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ESTIMATING RATIO OF PEAK TO UNIFORM VALUES OF VARIOUS PROFILES OF RELEVANCE TO PLASMA FOCUS PINCH COLUMNS

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Abstract

In the Lee Model code for radiative plasma focus computation, both the density profile and the temperature profile (versus anode radius) of the pinch column are approximated by step functions with uniform values across the column radius. This means that the computed density and temperatures will be lower than the physical situation where the density and temperature profiles will certainly have peak values higher than the uniform (with radius) values of the step function. It has been shown that the density profile is side-peaked (somewhat shell-shaped or like the shape of a volcanic crater) with the assumption of no reflected shock wave; whereas the temperature profile is centre-peaked somewhat like a Gaussian shape. The aim of this paper is to investigate the use of higher degree mathematical function, where the crater-shaped profile can be well represented, to approximate the plasma focus density profile. Two different approximated functions will be discussed: namely Gaussian distribution function and Bézier function. From these profiles we obtain the likely ratio of the profiled peak temperature to the step function uniform temperature and the peak density to the step function uniform density. In this manner we are able to suggest correction factors to the temperature and density computed by the Lee Model code.

Keywords: Plasma focus pinch, Density profile, Temperature profile, Bézier function.

1. Introduction

The Lee model code [1, 2] is useful for estimating key plasma experimental results, such as axial and radial dynamics, energy distributions and neutron yield and soft X-ray yields. Generating well estimated results is very important, which certainly provides better insight to manipulate the values of key parameters (e.g., pressure, voltage, etc.) when carrying out physical plasma experiment.

The radial phase of the plasma focus culminates in the formation of a column of hot dense plasma commonly described as a pinch column. The two quantities of importance in characterising the plasma are the temperature and density. The profiles of the temperature and density (with radius) depend on the process of pinch formation and also on the stage of the formation whether we are considering the early or late stage. Potter has shown that towards the end stage of pinch formation the temperature profile is centre-peaked whereas the density profile is shaped like a shell [3-5]. We prefer to liken the Potter density profile to that of the profile of a volcanic crater; hence we use the word crater-profile. The Potter profiles are considered without, or before, the reflected shock. Still it is instructive to consider the effect of the profile on the estimate of temperature and density compared to the case when the profile is taken simply as constant or uniform across the pinching column.

First we consider the density profile. For the purpose of comparing profiles we may write the line density as

$$N_{line} = \int_0^{r_p} 2\pi r f(r) dr$$

For a uniform function say $f(r) = m$, we have $N_{line} = \pi r_p^2 m$. Thus for the case of $f(r)$=Gaussian profile or $f(r)$=crater-profile, if we set:

$$N_{line} = \int_0^{r_p} 2\pi r f(r) dr = \pi r_p^2 m$$

We find the ratio of (peak value of $f(r)$)/m for each specified shape. The ratio found for the crater-profile is the correction factor we could apply to the density computed from the Lee Model code. Likewise the ratio found for the Gaussian profile is the correction factor for the temperature computed from the Lee code.

Although in finding the correction ratio for density we could call the integral a line density, we note that the method is a general one to find the ratio of (peak $f(r)$/uniform $f(r)$) for the case of a profiled $f(r)$ versus the case of uniform $f(r)$ whilst keeping the integral of the profile with respect to r a constant for both the cases of profiled $f(r)$ and uniform $f(r)$. Thus for both the cases of temperature and density we call the quantity $\pi r_p^2 m$ the line density.

The aim of this paper is to replace the step function by higher degree mathematical function to approximate the plasma density profile so that we may estimate the ratio of the profile-peaked density to the uniform density of the step-function; and hence obtain some information on the peak density.

We next find the functions that we can use to represent $f(r)$ for the temperature (Gaussian) and the density (crater-shape). Therefore, Gaussian distribution and Bézier functions [6] are considered in this paper. It is noted that both Gaussian distribution and Bézier functions are non-linear functions, so an iterative procedure may be required for approximating plasma density profile.

