Self-consistent stellar dynamical tori

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Abstract. We present preliminary results on a new family of distribution functions that are able to generate axisymmetric, truncated (i.e., finite size) stellar dynamical models characterized by *toroidal* shapes. The relevant distribution functions generalize those that are known to describe polytropic spheres, for which all the dynamical and structural properties of the system can be expressed in explicit form as elementary functions of the system gravitational potential. The model construction is then completed by a numerical study of the associated Poisson equation. We note that our axisymmetric models can also include the presence of an external gravitational field, such as that produced by a massive disk or by a central mass concentration (e.g., a supermassive black hole).

INTRODUCTION

Constructing self-consistent collisionless equilibrium models is a key step to understand the structure and the dynamics of stellar systems. The construction process usually starts with the assignment of a phase-space distribution function (DF) obeying the Jeans theorem (and thus a solution of the collisionless Boltzmann equation that describes such "gravitational plasma"), which leads to an expression for the density as a function of the potential; models are then calculated by solving the Poisson equation, as a non-linear, second-order partial differential equation for the potential. In the presence of special symmetries (for example, in the case of spherical models) the problem can be reduced to the study of an ordinary differential equation, and the model construction is relatively straightforward. For more general symmetries the procedure is inherently difficult. In general, solutions have to be found numerically and turn out to exist only for a finite range of the parameters appearing in the adopted DF.

Here we present the basic properties of the simplest member of a new family of DFs that are able to generate axisymmetric, truncated (i.e., finite size) stellar dynamical models characterized by *toroidal* shapes. The complete description of our family of DFs, together with the results of N-body simulations aimed at the study of the *stability* of these collisionless equilibrium configurations, will be presented in a separate paper (Ciotti, Bertin, Londrillo 2004). We recall that toroidal, self-consistent models are sometimes mentioned in the literature (e.g., see Lynden-Bell 1962; the Author expressed concerns about their stability). With our models we will investigate this matter further, also considering the possible stabilizing effect of a massive dark matter halo. Astrophysical applications will address the modeling of *peanut-shaped* bulges (e.g., see Aronica et al. 2003, and references therein) and the problem of the *central depression* recently discovered (Lauer et al. 2002) in the luminosity profiles of some elliptical galaxies hosting supermassive black holes.

THE MODELS

The particular set of axisymmetric models presented here is defined by the DF

$$f(Q,J_z^2) = f_0|J_z|^{2\alpha}(Q-Q_0)^{\beta}\Theta(Q-Q_0), \tag{1}$$

where f_0 is a (dimensional) physical scale, $Q \equiv \mathscr{E} - J_z^2/(2R_{\rm a}^2)$, Θ is the Heaviside step function, and α and β are two dimensionless constants; the natural coordinates are cylindrical, (R,z,φ). In addition, $\mathscr{E} \equiv \Psi_{\rm T} - ||\mathbf{v}||^2/2 = \Psi_{\rm T} - (v_R^2+v_z^2+v_\varphi^2)/2$ is the binding energy per unit mass, $J_z = Rv_\varphi$ is the axial component of the angular momentum (per unit mass), and Q_0 and $R_{\rm a}$ are a *truncation energy* and an *anisotropy radius*, respectively. The function $\Psi_{\rm T} = \Psi_{\rm T}(R,z)$ represents the total gravitational potential; for the moment, we only require that $\Psi_{\rm T} \sim {\rm O}(r^{-1})$ for $r \to \infty$, where $r = \sqrt{R^2+z^2}$. We allow for an external component also, i.e., $\Psi_{\rm T} = \Psi + \Psi_{\rm ext}$, where Ψ is the potential associated with the density distribution derived from eq. (1) as given by the following eq. (4), while $\Psi_{\rm ext}$ is taken to be a given function (equal to zero in the fully self-consistent case). Note that we can write

