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ENVELOPES OF COMETARY ORBITS

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SUMMARY: We discuss cometary orbits from the standpoint of Nonstandard (Leibnitz) analysis, a relatively new branch of mathematics. In particular, we consider parabolic cometary paths. It appears that, in a sense, every parabola is an ellipse.

Key words. Comets: general – Methods: analytical

1. INTRODUCTION

The standard approach to physics is based on mathematics over **R**, the field of real numbers, or **C**, the field of complex numbers. Both these structures are Archimedean, i.e. they do not admit explicitly infinite quantities. In discussing the real line **R** we have no way of knowing what a line in physical space is really like. It might be like the real line R, the hyperreal line $^*R$ which contains infinitesimals and infinite numbers, or neither. However, in applications of the mathematical analysis it is helpful to imagine a line in a physical space as $^*R$. The hyperreal line is, like the real line, a useful mathematical model for a line in the physical space. One of the aims of this paper is to advocate the use of methods from nonstandard analysis (also known as Leibnitz analysis, non-Archimedean analysis or Robinson's analysis) in studies of certain phenomena in astronomy. Here we shall discuss trajectories of comets.

2. NONSTANDARD ANALYSIS

It is generally accepted that Newton and Leibnitz, independently from each other, developed differential calculus. By infinitesimals Leibnitz assumed "infinitely small numbers", and he performed the usual algebraic operations over them in the same way as he did with real numbers. In particular, each positive infinitesimal ε in this contemplation was lesser than any ordinary real (standard) positive number, while $1/\varepsilon$ was greater than any standard positive number. The following rule was implicitly supposed:

Leibnitz principle: *Every mathematical proposition that is true for finite (real) numbers is also true for the extended system (i.e. system with infinite numbers), and vice versa.*

The major difficulty of Leibnitz's approach was a number of paradoxes and a lack of formal framework for consistent foundation of infinitesimal calculus. Introducing Weierstrass analysis the infinite quantities are expelled, for example the notion of the infinitesimal is replaced by the ε - δ formalism. In particular, zero-sequences (i.e. sequences that converge to 0) are seen as infinitesimals. However, this is only an auxiliary notion there, and they lack the use of all algebraic operations (such as division) over them.

Abraham Robinson (1961) solved the 300 years old problem of foundation of infinitesimal calculus. He founded Leibnitz analysis, i.e. introduced actual infinitely small and infinitely large numbers. They admit not only all algebraical operations, but also an application of usual functions from analysis (such as sin, cos, exp etc) on them. Robinson's solution was based on certain constructions and techniques from mathematical logic, such as the ultraproducts, the Compactness theorem and the saturated models. The reader can find details about these notions in Chang and Keisler (1990).

The nonstandard analysis is based on properties of $^*R$ and the transfer principle (Łoś theorem), the counterpart of the Leibnitz principle, which exchange propositions between $^*R$ and $\mathbf{R}$. The nonstandard analysis has been used since then in explaining certain phenomena in physics, in particular in statistical physics and quantum mechanics (e.g. S. Albeverio, J. E. Fenstad, T. Lindstrom, see Anderson 1976, Albeverio, et al. 1986).

Mathematical models of nonstandard analysis are non-Archimedean real fields enriched with nonstandard counterparts of notions of the mathematical analysis: elementary functions $\sin(x), \ln(x), \ldots$, sets: natural numbers N, integers Z, rational numbers Q, etc. As they are non-Archimedean, they contain infinitesimals and infinite quantities. The best of all is that we can do the same with more complex structures. For example, to construct the nonstandard enlargement of any infinite structure: complex numbers C, the space of real sequences R^N, the space of real functions R^R, each having the metric of our choice; then infinite functional, geometrical and topological spaces. This construction simply allows us to do nonstandard but consistent mathematics. Leibnitz transfer principle enables one to translate theorems expressed by special, so called internal formulas from nonstandard universe to the standard one. In particular, the Cauchy transfer principle is useful in such translations:

The Cauchy Principle Let $\varphi(x)$ be an internal formula. Then: *If $\varphi(x)$ holds for each infinitesimal x, then there is $r \in R, r > 0$, such that $\varphi(x)$ holds in R for all $x \in R, |x| < r$.*

Let us mention few facts about nonstandard analysis: by $^*R$ we shall denote some $\aleph_1$-saturated non-Archimedean (in the sense of order) elementary extension of the ordered field of reals R. Though $\aleph_1$-saturation provides uniqueness (up to isomorphism) at the given cardinal number of such structures, we do not have the canonical representation (such as decimal notation for reals) of nonstandard real numbers. Another useful property is expressed by the following theorem

