The European Journal of Comparative Economics
Vol. 8, n. 2, pp. 203-233
ISSN 1824-2979

Educational Systems, Intergenerational Mobility and Social Segmentation

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Abstract

We show that the very characteristics of educational systems generate social segmentation. A stylised educational framework is constructed in which everyone receives a compulsory basic education and can subsequently choose between direct working, vocational studies and university. There is a selection for entering the university which consists of a minimum human capital level at the end of basic education. In the model, an individual's human capital depends (i) on her/his parents' human capital, (ii) on her/his schooling time, and (iii) on public expenditure for education. There are three education functions corresponding to each type of study (basic, vocational, university). Divergences in total educational expenditure, in its distribution between the three studies and in the selection severity, combined with the initial distribution of human capital across individuals, can result in very different social segmentations and generate under education traps (situations in which certain dynasties remain unskilled from generation to generation) at the steady state. We finally implement a series of simulations that illustrate these findings in the cases of egalitarian and elitist educational systems. Assuming the same initial distribution of human capital between individuals, we find that the first system results in two-segment stratification, quasi income equality and no under education trap whereas the elitist system generates three segments, significant inequality and a large under education trap.

JEL Classification: E24, H52, I21.

Keywords: educational systems; intergenerational mobility; social segmentation; under-education trap

1. Introduction[3]

In the economic literature, the impact of human capital acquisition upon social segmentation has been analysed through the emergence of under education traps, i.e., situations in which a proportion of the population remains unskilled from generation to generation.

In the early approach of Becker and Tomes (1979) with a perfectly competitive credit market, all the dynasties converge towards the same human capital in the long term. Assuming credit market imperfections, Loury (1981) and Becker and Tomes (1986) show that this convergence still holds but it is slowed down, thereby creating a 'low mobility trap' (Piketty, 2000).

These rather optimistic diagnoses were subsequently questioned by a number of works that analysed the emergence of under education traps. Several determinants can cause the emergence of such traps: a credit constraint with a fixed cost of education (Galor and Zeira, 1993, Barham et al., 1995), an S-shaped education function (Galor and Tsiddon, 1997), a neighbourhood effect resulting from local externalities (Benabou,

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[3] We wish to thank the French Research National Agency (ANR) for its financial support.

1993, 1996a, 1996b; Durlauf 1994, 1996), limited parental altruism (Das, 2007) etc. In most of these works, the trap results from non convexities that make certain individuals select low education. However, these approaches typically suppose that the institutional access to education is equally guaranteed. Financial constraints, family and social characteristics and limited abilities are then the main factors that explain the differences in educational choices and the related social segmentation.

However, since Weber (1906), the sociological literature has drawn attention to the fact that the educational system itself can create social segmentation (Bidwell and Friedkin, 1988, for an early review). It has been underlined that the type of knowledge that is promoted corresponds to the cultural backgrounds of the children from the upper and middle classes (Sorokin, 1959; Bourdieu and Passeron, 1970; Baudelot and Establet, 1971) and that families from the lower classes overestimate the cost of and underestimate the return from education (Boudon, 1973, 1974). In addition, because of better information and network effects, the children from higher classes select better educational strategies, and they have access to better positions than children from lower classes even when they possess the same degree (Anderson, 1961; Boudon, 1973, 1974; Thelot, 1982). Finally, a number of analysts have emphasised the influence of the selection pattern, i.e. the very structure of the educational system, on the formation and the persistence of social segmentation (Bourdieu and Passeron, 1970; Bowles and Gintis, 1976). Several recent empirical studies confirm the impact of the educational system upon social stratification. Using data from an international survey, Shavit and Muller (2000) find that the institutional characteristics of the school systems partly explain the differences in educational and occupational attainment across countries. Similarly, by comparing the transition from school to work in France, Germany, the UK and the US, Kerckhoff (2000) concludes that the differences across these countries are partially due to the differences in their educational systems.

If sociologists have studied for a long time the impact of hierarchical educational systems upon social stratification, this has only recently been investigated by the economic theory. Driskill and Horowitz (2002) and Su (2004) analyse hierarchical educational systems by focusing on the allocation of public funding between basic and advanced education. They study the impacts upon growth, welfare and income distribution, but not on social stratification. Bertocchi and Spagat (2004) model a three-level educational system (basic education and secondary education divided between vocational and general studies) so as to analyse social stratification during the different stages of economic development. However, their approach does not generate lasting under education trap because workers without secondary education disappear with the vanishing of the traditional sector.

Our objective is to analyse the impact of the structure of the educational system, i.e., the way the courses of study are organised with their different stages, divisions, selection procedures and funding, upon the formation and the intergenerational persistence of social segmentation.

The starting point of the analysis consists in a simplified stylised picture of the education systems that exist in most of the countries. These educational patterns exhibit rather similar structures, with however significant differences in the weight and funding devoted to each stage and in the severity of the selection procedures (see Tavares, 1995, for a description of the European systems in the mid-nineties). Compulsory schooling is enforced until the age of 15-18 in advanced countries, and until 12-15 years old in most of the developing countries. After compulsory schooling, young adults can either join the labour market, or pursue their studies. In the latter case, they typically face two courses of study. They can firstly select vocational studies. If such studies do exist in all countries, their shape and entry conditions significantly differ between as well as within countries. Usually, the access to vocational study does not require the obtainment of a final degree that sanctions secondary schooling (A-Level, Abitur, Baccalaureat etc.), even if this is the case for certain technical studies. In addition, vocational studies typically begin at upper secondary school level and can be part of an apprenticeship system. A second course of study consists in going to university, i.e., the tertiary educational system. Entering a university typically requires the obtainment of a degree that sanctions secondary school, and additional selection procedures are often enforced.

We firstly construct a simple stylised model that can describe this general educational framework, and that can be declined into various configurations. From this general model, we derive several possible social segmentations depending on the characteristics of the educational system. We finally implement a series of simulations that illustrate different social segmentations resulting from different educational systems.

The article is original in several respects. It firstly develops an intergenerational theoretical framework that allows modelling the impact of the structure of the educational system upon social segmentation. Secondly, the model generates social segmentations that depend on both the educational system and the initial distribution of human capital between households. Finally, different educational systems result in different segmentations for the same initial distribution of human capital.

The main features of the educational general framework are presented in section 2. The educational choices of individuals are analysed in section 3. The characteristics of the educational systems and the related social segmentations are determined in section 4. Section 5 analyses the human capital dynamics and the resulting segmentation. A series of simulations are implemented in section 6. We conclude in section 7.

2. The model general framework

2.1. Production

The economy produces one good with technology $Y = \omega H$, $H = \sum_j t_j h_j$, h_j being the human capital of individual *j* and t_j her/his time spent in the production activity. By assuming perfect competition on the market for goods, the profit is nil at equilibrium and ω is the before tax wage per unit of human capital$\times$time. As a consequence, individual *j* earns the pre-tax income $\omega_j = \omega h_j t_j$.

2.2. Individuals and Education

We consider a succession of generations with the same number *M* of individuals. The successive generations linked by a parent-child relationship form a dynasty.

An individual's life comprises two periods. Being young, s/he receives a basic education. Being adult, s/he lives one period of length 1 that s/he can divide between higher education and work.

The government provides individuals with both basic and higher education. Public education is funded by a tax on the parents' income at rate τ. The after tax wage per unit of human capital$\times$time is thus $w=(1-\tau)\omega$.

Basic education is compulsory and this provides individuals with the human capital necessary to get access to the labour market. In contrast, pursuing higher education is a choice of the individual who takes her/his decision by comparing the related income benefit and cost. Albeit spending the same time in basic education, individuals differ in their human capital at the end of this time. This is because intra-family externalities make children from more educated families more able to acquire the provided education. In addition, it is assumed that the market for credit is perfect and that the interest rate is nil[4]. These assumptions are tailored so as to place individuals in the most favourable situation in their choice for higher education, and thereby to focus on the sole impacts (i) of the uneven distribution of human capital across parents and (ii) of the educational structure, on the emergence of social stratifications.

At the end of basic education, an individual can choose, either to join directly the labour market, or to pursue further education. In the latter case, two courses of study are open that are exclusive of each other. The individual can firstly choose vocational studies (denoted *V*) without any constraint in terms of human capital attainment at the end of basic education. S/He can also go to university (denoted *U*) if her/his human capital at the end of basic education is at least $\bar{\lambda}_U$.

2.3. Education functions

Basic education produces individual *j*'s human capital according to the following function:

$$\lambda_j = \delta_B \left(h_j(-1)\right)^{\eta} \tag{1}$$

δ_B denotes the productivity in basic education that depends on the government's educational policy. Expression $\left(h_j(-1)\right)^{\eta}$ depicts the intra-family externality, i.e., the impact of the parent's human capital $h_j(-1)$ on her/his child's human capital at the end of basic education. We suppose that the marginal impact of the intra-family externality is decreasing, i.e. $0<\eta<1$.

The education function in higher education *i*, $i=V,U$ is $h_j=\max\left\{\delta_i e_{ji}{}^{\varepsilon_i}\lambda_j,\lambda_j\right\}$, with δ_i being the productivity and e_{ji} individual *j*'s schooling

[4] Assuming a discount factor (non zero interest rate) would not change the model outcome.

time in higher education *i*, and $0 < \varepsilon_i < 1$. When effective, i.e. for $\delta_i e_{ji}^{\varepsilon_i} \lambda_j > \lambda_j$, higher education of type *i* thus depends:

1) on the productivity δ_i in higher education *i*, that is produced by the educational policy,
2) on the time e_{ji} spent for studying in *i* with decreasing returns because $0 < \varepsilon_i < 1$, and
3) on the already acquired basic education λ_j.

