

## WAS EINSTEIN A LAPLACEAN?

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It is surely a truism that the science and philosophy of an age influence one another, and this century has been no exception: the rise of the quantum theory profoundly threatened the most promising and universally respected conception of the physical world articulated since the demise of the Aristotelian doctrines of nature. In so doing, this bold theory precipitated one of natural philosophy's most dramatic disputes, between two of the century's most distinguished physicists, Niels Bohr and Albert Einstein.<sup>2</sup>

It is widely believed that the dynamics of this dialogue were dictated by an overview of "physical reality" held by Einstein. Such interpretations typically presume that Einstein's arguments in that exchange depend on a rather demanding constellation of epistemic and metaphysical theses reminiscent of the doctrines of Laplace. Central to these doctrines is the belief that every state variable of a physical system has a knowable, exact value at all times. The aim of this essay is to argue that even though there is something essentially correct in such an approach to understanding the Einsteinian Weltanschauung, the method is at least suspect and at best obscures the subtlety of Einstein's convictions.

### I

At the outset, we can sharply distinguish at least two approaches to the problem of understanding Einstein's view of quantum-mechanical (QM) reality. The first approach, taken by Hooker, Lenzen, and Furry,<sup>3</sup> is to hold that there are no interpretations of Einstein's pronouncements more epistemically or

metaphysically parsimonious relative to the entire body of his writings than the Laplacean. The second approach, in contrast, is to try to discover an interpretation which is epistemically and metaphysically more parsimonious.

Now the first of these approaches, though relatively easy, is problematic. For if there is an interpretation of Einstein's assertions which is more parsimonious than the Laplacean, any of Einstein's arguments which employ premises affected by this more parsimonious interpretation will be stronger under that interpretation than those same arguments under the Laplacean. Since it is a maxim of philosophic methodology to adopt that interpretation which extracts maximal cogency, I shall opt for the second method; though the more difficult, it is the more sympathetic.

## II

The most detailed formulation of Einstein's concerns over the quantum theory (QT), it is generally agreed, occurs in a paper published in Physical Review. The content of that paper, now known as the "EPR" (Einstein-Podolsky-Rosen) argument, is designed to show that the QT is not "complete," in the sense that there are describable physical situations which the QT must in principle ignore.

The story goes something like this. Let us suppose that there is a theory-independent "physical reality" which a "complete" physical theory must in some sense mirror (Einstein, p. 777):

- (C) Every element of physical reality must have a counterpart in the theory.

Naturally, in order to determine whether the QT in particular satisfies C, a means of determining the "elements of reality" independently of the theory must be provided. This task, the argument maintains, can be accomplished by an appeal to the results of experiment and measurement, and more specifically (Einstein, p. 777),

- (R) If without in any way disturbing a system we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical

reality corresponding to this physical quantity.

Using criteria C and R, respectively dubbed the "criterion of completeness" and the "criterion of reality," the argument then tries to show that there is a system in a physical state S which can be predicted with certainty even though S cannot be described by QT; hence the QT is reckoned "incomplete."

On the surface, at any rate, EPR certainly do appear to be committed to a neo-Laplacean theory of physical reality. To determine, however, whether this appearance is more than a cosmetic feature of the EPR story, it is necessary to look at the argument in more detail.

That detail is to be found in EPR's analysis of an apparently embarrassing QM system. Suppose that I and II designate two systems whose joint behavior is characterized by a wave function  $\Psi$ ; let I and II interact during a time interval  $[0, T]$  and be separated after T. If the states of I and II are known before  $T = 0$ ,  $\Psi$  allows a computation of the state of the complex system I + II at any time after T, but does not by itself afford a means of determining the states of the respective "component" systems (I and II) of this complex. Thus the states of the component systems can be determined, if at all, only with the help of measurement.

Now the QT asserts that the measuring process of the complex is to be analyzed in the following way. If  $a_1, a_2, a_3, \dots$  are the eigenvalues of a physical quantity A of system I and  $u_1(x_1), u_2(x_1), u_3(x_1), \dots$  are the corresponding eigenfunctions of A (where x stands for the variables used to describe system I), then  $\Psi$  takes the form

$$(1) \quad \Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1)$$

where the  $\psi_n(x_2)$  are merely coefficients of the expansion of  $\Psi$  into a series of orthogonal functions  $u_n(x_1)$ . If A is measured in I after T and found to have value  $a_k$ , the QT implies that I is left in the state given by  $u_k(x_1)$ , and II is left in the state given by  $\psi_k(x_2)$  with certainty.

