d\, x^2} + \left(c_i^V \omega_i + \frac{c_i^H}{\omega_i^{VH}} \omega_i + c_i^{H*} + k_i - m_L\, \omega_i^2\right) w_i = 0. \quad (4)$$

The general solution of the equation (4), for undamped vibrations (defined by the values $c_i^V = 0.$, $c_i^H = 0.$, $c_i^{H*} = 0.$), is of the form:

$$w_i(x) = C_{1i} \sin(\varepsilon_i \xi) + C_{2i} \cos(\varepsilon_i \xi) + C_{3i} \sinh(\varepsilon_i \xi) + C_{4i} \cosh(\varepsilon_i \xi). \quad (5)$$

In solution (5) $\xi = \mathrm{x / L}$ and $C_{ij},\ i = 1,...,4;\ j \in N$ are constants.

3. Euler-Bernoulli cable model

We use the author condition (6), performed by the cable wire in the cases studied in the literature, which represents our hypothesis that detaches the cable model of the Euler Bernoulli beam model:

$$\mathrm{e}^{-\delta_r} \approx 0. \quad (6)$$

The results of the Euler-Bernoulli beam model are applicable only for low frequencies of the beam and the results of the cable model are applicable for high frequencies because, in this condition, is respected the condition (6), such that the beam model is obliged substituted by the Euler-Bernoulli cable model.

The solution of Eq. (1) is searched [9]-[11] using the condition (6).

For the clamped cable, the equation of resonance frequencies is as follows.

$$\alpha^2 \sin \varepsilon_r - 2\beta_r^2 \cos \varepsilon_r = 0. \qquad (7)$$

The analytical expression of the free vibration modes for undamped free vibrations of the cable, in the case of clamped boundary conditions is shown in formula (8).

$$w_r(x) = C_r \left\{ \mathrm{e}^{-\delta_r \xi} + \frac{\delta_r}{\varepsilon_r} \sin \varepsilon_r \xi - \cos \varepsilon_r \xi - \right. \\ \left. - \frac{\delta_r}{\varepsilon_r} \mathrm{e}^{\delta_r (\xi - 1)} \sin \varepsilon_r + \mathrm{e}^{\delta_r (\xi - 1)} \cos \varepsilon_r \right\} \qquad (8)$$

The factor C_r, for each $r = 1, 2, \ldots$, is a constant and $\xi = x / L$.

Anyone can verify that Euler-Bernoulli beam model can be substituted by our cable model for sufficient high frequencies because the value $\delta_r \xrightarrow[r]{} \infty$ and thus $e^{-\delta_r} \xrightarrow[r]{} 0$.

We specify the following particular solutions $w_r(x)$ of the cable model that defines Eq. (2) in the case $q(x,t) = 0,\ c_r^V = 0,\ c_r^H = 0,\ c_r^{H*} = 0,$ solutions that are also the particular solutions of the beam model:

$$w_{1r}(x) = e^{-\delta_r \xi},\ \ w_{2r}(x) = e^{\delta_r \xi},\ \ \xi = \frac{x}{L}, \\ w_{3r}(x) = \sin\ \varepsilon_r \xi,\ \ w_{4r}(x) = \cos\ \varepsilon_r \xi. \qquad (9)$$

The relations from (3) can be used to justify the particular solutions (9).

The vibration mode of undamped vibration, expressed by relations (2) and (8), is a solution of Eq. (1), where $q(x,t) = 0,\ c_i^V = 0,\ c_i^H = 0,\ c_i^{H*} = 0,$ because $w_r(x)$, from (8), is a linear expression of the particular solutions from (9). The vibration mode (2) verifies also the imposed boundary conditions.

In the case of damped free vibrations described by equation (1), one searches the solution of the form $w_i(x,t) = X_i(x)\ T_i(t)$, where $X_i(x)$ define a vibrating mode of order i from (8). The equation deduced from Eq. (1) for the unknown function $T_i(t)$ is as below.

$$\frac{d^2T_i(t)}{dt^2}+2c_i^{VH}\ \frac{dT_i(t)}{dt}+c_i^{\Omega H*}T_i\ (t)=0.,$$
$$c_i^{VH}=c_i^{V}/m_L/2+c_i^{H}/m_L/\omega_i^{VH}/2, \qquad (10)$$
$$c_i^{\Omega H*}=\omega_i^2+c_i^{H*}/m_L-k_i/m_L,$$

The Eq. (10) is deduced using the formula:

$$EI\,\frac{d^4X_i(x)}{d\ x^4}-T\,\frac{d^2X_i(x)}{d\ x^2}=(\omega_i^2\ m_L-k_i)\ X_i(x). \qquad (11)$$

In Eq. (10) ω_i is circular frequency of free undamped vibration of the cable and ω_i^{VH} is circular frequency of free damped vibration of the cable.