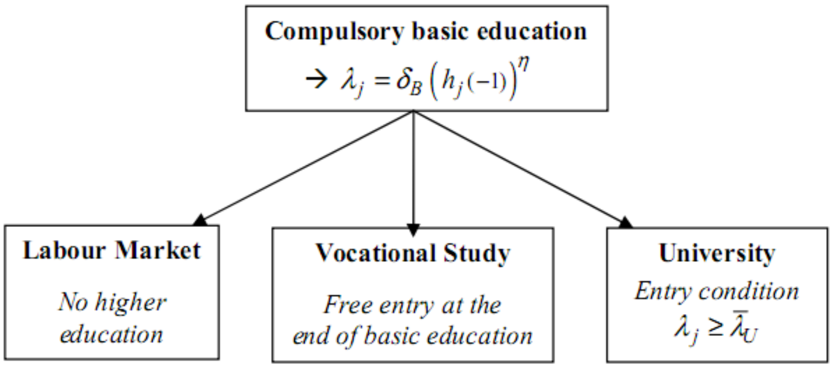
Pursuing higher education induces a fixed cost f paid by the individual and assumed to be identical for both types of higher education for the sake of simplicity. This cost consists in a fixed amount of goods, so that $f = \omega \times \bar{k} = w(1-\tau)^{-1}\bar{k}$, with $\bar{k}$ being the fixed amount of human capital utilised in terms of the fixed cost of education. There is a minimum schooling time for higher education to be effective ($\delta_i e_{ji}^{\varepsilon_i} \lambda_j > \lambda_j$): $e_{ji} > \delta_i^{-1/\varepsilon_i}$. This condition is always fulfilled at the individual's optimum because further education is only chosen if the related lifetime income is higher than that from direct working $w(1-e_{ji})\delta_i e_{ji}^{\varepsilon_i} \lambda_j - f > w\lambda_j$, which implies $e_{ji} > \delta_i^{-1/\varepsilon_i}$. The education function in study $i = V, U$ can thus be written:

$$h_j = \begin{cases} \delta_B \delta_i e_{ji}^{\varepsilon_i} \left(h_j(-1)\right)^{\eta} & \textit{iif} \quad e_{ji} > \delta_i^{-1/\varepsilon_i} \\ \lambda_j & \textit{otherwise} \end{cases} \tag{2}$$

A low ε_i signifies that the marginal gain from education decreases fast, and thus that the knowledge available from the study of type *i* can be captured rapidly. In return, with a high ε_i it is necessary to spend more time to acquire the human capital provided by study *i*. We suppose that $\varepsilon_V < \varepsilon_U$, i.e., that more time is necessary to acquire the knowledge provided by the university than that provided by vocational studies.

The set of possible educational choices is not the same for all individuals. All of them have access to vocational studies, whereas only these with a human capital higher than $\bar{\lambda}_U$ at the end of basic education can enter the university. Figure 1 depicts the individual's choices according to the entry conditions in each type of study.

Figure 1. The individual's choices at the end of basic education



As individual *j*' s human capital at the end of basic education is totally determined by relation $\lambda_j = \delta_B \left(h_j(-1)\right)^\eta$, it is possible to rewrite the entry condition in university in terms of her/his parent's human capital. As a consequence, only individuals whose parent's human capital is higher than $\bar{h}_U = \left(\bar{\lambda}_U / \delta_B\right)^{1/\eta}$ can enter the university.

3. Educational choices

Let us consider individual *j* endowed with human capital λ_j at the end of basic education. S/He chooses the educational pattern that maximises her/his lifetime income. If s/he chooses not to proceed with further education her/his lifetime income is $w\lambda_j$, and s/he earns $w(1-e_{ji})\delta_i e_{ji}{}^{\varepsilon_i} \lambda_j - f$ if s/he selects education *i*, *i* = *V*,*U*.

To determine the individual's educational choice, we firstly suppose that s/he pursues higher education and we calculate her/his related optimal schooling time. Afterwards, we compare the three possible choices (no further education, vocational study and university), and we select the strategy that maximises the individual's lifetime earning.

3.1. Schooling time

Lemma 1: An individual who selects higher education *i*, *i* = *V*,*U*, allows time $\hat{e}_i$ to this education, with:

$$\hat{e}_i = \varepsilon_i / (1 + \varepsilon_i) \tag{3}$$

Proof: By maximising $w(1-e_{ji})\delta_i e_{ji}{}^{\varepsilon_i} \lambda_j - f$ with respect to e_{ji} .

Since $\varepsilon_V < \varepsilon_U$, the time allowed for vocational study $\hat{e}_V = \varepsilon_V / (1+\varepsilon_V)$ is lower than that allocated to university $\hat{e}_U = \varepsilon_U / (1+\varepsilon_U)$.

3.2. Educational choices

Henceforth, we adopt the following notations:

$$\bar{\lambda} \equiv \frac{\bar{k}/(1-\tau)}{\delta_V (1-\hat{e}_V)\hat{e}_V{}^{\varepsilon_V} - 1} \tag{4}$$

$$\bar{\lambda}' \equiv \frac{\bar{k}/(1-\tau)}{\delta_U (1-\hat{e}_U)\hat{e}_U{}^{\varepsilon_U} - 1} \tag{5}$$

We denote $\bar{h}$ and $\bar{h}'$ the parents' human capital corresponding to the basic education levels $\bar{\lambda}$ and $\bar{\lambda}'$, i.e., $\bar{h} = \left(\bar{\lambda}/\delta_B\right)^{1/\eta}$ and $\bar{h}' = \left(\bar{\lambda}'/\delta_B\right)^{1/\eta}$.

Lemma 2: Individual *j* earns more (less) by working directly than by making vocational studies if $\lambda_j < \bar{\lambda}$ ($\lambda_j > \bar{\lambda}$) and s/he earns more (less) by working directly than by entering the university if $\lambda_j < \bar{\lambda}'$ ($\lambda_j > \bar{\lambda}'$).

Proof: Individual *j* earns more by working directly than by selecting study *i*, *i* = *V,U*, if the former provides her/him with a lifetime income higher than that given by study *i*, i.e. $w\delta_i(1-\hat{e}_i)\hat{e}_i^{\varepsilon_i}\lambda_j - f < w\lambda_j$. The conditions on λ_j are obtained by inserting $f = w(1-\tau)^{-1}\bar{k}$ into this inequality.

The related conditions on individual *j*'s parent human capital $h_j(-1)$ are (i) $h_j(-1) < \bar{h}$ for higher earnings from direct working than from vocational studies, and (ii) $h_j(-1) < \bar{h}'$ for higher earnings from direct working than from entering the university.

Lemma 3: All individuals prefer the university to vocational studies if $\bar{\lambda} > \bar{\lambda}'$ (equivalently $\bar{h} > \bar{h}'$), they all prefer vocational studies to the university if $\bar{\lambda} < \bar{\lambda}'$ ($\bar{h} < \bar{h}'$), and both choices are equivalent for everyone if $\bar{\lambda} = \bar{\lambda}'$ ($\bar{h} = \bar{h}'$).

Proof: Individuals prefer university to vocational studies if the former provides a higher lifetime income than the latter: $w(1-\hat{e}_U)\delta_U\hat{e}_U{}^{\varepsilon_U}\lambda_j > w(1-\hat{e}_V)\delta_V\hat{e}_V{}^{\varepsilon_V}\lambda_j$, i.e. $\delta_U(1-\hat{e}_U)\hat{e}_U{}^{\varepsilon_U} > \delta_V(1-\hat{e}_V)\hat{e}_V{}^{\varepsilon_V} \Leftrightarrow \bar{\lambda} > \bar{\lambda}' \Leftrightarrow \bar{h} > \bar{h}'$. Similarly, they prefer vocational studies to university if $\overline{\lambda} < \overline{\lambda}" \Leftrightarrow \overline{h} < \overline{h}'$, and both choices provide the same lifetime income if $\overline{\lambda} = \overline{\lambda}' \Leftrightarrow \overline{h} = \overline{h}'$.

Proposition 1: Consider individual j with human capital λ_j at the end of basic education. Then:

1) Individual *j* joins directly the labour market (i) when $\lambda_j < \overline{\lambda}$ and $\lambda_j < \overline{\lambda}'$, or (ii) when $\overline{\lambda}' \leq \lambda_j < \overline{\lambda}$ and $\lambda_j < \overline{\lambda}_U$.

2) Individual *j* selects vocational studies (i) when $\overline{\lambda} \leq \lambda_j < \overline{\lambda}'$, or (ii) when $\lambda_j \geq \overline{\lambda} > \overline{\lambda}'$ and $\lambda_j < \overline{\lambda}_U$.

3) Individual *j* enters the university when $\lambda_j \geq \overline{\lambda}'$, $\overline{\lambda} > \overline{\lambda}'$ and $\lambda_j \geq \overline{\lambda}_U$.

Case (1) corresponds to the two situations in which the individual directly joins the labour market. In the first, doing this provides her/him with higher income than selecting, either vocational studies or the university (because $\lambda_j < \overline{\lambda}$ and $\lambda_j < \overline{\lambda}'$, see Lemma 2). In the second, the individual's income is higher when working directly than when pursuing vocational studies and lower when working directly than when studying in the university ($\lambda_j < \overline{\lambda}$ and $\lambda_j \geq \overline{\lambda}'$, see Lemma 2), but this last option is impossible because her/his human capital at the end of basic education is lower than $\overline{\lambda}_U$.

Case (2) depicts the two situations in which the individual selects vocational studies. In the first, vocational studies provide her/him with a higher income than both direct working and university ($\lambda_j \geq \overline{\lambda}$ and $\lambda_j < \overline{\lambda}'$, Lemma 2). In the second, vocational studies is preferred to direct working ($\lambda_j \geq \overline{\lambda}$, Lemma 2), but less profitable than university ($\overline{\lambda} > \overline{\lambda}'$, Lemma 3), with this last choice being unachievable because of a lack of human capital at the end of basic education ($\lambda_j < \overline{\lambda}_U$).

Finally, case (3) describes the situation in which the individual chooses the university. This is when this choice provides her/him with a higher income than both direct working ($\lambda_j \geq \overline{\lambda}'$, Lemma 2) and vocational studies ($\overline{\lambda} > \overline{\lambda}'$, Lemma 3), with her/his human capital at the end of basic education being enough to enter this type of study.

By replacing the λ *s* by the related parent's human capital $h(-1) = \left(\lambda / \delta_B\right)^{1/\eta}$, we obtain the corollary proposition:

Corollary: Consider individual *j* whose parent's human capital is $h_j(-1)$. Then:

1) Individual *j* joins directly the labour market (i) when $h_j(-1) < \overline{h}$ and $h_j(-1) < \overline{h}'$, or (ii) when $\overline{h}' \leq h_j(-1) < \overline{h}$ and $h_j(-1) < \overline{h}_U$.

