View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Repository of Faculty of Science, University of Zagreb

PHYSICAL REVIEW D **88,** 077501 (2013)

Physical interpretation of the dressed Polyakov loop in the Nambu–Jona-Lasinio model

S. Benić*

Physics Department, Faculty of Science, University of Zagreb, Zagreb 10000, Croatia

(Received 9 July 2013; published 4 October 2013)

We investigate the rapid rise of the dressed Polyakov loop in the Nambu–Jona-Lasinio (NJL) model as a function of temperature. In QCD such a behavior is interpreted as a confinement-deconfinement phase transition. However, we demonstrate that in the NJL model this is simply a remnant of the chiral transition.

DOI: 10.1103/PhysRevD.88.077501 PACS numbers: 12.38.Aw, 11.30.Rd, 12.38.Mh

I. INTRODUCTION

The Polyakov loop [1] is a particular representation of a static source of color, propagating in compact Euclidean time τ. When the color source has a finite mass, the path in Euclidean space need not be any more a straight line in τ - a possibility to deviate in space directions goes as $\propto 1/m^{|l|}$, where m is the quark mass, and $|l|$ is the number of links in a loop [2–4]. In this case we say that the Polyakov loop is "dressed" [2–4].

Because of its transformation with respect to the center of the color $SU(N_c)$ group of quantum chromodynamics (QCD) in the static limit $m \to \infty$, it can be used as an order parameter for a confinement-deconfinement phase transition, just like the ordinary Polyakov loop [4]. The dressed Polyakov loop (dPL) has proven to be a valuable tool in continuum studies of QCD, with the confinement-deconfinement crossover studied in the Dyson-Schwinger framework [5–7].

Model calculations of the dPL were performed in the NJL model [8,9], as well as in the Polyakov NJL (PNJL) model [10] with a magnetic field [11]. In particular, one of the results of Ref. [8] shows that even in the NJL model the dPL shows a rapid rise when proceeding from low to high temperatures. As the NJL model is not confining,[1] it should be clear then that a physical interpretation of the behavior of the dPL in the NJL model will not be the same as in QCD.

In this paper we consider a natural interpretation of the dPL in the NJL model. By simple Landau analysis we will understand how it comes to be that the dPL in the NJL model exhibits a rapid change as a function of the temperature in the first place. We will analytically show that the temperature at which the change is most pronounced is, irrespective of the model details, the chiral restoration temperature. With this result we can understand that the crossover behavior in the dPL calculated in the NJL model should be interpreted merely as an imprint of the chiral phase transition.

II. NAMBU–JONA-LASINIO MODEL WITH TWISTED BOUNDARY CONDITIONS

We work in the chiral limit with $N_f = 2$, and zero real chemical potential, $\mu = 0$. In order to calculate the dPL one has to distort the fermionic boundary conditions by introducing a twisted angle ϕ.[2] Alternatively, one can start from the imaginary chemical potential, so that the thermodynamic potential in the NJL model is given as [8]

$$\Omega = \Omega_{\rm cond} + \Omega_{\rm vac}^{\rm kin} + \Omega_{\rm th}^{\rm kin}, \tag{1}$$

where the condensation potential is $\Omega_{\rm cond} = \sigma^2/2G$, and the vacuum and thermal one-loop contributions read

$$\Omega_{\rm vac}^{\rm kin} = -d_q \int \frac{d^3p}{(2\pi)^3}\frac{E}{2}, \tag{2}$$

$$\Omega_{\rm th}^{\rm kin} = -\frac{d_q}{2}T\int \frac{d^3p}{(2\pi)^3}[\log(1+e^{-\beta(E+i\mu_I)}) + (\mu_I \to -\mu_I)], \tag{3}$$

where $E = \sqrt{\mathbf{p}^2+\sigma^2}$, and $d_q = 2\times 2\times N_f \times N_c$. Divergence of the vacuum energy (2) is regulated by a cutoff Λ. The mean field σ is obtained by minimizing the thermodynamic potential (1). A nonzero value of the mean field σ signals chiral symmetry breaking. Equivalently, one considers the quark condensate $\langle\bar{q}q\rangle = -\sigma/G$ as an order parameter.

Twisted boundary conditions for the fermion field $\psi(\mathbf{x},\tau) = e^{i\phi}\psi(\mathbf{x},\tau+\beta)$ are equivalent to setting $\mu_I = T(\phi - \pi)$, with $\phi \in [0, 2\pi\rangle$. The statistically correct fermion degrees of freedom are obtained with $\phi = \pi$. However, it is important to notice that using $\phi = 0$ does not make these fields bosons. Only by altering the overall sign in the vacuum and thermal one-loop contributions does one obtain a true Bose potential.