Theorem (Extension property) *Every function $f\colon R \longrightarrow R$ can be extended to $^*f\colon {}^*R \longrightarrow {}^*R$ which preserves all first order properties of f.*

For example, if $f(x) = \sin(x)$, $g(x) = \cos(x)$, since

$$sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y),$$

the same identity holds for $^*f(x) = {}^*\sin(x)$ and $^*g(x) = {}^*\cos(x)$. Something similar is true for analytical continuations of real functions, but only for identities. In nonstandard analysis all the first order properties are preserved, including monotonicity, properties of zeros, etc. It is customary that the asterisk * is omitted in the case of elementary functions. Thus, $\sin(x)$ will denote $^*\sin(x)$ if $x \in {}^*R$, otherwise it is the "ordinary" sinus function.

Another useful notion in nonstandard analysis are monads. An element $a \in {}^*R$ is *finite* if there is a positive integer n such that $-n < a < n$. By $^*R_{fin}$ we shall denote the set of all finite elements of $^*R$. The *galaxy* of a is the set $g(a)$ of all nonstandard reals b such that $a - b$ is finite. In particular, $^*R_{fin} = g(0)$. It is easy to see that the mapping $st\colon {}^*R_{fin} \longrightarrow R$ (*standard part*) defined by

$$st(x) = \sup_R \{y \mid y \leqslant x\}$$

is an epimorphism. In particular, $^*R_{fin}/ker(st) \cong R$. An *infinitesimal* is each finite ε such that $st(\varepsilon) = st(0) = 0$. The *monad* of 0 is the set $\mu(0)$ of all infinitesimals. Notice that $\mu(0)$ is closed under addition and multiplication. Further, we say that a and b are *infinitely close*, denoted by $a \approx b$, if $a - b \in \mu(0)$. In fact, $\mu(0)$ is a kernel of epimorphism st and it is a maximal ideal of the ring $^*R_{fin}$. We get the other monads by translations, i.e. $\mu(a) = a + \mu(0)$.

By use of homomorphism st one can replace the ε-δ formalism by algebraic identities. Let us illustrate this with several examples:

Examples from mathematics

1. If ε is an infinitesimal, then $st(a+\varepsilon) = st(a)$.
2. $f\colon R \longrightarrow R$ is continuous iff (if and only if) for all $a \in {}^*R_{fin}$, $st({}^*f(a)) = f(st(a))$.
3. $f\colon R \longrightarrow R$ is uniformly continuous iff for all $a, b \in {}^*R$, $a \approx b \to {}^*f(a) \approx {}^*f(b)$.
4. Let $f\colon R \longrightarrow R$ be a differentiable function and let $\varepsilon \neq 0$ be an infinitesimal. Then $f'(x) = st\left(\frac{{}^*f(x+\varepsilon)-f(x)}{\varepsilon}\right)$. For example,

$$(x^2)' = st\left(\frac{x^2+2x\varepsilon+\varepsilon^2-x^2}{\varepsilon}\right) = st(2x+\varepsilon) = 2x.$$

5. Every subset S of R has the nonstandard enlargement ${}^*S$. If S is finite then ${}^*S = S$, but if S is infinite then ${}^*S \setminus S$ is also infinite.
6. If f is continuous, then the Riemann integral may be defined, for example $\int\limits_0^1 f(x)dx = \frac{1}{H}\sum\limits_{i=0}^{H} {}^*f(i/H)$, where H is an infinite number, i.e. $H \in {}^*N \setminus N$.

One can find detailed development of nonstandard analysis in Stroyan and Luxemburg (1976).

Examples in Geometry and Astronomy. Nonstandard analysis, based on ${}^*R$, introduces a specific mathematical method, as well as a way of thinking. As we saw, it introduces *actual* infinitely small quantities and infinitely large quantities. Therefore, it gives good ground in considering physical systems which in idealized form have infinitely many degrees of freedom. Definitions and proofs are more intuitive, and its use is natural and intuitive whenever the considered (idealized) physical system is composed of infinitely many particles. There are a lot of applications of nonstandard analysis based on this assumption in mathematical physics, in particular in quantum mechanics, fluid mechanics, dynamical systems, etc. As an example, let us first consider Dirac delta function.