$$Q = \Psi_{\rm T} - \frac{v_{\rm m}^2}{2} - \frac{v_\varphi^2}{2}\left(1+\frac{R^2}{R_{\rm a}^2}\right), \tag{2}$$

where $v_{\rm m}^2 \equiv v_R^2 + v_z^2$ is the square of the meridional (or poloidal) velocity component. Some dynamical properties of the systems associated with eq. (1) are the following: (i) no net rotation is present, and (ii) the two components σ_R^2 and σ_z^2 of the velocity dispersion tensor are equal. In addition, note that for $\alpha = 0$ if we formally consider $R_{\rm a} \to \infty$ we recover the polytropic spheres, while for integer α and β the distribution function is a *sum* of Fricke's (1952) DFs. The functional dependence of the system structural and dynamical properties on $\Psi_{\rm T}$ is obtained by integration over $\Omega(\mathbf{x})$, the velocity section of phase–space over which $f \geq 0$ at given $\mathbf{x}$. The coordinates (v,ζ,ξ) to be used to integrate the various quantities over the rotationally symmetric (ellipsoidal) region $\Omega(\mathbf{x})$ are naturally defined by

$$v_R = v\sin\zeta\cos\xi, \quad v_z = v\sin\zeta\sin\xi, \quad v_\varphi = \frac{v\cos\zeta}{\sqrt{1+R^2/R_{\rm a}^2}}, \tag{3}$$

where $0 \leq \zeta \leq \pi$, $0 \leq \xi \leq 2\pi$. With this choice, the ellipsoidal radius v of the velocity section $\Omega(\mathbf{x})$ runs in the range $0 \leq v \leq \sqrt{2[\Psi_{\rm T}(\mathbf{x})-Q_0]}$. Integration over the two angular coordinates gives

$$\rho = 4\pi f_0 g(R)\frac{B(\alpha+3/2,\beta+1)2^{\alpha+1/2}}{2\alpha+1}(\Psi_{\rm T}-Q_0)^{\alpha+\beta+3/2}\Theta(\Psi_{\rm T}-Q_0), \tag{4}$$

where $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ is the Euler Beta function,

$$g(R) = \frac{R^{2\alpha}}{(1+R^2/R_{\rm a}^2)^{\alpha+1/2}}, \tag{5}$$

and for convergence it is required that $\alpha > -1/2$, $\beta > -1$. Moreover,

$$\rho\sigma_{\rm m}^2 = 4\pi f_0 g(R) \frac{B(\alpha+5/2,\beta+1)2^{\alpha+5/2}}{(2\alpha+1)(2\alpha+3)}(\Psi_{\rm T}-Q_0)^{\alpha+\beta+5/2}\Theta(\Psi_{\rm T}-Q_0) \tag{6}$$

and

$$\rho\sigma_\varphi^2 = \frac{4\pi f_0 g(R)}{1+R^2/R_{\rm a}^2}\frac{B(\alpha+5/2,\beta+1)2^{\alpha+3/2}}{2\alpha+3}(\Psi_{\rm T}-Q_0)^{\alpha+\beta+5/2}\Theta(\Psi_{\rm T}-Q_0). \tag{7}$$

The number of free parameters (excluding those associated with the external potential) needed to describe a model completely is five: the three dimensional quantities f_0, $R_{\rm a}$, and Q_0, and the two dimensionless constants α and β. The relation between the meridional velocity dispersion $\sigma_{\rm m}^2$ and the azimuthal (toroidal) velocity dispersion σ_φ^2, can be described by the anisotropy distribution

$$a(R,z) \equiv 1 - \frac{2\sigma_\varphi^2}{\sigma_{\rm m}^2} = \frac{R^2/R_{\rm a}^2 - 2\alpha}{R^2/R_{\rm a}^2+1}. \tag{8}$$