*sanjinb@phy.hr

[1]A simple reasoning behind this statement is that the quark propagator in the NJL model is just the tree-level fermion propagator, albeit with a constituent (~300 MeV) rather than current quark mass (~5 MeV). Therefore, a positive-definite spectral representation is available, allowing the excitation of these states.

[2]Details are omitted for simplicity; the interested reader is invited to consult [2] or [5].

 © 2013 American Physical Society

A. No symmetry restoration at the boundary

We employ a $\sigma/\Lambda \ll 1$ and a $\sigma/T \ll 1$ expansion in the vacuum and thermal parts, respectively. The relevant expressions are well known in the literature: The vacuum part can be found in [12], and the thermal part e.g., in [13]. To discuss the second order chiral phase transition, the quadratic part has to contain vacuum and thermal fluctuations, while for the quartic part one can just use the vacuum contribution,

$$\Omega(\sigma) \simeq -\frac{1}{2}a(T,\phi)\sigma^2 - \frac{d_q}{64\pi^2}\log\left(\frac{\sigma^2}{4\Lambda^2}\right)\sigma^4, \quad (4)$$

where

$$a(T,\phi) = a_0 + \frac{d_q T^2}{2}B_2\left(\frac{\phi}{2\pi}\right) = a_0 + \frac{d_q T^2}{8\pi^2}\left[(\phi-\pi)^2 - \frac{\pi^2}{3}\right] \quad (5)$$

and $B_2(x)$ is the second Bernoulli polynomial. The factor a_0 is just the quadratic vacuum contribution

$$a_0 = \frac{1}{G_c} - \frac{1}{G},$$

where $G_c\Lambda^2 = 8\pi^2/d_q$. For fermion boundary conditions $\phi = \pi$, the usual role of thermal fluctuations is to flip the sign of the quadratic term, marking the critical temperature.

In the case of general ϕ it is interesting that the thermal contribution of $a(T,\phi)$ itself can change sign. This happens at

$$\phi_\pm = \pi \pm \frac{\pi}{\sqrt{3}}.$$

Namely, in the region $\phi_- < \phi < \phi_+$, which we call fermionlike, the model can be subjected to a standard symmetry-breaking-restoration scenario, provided that the symmetry is broken in the vacuum, i.e., $G > G_c$. This is the usual case in the NJL model. However, at bosonlike twisted angles $0 \leq \phi < \phi_-$, $\phi_+ < \phi < 2\pi$, the quark condensate will not respond to arbitrary high thermal excitations. In other words, the critical temperature obtained from the condition $a(T,\phi) = 0$,

$$T_c(\phi) = \frac{8\pi^2}{d_q a_0}\frac{1}{\frac{\pi^2}{3} - (\phi-\pi)^2}, \quad (6)$$

diverges at the boundaries. For convenience we denote $T_\chi = T_c(\pi)$.

Thus, the only way for the mean field σ to be zero at bosonlike angles is by altering the theory by hand. For example, if we choose $a_0 < 0$, i.e., $G < G_c$, then we find ourselves in a weird position where the model has a restored phase at low temperatures and a broken phase at high temperatures. The other possibility would be to acknowledge the fact that bosonic theories are capable to break and restore symmetries just as fermionic ones are perfectly well. More precisely, if we understand the vacuum contribution in (1) as a potential term in a classical bosonic $Z(2)$ theory, then the thermal contribution has to have an *ad hoc* sign change if the thermal fluctuations are also understood as coming from bosonic fields. Only then does this bosonic theory break the symmetry at low temperatures and restore it at high temperatures.

B. Qualitative behavior of the dressed Polyakov loop

Strictly speaking, the dPL can be defined only when the quark mass is nonzero [4]. Naively speaking, one can still calculate this quantity by using its definition [4] as a first Fourier mode of the quark condensate at the nontrivial twisted angle

$$\Sigma_1(T) = \int_0^{2\pi}\frac{d\phi}{2\pi}e^{i\phi}\langle\bar{q}q\rangle(T,\phi). \quad (7)$$

We will now use arguments of the previous subsection to qualitatively understand that $\Sigma_1(T)$ has to rapidly change across the chiral phase boundary.

First of all, by letting $T \to 0$, the condensate does not depend on ϕ. This is because the generalized boundary conditions modify only the thermal part (3).[3] Therefore, at $T \simeq 0$, we conclude that $\Sigma_1 \simeq 0$.