The remaining sections of this paper are organized as follows. Section 2 introduces the algorithm of approximating density profile which involves iterative process. Section 3 discusses the use of Gaussian distribution to approximate temperature profile, which mainly explains how the parameters of Gaussian distribution function relate to the shape change of temperature profile. Section 4 discusses the use of Bézier function to approximate density profile, which mainly highlights the constraint of Bézier function parameters for approximation of density profile. Section 5 presents the graphical results of density profile, and highlights the significance of the results. Section 6 summarize the overall achievement of the research and reviews critically the performance of the approximation method.

2. Algorithm of Approximating Density

This section presents an algorithm for approximating density profile as shown in Fig. 1. The input of the algorithm is the line density and the specified mathematical function. Whereas, the final output is the density profile defined by the specified function.

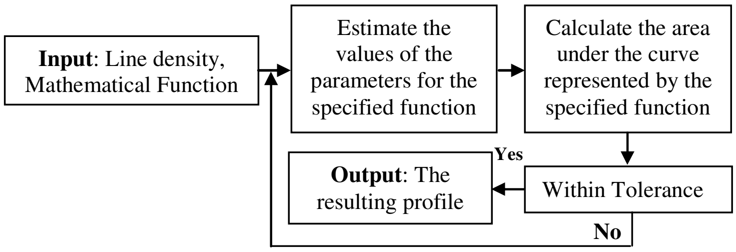

Fig. 1. Algorithm of Approximating Profile.

Initially, the desired line density (which is arbitrary but just needs to be fixed to enable shape comparison) and the mathematical function are specified, where the function can be either Gaussian distribution function (for temperature) or Bézier function (for density). It is noted that there are certain parameters that govern the shape change of the curve defined by Gaussian distribution/Bézier function, and how the parameters relate to shape change will be detailed in Section 3 and Section 4 respectively.

Subsequently, the parameter values are estimated and then the configuration of the curve defined by specified function can then be generated. To determine the line density (already arbitrarily fixed), the area under the curve is then calculated by trapezoidal rule. If the resulting line density (i.e., area under the curve) is close to the desired line density within an acceptable tolerance, then the curve is considered as the desired profile. Otherwise, the change of parameter values is carried out iteratively until the curve (profile) matches the desired line density within an acceptable tolerance.

3. Approximation Method with Gaussian Distribution Function

This section discusses the use of Gaussian distribution function to approximate the temperature profile. The Gaussian distribution is defined as follow:

$$f(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(r-\mu)^2}{2\sigma^2}}$$

where $f(r)$ is plasma column density at r,

r is the anode radius, where $0 \le x \le r_p$,

μ and σ are shape change parameters.

By specifying the values of μ and σ, and tabulating a set of values x, where $x \epsilon$ [0, R], the density profile can be obtained, as shown in Fig. 2.

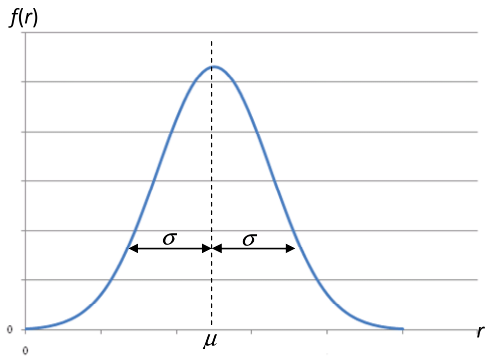

Fig. 2. Density Profile Defined by Gaussian Distribution Function.

The shape change parameter μ is symmetrical axis, which purely controls the position r of peak density without changing the peak density value $f(r)$. For instance, the peak density is at $r = \mu$ as shown in Fig. 2, by changing the value of μ to $\mu + \Delta\mu$, the peak density will be shifted to $x = \mu + \Delta\mu$. Whereas, the shape change parameter σ controls the width and peak density value of the curve as shown in Fig. 2. For instance, by increasing the value of σ to $\sigma + \Delta\sigma$, the curve will become flatter, then the peak density value will be decreased accordingly.