2) Individual *j* selects vocational studies (i) when $\overline{h} \leq h_j(-1) < \overline{h}'$, or (ii) when $h_j(-1) \geq \overline{h} > \overline{h}'$ and $h_j(-1) < \overline{h}_U$.

3) Individual *j* enters the university when $h_j(-1) \geq \overline{h}'$, $\overline{h} > \overline{h}'$ and $h_j(-1) \geq \overline{h}_U$.

Finally, the following feature can be established when both *V* and *V* are concurrently chosen inside a generation.

Lemma 4: Within the same generation, the coexistence of individuals who prefer *V* to *V* and individuals who prefer *V* to *V* is impossible. Consequently, if both *V* and *V* are selected within the same generation, then, either all the individuals who select *V* are constrained by the entry threshold $\overline{\lambda}_U$, or all individuals have the same lifetime earning when choosing *V* and *U*.

Proof: see Appendix 1.

4. Educational systems and segmentation

4.1. Educational Systems

The education functions (1) and (2) and the individuals' educational choices depend on the values δ_i, $i = B, V, U$. The educational policy determines these values according to the functions:

$$\delta_B = \overline{\delta}_B e_B{}^{\varepsilon_B} \left(\frac{q_B \tau y_{-1}}{e_B} \right)^{\beta} \tag{6}$$

$$\delta_i = \overline{\delta}_i \left(\frac{q_i \tau y_{-1}}{\mu_i \hat{e}_i} \right)^{\beta}, \quad i = V, U \tag{7}$$

q_i is the proportion of total levies allocated to education *i* ($q_B + q_V + q_U = 1$), y_{-1} the total income per capita in the parents' generation, μ_i the proportion of the current generation involved in study *i* (since all the individuals follow basic education, $\mu_B = 1$), e_B the compulsory schooling time, and coefficients $\overline{\delta}_i$ indicate the efficiency of public spending in the *i*-study. It can be noted that, unlike δ_V and δ_U, δ_B integrates the compulsory schooling time in basic education because this is part of the educational policy and because it was not accounted for in the basic education function (1). We also assume $0 < \beta < 1$, which indicates that the marginal impact of public spending on education is decreasing.

To provide an interpretation of function (7), let us rewrite it $\delta_i = \overline{\delta}_i \left(q_i \tau Y_{-1} / \mu_i M \hat{e}_i \right)^{\beta}$, where Y_{-1} is the total income of the parents' generation and *M* the number of dynasties. The productivity in the *i*-study depends (i) positively on the amount of levies $q_i \tau Y_{-1}$ allocated to *i*, and (ii) negatively on the number $\mu_i M$ of students involved in *i* and on the length $\hat{e}_i$ of this course of study. Expression $\left(q_i \tau Y_{-1} / \mu_i M \hat{e}_i \right)$ is thus the public expense for education *i* per pupil$\times$schooling time.

Definition 1: We call *'Educational System* $\left(\tau, q_B, q_V, q_U, \overline{\lambda}_U, e_B \right)$' the educational pattern that allocates levies $\tau Y_{-1} > 0$ to the three types of studies (*B,V,U*) in the proportions $q_B > 0$, $q_V \geq 0$, and $q_U \geq 0$, with the entry conditions $\overline{\lambda}_U$ in the university and the compulsory schooling time in basic education $e_B > 0$, the public education productivity functions (6) and (7) being given.

Features $\tau Y_{-1} > 0$, $q_B > 0$ and $e_B > 0$ are necessary for the existence of the educational system. In addition, a necessary condition for an educational system to be efficient is $\mu_i = 0 \Rightarrow q_i = 0$ for *i = V,U*. This condition signifies that there is no waste of public spending: if nobody chooses study *i,* then the social planner does not allocate funds to this study. Henceforth, we suppose that the social planner never allocates funds to a study which is chosen by no-one.

We denote $A_T \equiv \overline{\delta}_B e_B{}^{\varepsilon_B - \beta} \left(q_B \tau \right)^{\beta}$, $A_V \equiv A_T \overline{\delta}_V \hat{e}_V{}^{\varepsilon_V - \beta} \left(q_V \tau \right)^{\beta}$, and $A_U \equiv A_T \overline{\delta}_U \hat{e}_U{}^{\varepsilon_U - \beta} \left(q_U \tau \right)^{\beta}$. These values are constant for a given educational system.

We also denote $h_{j,k}$, $k = T, V, U$, the human capital of individual *j* whose highest skill is *k*, where *T* (for *Trap*, see hereafter) denotes that the individual has only received basic education, i.e., that s/he joins directly the labour market after compulsory schooling. Consequently, the education functions are (by inserting (6) and (7) into (1) and (2)):

$$h_{j,T} = A_T y_{-1}{}^{\beta} \left(h_j(-1)\right)^{\eta} \tag{8}$$

$$h_{j,i} = A_i \left(\mu_i(t)\right)^{-\beta} y_{-1}{}^{2\beta} \left(h_j(-1)\right)^{\eta}, \; i = V, U \tag{9}$$

Lemma 5: Everyone prefers the university to vocational studies if and only if $\frac{\mu_V}{\mu_U} > \rho = \left(\frac{(1-\hat{e}_V)A_V}{(1-\hat{e}_U)A_U}\right)^{1/\beta}$; everyone prefers vocational studies to university if and only if $\frac{\mu_V}{\mu_U} < \rho$; both studies bring the same lifetime income if and only if $\frac{\mu_V}{\mu_U} = \rho$.

Proof: see Appendix 2.

4.2. Segmentation and under-education trap

Segmentation occurs when the individuals inside a generation are divided between several segments (groups) in terms of educational choice and there is an *under education trap* if certain individuals do not pursue higher education. In the model developed here, there are three possible segments, T (under education trap), V (individuals selecting vocational studies) and V (individuals entering the university). The segmentation can be transitory or lasting. In the first case, certain segments tend to disappear with time.

Definition 2: There is a *permanent segmentation* if the proportions μ_i, $i = T, V, U$, are constant over time.

The case of permanent segmentation refers to a segmentation that is unchanged over time, which signifies that the number of segments and the number of dynasties in each segment remain constant. Permanent segmentation covers two cases. In the first, the economy tends towards a steady state in terms of human capital distribution and there is thus steady state segmentation. The population is then divided between several skill groups, the number of groups and the number of individuals inside each group remaining unchanged, and each group being characterised by one constant level of human capital at the steady state. In the second case, the human capital and the related income per capita increase or decrease from generation to generation (there is thus no steady state), but the distribution of the dynasties between the existing segments remains unchanged over time.

There is thus a *permanent under education trap* if there is a permanent social segmentation such that $\mu_T > 0$. In a situation of permanent under education trap, a constant number of dynasties remain at the basic education level from generation to generation. In the case of steady state segmentation, all these dynasties possess the same constant human capital.

4.3. Segmentation characteristics

A segmentation pattern is characterised by three different dimensions:

1) The number of segments;
2) The weight of each segment as a percentage of the working population;
3) The gaps between the segments in terms of human capital and income.

Acting on these three dimensions through the determinants of the educational system participates in a policy that concerns social inequality. We do not directly address this issue, which would require the choice of a social welfare function, and thus a level of inequality aversion and a time preference (since we have a succession of generations). We just analyse the impact of the educational system determinants $\left(\tau, q_B, q_V, q_U, \bar{\lambda}_U, e_B\right)$ and of the real product per capita on the aforementioned characteristics.

Firstly, the number of individuals inside the under education trap tends (i) to decrease with a rise in shares q_B and q_V of the levies allowed for basic education and vocational studies, (ii) to decrease with the income per capita y_{-1}, and (iii) to increase with the fixed cost of education $\bar{k}$ (proofs in Appendix 3). It can be noted that the increase in q_V is more efficient to reduce the trap that the increase in q_B. Finally, the impact of the tax rate τ is ambiguous because a rise in τ firstly increases δ_B and δ_V, which reduces the trap, but it also makes further education less profitable by raising its real fixed cost $\bar{k}/(1-\tau)$.

Secondly, the number of individuals that would like to enter the university and are prevented from this by the level of their human capital at the end of basic education is (i) reduced by a decrease in the selection threshold $\bar{\lambda}_U$, (ii) reduced by an increase in the share of total levies allocated to basic education q_B , and (iii) reduced by an increase in the tax rate τ and in the gross income per dynasty y_{-1} of the parents' generation (see proofs in Appendix 4).

Finally, it can be noted that an increase in q_B tends both to reduce the under education trap and to increase the number of young that are allowed to enter the university, which underlines the crucial impact of basic education.

In terms of income gap, it is clear that the individuals inside the under education trap have a lower lifetime income than these who pursue vocational studies. We show hereafter that the individuals from vocational studies always have a lower lifetime income than these from the university.

5. Dynamics, steady states and resulting segmentation

5.1. Steady States

We suppose that the number of individuals M is large enough so that the impact of any individual on total human capital H, and thus on the income per capita y, is negligible. From (8) and (9), we can write the human capital intergenerational mobility functions corresponding to the three possible choices of the individuals:

- If the individual does not pursue further education and stands in the under education trap:

$$h_j(t) = A_T y_{t-1}{}^{\beta} \left(h_j(t-1)\right)^{\eta} \tag{10}$$

- If the individual selects vocational studies:

$$h_j(t) = A_V y_{t-1}{}^{2\beta} \left(\mu_V(t)\right)^{-\beta} \left(h_j(t-1)\right)^{\eta} \tag{11}$$

- If the individual enters the university:

$$h_j(t) = A_U y_{t-1}{}^{2\beta} \left(\mu_U(t)\right)^{-\beta} \left(h_j(t-1)\right)^{\eta} \tag{12}$$

In addition, thresholds $\overline{h}$ and $\overline{h}_U$ are varying over time and decreasing with the income per capita[5]:

$$\overline{h}(t) = \left(\frac{\overline{k}/(1-\tau)}{A_V \left(\mu_V(t)\right)^{-\beta} (1-\hat{e}_V) y_{t-1}{}^{2\beta} - A_T y_{-1}{}^{\beta}} \right)^{1/\eta} \tag{13}$$

$$\overline{h}_U(t) = \left(\frac{\overline{\lambda}_U}{A_T y_{t-1}{}^{\beta}} \right)^{1/\eta} \tag{14}$$