If on the other hand  $B \neq A$  is a physical quantity with eigenvalues  $b_1, b_2, b_3, \dots$  with corresponding

eigenfunctions  $v_1(x_1), v_2(x_1), v_3(x_1), \dots$  takes the form

$$(2) \quad \Psi(x_1, x_2) = \sum_{r=1}^{\infty} \Phi_r(x_2) v_r(x_1)$$

where again the  $\Phi_r(x_2)$  are coefficients of the expansion of  $\Psi$  into a series of orthogonal functions  $v_r(x_1)$ . And again, if B is measured in I after T and found to have value  $b_r$ , the QT implies, as in (1), that system I is in state  $v_r(x_1)$  and II is in state  $\Phi_r(x_2)$  with certainty.

Thus, depending on what physical quantity in I is measured, II is assigned two wave functions,  $\Psi_k(x_2)$  and  $\Phi_r(x_2)$ , respectively, with certainty. Since I and II are separated after T, measurement in I does not disturb II, and the criterion of reality implies that  $\Psi_k(x_2)$  and  $\Phi_r(x_2)$  have simultaneous reality.

In particular, then, suppose that the two wave functions  $\Psi_k(x_2)$  and  $\Phi_r(x_2)$  are eigenfunctions of two noncommuting physical quantities, P and Q, respectively. The above remarks imply (Einstein, p. 779-80) that by measuring the value of P or Q in system I, the value of P and Q in II can be determined to have simultaneous reality.

And now, the rub. QM implies that if the operators corresponding to two physical quantities A and B do not commute, then precise knowledge of one of them forbids precise knowledge of the other. Since the analysis of I + II shows that the values of P and Q can be determined with certainty, they can, by R, have simultaneous reality. But the QT precludes this possibility. Hence, the QT is by C incomplete.

On the surface the EPR analysis looks very tight. It respects the QM formalism and adheres to the QM theory of measurement. Yet in spite of this, it seems to show that there are "elements of reality" whose existence the QT ignores. We thus seem faced with two rather awkward choices: to admit an embarrassing lacuna in QM or to suspect that there is an evasive non-QM assumption in the argument. In the following, I will try to show there is a problem of the latter sort and that this problem discloses a subtlety of the Einsteinian view badly obscured by the Laplacean analysis.

## III

In the heart of the EPR argument lies an apparently innocent, but in fact quite troublesome assumption (Einstein, p. 778):

- (S) If two quantum systems have interacted and have been subsequently separated, measurement of one of them does not disturb the other in ways which in principle assign a probability of less than unity to the prediction of the value of some physical quantity.

This thesis, which will be called the "separability" postulate, is essential for the success of the EPR argument. For if S did not hold, measurement on I could influence physical magnitudes in II in such a way that in principle they could not be predicted with certainty. A condition of the criterion of reality would then fail to be met and the argument would collapse.

Now it has frequently been asserted<sup>7</sup> that, more than any other feature of the EPR argument, the separability postulate reveals or depends on the Laplacean conviction that

- (L) Every physical magnitude has a knowable, exact value at all times.

The argument connecting L and S goes something like this. Einstein frequently asserted that every physical magnitude has a knowable, exact value at all times (L). Thus an adequate theory of interaction cannot in principle forbid the meaningful ascription, with certainty, of exact values to physical magnitudes (S).

Although it is probably true that there is some interpretation of L which would imply the separability postulate, it is hardly so clear that the EPR argument need rest on anything so strong as that Laplacean canon. And if not, then the claim that the EPR argument must turn on such a view simply does not stand.

Let me first argue, then, that EPR can get by on an interpretation of S much weaker than L. In particular, it would suffice for EPR to assume merely that:

- (S<sub>p</sub>) If P and Q are Noncommuting parameters, the probability of finding that P has value p in II, given that Q has been measured in I and found to have value q, is just the classical conditional probability. Moreover, the probability that Q has value q in II, given that P has been measured in I and found to have value p, is likewise the classical conditional one.