1. Dirac δ function Let $a(t) = e^{-1/(1-|t|^2)}$ if $|t| < 1$, $a(t) = 0$ otherwise. This is a simple variation of Cauchy's flat function, and it belongs to the space $\mathcal{E}^\infty$ of infinitely many differentiable functions. Let ε be a positive infinitesimal, and let $b(t) = a(t/\varepsilon)$. Finally, let $k = \int_{-\infty}^{\infty} b(t)dt$ and let $\delta(t) = b(t)/k = a(t/\varepsilon)/k$. Then $\delta(t)$ belongs to ${}^*\mathcal{E}^\infty$, it is positive, and has integral one. In fact, this is what is expected, $\delta(t)$ is a finite compact distribution and it has all properties attributed to the Dirac function.

2. Tiling the Euclidean plane, Hao-Wang dominoes problem: if there is a covering by the certain pattern of the finite type τ of each bounded domain in the plain such as squares and circles, prove that there is a cover of the type τ of the entire plane. One solution goes like this: by the extension principle, we can find the covering $\mathcal{C}$ of the type ${}^*\tau$ of a square with edges having the infinite length H, i.e. $H \in {}^*N \setminus N$. Since τ is finite, we have ${}^*\tau = \tau$. Therefore, this particular nonstandard cover induces the covering of the entire Euclidean plane by restricting $\mathcal{C}$ to the standard (finite) part of ${}^*R \times {}^*R$.

In this example we have seen how to extend certain local property to the global one. We can try to interpret this covering property to the foundation of fundamental cosmological principles. Namely, all observations from the Earth are local, even on the large scale. But observations on the large scale show that the Universe is homogeneous and isotropic. Identifying observations with tiling, we see at once that we may assume two basic cosmological principles: homogeneity and isotropy of the Universe. Therefore, from the mathematical point of view at least it is consistent to assume so.

Consistency and conservativeness of nonstandard analysis. Nonstandard analysis is a consistent and conservative extension of classical analysis. This follows from the ultraproduct construction (consistency), and Łoś theorem (conservativeness). Nonstandard analysis cannot produce propositions in the classical mathematical analysis that are not possible to prove by means of classical mathematics. Thus, nonstandard analysis is a method of proving, first of all. However, we should mention that some problems, such as Bernstein-Robinson theorem on polynomial operators, were first solved by means of nonstandard analysis.

There are other mathematical non-Archimedean methods that are used in physics. Particularly popular in last two decades became p-adic physics, which is based on the so called p-adic mathematics. It admits counterparts of all basic notions of classical analysis, but they are not true extensions of standard functions of classical analysis. Also, it lacks the transfer principles such as the Leibnitz transfer principle, or they are much weaker, such as the Hasse-Minkowski theorem for Henselian fields. Without any intention to doubt the trustiness on p-adic physics, it certainly gives interpretation of physical phenomena that differs from those in the main-stream physics. Detailed discussion on this topic one can find in Mijajlović et al. (2006), Mijajlović and Pejović (2007), Mijajlović et al. (2007).

3. ELLIPSE IN THE NONSTANDARD UNIVERSE

Let E be an ellipse having foci at the points $(p, 0)$ and $(q, 0)$ where $p > 0$ is a fixed positive real number and $q > 0$ is an infinite real number. Then all standard points of E, i.e. the points lying in the real plane R^2, are the points of loci of an "ordinary" parabola P having the focus at $(p, 0)$. We show that P is actually the envelope of the family of all (standard) ellipses having one focus in $(p, 0)$, the other one in $(b, 0)$, $b > p$ is a positive real number.

From the stated assumptions on the ellipse E, see Fig. 1, we infer the following formulas:

$$\left\{\begin{array}{rcl} l_1 + l_2 & = & d \\ l_1^2 & = & (x-p)^2 + y^2, \\ l_2^2 & = & (x-q)^2 + y^2. \end{array}\right. \qquad (1)$$

From these equations it follows

$$\begin{aligned} l_1 &= ((q-p)x+pd)/d, \\ l_2 &= ((p-q)x+qd)/d. \end{aligned}$$

By eliminating l_1 and l_2 from the set of formulas (1), we obtain finally the equation of the ellipse E:

$$y^2 = 4px - 4p\frac{p(p+q)x+qx^2}{(p+q)^2} \tag{2}$$

We can interpret the formula (2) in the following two ways.