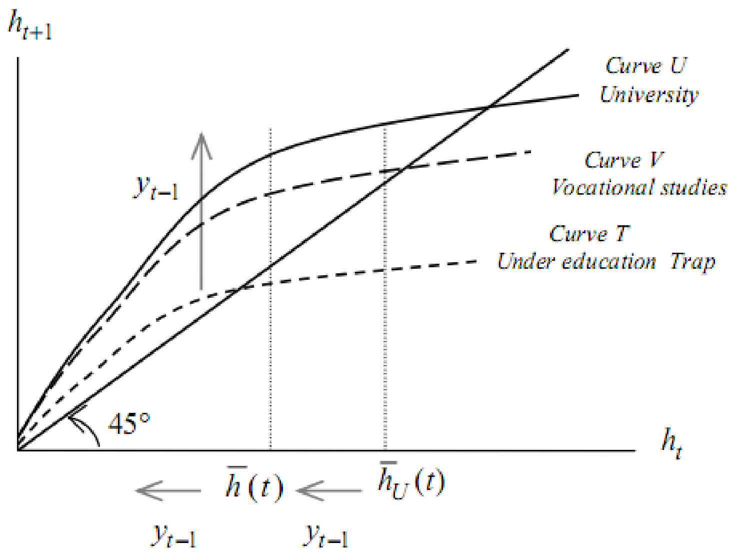
Finally, the product per capita at time t is:

[5] These functions are built by inserting (6) and (7) into $\overline{h} = \left(\overline{\lambda} / \delta_B\right)^{1/\eta}$ and $\overline{h}_U = \left(\overline{\lambda}_U / \delta_B\right)^{1/\eta}$.

$$y(t)=\frac{\omega}{M}\left(\sum_{j\in T}h_j(t)+\sum_{k\in V}(1-\hat{e}_V)h_k(t)+\sum_{l\in U}(1-\hat{e}_U)h_l(t)\right) \tag{15}$$

The three intergenerational dynamics are depicted on Figure 2.

Figure 2. The three intergenerational dynamics



Let us suppose that $\bar{h}'(t) < \bar{h}(t)$ so that the university brings a higher lifetime income than vocational studies. On Figure 2, all the individuals whose parents human capital stands beneath $\bar{h}(t)$ follow the dynamics situated on curve *T*, all these with a human capital of their parents located between $\bar{h}(t)$ and $\bar{h}_U(t)$ follow dynamics *V*, and those with parents above $\bar{h}_U(t)$ follow dynamics *U*. At the next generation (*t*+1) the curves *T, V* and *V* as well as the thresholds $\bar{h}(t)$ and $\bar{h}_U(t)$ are shifting. These moves depend on the product per capita at the preceding generation y_{t-1} that varies as long as the economy has not reached a steady state. When y_{t-1} increases (decreases), curves *T, V* and *V* move upwards (downwards) and $\bar{h}(t)$ and $\bar{h}_U(t)$ decrease (increase). In addition, these directly depend (except curve *T* and $\bar{h}_U(t)$) on the distribution of the dynasties across the segment in the current generation, which changes over time when certain dynasties pass from one segment to another because of the changes in the curves and the thresholds.

Proposition 2: Consider Educational System $\left(\tau, q_B, q_V, q_U, \overline{\lambda}_U, e_B\right)$ and the set of values $\left\{\left(\hat{h}_i, \hat{\mu}_i\right),\ i = T, V, U\right\}$ $\hat{\mu}_i, \hat{h}_i \geq 0$, $\sum \hat{\mu}_i = 1$, $q_k > 0$ if $\hat{\mu}_k > 0$, $k = V, U$, and such that:

1) $\hat{h}_T = \left(A_T \hat{y}^{\beta}\right)^{1/1-\eta}$, $\hat{h}_V = \left(A_V \hat{y}^{2\beta} \hat{\mu}_V{}^{-\beta}\right)^{1/(1-\eta)}$, and
$\hat{h}_U = \left(A_U \hat{y}^{2\beta} \hat{\mu}_U{}^{-\beta}\right)^{1/(1-\eta)}$ with
$\hat{y} = \omega\left(\hat{\mu}_T \hat{h}_T + \hat{\mu}_V (1-\hat{e}_V)\hat{h}_V + \hat{\mu}_U (1-\hat{e}_U)\hat{h}_U\right)$;

2) $\hat{h}_T \leq \hat{\overline{h}} = \left(\dfrac{\overline{k}/(1-\tau)}{A_V \hat{\mu}_V{}^{-\beta}(1-\hat{e}_V)\hat{y}^{2\beta} - A_T \hat{y}^{\beta}}\right)^{1/\eta}$;

3) $\hat{h}_U > \hat{\overline{h}}_U = \left(\dfrac{\overline{\lambda}_U}{A_T \hat{y}^{\beta}}\right)^{1/\eta}$ if $\hat{\mu}_U > 0$;

4) if both $\hat{\mu}_V, \hat{\mu}_U > 0$, then (i) $\dfrac{\hat{h}_U}{\hat{h}_V} \geq \left(\dfrac{1-\hat{e}_V}{1-\hat{e}_U}\right)^{\frac{1}{1-\eta}}$ and
$\dfrac{\mu_U}{\mu_V} \leq \left(\dfrac{1-\hat{e}_U}{1-\hat{e}_V}\dfrac{A_U}{A_V}\right)^{1/\beta}$ if $\hat{h}_V < \hat{\overline{h}}_U$, and (ii) $\dfrac{\hat{h}_U}{\hat{h}_V} = \left(\dfrac{1-\hat{e}_V}{1-\hat{e}_U}\right)^{\frac{1}{1-\eta}}$ and
$\dfrac{\mu_U}{\mu_V} = \left(\dfrac{1-\hat{e}_U}{1-\hat{e}_V}\dfrac{A_U}{A_V}\right)^{1/\beta}$ if $\hat{h}_V \geq \hat{\overline{h}}_U$.

Then, $\left\{\left(\hat{h}_i, \hat{\mu}_i\right),\ i = T, V, U\right\}$ define a steady state of Educational System $\left(\tau, q_B, q_V, q_U, \overline{\lambda}_U, e_B\right)$.

Proof: Feature (1): $\hat{h}_T$, $\hat{h}_V$ and $\hat{h}_U$ are respectively the steady states of dynamics (10), (11) and (12). Feature (2) stipulates that all the individuals inside the trap have no interest to pursue vocational studies and Feature (3) that all these who enter the university have enough human capital to do this. Features (4) give the conditions for the dynasties inside *V* to stay inside *V* and the dynasties inside *V* to stay inside *V* (see the proof in Appendix 5).

Lemma 6: At the steady state, the lifetime income is always lower for the dynasties in segment *V* than for these in segment *U*.

Proof: See Appendix 5.

It must be noted (i) that the same educational system typically results in several steady states depending on the initial distribution of human capital across dynasties, and (ii) that it can lead to no steady state.

5.2. Possible steady states and resulting segmentations

From the dynamics (10)-(12), several features may be identified:

(i) The functions defining the three dynamics being concave, there is a convergence of the different individuals towards the same human capital level inside each segment *T*, *V* and *U*.

(ii) During the dynamics, certain individuals typically pass from one segment to another. Consequently, for an initial distribution of human capital across the individuals, there are educational systems that cannot be maintained. This is because, after a number of generations, certain studies disappear as no-one chooses them any longer. Then, the efficiency condition ($q_i = 0$ if $\mu_i = 0$, $i = V, U$) makes that the initial educational system must be cancelled.

(iii) The long term evolution critically depends on the variation in the product per capita y_{t-1}. For the individuals who select vocational studies or university, they also depend on the changes in the proportions μ_V and μ_U of individuals in each type of study.

The analysis of the different dynamics and of their possible outcomes is described in Appendix 6. These dynamics depend on the values of $2\beta + \eta$ and $\beta + \eta$. From this analysis, we derive the following three propositions.

Proposition 3: Assume Educational System $\left(\tau, q_B, q_V, q_U, \bar{\lambda}_U, e_B\right)$ with $q_i > 0$, $i = V, U$, and education functions such that $2\beta + \eta < 1$. Then, the human capital dynamics can lead to the following outcomes:
1) Stable Steady states with three segments.
2) Stable steady states with the two segments *V* and *U*.
3) A withdrawal of the Educational System.

Proposition 4: Assume an Educational System $\left(\tau, q_B, q_V, q_U, \bar{\lambda}_U, e_B\right)$ and education functions such that $2\beta + \eta = 1$. Then, the human capital dynamics can lead to:
1) A stable steady state if and only if $q_i = 0,\ i = V, U$
2) A two-group (*V* and *U*) permanent segmentation, with the same steady growth rate of the product per capita.
3) A withdrawal of the educational system.
4) *U*nstable steady states, this outcome being nevertheless very unlikely.

Proposition 5: Assume an Educational System $\left(\tau, q_B, q_V, q_U, \bar{\lambda}_U, e_B\right)$ and education functions such that $2\beta + \eta > 1$. Then, the human capital dynamics can

lead to:
1) A stable steady state if and only if $q_i = 0,\ i = V, U$ and $\beta + \eta < 1$
2) If $q_i = 0,\ i = V, U$ and $\beta + \eta > 1$, either a collapse of the economy (i.e. a continuous decrease of human capital and the product per capita), or an unstable steady state, or explosive growth.
3) If $q_i > 0,\ i = V$ *and / or* U, either an unstable steady state, or explosive growth, or a withdrawal of the educational system.
4) In the case of explosive growth with $q_i > 0,\ i = V, U$, a permanent two-group segmentation with the same growth rates and the same lifetime income in both segments.

6. Simulations

We now implement a series of simulations that illustrate the divergent impacts of different educational patterns upon social segmentation. In this purpose, we apply three educational systems to the same economy. These must not be seen as depicting existing systems but rather as portraying two ideal-types, i.e., one egalitarian system (E) and one inequality-oriented and elitist system (I), and a system in-between (M for medium). The egalitarian system E is pro-education (high public funding), with a rather slight selection and a balanced distribution of taxes across the three courses of study (*B*, *V* and *U*). In the elitist system I, public funding for education is rather low, selection is severe and a substantial part of educational funding goes to a narrow elite. System M is in-between. We start from a certain distribution of human capital and we simulate the intergenerational impact of these three systems.