How does the argument go with S<sub>p</sub> substituted for S? Well, whatever we might mean by "isolating" a physical system K from another, a minimal (necessary) condition of such, it would seem, would be that the conditional probability of some observable Q having value q in K would simply be the classical one. S<sub>p</sub> precludes the only problematic possibility along these lines in the context of the EPR argument; hence the argument follows, provided it did before.

Notice that S<sub>p</sub> does not require that the noncommuting observables P and Q have definite values at all times. Indeed, even if P and Q did not have definite values at any time, the conclusion of EPR would still stand under S<sub>p</sub>. Moreover, L entails S<sub>p</sub> but not the converse. And clearly, the epistemic and metaphysical commitments of S<sub>p</sub> are fewer than those of L. Thus S<sub>p</sub> is an epistemically and metaphysically more parsimonious interpretation of S than L, relative to the body of Einstein's beliefs and the conclusion of the EPR argument.

Yet even though S can be interpreted in such a way (S<sub>p</sub>) that the metaphysical and epistemic promiscuity of L is avoided, it seems that there must nevertheless be some justice in the view that S in a significant presupposition of the EPR argument about the nature of physical reality. But however obvious on the surface, this remark requires careful formulation and defense because some authors have raised serious questions about whether it makes sense to say such a thing.

In particular, there is a very serious problem which must be avoided if we are to assert that the claim S<sub>p</sub> makes an assertion about physical reality. For the notion of "saying something about physical reality" seems to entail, if not be entailed by, "having empirical content." And, as Hempel and others have shown, there are grave difficulties in attempting to say that a sentence simpliciter has empirical

content. For, given any sentence  $A$ , it is always possible to construct a theory  $T$  under which we would want to assert that  $A$  had no empirical content and a theory  $T'$  under which we would want to assert that  $A$  did have empirical content.

Hempel's arguments strongly intimate that by relativizing the notion of empirical content to a set of sentences, we can get at least a sufficient criterion for it. I shall accordingly, and I hope unproblematically, assume that a sentence  $A$  has empirical content relative to a set  $EU B$  of sentences if  $EU B$  and  $EU \{A\}$  are each consistent,  $\overline{B} \vee A$ ,  $EU B \neq EU \{A\}$  and if the in-principle testable consequences of  $EU \{A\}$  differ from the in-principle testable consequences of  $EU B$ . I take to be "testable" whatever the scientific community claims is testable. In this sense, then, I wish to argue that the separability postulates  $S_p$  has empirical content relative to the remainder of the EPR argument and the QT theory of probability, and hence discloses something about Einstein's implicit commitments concerning the nature of physical reality.

The argument given here is inspired by Furry,<sup>9</sup> who ironically believed that it showed that Einstein was (mistakenly) committed to  $L$ . Consider in particular two once-interacting systems I and II, whose joint state (i.e., the state of the complex I + II) can be characterized by

$$(3) \quad \Psi(x_1, x_2) = \sum_k \omega_k \phi_{\lambda_k}(x_1) \xi_{\rho_k}(x_2)$$

where  $\phi_{\lambda_k}(x_1)$  are eigenfunctions of an observable  $L$  with corresponding eigenvalues  $\lambda_k$ ,  $x_1$  are the variables describing I, the  $\xi_{\rho_k}(x_2)$  are the eigenfunctions of an observable  $R$  with corresponding eigenvalues  $\rho_k$ , and  $x_2$  are the variables describing II. Let  $M$  and  $S$  be two observables, neither of which is equivalent to  $L$  or  $S$ . Let  $M$  have eigenvalues  $\mu$  and eigenfunctions  $\psi_{\mu}$  and let  $S$  have eigenfunctions  $\eta_{\sigma}$  with eigenvalues  $\sigma$ . Suppose that  $M$  has been measured in I in a large number of similarly prepared pairs of systems and found to have value  $\mu'$  in some of them.