The characteristic equation attached to Eq. (10) is as follows:

$$Z_i^{\ 2}+2c_i^{VH}\ Z_i+c_i^{\Omega H*}=0, \qquad (12)$$

If $c_i^{\Omega H*}\le\left(c_i^{VH}\right)^2$ or $\omega_i^2\le\left(c_i^{VH}\right)^2+(k_i-c_i^{H*})/m_L$ then the general solution of Eq. (10) is:

$$T_i(\ t\ )\ =\ C_1\ \mathrm{e}^{Z_{i_1}t}+\ C_2\ \mathrm{e}^{Z_{i_2}t} \qquad (13)$$

The solution (13) does not describe our physical model. It is necessary to take into account the inequality $c_i^{\Omega H*}>\left(c_i^{VH}\right)^2$ or $\omega_i^2+c_i^{H*}/m_L-k_i/m_L>\left(c_i^{VH}\right)^2$.

In the above case there exists the solution described below.

$$T_i(t)=e^{-\,c_i^{VH}t}\left\{C_{1i}\sin\omega_i^{VH}t+C_{2i}\cos\omega_i^{VH}t\right\},$$
$$\omega_i^{VH}=\left(\omega_i^{\ 2}+c_i^{H*}/m_L-\left(c_i^{VH}\right)^2\right)^{\frac{1}{2}},\ i=1,2,... \qquad (14)$$

If the initial conditions for searched solution of the form $w_i(x,t)=X_i(x)\ T_i(t)$ (with fixed index i) for Eq. (1) (where $q(x,t)=0.$) are chosen as $w_i(x_o,t_o)=D_{oi}$, $\frac{\partial\ w_i}{\partial\ t}(\ x_o,t_o)=V_{oi}$, where $X_i(x)$ is a vibrating mode defined by formula (8), then we take the expression of the vibration mode as follows.

$$w_i(x,t)=\frac{X_i(x)}{X_i(x_o)}e^{-c_i^{VH}\ (t-t_o)}\left\{\left(c_i^{VH}\frac{D_{oi}}{\omega_i^{VH}}+\frac{V_{oi}}{\omega_i^{VH}}\right)\sin\omega_i^{VH}(t-t_0)+\right.$$

$$\left.+D_{oi}\cos\omega_i^{VH}(t-t_o)\right\},\ \omega_i^{VH}=\left\{\omega_i^{\ 2}+c_i^{H*}/m_L-\left(c_i^{VH}\right)^2\right\}^{\frac{1}{2}},\ i=1,2,...$$

For $t_0=0$ and $v_0=0$, we can write:

$$w_i(x,t)=X_i(x)\frac{D_{oi}}{X_i(x_o)}e^{-c_i^{VH}t}\times\left\{\frac{c_i^{VH}}{\omega_i^{VH}}\sin\omega_i^{VH}t+\cos\omega_i^{VH}t\right\},\ i=1,2,... \quad (15)$$

The following notation and formulas are used in the relation (15):

$$c_i^{VH}/\omega_i^{VH}=\text{ctg}(\alpha_i),\ \alpha_i=\text{arcctg}(c_i^{VH}/\omega_i^{VH}),$$

$$\alpha_i\in(0,\frac{\pi}{2}),\ \sin^2(\alpha_i)=(\omega_i^{VH})^2/(\omega_i^2+c_i^{H*}/m_L),$$

$$\cos^2(\alpha_i)=(\text{c}_i^{VH})^2/(\omega_i^2+c_i^{H*}/m_L)$$

Hence, the form deduced for the function $T_i(t)$ is:

$$T_i(t)=D_i^{H*}e^{-c_i^{VH}t}\sin(\omega_i^{VH}t+\alpha_i),\ i=1,2,...;\ D_i^{H*}=\frac{D_{oi}(\omega_i^2+c_i^{H*}/m_L)^{1/2}}{X_i(x_o)\omega_i^{VH}} \quad (16)$$