The egalitarian system results in the disappearing of the under-education trap (i.e., two segments) and in quasi income equality at the steady state. The second system generates three segments with a large under education trap and high inequalities in terms of skill and income. Finally, the third system generates two segments (no trap), quasi equality in the long term, and educational levels that stand in between the two first.

6.1. Initial characteristics and the educational systems parameters

We consider an education function such that $2\beta + \eta < 1$. This can lead, either to steady states of different shapes (2 or 3 segments), or to the withdrawal from the educational system (Proposition 3).

The education functions parameters are depicted in Table 1 and the educational system characteristics in Table 2.

Table 1. The economy's parameters

ω	η	β	ε_B	ε_V	ε_U	$\bar{k}$	$\bar{\delta}_B$	$\bar{\delta}_V$	$\bar{\delta}_U$
1	0.3	0.2	0.4	0.1	0.2	0.025	2.5	2	3

Table 2. Characteristics of the Educational Systems

System	τ	q_B	q_V	q_U	e_B	μ_U	$\hat{e}_V$	$\hat{e}_U$	$\bar{\lambda}_U$
E	0.07	0.7	0.1	0.2	0.2	0.2	0.091	0.166	0.9212
I	0.04	0.6	0.1	0.3	0.2	0.05	0.091	0.166	0.8723
M	0.055	0.7	0.1	0.2	0.2	0.1	0.091	0.166	0.9336

Coefficient η is chosen to correspond to the lower values given by the estimations of the elasticity of individual skill with respect to the parents' skill (see Solon, 1999). The parameters ε_B, ε_V, ε_U, $\bar{k}$ are selected so as to produce results consistent with observed facts in terms of schooling time for each stage of education (8-10 years for basic education, 2-3 years for vocational studies and about 5 years for university) and in terms of fixed cost (Table 1).

The egalitarian system E allocates 7% of total income to education, and its selection procedure is rather slight since it allows 20% of the first generation to enter the university. System E distributes levies between the three types of studies in the proportion $q_B =$ 0.7, $q_V =$0.1 and $q_U =$ 0.2, i.e. a large proportion of public expenditures allowed for basic education.

The elitist system I allocates 4% of total income to education, restricts the entry to the universities to 5% of the first generation (time 1), and it allows a rather large part of the levies to the universities (30%) at the expense of basic education.

Finally, system M allocates 5.5% of the total income for education, the levies being distributed in the same proportions as in case E with however only 10% of the first generation entering the university because of the selection threshold $\bar{\lambda}_U$.

It can be noted that these proportions are in line with those observed in Europe and the US, in which (i) public expenditure for education represents between 4-5% of the GDP (Greece, Italy, Spain) and 7-8% (Denmark, Sweden), and (ii) the share of tertiary education in total education expenditure is between 19% (Italy) and 33% (Denmark, Finland)[6].

The levels of $\bar{\delta}_B$, $\bar{\delta}_V$ and $\bar{\delta}_U$ are selected to obtain 70% of the individuals with basic education only, 20% in vocational studies and 10% in the university in the system M at the initial time 1.

[6] UNESCO database http://stats.uis.unesco.org/unesco.

We assume a constant number of 1000 dynasties. We start from an initial situation in which 80% of the parents are uniformly distributed over the interval $]0,1.029]$ and 20% are uniformly distributed over the interval $]1.029,1.49746]$. These values are chosen to have a human capital distribution consistent with the European situation in the early seventies.

The initial distribution of human capital once determined, the selection thresholds $\bar{\lambda}_U$ for each of the three systems (E, I and M) can be calculated to generate the desired proportion of a generation going to the university at the initial time (see Table 2).

The egalitarian system leads to the following situation at the initial time: 54% of people with basic education only, 26% of people who pursue vocational studies, and 20% of people going to university. In the elitist system, 80% of the population have basic education only, 15% pursue vocational studies and 5% go to university at the initial time. Finally, these proportions are respectively 69%, 21% and 10% in system M.

6.2. The results

Table 3 describes the characteristics of the three steady states, and Figures 3-5 the corresponding human capital dynamics for the 1000 dynasties.

The education trap vanishes in both the egalitarian and the medium systems, at generation 4 for the former and generation 7 for the latter. In contrast, the under education trap is maintained in the elitist system, and it still accounts for 76% of the dynasties at the steady state. Vocational studies and the university respectively represent 25% and 75% of a generation at the steady state in both scenarios E and M, whereas these proportions are 19% and 5% in I. Finally, E and M lead to a quasi equality of the lifetime incomes for all individuals at the steady state (individuals with a university level have a higher human capital than those from vocational studies, but this is almost fully offset by their working time that is lower).

Table 3. The steady states characteristics

	Egalitarian	**Elitist**	**In-between**
Number of segments at the steady state	2	3	2
$(\hat{\mu}_T, \hat{\mu}_V, \hat{\mu}_U)$	(0, 0.253, 0.747)	(0.763, 0.187, 0.05)	(0, 0.253, 0.747)
$(\hat{h}_T, \hat{h}_V, \hat{h}_U)$	(-, 1.82, 2.06)	(0.765, 0.919, 2.325)	(-, 1.322, 1.497)
Generation when the trap disappears	4th	*never*	7th
Product per capita at the steady state ($\hat{y}$)	1.705	0.837	1.236
$(\hat{I}_U / \hat{I}_T, \hat{I}_U / \hat{I}_V)$*	(-, 1.04)	(2.53, 2.32)	(-, 1.04)

* $\hat{I}_i, i = T, V, U,$ *is the steady state lifetime income corresponding to the educational choice i.*

Figure 3. Segmentation in the Egalitarian educational system (E)

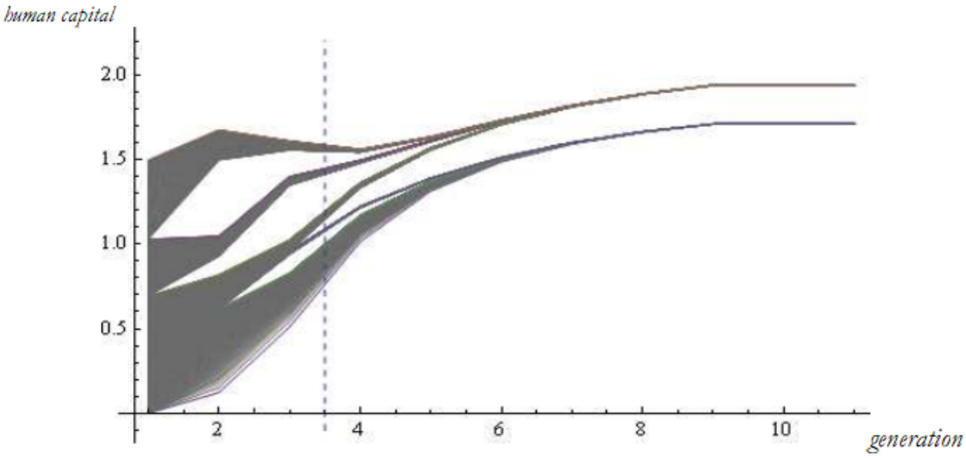


Figure 4. Segmentation in the Elitist educational system (I)

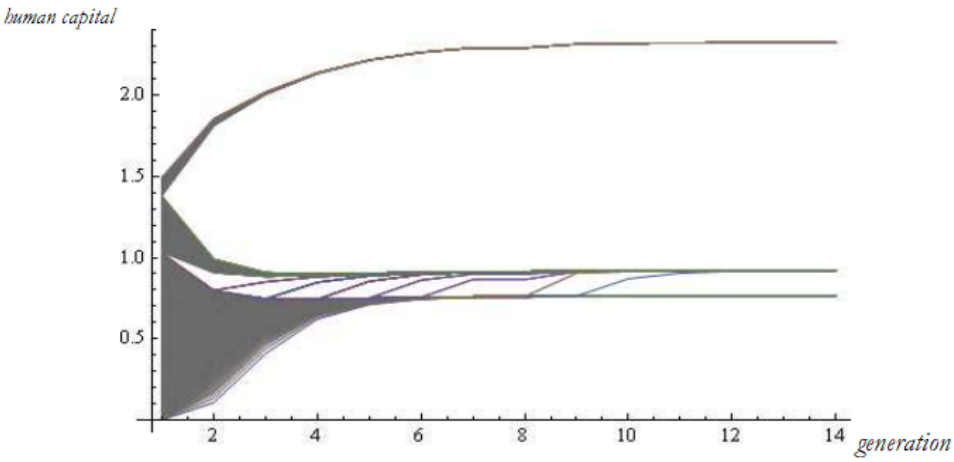
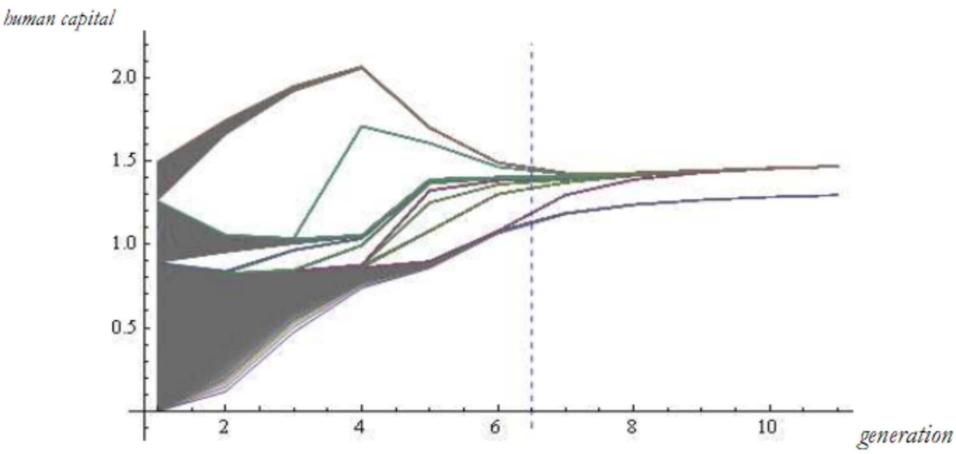


Figure 5. Segmentation in the system in-between (M)



7. Conclusion

We have constructed a stylised model that portrays the main features of educational systems and can be declined in several configurations. We have determined the characteristics of the related possible intergenerational dynamics and shown that the educational system characteristics combined with the initial distribution of human capital across individuals can generate very different social stratifications, with under education traps. This is because individuals from unskilled families attain a low human capital level at the end of basic education, this level being a key factor determining their performance in higher education. They therefore have no incentive to pursue further education because the related cost is higher than the related income benefit. Simulations have finally been implemented to illustrate these findings. The simulations portray two ideal-type systems, one egalitarian and the other elitist, and we also analyse a system in-between. The egalitarian system results in two-segment stratification, quasi income equality and no under education trap whereas the elitist system generates three segments, significant inequality and a large under education trap. This modelling could be extended (e.g., by inserting a larger choice for individuals so as to be closer to reality) and applied to the analysis of the existing systems. In particular, a distinction could be made between Scandinavian systems that provide between 7 and 8% of their GDP for education with a rather large share allocated to higher education and a rather low selection, and Southern European systems allocating 4-5% of their GDP to education, the expenses being centred on primary education. The model can also generate endogenous growth even for $2\beta+\eta<1$ if we assume human capital externalities in the production and/or the education function, or by adding an R&D activity that utilises human capital. Finally, the model could be extended to compare the welfare impacts of different educational systems and to analyse public strategies that combine successive educational systems.