The resulting curve generated by using Gaussian distribution function is symmetrical in nature. Therefore the peak value of the resulting curve is always at the center regardless of the values of μ and σ. However, the density profile as pointed out by potter could be somewhat side-peak rather than center-peak. Therefore, it is not suitable to use Gaussian distribution function.

In addition, there are only two shape change parameters μ and σ. The parameter μ only controls the location of the curve with respect to center peak. The parameter σ controls the width of the curve, which can change the curve shape entirely. In this sense, the shape change of the curve is mainly governed by parameter σ.

Here we estimate what the Gaussian curve estimates the correction factor for Gaussian profiled temperature should be compared to uniformed (across the column) temperature in a similar fashion as for section 5; where we estimate the correction factor for density.

Whilst the curve shape is suitable to simulate the temperature profile, there seems to be insufficient degree of freedom to manipulate the curve shape to fit the crater shape required for Potter's density profile. Consequently, Bézier function is implemented, which allows more degrees of freedom for shape manipulation. The approximation method with Bézier function will be detailed in Section 4.

4. Approximation Method with Bézier Function

This section discusses the use of Bézier function to approximate the density profile of plasma column. The Bézier function is defined by vector function of single variable u, such that $\mathbf{R}(u) = (x(u), y(u))$, where $0 \le u \le 1$. The general form of Bézier function is given as follow:

$$\mathbf{R}(u) = \sum_{i=0}^{n} \frac{n!}{n!(n-i)!} u^n (1-u)^{n-i} \mathbf{V}_i \quad \text{for } 0 \le u \le 1$$

where n is a degree of the polynomial

$\mathbf{V}_i$ are control points with the components of (x_i , y_i)

$\mathbf{R}(u)$ are resulting points of $(x(u), y(u))$ for $0 \le u \le 1$

For density profile approximation, it is crucial to have sufficient degree of freedom in shape control, which is dependent on the degree of polynomial n. It should be noted that the curve shape characterization is governed by $\frac{d^2\mathbf{R}(u)}{du^2}$, which is essentially the rate of change of tangent vector. Consequently, 5th Degree (n = 5) of Bézier function is used, which is the lowest degree of Bézier function that allows the control of $\frac{d^2\mathbf{R}(u)}{du^2}$. The expression of 5th degree Bézier function is given as follow:

$$\mathbf{R}(u) = (1-u)^5 \mathbf{V}_0 + 5u(1-u)^4 \mathbf{V}_1 + 10u^2(1-u)^3 \mathbf{V}_2 + 10u^3(1-u)^2 \mathbf{V}_3 + 5u^4(1-u)\mathbf{V}_4 + u^5 \mathbf{V}_5$$

By specifying the values of control points $\mathbf{V}_i$ (i = 0,1,…,5), and tabulating a set of values $u \in [0\ ,1]$, the curve and the respective control points can be generated as shown in Fig. 3.

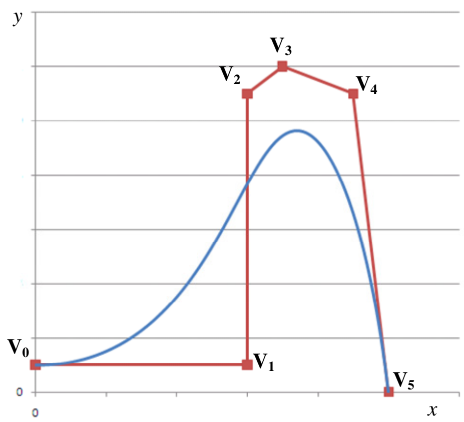

Fig. 3. Density Profile Defined by Bézier Function.

To approximate the density profile, the first step is to fix the points $\mathbf{V}_0$ and $\mathbf{V}_5$, which coincide with the end points of the curve. Once the end points are fixed, which is essentially to incorporate the curve with the respective anode radius. In order to obtain a trend that approach to somewhat crater-shape with side peak, the end tangent vectors are specified, which are mainly control by $\mathbf{V}_1$ and $\mathbf{V}_4$. Subsequently, the $\mathbf{V}_2$ and $\mathbf{V}_3$ are adjusted in a way that a side-peak can be generated as shown in Fig. 3 (showing one side of the crater profile, the other side is assumed to be symmetrical). It is noted that $\mathbf{V}_2$ and $\mathbf{V}_3$ controls the second derivatives of the end points, which provides four degree of freedom (2 control points have 2 sets of (x, y) components in total) in manipulating the curve shape.