The function $\frac{dT_i(t)}{dt}$, deduced from (15), has the form:

$$\frac{dT_i(t)}{dt}=-D_i^{H*}e^{-c_i^{VH}\ t}\sin(\omega_i^{VH}t),\ i=1,2,... \quad (17)$$

Between the parameters of the mathematical model of the cable the following condition of compatibility arises for $i=1,2,...$:

$$(\omega_i^{VH})^2=\left\{\omega_i^{\ 2}-\frac{k_i}{m_L}+\frac{c_i^{H*}}{m_L}-\left(\frac{c_i^V}{2m_L}+\frac{c_i^H}{2m_L\omega_i^{VH}}\right)^2\right\},$$

The condition of compatibility has also the following form:

$$\begin{aligned}&4m_L^2(\omega_i^{VH})^4-(4m_L^2\omega_i^2-4m_Lk_i+4m_Lc_i^{H*}-\\&-(c_i^V)^2)(\omega_i^{VH})^2+2c_i^Vc_i^H\omega_i^{VH}+(c_i^H)^2=0.\end{aligned} \quad (18)$$

The infinite set of coefficients c_i^V, c_i^H, c_i^{H*}, $i = 1, 2, ...$ (with possibility of repetition), is supposed to be bounded.

The formulas (14) and (16) specify that the influence of the hysteretic and dry friction damping are negligible for vibration mode, if this mode is sufficient high, because $c_i^{VH} \approx c_i^V / m_L / 2,$ and $\omega_i^{VH} \approx \omega_i$, for sufficient high frequencies. However, the influence of viscous damping is maintained. This property of the cable is confirmed experimentally too.

The formula (14) can be used also for performing the objective function, referred to unknown parameters $EI,\ k_i,\ c_i^H,\ c_i^{H*},\ c_i^V$, and expressed by theoretical and experimental way.

We use cable displacements and the weighted least square method to identify the specified parameters. The expression of the objective function is as below.

$$f(EI, c_i^V, c_i^H, c_i^{H*}) = \sum_{i,x,t} w_i^{cof} \left\{ w_i^c(x,t) - w_i^{\exp}(x,t) \right\}^2 \qquad (19)$$

The theoretical $w_i^c(x,t)$ and experimental $w_i^{\exp}(x,t)$ displacements, theoretical calculated or measured in some points of abscises x, also for some moments of time and for some frequencies, in the domain of interest, and the weight $w_i^{cof} = 1./(w_i^{\exp}(x,t))^2$ assure the objective function dimensionless.

For performing of the parameters we used the model of cable defined by Eq. (1) on the hypothesis of viscous free damping ($c_i^H = 0,\quad c_i^{H*} = 0$) only. The searched parameters are the bending rigidity, EI, of the cable and the coefficient, c^V, of viscous damping referred to some frequencies in the domain of interest. The experimental values are measured by using an experimental stand with single overhead cable. The span is $L = 33m$, the ends of the cable are well fixed, $T = 1333N$, and $m_L = 0.757 kg/m$.

The analyzed frequencies, according to experimental values, are $f_9 = 11.89Hz,\ f_{15} = 19.5Hz,\quad f_{19} = 24.98Hz$. The points $x_1 = 0.089\ m$ $x_2 = 2.00\ m,\ x_3 = 16.20\ m$ are used. The moments of time correspond to the main values of the displacements. The values of the bending rigidity, $EI\ (40Nm^2)$ and of viscous damping, $c^V(2.675\ Ns/m)$, are determined by the minimization of the function (19) adapted to this case [15]-[18].

The diagram of the damped displacement of the cable for the resonance frequency of $19.5\ Hz$, in the point of abscissa $x = 16.2\ m$ on the cable span, is plotted in Fig. 1.