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Appendix 1

Proof of Lemma 4: Individual *j* prefers *V* to *V* if this provides her/him with a higher lifetime income, i.e.
$w\delta_U\delta_B(1-\hat{e}_U)\hat{e}_U^{\varepsilon_U}\left(h_j(-1)\right)^\eta - f > w\delta_V\delta_B(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V}\left(h_j(-1)\right)^\eta - f$, which gives after simplifying $\delta_U(1-\hat{e}_U)\hat{e}_U^{\varepsilon_U} > \delta_V(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V}$. Identically, the condition for individual *l* to prefer *V* to *V* is
$w\delta_V\delta_B(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V}\left(h_l(-1)\right)^\eta > w\delta_U\delta_B(1-\hat{e}_U)\hat{e}_U^{\varepsilon_U}\left(h_l(-1)\right)^\eta$, which gives after simplifying $\delta_V(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V} > \delta_U(1-\hat{e}_U)\hat{e}_U^{\varepsilon_U}$. The coexistence of individuals who prefer *V* to *V* with individuals who prefer *V* to *U* is thus impossible. Hence, the coexistence of individuals who select *U* with individuals who select *V* is possible in two cases only: (i) if the individuals who select *V* prefer *U* but are impeached to enter the university because of threshold $\overline{\lambda}_U$, and (ii) if, for everyone, choosing *U* and *V* provides the same lifetime earning, which implies $\delta_U(1-\hat{e}_U)\hat{e}_U^{\varepsilon_U} = \delta_V(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V}$. In the later case, the individuals who select *V* can be or no be constrained by threshold $\overline{\lambda}_U$ for their entry into the university. Thus, when at least one individual who selects *V* is above threshold $\overline{\lambda}_U$, *U* and *V* must be equally profitable to each individual.

Appendix 2

Proof of Lemma 5: Everyone prefers the university to vocational studies when $w(1-\hat{e}_U)h_{j,U} - f > w(1-\hat{e}_V)h_{j,V} - f \Leftrightarrow (1-\hat{e}_U)h_{j,U} > (1-\hat{e}_V)h_{j,V},\quad \forall j$. By inserting $h_{j,U}$ and $h_{j,V}$ as defined by (9) into this inequality, it comes:
$\dfrac{\mu_V(t)}{\mu_U(t)} > \left(\dfrac{(1-\hat{e}_V)A_V}{(1-\hat{e}_U)A_U}\right)^{1/\beta} \equiv \rho$. Identically, everyone prefers vocational studies to the university if $\mu_V/\mu_U < \rho$, and both studies are equally profitable if $\mu_V/\mu_U = \rho$.

Appendix 3

Reducing the number of individuals inside the under education trap consists in lowering $\overline{h} = \overline{k}^{1/\eta}\left((1-\tau)\delta_B(\delta_V(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V} - 1)\right)^{-1/\eta}$. By inserting (6) and (7) into this expression we obtain:

$$\overline{h} = \overline{k}^{1/\eta}\left((1-\tau)\overline{\delta}_B e_B^{\varepsilon_B-\beta} q_B^{\beta}\right)^{-1/\eta}\left(\overline{\delta}_V(1-\hat{e}_V)\hat{e}_V^{\varepsilon_V-\beta}(q_V/\mu_V)^\beta\left(\tau y_{-1}\right)^{2\beta} - \left(\tau y_{-1}\right)^\beta\right)^{-1/\eta}.$$

The signs of the derivatives are $\dfrac{\partial\overline{h}}{\partial q_B} < 0$, $\dfrac{\partial\overline{h}}{\partial q_V} < 0$, $\dfrac{\partial\overline{h}}{\partial y_{-1}} < 0$ and $\dfrac{\partial\overline{h}}{\partial\overline{k}} > 0$. In addition $\partial\overline{h}/\partial q_V < \partial\overline{h}/\partial q_B < 0$ and the sign of $\partial\overline{h}/\partial\tau$ is ambiguous.

Appendix 4

From $\overline{h}_U = \left(\lambda_U / \delta_B\right)^{1/\eta} = \dfrac{\lambda_U^{\;1/\eta}}{\overline{\delta}_B^{\;1/\eta} e_B^{\;(\varepsilon_B-\beta)/\eta} \left(q_B \tau y_{-1}\right)^{\beta/\eta}}$, it is clear that

$\dfrac{\partial \overline{h}_U}{\partial \lambda_U} > 0$, $\dfrac{\partial \overline{h}_U}{\partial q_B} < 0$, $\partial \overline{h}_U / \partial \tau < 0$ and $\partial \overline{h}_U / \partial y_{-1} < 0$.

Appendix 5

Proof of Feature (4) of Proposition 2: As $\hat{h}_V = \left(A_V \hat{y}^{2\beta} \hat{\mu}_V^{\;-\beta}\right)^{1/(1-\eta)}$ and $\hat{h}_U = \left(A_U \hat{y}^{2\beta} \hat{\mu}_U^{\;-\beta}\right)^{1/(1-\eta)}$ (Proposition 2, feature 1), then $\dfrac{\hat{h}_V}{\hat{h}_U} = \left(\dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}\right)^{1/(1-\eta)}$.

The dynasties inside segment *V* remain in *V* and those inside *U* remain in *U* in two cases:

1) when $\hat{h}_V < \hat{\overline{h}}_U < \hat{h}_U$ and $w(1-\hat{e}_U) A_U y_{t-1}^{\;2\beta} \hat{\mu}_U^{\;-\beta} \left(\hat{h}_i\right)^{\eta} - f \geq w(1-\hat{e}_V) A_V y_{t-1}^{\;2\beta} \hat{\mu}_V^{\;-\beta} \left(\hat{h}_i\right)^{\eta} - f$, $i = V, U$, i.e.,

$\dfrac{1-\hat{e}_U}{1-\hat{e}_V} \geq \dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}$, and thus $\dfrac{\hat{h}_V}{\hat{h}_U} = \left(\dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}\right)^{1/(1-\eta)} \leq \left(\dfrac{1-\hat{e}_U}{1-\hat{e}_V}\right)^{1/(1-\eta)}$.

2) when $\hat{h}_V > \hat{\overline{h}}_U$ and selecting *V* or *U* is indifferent for both the individuals in *V* and *U*, i.e. $w(1-\hat{e}_U) A_U y_{t-1}^{\;2\beta} \hat{\mu}_U^{\;-\beta} \left(\hat{h}_i\right)^{\eta} = w(1-\hat{e}_V) A_V y_{t-1}^{\;2\beta} \hat{\mu}_V^{\;-\beta} \left(\hat{h}_i\right)^{\eta}$, $i = V, U$ (Lemma 4), i.e., $\dfrac{1-\hat{e}_U}{1-\hat{e}_V} = \dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}$, and thus $\dfrac{\hat{h}_V}{\hat{h}_U} = \left(\dfrac{1-\hat{e}_U}{1-\hat{e}_V}\right)^{1/(1-\eta)}$.

Inequality $w(1-\hat{e}_U) A_U y_{t-1}^{\;2\beta} \hat{\mu}_U^{\;-\beta} \left(\hat{h}_i\right)^{\eta} \geq w(1-\hat{e}_V) A_V y_{t-1}^{\;2\beta} \hat{\mu}_V^{\;-\beta} \left(\hat{h}_i\right)^{\eta}$ always holds, and thus: $\dfrac{1-\hat{e}_U}{1-\hat{e}_V} \geq \dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}$.

Proof of Lemma 6: The steady state lifetime income is lower for the dynasties in segment *V* than for these in segment *U* if and only if $(1-\hat{e}_U)\hat{h}_U > (1-\hat{e}_V)\hat{h}_V$. In all cases we have (see above): $\dfrac{1-\hat{e}_U}{1-\hat{e}_V} \geq \dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}$. Since $\dfrac{\hat{h}_V}{\hat{h}_U} = \left(\dfrac{A_V}{A_U}\left(\dfrac{\hat{\mu}_U}{\hat{\mu}_V}\right)^{\beta}\right)^{1/(1-\eta)}$, then

$\frac{\hat{h}_V}{\hat{h}_U} \le \left(\frac{1-\hat{e}_U}{1-\hat{e}_V}\right)^{1/(1-\eta)}$, and after rearranging $\frac{(1-\hat{e}_V)\hat{h}_V}{(1-\hat{e}_U)\hat{h}_U} \le \left(\frac{1-\hat{e}_U}{1-\hat{e}_V}\right)^{\eta/(1-\eta)}$, with $\left(\frac{1-\hat{e}_U}{1-\hat{e}_V}\right)^{\eta/(1-\eta)} < 1$ since $1 > \hat{e}_U > \hat{e}_V$. Hence $(1-\hat{e}_U)\hat{h}_U > (1-\hat{e}_V)\hat{h}_V$.