Comparing to Gaussian distribution function, 5th degree Bézier function has more degree of freedom for curve shape modification. Therefore, it is implemented for the approximation of density profile of plasma column. The result of the approximation will be discussed in Section 5.

5. Results and Discussion

This section presents the result of a set of density profiles in similar pattern that approximated by Bézier function. As an example, a line density with 200 is specified (which is defined by area under the curve), the respective set of density profiles are shown in Fig. 4.

Referring to Fig. 4, it can be seen that lowest peak density value is 87 units/ unit length and the highest peak density value is 112 numbers of particles/unit length. Comparing to the use of step function, where the density value is at 40 units/unit length (line density = 40 × 5 =200), the ratio of peak density values between the Bézier function and step function is ranging from 2.175 to 2.8. In this sense, the use of step function likely estimates a density value up to 2.8 times less than actual peak value. It is also noted that if the crater rim is sharper the underestimation would be more severe. This is because the decrease of width has to be compensated by the increase of peak value in order to maintain the same area under the curve. So the underestimation of density can even be more than a factor of 3. In addition, it can also be observed that the higher the peak value, the sharper the bend of the peak region. In this sense, the Bézier function has a characteristic of maintaining a smooth transition, where there is no unnecessary oscillation.

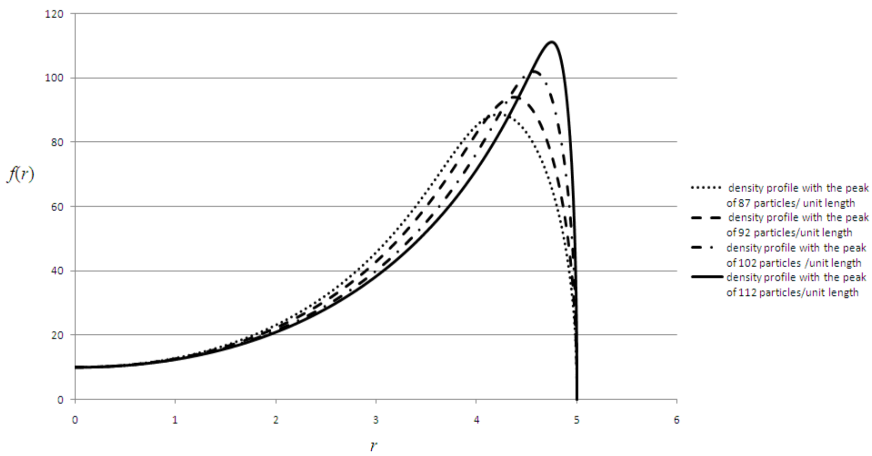

Fig. 4. Density Profiles for Different Peak Values.

6. Conclusions

In this paper, we suggest using Gaussian profile to represent the temperature profile of the pinch column. To simulate the Potter density profile for the plasma focus pinch we suggest the Bézier function which we demonstrate to be able to generate the desired crater-shape profile.

We conclude that for moderately shaped Gaussian profile as compared with Potter's results [4] the Lee Model code underestimates the peak temperature by a factor of 3.125. For moderately shaped crater-like profile the Lee Model code underestimates the peak density by a factor up to 2.8. In both case the correction factors to peak values could be much greater if the profiles are more peaked than the moderate curves considered.

To go further along these lines it is necessary to use Potter's methods to evaluate more precisely the profiles of density and temperature across the pinch columns. Alternatively laboratory measurements may be made for density and temperature profile across the pinch column. In those manners either directly the peak values of density and temperatures are obtained or if only relative profiles are obtained then Bézier functions may be suitable, to provide the correction factors to models using uniform density and temperature.

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