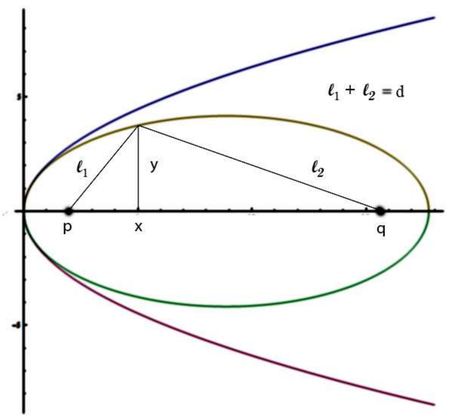

Fig. 1. *The ellipse having the foci respectively in $(p,0)$ and $(q,0)$ with the vertex at the coordinate origin and $l_1 + l_2 = d$, $d = p + q$, where l_1 and l_2 are distances of a point on the ellipse from the foci.*

1. Ellipse in the nonstandard plane. Assume that $p \in R$ and that $x \in {}^*R$ is finite and $q \in {}^*R$ is infinite. Then the term

$$4p\frac{p(p+q)x+qx^2}{(p+q)^2} \tag{3}$$

is an infinitesimal, while $4px$ is finite. Hence y is also finite and $y^2 \approx 4px$. Thus, $\mathrm{st}(y)^2 = 4p\mathrm{st}(x)$, so by replacing $\mathrm{st}(x)$ by x and $\mathrm{st}(y)$ by y we obtain the equation $y^2 = 4px$ of a parabola. Therefore, the standard part of the ellipse E (see Fig. 2) in the nonstandard plane with the finite focus $(p,0)$ and the infinite focus $(q,0)$ is the confocal parabola P determined by the equation $y^2 = 4px$. Observe that P does not depend on the choice of the infinite focus $(q,0)$.

All geometric and differential properties of the parabola P can be derived from the properties of the ellipse E. For example, the optical property that if a ray of light travels parallel to the symmetry axis of a parabola and strikes the concave side of the parabola, then it will be reflected to the focus follows immediately from the corresponding optical property of the ellipse E. Just note that if a ray r' is coming from

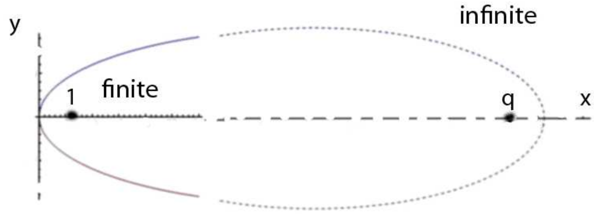

Fig. 2. *The ellipse E in the nonstandard plane.*

the infinite focus $(q,0)$ it reflects from the ellipse to the focus $(p,0)$. Then the standard part $r = st(r') = \{(st(x), st(y)) \colon (x,y) \in r', x, y \in {}^*R_{fin}\}$ is a (standard) ray parallel to the x-axis and as r is infinitely close to r' it also enters into the focus.

2. Family of confocal ellipses. We may take (2) as the equation of the family of (standard) ellipses sharing the fixed focus $(p,0)$, while the second focus $(q,0)$ runs over the x-axis. Observe that from the astrodynamics point of view this family of ellipses may be regarded as Hohmann-Vetchinkin transfer orbits connecting co-planar circular orbits.

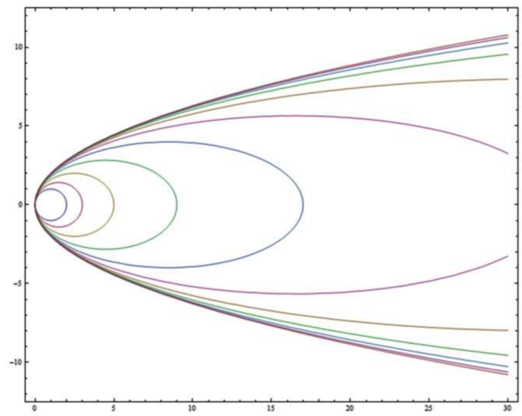

Fig. 3. *Family of confocal ellipses.*

The parabola P is the limit curve enveloping ellipses (see Fig. 3) from this family. In the classical approach in mathematical analysis, the existence of the limit curve is guaranteed by the Arzelà-Ascoli theorem applied in the L^2 space.

However, it should be mentioned that P is not the envelope of the family of ellipses given by the Eq. (2) as it is defined in mathematical analysis. Namely, if a family of plane curves are given by a formula $F(x,y,q) = 0$, q is a parameter, then the envelope of this family is a curve osculating each member of the family. The equation of the envelope is obtained by elimination of q from the system of equations

$$F(x,y,q) = 0, \quad \partial F(x,y,q)/\partial q = 0.$$

In our case,

$$F(x,y,q) = y^2 - 4px + 4p(p(p+q)x + qx^2)/(p+q)^2,$$

and it is easily found that the envelope is in fact the critical point $x = 0, y = 0$, the perihelion of q-ellipses.