Appendix 6

Equations (10)-(12) and Figure 2 in the text describe the dynamics of the dynasties depending on their segment (*T*, *V*, or *U*). In each of the possible three segments, the individual dynamics converge towards the same human capital level. To analyse the different possible outcomes of these dynamics, let us suppose that all the dynasties belonging to one segment have the same human capital at each period of time (this human capital changing with time). Assuming this, there are three dynamics only which are as follows:

- For the dynasties inside the under education trap:

$$h_T(t) = A_T {y_{t-1}}^{\beta} \left(h_T(t-1)\right)^{\eta} \tag{A1}$$

- For the dynasties following vocational studies:

$$h_V(t) = A_V {y_{t-1}}^{2\beta} \left(\mu_V(t)\right)^{-\beta} \left(h_V(t-1)\right)^{\eta} \tag{A2}$$

- For the dynasties having a university degree:

$$h_U(t) = A_U {y_{t-1}}^{2\beta} \left(\mu_U(t)\right)^{-\beta} \left(h_U(t-1)\right)^{\eta} \tag{A3}$$

with:

$$y_{t-1} = \omega\left(\mu_T(t-1)h_T(t-1) + (1-e_V)\mu_V(t-1)h_V(t-1) + (1-e_U)\mu_U(t-1)h_U(t-1)\right) \tag{A4}$$

By inserting (A4) into (A1)-(A3), and after rearranging, it comes:

$$\frac{h_T(t)}{h_T(t-1)} = B_T\left(a_T + \frac{a_V h_V(t-1) + a_U h_U(t-1)}{h_T(t-1)}\right)^{\beta} \left(h_T(t-1)\right)^{\eta+\beta-1} \tag{A5}$$

$$\frac{h_V(t)}{h_V(t-1)} = B_V\left(a_V + \frac{a_T h_T(t-1) + a_U h_U(t-1)}{h_V(t-1)}\right)^{2\beta} \left(h_V(t-1)\right)^{2\beta+\eta-1} \tag{A6}$$

$$\frac{h_U(t)}{h_U(t-1)} = B_U \left(a_U + \frac{a_T h_T(t-1) + a_V h_V(t-1)}{h_U(t-1)} \right)^{2\beta} \left(h_U(t-1)\right)^{2\beta+\eta-1} \tag{A7}$$

with $B_T = A_T \omega^{\beta}$, $B_V = A_V \omega^{2\beta} \left(\mu_V(t)\right)^{-\beta}$, $B_U = A_U \omega^{2\beta} \left(\mu_U(t)\right)^{-\beta}$, $a_T = \mu_T(t-1)$, $a_V = (1-\hat{e}_V)\mu_V(t-1)$, and $a_U = (1-\hat{e}_U)\mu_U(t-1)$.

Combining (A2) and (A3) yields:

$$\frac{h_V(t)}{h_U(t)} = \frac{B_V}{B_U} \left(\frac{h_V(t-1)}{h_U(t-1)} \right)^{\eta} \tag{A8}$$

From (A8), it is clear that $h_V(t)/h_U(t)$ tends towards the steady value $\left(B_V / B_U\right)^{1/1-\eta}$. Hence, any steady state is such that:

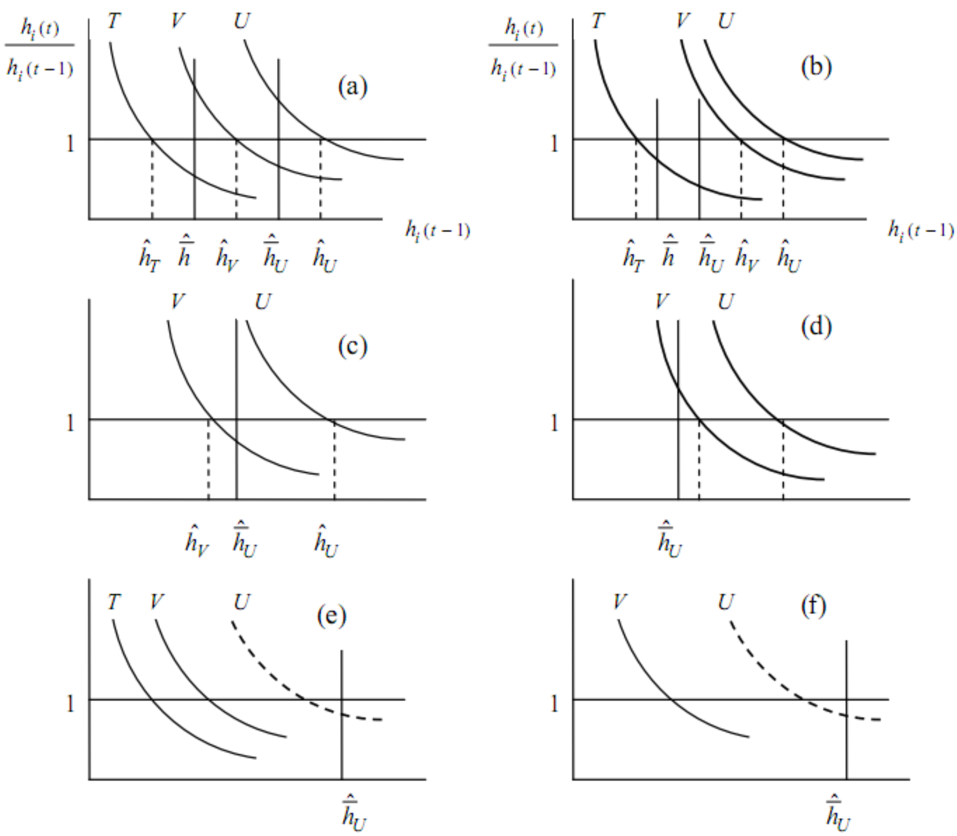
$$\frac{\hat{h}_V}{\hat{h}_U} = \left(\frac{B_V}{B_U} \right)^{\frac{1}{1-\eta}} = \left(\frac{\bar{\delta}_V \hat{e}_V{}^{\varepsilon_V - \beta} q_V{}^{\beta}}{\bar{\delta}_U \hat{e}_U{}^{\varepsilon_U - \beta} q_U{}^{\beta}} \right)^{\frac{1}{1-\eta}} \left(\frac{\hat{\mu}_U}{\hat{\mu}_V} \right)^{\frac{\beta}{1-\eta}} \tag{A9}$$

In addition, suppose that at the steady state the dynasties in segment *V* are not constrained by the barrier to entry in the university, i.e. $\hat{h}_V \geq \hat{\bar{h}}_U$. Then, we know from Proposition 2 that $\frac{\hat{h}_U}{\hat{h}_V} = \left(\frac{1-\hat{e}_V}{1-\hat{e}_U} \right)^{1/(1-\eta)}$ and $\frac{\mu_U}{\mu_V} = \left(\frac{1-\hat{e}_U}{1-\hat{e}_V} \frac{A_U}{A_V} \right)^{1/\beta}$. Relations (A5)-(A7) form a three-equation dynamic system that depends on the values $2\beta+\eta$ and $\beta+\eta$.

1) First case: $2\beta+\eta<1$. As $2\beta+\eta-1<0$, functions (A5)-(A7) are respectively decreasing in $h_i(t-1)$, $i=T,V,U$. The possible outcomes of these dynamics are depicted on Figures A1.

In Cases (a) and (b), the dynamics result in three-segment steady states ($\hat{h}_T < \hat{\bar{h}}$). In the case (a), certain dynasties would select the university if they were not prevented for this by the level of the barrier to entry ($\hat{h}_V < \hat{\bar{h}}_U$). As a consequence, for the values $\hat{h}_V$ and $\hat{h}_U$, selecting the university would provide higher earnings to the individuals in *V* and $\frac{\mu_U}{\mu_V} < \left(\frac{1-\hat{e}_U}{1-\hat{e}_V} \frac{A_U}{A_V} \right)^{1/\beta}$ (Proposition 2). In case (b), the dynasties who select vocational studies are not constrained by the selection for university ($\hat{h}_V > \hat{\bar{h}}_U$). Then, individuals are indifferent between *V* and *U*[7]; hence $\frac{\hat{h}_U}{\hat{h}_V} = \left(\frac{1-\hat{e}_V}{1-\hat{e}_U}\right)^{1/(1-\eta)}$ and $\frac{\mu_U}{\mu_V} = \left(\frac{1-\hat{e}_U}{1-\hat{e}_V}\frac{A_U}{A_V}\right)^{1/\beta}$ (Proposition 2).

Figures A1. Phase diagrams when $2\beta + \eta < 1$



In cases (c) and (d), the total income per capita is high enough to make all dynasties go out of the under education trap. Then, two segments only subsist. Case (c) corresponds to the situation in which certain dynasties would leave vocational studies for the university if they were not constrained by the barrier $\hat{\bar{h}}_U$, and case (d) to the situation where they are not constrained $\hat{h}_V \geq \hat{\bar{h}}_U$ and thus indifferent between *V* and *U*. So, case (d) corresponds (Proposition 2) to $\frac{\hat{\mu}_U}{\hat{\mu}_V} = \left(\frac{1-\hat{e}_U}{1-\hat{e}_V}\frac{A_U}{A_V}\right)^{1/\beta}$, $\hat{\mu}_U = 1 - \hat{\mu}_V$,

[7] The individuals in *V* are indifferent between *V* and *U*, but the individuals in *U* earn more that those in *V* because their parents have higher human capital (Lemma 6).

$\hat{h}_V = \left(A_V \hat{y}^{2\beta} \hat{\mu}_V{}^{-\beta} \right)^{1/(1-\eta)}$, $\hat{h}_U = \left(A_U \hat{y}^{2\beta} \hat{\mu}_U{}^{-\beta} \right)^{1/(1-\eta)}$ and $\hat{y} = \omega\left((1-\hat{e}_V)\hat{\mu}_V \hat{h}_V + (1-\hat{e}_U)\hat{\mu}_U \hat{h}_U \right)$. This defines one unique possible two-segment steady state $\left\{ \left(\hat{h}_V, \hat{\mu}_V \right), \left(\hat{h}_U, \hat{\mu}_U \right) \right\}$ with $\hat{h}_V \geq \hat{\bar{h}}_U$.

In Cases (e) and (f), nobody enters the university. This corresponds to $\hat{h}_U < \hat{\bar{h}}_U$ for the smallest possible value of μ_U, i.e. $\mu_U = 1/M$. The educational system must thereby be waived. This rather unlikely case corresponds to an extremely high barrier to entry $\bar{\lambda}_U$ or/and an extremely low allowance to the university.