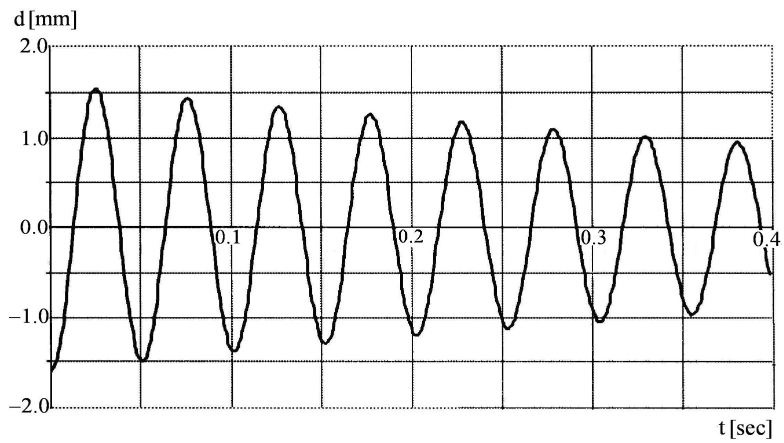

Figure 1. The diagram of damped displacement for resonance frequency

4. Possible equivalence between damping coefficients of cable

In this paragraph, the cases of cable with only dry friction, viscous or hysteretic internal damping are separately studied and the possible equivalence between damping coefficients of the cable by equalizing the absolute values of the energies dissipated by the damping forces of the cable is analyzed.

Cable with dry friction internal damping

In this case the equilibrium equation is:

$$m_L \frac{\partial^2 w_i}{\partial t^2} = -(c_i^{H*} + k_i) w_i + T \frac{\partial^2 w_i}{\partial x^2} - EI \frac{\partial^4 w_i}{\partial x^4} + q \qquad (20)$$

The free vibrations of the cable, with initial conditions $w_i(x_o, 0) = D_{oi}$, $\dot{w}_i(x_o, 0) = 0.$, are:

$$w_i(x,t) = \frac{X_i(x)}{X_i(x_o)} D_{oi} \cos \omega_i^{H*} t \; ; \; \left(\omega_i^{H*}\right)^2 = \omega_i^2 - \frac{k_i}{m_L} + \frac{c_i^{H*}}{m_L} > 0,\; i = 1, 2, \ldots \qquad (21)$$

The damping force of the cable is here of the form $c_i^{H*} w_i(x,t) = c_i^{H*} X_i(x) T_i(t)$, where $X_i(x)$ is defined through relation (8) and $T_i(t)$ through relation (21). The sign of the term c_i^{H*}, constant on piecewise, is such that the expression $c_i^{H*} w_i(x,t)\dot{w}_i(x,t) < 0$. There are denoted $c_{i0}^{H*} = \left|c_i^{H*}\right|$, $T_1 = \dfrac{\pi}{2\omega_i^{H*}}$, $T_2 = 2T_1, T_3 = 3T_1$, $T_4 = 4T_1 = \dfrac{2\pi}{\omega_i^{H*}}$. The sign of c_i^{H*} is positive for $t \in [0,T_1) \cup$ $\cup [T_2,T_3)$ and negative on the complementary time interval from the time interval cycle $[0,T_4) = [0,(2\pi/\omega_i^{H*}))$.

The corresponding dissipated energy of the cable, per cycle of vibration, referred to the above damping force, has the value below:

$$E_i^{H*} = \int_0^{T_4} c_i^{H*} T_i(t)(dT_i(t)/dt)\left(\int_0^L X_i^2(x)dx\right)dt - 2D_o^2\left(\int_0^L X_i^2(x)dx\right)c_{io}^{H*} / X_i^2(x_o)$$

Cable with viscous internal damping

In this case the equilibrium equation is:

$$m_L \frac{\partial^2 w_i}{\partial t^2} = -k_i w_i - c_i^V \frac{\partial w_i}{\partial t} + T\frac{\partial^2 w_i}{\partial x^2} - EI\frac{\partial^4 w_i}{\partial x^4} + q \qquad (22)$$

The free vibrations of the cable, in initial conditions $w(x_o,0) = D_o$, $\dot{w}(x_o,0) = 0.$, are:

$$w_i(x,t) = X_i(x)\frac{D_{oi}}{X_i(x_o)} e^{-c_i^{VH} t} \times \left\{\frac{c_i^{VH}}{\omega_i^V}\sin \omega_i^{VH} t + \cos \omega_i^{VH} t\right\},\ i = 1,2,...$$
$$c_i^{VH} = c_i^V / m_L / 2,\ \left(\omega_i^V\right)^2 = \omega_i^2 - k_i / m_L - \left(c_i^{VH}\right)^2 > 0. \qquad (23)$$