4. COMETARY TRAJECTORIES

First studies of cometary orbits serve as a historical introduction to astrodynamics. Namely, their trajectories are also influenced by non-gravitational forces, for example by the acceleration resulting from ejection of a jet of a material from the comet. Most cometary orbits are very elongated. Many physical quantities related to the very elongated cometary orbits change by several orders of magnitude. Every cometary orbit which is observed as parabolic actually is elliptical as further calculations usually show. If this is not the case, this is due to the fact that the second focus is too remote to measure it. Therefore, nonstandard analysis could be the appropriate mathematical tool in the study of cometary trajectories. In the rest of this article we shall use the terminology of nonstandard analysis and the words *standard* and *infinitesimal* will have meaning as explained in the previous sections. For example, if the value of the velocity at the perihelion is assumed to be standard, then the velocity at aphelion may be taken as an infinitesimal. In particular, we shall discuss parabolic paths. By our consideration in the previous section we may assume that every parabolic trajectory is an ellipse. Our discussion is relied on available cometary data, so we shall first shortly review them.

The number of observed comets is rapidly growing due to the development of space technology. For example The ESA/NASA SOHO spacecraft, `http://sohowww.nascom.nasa.gov`, discovered exactly 1500 comets since 1995, the last one 27. June 2008 when this paper was submitted. About 2300 comets are catalogued, even if it is believed that there are more than 10^9 of them. As very few comets have periods less than 12 years, their trajectories are good illustration for very elongated or nearly parabolic ellipses. Here is the short history on comet discoveries in the last four decades.

According to Baker and Makemson (1960), in 1960, of the 1000 comets for which orbits have been computed, fewer than 100 had periods of revolution less than 100 years. Some 40 or 50 had periods between 100 and 1000 years, and the periods of the rest were very uncertain. Fewer than 30 comets were known to have been observed on two or more returns to the Sun. About 50 comets had periods less than 12 years (Jupiter family).

The Catalog of Cometary Orbits, compiled by Marsden, 1989 edition, lists 1292 computed orbits from 239 BC to AD 1989; only 91 of them were computed using the rare accurate historical data from before the 17th century. More than 1200 are, therefore, derived from cometary passages during the last three centuries. Sets of orbital elements in Marsden's catalog involve only 810 individual comets; the remainder represents the repeated returns of periodic comets. Four of these comets had been definitely lost, and three more were probably lost, presumably because of their decay due to the solar heat. Of the 155 short-period comets, 93 have been observed at two or more perihelion passages.

The 16th edition of the Catalogue of Cometary Orbits of Smithsonian Astrophysical Observatory issued in 2005 contains 3031 sets of orbital elements (in the J2000.0 system) for 2991 cometary apparitions of 2221 different comets through mid-August 2005. There is a special tabulation giving osculating elements for the 170 designated periodic comets, excluding seven deemed to be lost.

According to the list of periodic comets on the Planetary Data System Small Bodies Node, NASA, last update on 10 April 2008, `http://pdssbn.astro.umd.edu/comet_data`, there are 420 designated periodic comets. According to Seiichi Yoshida's Comet Catalog, `http://www.aerith.net/`, there are 243 non-designated periodic comets (the last discovered C/2008 L3, 13. Jun 2008) and 200 designated periodic comets.

In discussion of very elongated cometary orbits we shall rely on Marsden Catalog of Cometary Orbits. Of the 655 comets of long period contained in the Catalog, 192 have osculating elliptic orbits, and 122 have osculating orbits that are very slightly hyperbolic. Finally, 341 are listed as having parabolic orbits, but this is false because either it has not been possible to detect unequivocal deviations from a parabola on the usually very short arc over which the comets have been observed or the final calculations have never been made. However, the parabola is always assumed first in the computation of the preliminary orbit. If the osculating orbit is computed backwards to when the comet was still far beyond the orbit of Neptune and if the orbit is then referred to the centre of mass of the solar system, the original orbits almost always prove to be elliptic.

These data show that the methods of nonstandard analysis are suitable in studying of cometary trajectories. For example, in the computation of the preliminary orbits, the value of the parameter p is computed. Simply, the second term (3) in Eq. (2) may be omitted as we may consider it as an infinitesimal. It also shows that the formula (2) could be very appropriate in calculation of cometary orbits in the sense that it represents better starting point for the method of differential corrections than the simple parabola.