2) Second case: $2\beta + \eta = 1$. Since $2\beta + \eta - 1 = 0$, equations (A5)-(A7) become:

$$\frac{h_T(t)}{h_T(t-1)} = B_T \left(a_T + \frac{a_V h_V(t-1) + a_U h_U(t-1)}{h_T(t-1)} \right)^{\beta} \left(h_T(t-1) \right)^{\eta+\beta-1} \tag{A10}$$

$$\frac{h_V(t)}{h_V(t-1)} = B_V \left(a_V + \frac{a_T h_T(t-1) + a_U h_U(t-1)}{h_V(t-1)} \right)^{2\beta} \tag{A11}$$

$$\frac{h_U(t)}{h_U(t-1)} = B_U \left(a_U + \frac{a_T h_T(t-1) + a_V h_V(t-1)}{h_U(t-1)} \right)^{2\beta} \tag{A12}$$

It is clear that, in the case $q_V = q_U = 0$, equations (A6) and (A7) disappear and equation (A5) becomes $\dfrac{h_T(t)}{h_T(t-1)} = B_T a_T{}^{\beta} \left(h_T(t-1) \right)^{\beta+\eta-1}$, which generates the unique stable steady state $\left\{ \hat{h}_T = \left(a_T{}^{\beta} B_T \right)^{1-\beta-\eta}, \hat{\mu}_T = 1 \right\}$.

We now analyse the cases with $q_i > 0,\ i = V, U$ ($q_B > 0$ by definition).

1. Let us firstly suppose that the distribution of human capital across the dynasties makes the income per capita increase. Hence, (i) all the dynasties get out of the under education trap resulting in a two-segment stratification , and (ii) all the dynasties inside segment *V* (these selecting vocational studies) are no longer constrained by the barrier to university $\bar{\lambda}_U$. In such a situation, the system (A10)-(A12) becomes:

$$\frac{h_V(t)}{h_V(t-1)} = B_V \left(a_V + a_U \frac{h_U(t-1)}{h_V(t-1)} \right)^{2\beta} \quad \text{(A13)}$$

$$\frac{h_U(t)}{h_U(t-1)} = B_U \left(a_U + a_V \frac{h_V(t-1)}{h_U(t-1)} \right)^{2\beta} \quad \text{(A14)}$$

Since there are two segments and the dynasties are not constrained by threshold $\bar{\lambda}_U$, we have $\mu_U(t) = 1 - \mu_V(t)$ and $\frac{\mu_U(t)}{\mu_V(t)} = \left(\frac{A_U(1-\hat{e}_U)}{A_V(1-\hat{e}_V)} \right)^{1/\beta}$ (Lemma 5), which yields:

$$\hat{\mu}_U = \frac{\left(A_U(1-\hat{e}_U) \right)^{1/\beta}}{\left(A_V(1-\hat{e}_V) \right)^{1/\beta} + \left(A_U(1-\hat{e}_U) \right)^{1/\beta}}$$

$$\hat{\mu}_V = \frac{\left(A_V(1-\hat{e}_V) \right)^{1/\beta}}{\left(A_V(1-\hat{e}_V) \right)^{1/\beta} + \left(A_U(1-\hat{e}_U) \right)^{1/\beta}}$$

Consequently, there is a permanent segmentation with proportions $\left(\hat{\mu}_V, \hat{\mu}_U \right)$ and this fully determines the values B_i and a_i , $i = V, U$.

By combining (A13) and (A14), we generate the following implicit function:

$$\left(\frac{h_V(t)}{h_V(t-1)} \right)^{\frac{1}{2\beta}} \left(\frac{h_U(t)}{h_U(t-1)} \right)^{\frac{1}{2\beta}} - a_U \left(B_U \frac{h_V(t)}{h_V(t-1)} \right)^{\frac{1}{2\beta}} - a_V \left(B_V \frac{h_U(t)}{h_U(t-1)} \right)^{\frac{1}{2\beta}} = 0 \quad \text{(A15)}$$

Because of (A8), we know that the only solution consistent with the dynamics is such that $\frac{h_V(t)}{h_V(t-1)} = \frac{h_U(t)}{h_U(t-1)}$. Inserting $\lambda \equiv \frac{h_i(t)}{h_i(t-1)}$, $i = V, U,$ into (A15), the implicit function becomes $\lambda^{1/\beta} - a_U \left(B_U \lambda \right)^{1/2\beta} - a_V \left(B_V \lambda \right)^{1/2\beta} = 0$, with the solution:

$$\lambda = \left(a_U {B_U}^{1/2\beta} + a_V {B_V}^{1/2\beta} \right)^{2\beta}$$

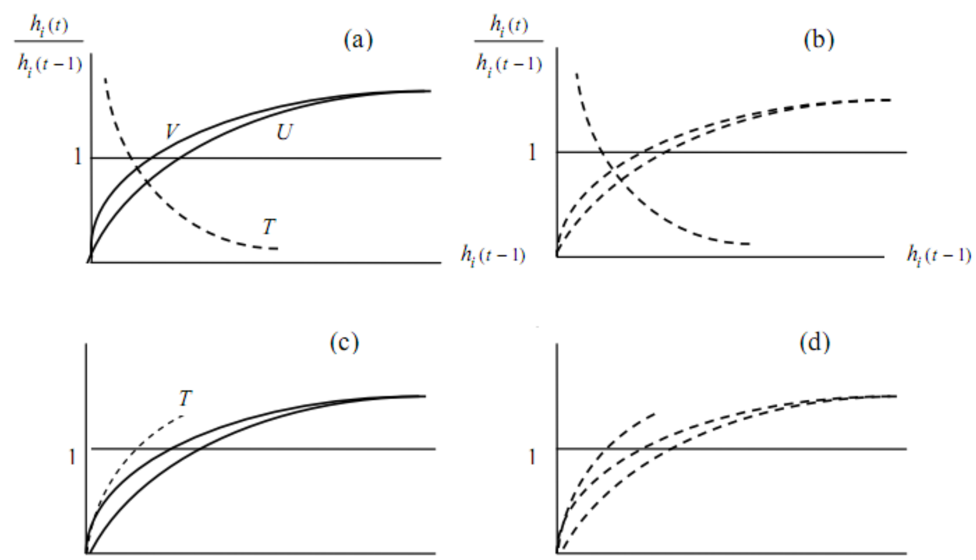
Both $h_V(t)$ and $h_U(t)$ grow at the same steady rate $\hat{\gamma} = \left(a_U {B_U}^{1/2\beta} + a_V {B_V}^{1/2\beta} \right)^{2\beta} - 1$, as well as the income per capita $y_t = \omega\left((1-\hat{e}_V)\hat{\mu}_V h_V(t) + (1-\hat{e}_U)\hat{\mu}_U h_U(t) \right)$. Finally for such a dynamics to occur, condition $\left(a_U {B_U}^{1/2\beta} + a_V {B_V}^{1/2\beta} \right)^{2\beta} > 1$ must be fulfilled.

2. When the distribution of human capital across dynasties makes the income per capita decrease, this moves upwards the entry threshold $\overline{h}_U$ and downwards the human capital in segment *U*, $h_U(t)$. From a certain time, nobody can enter the university, making this segment disappear as well as the Educational system (since it is no longer possible to maintain $q_U > 0$ without violating the efficiency condition).

3. Finally the dynamic system (A10)-(A12) may theoretically generate unstable steady states with two (*V* and *U*) or three (*T*, *V* and *U*) segments, but this outcome is very unlikely.

3) Third case: $2\beta + \eta > 1$. In this case, the functions $h_i(t)/h_i(t-1),\ i = V, U$, are increasing in $h_i(t-1)$ and equal to 0 for $h_i(t-1) = 0$ (relations A6 and A7). In contrast, function $h_T(t)/h_T(t-1)$ can be either decreasing or increasing in $h_T(t-1)$, depending on the sign of $\beta + \eta$. Figures A2(a) and A2(b) depict the former case, and A2(c) and A2(d) the latter.

Figures A2. Phase diagrams when $2\beta + \eta > 1$



1. Figure A2(a) depicts the case $2\beta + \eta > 1$ and $\beta + \eta < 1$ when the distribution of human capital across the dynasties results in a growing income per capita, making thereby the under education trap empty and the barrier to university inoperative. Since there are two segments and the dynasties are not constrained by threshold $\overline{\lambda}_U$, we have for the same reason as in case $2\beta + \eta = 1$:

$$\hat{\mu}_U = \frac{\left(A_U(1-\hat{e}_U)\right)^{1/\beta}}{\left(A_V(1-\hat{e}_V)\right)^{1/\beta} + \left(A_U(1-\hat{e}_U)\right)^{1/\beta}}$$

$$\hat{\mu}_V = \frac{\left(A_V(1-\hat{e}_V)\right)^{1/\beta}}{\left(A_V(1-\hat{e}_V)\right)^{1/\beta} + \left(A_U(1-\hat{e}_U)\right)^{1/\beta}}$$

In addition, for the same reason as in case $2\beta+\eta=1$, both $h_V(t)$ and $h_V(t)$, and thus the income per capita *y*(*t*), grow at the same rate. Since $h_i(t)/h_i(t-1),\ i=V,U$ is increasing in $h_i(t-1)$, the growth rate is increasing with time (growth is explosive).

2. Figure A2(b) depicts the case $2\beta+\eta>1$ and $\beta+\eta<1$ when the distribution of human capital across the dynasties results in a declining income per capita, making thereby threshold $\overline{h}_U$ increase and the human capital in segment *U* decrease, emptying thereby the university. When there is no student in the university, the educational system cancels.

3. Figure A2(c) depicts the case $\beta+\eta>1$ when the distribution of human capital across the dynasties results in a growing income per capita. The dynamics is the same as in case (a).

4. Figure A2(d) examines the case $\beta+\eta>1$ when the distribution of human capital across the dynasties results in a decreasing income per capita. This typically results in the collapse of the educational system because segment *U* turns out to be empty sooner or later. It can be noted that, once the levies have been redistributed between vocational studies and basic education, the decline in *y*(*t*) may well continue, leading to the disappearing of vocational studies and subsequently to the fall of everyone into the under education trap.

5. Finally, as in the case $2\beta+\eta=1$, the only stable steady state occurs when $q_V=q_U=0$ and $\beta+\eta<1$, and this is characterised by the couple $\left\{\hat{h}_T=\left(a_T{}^{\beta}B_T\right)^{1-\beta-\eta}, \hat{\mu}_T=1\right\}$.