The damping force of the cable is here of the form $c_i^V \dot{w}_i(x,t) = c_i^V X_i(x)\dot{T}_i(t)$, where $X_i(x)$ is defined by the relation (8) and $T_i(t)$ through the relation (23). The corresponding dissipated energy of the cable, per cycle of vibration, referred to the above damping force, has the value below:

$$E_i^V = D_o^2\left(\int_0^L X_i^2(x)dx\right)\omega_i^2 \times m_L\left(1 - e^{-4\pi R}\right)/ X_i^2(x_o)/2;\ R = c_i^{VH} / \omega_i^V. \qquad (24)$$

For sufficiently high frequency we can approximate $\left(1 - e^{-4\pi R}\right) \approx 4\pi R$ and $\omega_i^V \approx \omega_i$ so that is respected the below relation.

$$E_i^V \approx \pi c_i^V \omega_i D_o^2 \left(\int_0^L X_i^2(x)dx \right) / X_i^2(x_o). \qquad (25)$$

Cable with hysteretic internal damping

In this case the equilibrium equation is:

$$m_L \frac{\partial^2 w_i}{\partial t^2} = -k_i w_i - \frac{c_i^H}{\omega_i^H} \frac{\partial w_i}{\partial t} + T \frac{\partial^2 w_i}{\partial x^2} - EI \frac{\partial^4 w_i}{\partial x^4} + q. \qquad (26)$$

The free vibrations of the cable, in initial conditions $w_i(x_o, 0) = D_{oi}$, $\dot{w}_i(x_o, 0) = 0.$, are:

$$w_i(x,t) = X_i(x) \frac{D_{oi}}{X_i(x_o)} e^{-c_i^{VH} t} \times \left\{ \frac{c_i^{VH}}{\omega_i^H} \sin \omega_i^{VH} t + \cos \omega_i^{VH} t \right\},\ i = 1, 2, ...$$
$$c_i^{VH} = c_i^H / \omega_i^H / m_L / 2,\ \left(\omega_i^H\right)^2 = \omega_i^2 - k_i / m_L - \left(c_i^{VH}\right)^2 > 0. \qquad (27)$$

The damping force of the cable is here of the form $\frac{c_i^H}{\omega_i^H} \dot{w}_i(x,t) = \frac{c_i^H}{\omega_i^H} X_i(x) \dot{T}_i(t)$, where the function $X_i(x)$ is defined by the relation (8) and $T_i(t)$ through the relation (27). The corresponding dissipated energy of the cable, per cycle of vibration, referred to the above damping force, has the value below:

$$E_i^H = D_o^2 \left(\int_0^L X_i^2(x)dx \right) \omega_i^2 m_L \times \left(1 - e^{-4\pi R}\right) / X_i^2(x_o) / 2;\ R = c_i^{VH} / \omega_i^H. \qquad (28)$$

For sufficiently high frequency we can approximate $\left(1 - e^{-4\pi R}\right) \approx 4\pi R$ and $\omega_i^H \approx \omega_i$ so that:

$$E_i^H \approx \pi c_i^H D_o^2 \left(\int_0^L X_i^2(x)dx \right) / X_i^2(x_o). \qquad (29)$$

The possible equivalence between internal damping coefficients of cable, in free vibration mode, is easily established for sufficiently high frequencies. The corresponding relations are as below.

$$c_{io}^{H*} = \frac{\pi}{2} \omega_i c_i^V;\ c_i^V = \frac{c_i^H}{\omega_i};\ c_{io}^{H*} = \frac{\pi}{2} c_i^H. \qquad (30)$$

For low frequencies, the equivalence is established by solving the transcendental algebraic equation, in the case when the solution exists.

5. Conclusion

The original analytical results, concerning the definition of the cable in viscous, hysteretic or dry friction internal damping hypothesis, using our cable model detached from the Euler-Bernoulli beam model, permit us to perform the analytical vibration modes of the cable, using a large number of model parameters that include the cable rigidity.

We remark analytically that there is the possibility for the suspended cable to consider simultaneously influence of viscous, hysteretic and dry friction internal damping, where the hysteretic and dry friction damping are negligible for a sufficiently high vibration mode of the cable while the influence of viscous damping is maintained. The simple algebraic relations of equivalence between internal damping coefficients of the cable are established for sufficiently high frequencies. For low frequencies, numerical methods for solving transcendental algebraic equations are needed.

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