However, slightly above the chiral restoration $T \gtrsim T_\chi$, chiral symmetry is first restored in a small region around $\phi = \pi$. This allows for a nontrivial Fourier transform (7), establishing a nonzero Σ_1. Therefore, it appears that as long as chiral symmetry is broken in the vacuum, i.e., $a_0 > 0$, the dPL will inevitably display a significant change, proceeding from low ($T \ll T_\chi$) to high ($T \gg T_\chi$) temperatures.

III. DIVERGENCE OF THE TEMPERATURE DERIVATIVE OF THE DRESSED POLYAKOV LOOP

From a general thermodynamical point of view it is known that the phase transition leaves its mark on all thermodynamic quantities calculated from the partition function. For example, a second order chiral phase transition leads to a nonanalyticity in the second derivatives of the thermodynamic potential, e.g., chiral or thermal susceptibility, heat capacity, and so on. Therefore, it is not unreasonable to expect that the dPL in the NJL model should also display similar properties, not because the NJL model describes the confinement as well as the confinement-deconfinement phase transition, but simply because it does a good job at describing the chiral one.

[3] Actually, this is tantamount to saying that the Polyakov loop Φ itself will be zero strictly at $T = 0$ regardless of whether the theory is confining or not. That is, even if the free energy F of a static quark is finite, we have that $\Phi = e^{-F/T} = 0$ since $T = 0$.

Let us now look for the temperature where the value of $d\Sigma_1/dT$ has a maximum. By acknowledging the fact that $\langle\bar{q}q\rangle$ can be zero at temperatures $T > T_\chi$, we have

$$\Sigma_1(T) = \int_0^{\phi_c(T)} \frac{d\phi}{\pi} \cos\phi \langle\bar{q}q\rangle(T,\phi), \tag{8}$$

where the upper limit of integration depends on the temperature; the specific values are given by solving $a(T,\phi) = 0$ for ϕ,

$$\phi_c(T) = \pi - \frac{\pi}{\sqrt{3}}\left(1 - \frac{T_\chi^2}{T^2}\right)^{1/2}. \tag{9}$$

Using simple algebra, the quantity $d\Sigma_1(T)/dT$ is given as

$$\frac{d\Sigma_1}{dT} = \int_0^{\phi_c(T)} \frac{d\phi}{\pi} \cos\phi \frac{\partial\langle\bar{q}q\rangle(T,\phi)}{\partial T} + \frac{d\phi_c(T)}{dT}\frac{\partial}{\partial\phi}\left[\frac{1}{\pi}\langle\bar{q}q\rangle(\phi,T)\cos\phi\right]_{\phi=\phi_c(T)}. \tag{10}$$

Here

$$\frac{d\phi_c(T)}{dT} = -\frac{\pi}{\sqrt{3}}\frac{1}{T}\frac{T_\chi^2}{T^2}\left(1 - \frac{T_\chi^2}{T^2}\right)^{-1/2}. \tag{11}$$

We now realize that (11) diverges as $T \to T_\chi$ from above. If we naively assume that the first term in (10), as well as the one multiplying (11), is smooth across the phase transition, then $d\Sigma_1/dT$ would have a maximum or, more precisely, would diverge exactly at $T = T_\chi$.

In an actual calculation, it turns out that the critical behavior itself is "one level" milder. The solution of the gap equation $\partial\Omega/\partial\sigma = 0$ with the truncated thermodynamic potential (4) is given in terms of the well-known Lambert W_{-1} function [12,14]. Thus, the thermal behavior of the condensate for general ϕ can be approximated as

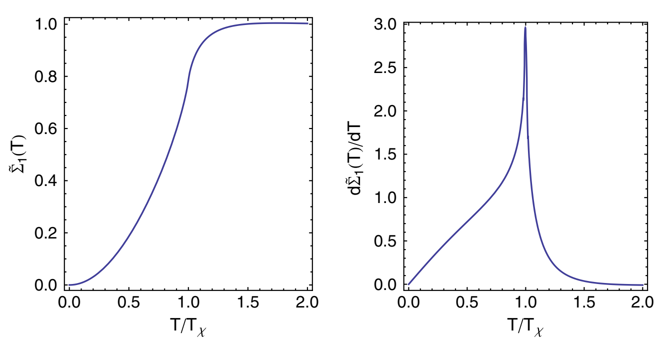

FIG. 1 (color online). On the left panel the dPL as a function of temperature is shown as calculated from the approximation to the condensate (12), while the right panel provides the temperature derivative. The explicit value is normalized to the high temperature behavior, i.e., $\tilde{\Sigma}_1(T) = \Sigma_1(T)/\Sigma_1(\infty)$.