Let us consider very-long-period comets and comets having orbits not significantly different from a parabola. It is believed that these comets originate in the Oort cloud which is located 10000 and 100000 AU from the Sun. By our previous discussion it is appropriate to use here methods of nonstandard analysis. So let us assume that a hypothetical comet C is moving along an ellipse E in the nonstandard plane having the second focus at $(q, 0)$ where q is an infinite number. Therefore, the aphelion of E is at infinity, and by the second Kepler's law the velocity v of the comet near the aphelion (i.e. at the finite distance from aphelion in terms of nonstandard analysis) is an infinitesimal. Otherwise, the surface swept by the comet for the finite time $\triangle t$ would be infinite due to the infinite distance of the comet from the Sun, and that would contradict the Second Kepler's law. In reality, a simple calculation shows that the velocity v of the comet C near aphelion would be around 100 m/sec, negligibly small comparing to the velocity at the perihelion. Therefore, the momentum $p = mv$ of the comet C is an infinitesimal too;

we could say that the comet C floats in the Oort cloud instead of travelling around the Sun. Hence the trajectory of the comet C is subject to a very small perturbation. We show that an infinitely small force, or impulse, would change significantly its trajectory. Simply saying, parabolic orbits at large distances are very unstable. We can see that using the following formula from astrodynamics (see Andjelić 1983, pages 132-134):

$$v_{a_t} = \sqrt{\frac{2\mu}{\rho_{a_2}} \frac{\rho_{p_1}}{\rho_{p_1} + \rho_{a_2}}} \tag{4}$$

which determines the velocity v_{a_t} needed for the body C to continue travelling along the other orbit which is coplanar and confocal to E.

So let O_2 be the orbit E with the apoapsis ρ_{a_2} along which the body C travels so that C is at the aphelion (i.e. in the Oort cloud) with the infinitely small velocity v_C. Further, let O_1 be the orbit, coplanar and confocal to O_2, with the periapsis ρ_{p_1} to which the body C will be transferred under an action $\mathcal{A}$. Observe that ρ_{a_2} is infinite, while ρ_{p_1} is finite. According to the Maupertuis' principle of the least action, we may assume that the transition path from O_2 to O_1 will be the Hohmann transfer orbit H touching the orbit O_1 at the aphelion, so the equation (4) can be applied. Here, μ is the gravitational parameter and v_{a_t} is the velocity resulting from the action $\mathcal{A}$. Under these assumptions, we have

$$v_{a_t} = \frac{\sqrt{2\mu\rho_{p_1}}}{\rho_{a_2}}\left(1 - \frac{\rho_{p_1}}{2\rho_{a_2}} + \frac{\varepsilon}{\rho_{a_2}}\right)$$

where ε is an infinitesimal. Simplifying the previous formula, we find $v_{a_t} = \sqrt{2\mu\rho_{p_1}}/\rho_{a_2} - \kappa/\rho_{a_2}^2$, where κ is a standard finite value. Observe that v_{a_t} is an infinitesimal of the order $1/\rho_{a_2}$.

Let $v_{a_2} = v_C$ be the velocity of the body C at the aphelia on the orbit O_2. Then $v_\delta = v_{a_2} - v_{a_t}$ is the velocity needed for transition from the orbit O_2 to the transfer ellipse which would carry the comet C to the orbit O_1. Since v_{a_2} and v_{a_t} are infinitesimals, it follows that v_δ is an infinitesimal, too. The transfer ellipse H with the second focus at infinity will be seen from the near neighborhood of the Sun as a parabola.

There are astronomical evidences that support our discussion. Namely, according to Delsemme (2008) among the very-long-period comets, there is a particular class of comets that Oort showed as having never passed through the planetary system before, notwithstanding the fact that their original orbits were elliptic, which implies repeated passages. This paradox vanishes when it is understood that their perihelia were outside of the planetary system before their first appearance but that their orbits have been perturbed while they resided near aphelia, either by stellar or dark interstellar-cloud passages or by galactic tides, in such a way that their perihelia were lowered into the planetary system.

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ОБВОЈНИЦЕ КОМЕТСКИХ ОРБИТА

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Оригинални научни рад

У раду се разматрају кометске орбите са становишта нестандардне анализе, релативно нове области математике. Посебно се изучавају параболичне кометске орбите. Показује се да је, у одређеном смислу, свака парабола заправо елипса.