Assume now that the probability of  $S$  having value  $\sigma'$  in II, given that  $M$  has value  $\mu'$  in I, is just the classical conditional probability ( $S_p$ ). To compute this, we note that the fraction having value  $\mu'$  for  $M$  in I is given by

$$(4) \quad \sum_k \omega_k |(\phi_{\lambda_k}, \psi_{\mu'})|^2$$

The fraction giving  $\mu'$  for M in I and having II in state  $\xi_{\rho_k}$  is  $\omega_k |\phi_{\lambda_k}, \psi_{\mu'}|^2$ . Then the fraction giving values  $\mu'$  for M and  $\sigma'$  for S is

$$(5) \quad \sum_k \omega_k |\phi_{\lambda_k}, \psi_{\mu'}|^2 |(\xi_{\rho_k}, \eta_{\sigma'})|^2$$

Dividing (5) by (4), we obtain the classical conditional probability that S has value  $\sigma'$  in II, given that M has been found to have value  $\mu'$  for M in I, i.e.,

$$(6) \quad \frac{\sum_k \omega_k |\phi_{\lambda_k}, \psi_{\mu'}|^2 |(\xi_{\rho_k}, \eta_{\sigma'})|^2}{\sum_k \omega_k |\phi_{\lambda_k}, \psi_{\mu'}|^2}$$

On the other hand, if we suppose that the probability of S having value  $\sigma'$  in II, given that M has value  $\mu'$  in I, is not the classical conditional probability (thus denying  $S_p$ ), we have to find some other way to calculate the requisite probability. QM supplies the only natural answer in this context, to wit

$$(7) \quad \int \psi_{\mu'}^*(x_1) \psi(x_1, x_2) dx_1 = \sum_k \omega_k^{\frac{1}{2}} (\psi_{\mu'}, \phi_{\lambda_k}) \xi_{\rho_k}(x_2)$$

Normalizing this function and taking the square root of its inner product with  $\eta_{\sigma'}$ , we obtain the desired nonclassical conditional probability

$$(8) \quad \frac{|\sum_k (\omega_k)^{\frac{1}{2}} (\phi_{\lambda_k}, \psi_{\mu'}) (\xi_{\rho_k}, \eta_{\sigma'})|^2}{\sum_k \omega_k |\phi_{\lambda_k}, \psi_{\mu'}|^2}$$

The discrepancy between the probability given by (6) and that given by (8) arises from the fact that after M is measured in I, the QT formally assigns a pure state to II which in general is not one of the  $\xi_{\rho_k}$ . And, since the only pure states generable by manipulation of the coefficients  $\omega_k$  are contained in the set of  $\xi_{\rho_k}$ , there is no way that the formal statistics predicted under assumption  $S_p$  substituted for S in the EPR argument in general will agree with the wave-function determination of probability substituted for S. On the criterion of empirical content defined above, then, if the statistics characterized by (6) and (8) testably differ,  $S_p$  has empirical content relative to the rest of the EPR argument and the QM theory of probability.

The question thus arises whether the statistics determined by (6) and (8) testably differ. This is no mere pedantic worry, or it may be that the formal discrepancies between (6) and (8) present no testable difference.



To parody Mill, I know of no better way to show that something is testable than to show that it has been tested. A test of an EPR experiment has been made by Wu and Shaknov,<sup>11</sup> among others, and it goes like this. When two photons are emitted in opposite directions in the annihilation of a positron-electron pair, the QT predicts that they will have perpendicular polarizations and that these polarizations cannot be simultaneously determined with precision.

To confirm these predictions, a source of slow positrons is covered with a foil sufficiently thick to guarantee annihilation of them all. The foil-covered source is placed at the center of a lead sphere which has had a narrow hole drilled along one of its diameters. Aluminum or carbon scatterers are placed at the ends of this channel. Photons scattered through about  $90^\circ$  by one of these scatterers are recorded by a counter at a known azimuth, while those scattered at the other end are recorded by a counter in coincidence with the first. By measuring the coincidence counting rates with different relative azimuths, the perpendicular correlation of polarization can be determined. These correlations, Wheeler<sup>12</sup> has theoretically shown, discriminate rather sharply between (6) and (8).