$$\langle\bar{q}q\rangle(T,\phi) \simeq -\frac{2\Lambda}{G}\exp\left[-\frac{1}{4} - \frac{1}{2}W_{-1}\left(-\frac{4\pi^2 e^{1/2} a(T,\phi)}{d_q\Lambda^2}\right)\right], \tag{12}$$

which is to be used only in the fermioniclike region. In the bosoniclike region the mass gap is finite, so the σ/Λ, $\sigma/T \ll 1$ expansion is no longer applicable, but we might just approximate the true solution in this region with its vacuum value. This is simply (12) with a replacement $a(T,\phi) \to a_0$. Then we can use this in order to integrate (8). We use the parameters of Ref. [15], where $G\Lambda^2 = 4.636$ and $\Lambda = 602.472$ MeV. Figure 1 shows the result, where in the derivative of dPL, instead of the naive divergence, there is a sharp cusp structure.

We stress that a similar cusp behavior was seen in lattice QCD calculations in the strong coupling limit [16]. Whereas the NJL model is nonconfining, it is interesting that in Ref. [16] one deals with a completely opposite situation: Thanks to the fact that the system is strongly coupled, deconfinement does not occur, so the change in the Polyakov loop, and, in particular, the cusp, is indeed interpreted as an imprint of the chiral transition; see Figs. 2 and 3 in [16].

IV. CONCLUSIONS

Because the NJL interaction dresses the quark with a momentum independent mass, the singularity structure of the quark propagator is very simple, which is usually interpreted as a lack of confinement.[4] However, as shown here, and numerically by Ref. [8], calculation of the dPL within the NJL model leads to a order parameterlike behavior.

The semi-analytic study performed here demonstrated that the change in the temperature behavior of the dressed Polyakov loop in the NJL model is entirely dictated by the chiral transition. Our second result is that the temperature where $d\Sigma_1/dT$ diverges and the chiral transition temperature coincide exactly in the chiral limit. In Ref. [8] the latter result was obtained numerically for a particular set of parameters. We have shown that their result is more general, i.e., independent of the model parameters.

ACKNOWLEDGMENTS

We thank D. Horvatić for useful comments and a critical reading of the manuscript. Helpful discussions with B. Klajn, O. Philipsen and W. Unger are acknowledged. S. B. is supported by the Ministry of Science, Education and Sports of Croatia under Contract No. 119-0982930-1016, and by the CompStar Network.

[4]For example, in NJL model calculations this lack of confinement usually leads to the ρ meson mass lying above the kinematic threshold for $\bar{q}q$ decay.

[1] A. M. Polyakov, Phys. Lett. **72B**, 477 (1978).
[2] C. Gattringer, Phys. Rev. Lett. **97**, 032003 (2006).
[3] F. Synatschke, A. Wipf, and C. Wozar, Phys. Rev. D **75**, 114003 (2007).
[4] E. Bilgici, F. Bruckmann, C. Gattringer, and C. Hagen, Phys. Rev. D **77**, 094007 (2008).
[5] C. S. Fischer, Phys. Rev. Lett. **103**, 052003 (2009).
[6] C. S. Fischer and J. A. Mueller, Phys. Rev. D **80**, 074029 (2009).
[7] M. Hopfer, M. Mitter, B.-J. Schaefer, and R. Alkofer, Acta Phys. Pol. B Proc. Suppl. **6**, 353 (2013).
[8] T. K. Mukherjee, H. Chen, and M. Huang, Phys. Rev. D **82**, 034015 (2010).
[9] A. Flachi, Phys. Rev. D **88**, 041501 (2013).
[10] K. Kashiwa, H. Kouno, and M. Yahiro, Phys. Rev. D **80**, 117901 (2009).
[11] R. Gatto and M. Ruggieri, Phys. Rev. D **82**, 054027 (2010).
[12] K. Yagi, T. Hatsuda, and Y. Miake, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **23**, 1 (2005).
[13] A. Actor, Nucl. Phys. **B265**, 689 (1986).
[14] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, Adv. Comput. Math. **5**, 329 (1996).
[15] H. Grigorian, Phys. Part. Nucl. Lett. **4**, 223 (2007).
[16] M. Fromm, J. Langelage, O. Philipsen, P. de Forcrand, W. Unger, and K. Miura, Proc. Sci., LATTICE 2011 (2011) 212.