The velocity dispersion tensor is isotropic when $a = 0$, tangentially anisotropic when $a < 0$, and meridionally anisotropic when $a > 0$. Note that the velocity dispersion anisotropy is constant on *cylinders*. It can be proved that eq. (8) holds also when the factor $(Q-Q_0)^\beta$ in eq. (1) is replaced by a generic function $h(Q-Q_0)$ (Ciotti et al. 2004). We now impose the system consistency, i.e., we require that

$$\triangle\Psi = -4\pi G\rho, \tag{9}$$

with Ψ a nowhere negative function. In order to obtain the numerical solution of eq. (9), the Poisson equation is first recast in dimensionless form, by referring to the physical scales $R_{\rm a}$ for lengths and Q_0 for potentials. After rescaling, we are then led to the solution of the following problem:

$$\tilde{\triangle}\phi = -\lambda\tilde{\rho}, \qquad \tilde{\rho} \equiv g(\tilde{R})(\phi_{\rm T}-1)^\gamma\Theta(\phi_{\rm T}-1), \tag{10}$$

where $\gamma \equiv \alpha+\beta+3/2 > 0$, and the dimensionless quantities are defined as $\phi_{\rm T} \equiv \Psi_{\rm T}/Q_0$, $\tilde{R} \equiv R/R_{\rm a}$, and $g(\tilde{R}) \equiv \tilde{R}^{2\alpha}/(1+\tilde{R}^2)^{\alpha+1/2}$, with

$$\lambda \equiv \frac{16\pi^2 G f_0 R_{\rm a}^{2\alpha+2} Q_0^{\alpha+\beta+1/2} B(\alpha+3/2,\beta+1)2^{\alpha+1/2}}{2\alpha+1}. \tag{11}$$

It follows that

$$\rho = \frac{Q_0}{GR_{\rm a}^2}\frac{\lambda\tilde{\rho}}{4\pi} \tag{12}$$

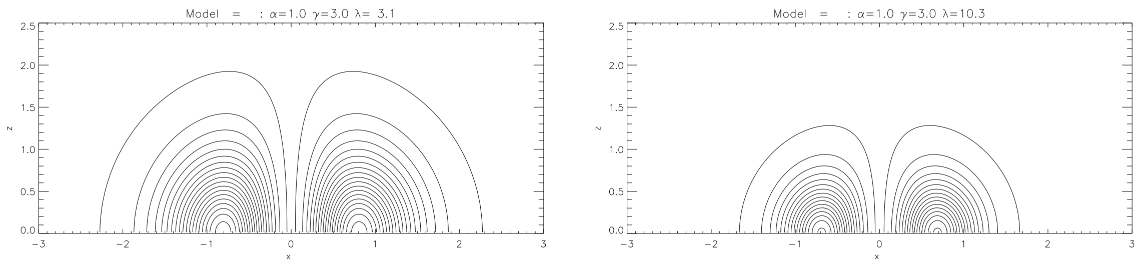

FIGURE 1. Isodensity contours of a meridional section of the dimensionless density distribution associated with the models $(\alpha,\gamma)=(1,3)$ for $\lambda\simeq 3.1$ (left) and $\lambda\simeq 10.3$ (right).

and

$$\tilde{\sigma}_{\rm m}^2=\frac{2}{\gamma+1}(\phi_{\rm T}-1)\Theta(\phi_{\rm T}-1),\quad \tilde{\sigma}_{\varphi}^2=\frac{2\alpha+1}{(\gamma+1)}\frac{(\phi_{\rm T}-1)\Theta(\phi_{\rm T}-1)}{1+\tilde{R}^2}. \qquad (13)$$

Equation (10), a classical non-linear elliptic partial differential equation, is then solved by using a Newton iteration scheme, described in detail by Ciotti et al. (2004). In general, for assigned model parameters we found convergence only for finite intervals of λ. In Figure 1 we show the meridional sections of the density distribution of two representative truncated and fully self-consistent ($\Psi_{\rm ext}=0$) toroidal models, obtained by fixing $\alpha>0$ (see eqs. [5] and [10]), for two different values of λ.

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