The relation of this experiment to the situation described by EPR is as follows. If we assume that the two photons (say, I and II, respectively) are moving in the +z and -z directions, the conservation laws of parity and momentum imply that the state of I + II may be written as

$$(9) \quad \psi = \frac{1}{\sqrt{2}} \left( \psi_r^I \psi_r^{II} + \psi_l^I \psi_l^{II} \right)$$

where  $\psi_r^k$  denotes that the  $k^{\text{th}}$  photon is right circularly polarized and  $\psi_l^k$  denotes that the  $k^{\text{th}}$  photon is left circularly polarized.

We may also write  $\psi$  as

$$(10) \quad \psi = \frac{1}{\sqrt{2}} \left( \psi_x^I \psi_y^{II} - \psi_y^I \psi_x^{II} \right)$$

where  $\psi_x^k$  denotes that the linear polarization of the  $k^{\text{th}}$  photon is along the x-axis.

If the circular polarization of photon I is measured and found to be in  $\psi_r^I$ , then the QT implies that photon II is in state  $\psi_r^{II}$ . But if the linear polarization of photon I is measured and I is found to be state  $\psi_x^I$ , photon II is in state  $\psi_y^{II}$ . Thus,

depending on whether the linear or circular polarization of photon I was measured, we can predict either the linear or circular polarization of photon II, respectively. QM says that the linear and circular polarizations are non-commuting quantities and hence cannot have simultaneously definite values for both kinds of polarization. Thus polarization-correlation experiments are a particular kind of the class of EPR systems.

The correlation results obtained in the Wu-Shaknov experiment and others like it in fact sustain the QM picture and disconfirm the EPR prediction under assumption  $S_p$ . Hence  $S_p$  has empirical content relative to the rest of the EPR argument and the QM theory of probability.

A little caution is in order here. The above considerations establish at best only that  $S_p$  says something about Einstein's conception of physical reality relative to the rest of the EPR argument and the QM theory of probability. These considerations may say nothing about whether  $S_p$  has empirical content relative to theories which diverge from the EPR view, and there are obviously several of these.<sup>13</sup>

I have tried to argue, then, that contrary to a popular view of the Einsteinian Weltanschauung, the separability postulate of the EPR argument need not rest on anything so strong as a Laplacean view of physics. Nevertheless, that postulate -- a crucial feature of the EPR view -- does have relative empirical content and hence in some sense reflects Einstein's convictions concerning the physical world.

Unfortunately, the argument used to support this latter claim appealed to research which challenges the correctness of the postulate. But though contemporary investigations should crush the credibility of EPR's conclusions, the weakness of the premises under which that argument still stands remains testimony to the subtlety of the mind which created it.

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## NOTES

<sup>1</sup>Niels Bohr, "Can Quantum Mechanical Description of Physical Reality Be Considered Complete?" Physical Review 46 (1935), pp. 696-702.

<sup>2</sup>Albert Einstein, B. Podolsky, and N. Rosen, "Can Quantum Mechanical Description of Physical Reality Be Considered Complete?" Physical Review 47 (1935), pp. 777-80.

<sup>3</sup>Clifford Hooker, "The Nature of Quantum Mechanical Reality: Einstein vs. Bohr," in R. Colodny, ed., Paradigms and Paradoxes, Pittsburgh, 1972, pp. 67-302.

<sup>4</sup>Victor F. Lenzen, "Einstein's Theory of Knowledge," in P. A. Schilpp, ed., Albert Einstein: Philosopher-Scientist, Open Court, 1970, pp. 357-82.

<sup>5</sup>W. H. Furry, "Note on the Quantum-Mechanical Theory of Measurement," Physical Review 49 (1936), pp. 393-99; W. H. Furry, "Remarks on Measurement in Quantum Theory," Physical Review 49 (1936), p. 476.

<sup>6</sup>See note 2.

<sup>7</sup>See nn. 3 - 5.

<sup>8</sup>C. Hempel, "Empiricist Criteria of Cognitive Significance," in Aspects of Scientific Explanation, The Free Press, pp. 101-20.

<sup>9</sup>See note 5.

<sup>10</sup>See note 5.

<sup>11</sup>C. S. Wu and I. Shaknov, "The Angular Correlation of Scattered Annihilation Radiation," Physical Review 77 (1950): 136.

<sup>12</sup>J. A. Wheeler, Ann. New York Acad. Sci. 48  
(1946): 219.

<sup>13</sup>Certain hidden variable theories, for example.