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The Fusion of Point and Linear Objects in Navigation

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ABSTRACT: There are great many human activities where problems dealt with are based on data from a number of sources or where we lack certain data to solve a problem correctly. Such situations also occur in navigation, where we have to combine data from diverse navigational devices with archival data, including images. This article discusses a problem of the fusion of position data from shipboard devices with those retrieved from a hydrographic data base, the data being of varying accuracy. These considerations are illustrated with examples of the fusion of shipboard measurements with the pier line (or another cartographic object).

1 INTRODUCTION

In a general case data fusion is a process of combining data for the purpose of:

- supplementing data to get a complete mathematical model of an examined process,
- data verification and consistency,
- estimation and prediction.

Data fusion in navigation is mostly associated with the top level of fusion. However, with modern navigational and computer technologies, data fusion can be applied at all levels, not only for the estimation of navigational measurements.

Navigation makes use of many engineering and computing methods to determine position coordinates in an established reference system. Basically, these methods can be divided into three types:

- model, based on a model of navigating object movement – dead reckoning (DR) and inertial navigation system (INS),
- parametric, in which a position is determined from a measurement of navigational parameters, that is spatial relations between navigating object coordinates and navigational marks,
- comparative navigation, in which images of measured Earth's physical fields are compared with cartographic images (databases).

Cartographic data are directly used for object position determination in the last mentioned method only. However, these measurements are not combined with other position determination methods. We present herein possibilities of the fusion of data from cartographic database with a running fix (parametric navigation).

The authors got inspired to deal with the issue by the fact that there occurs a statistical incompatibility of ship' position with cartographic data in cases of vessels berthing, docking or proceeding along a fairway.

2 FORMULATION OF THE PROBLEM

All combined navigational data should always be brought to a joint reference system. At present, WGS-84 fulfills this function due to a wide use of the satellite navigational GPS and ECDIS system. For this reason all navigational measurement and cartographic data from a navigational-hydrographic database should be brought to this reference system unless original data have been determined in this system. Failing to satisfy this condition results in a systematic error substantially exceeding random errors of the data.

The following assumptions have been made in the measurement (position) and cartographic data fusion problem to be solved:

– data are determined in the same reference system,
– data are of random character with a specific probability distribution,
– data are not burdened with systematic errors,
– data will undergo fusion by means of the least squares method with or without measurement covariance matrix being considered.

The relative positions of a ship and the pier (chart feature) are shown in Figure 1.

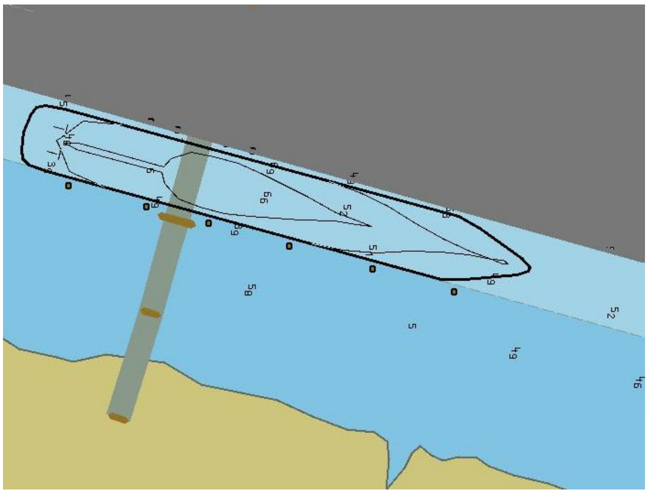

Figure 1. A ship berthing along a pier.

The ship is lying alongside, so the pier line can be regarded as a conventional line of position parallel to the ship's plane of symmetry shifted by a vector from a conventional ship's point, to which all navigational measurements are brought. The vector can be determined by direct measurement or indirectly, calculating its elements on the basis of a known position of conventional point on ship's plane and distance of ship's side to the pier line.

3 DATA FUSION

High accuracy of satellite navigational systems and autonomous shipboard systems (dead reckoning, inertial navigational systems) creates high standard requirements for methods of navigational data processing.

We will perform a fusion of navigational and cartographic data using the method of least squares. In the method, we will regard the line of a cartographic object (chart feature) as an additional line of position. A Kalman filter can be used if a ship is proceeding. There is also a possibility of measuring the relative position of cartographic objects.

If we do not take data accuracy into account, the method of least squares (LS) can be written in this form [6], [8], [9], [10]:

$$\mathbf{x} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{z}, \qquad (1)$$

where

$\mathbf{x}$ – m -dimensional state vector (of ship's coordinates, searched-for position),

$\mathbf{z}$ – n - dimensional vector,

$\mathbf{u}$ – n- dimensional vector of measured navigational parameters,

$\mathbf{G} = \mathbf{f}'(\mathbf{x})$ – Jacobian matrix of the function $\mathbf{f}$ in respect to $\mathbf{x}$.

$$\mathbf{G} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_m} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_m} \end{bmatrix}, \qquad (2)$$

$\mathbf{f}$ – n- dimensional vector function,

$\mathbf{u}$ – vector of direct measurements,

$\mathbf{z} = \mathbf{u} - \mathbf{f}(\mathbf{x})$ – generalized vector of measurements.

The position $\mathbf{x}$ coordinates vector covariance matrix is expressed by this formula [6], [9], [10], [11]:

$$\mathbf{P}_{\mathbf{x}} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}\right)^{-1} \qquad (3)$$

When we take data accuracy into account, we deal with the method of weighted least squares (*WLS*)

$$\mathbf{x} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{z}, \qquad (4)$$

where

$$\mathbf{R} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 & 0 \\ \sigma_{xy}^2 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_{pier}^2 \end{bmatrix}$$ - navigational data covariance matrix.

We make a fusion of positions or lines of position with the pier line following this procedure:

– determine the position coordinates (or lines of position) together with their accuracy assessment (variances and covar iances),

- determine the direction and accuracy of berth line (using a relevant chart, or database, and possibly, using the relative error to establish the accuracy of that line,
- shift the berth line parallel towards the ship's position by the vector representing the distance of that line from the assumed reference point connected with ship's position – centre of masses, geometric centre, GPS antenna position or another),
- calculate the ship's position coordinates, regarding the berth line as an additional position line.

The covariance matrix of the running fix in case of a GPS is calculated from a series of positions or from Kalman filter. If there are terrestrial navigational systems, we can use the following relations [2].

An average error of geographic latitude determination for the common middle station

$$\sigma_\phi = 0,5\sigma_{\Delta D} \cos\mathrm{ec}\theta \sqrt{\cos^2 A_{12} \cos\mathrm{ec}^2 \frac{\omega_{23}}{2} + \cos^2 A_{23} \cos\mathrm{ec}^2 \frac{\omega_{12}}{2}} \quad (5)$$

where

A_{ij} – average azimuth between the *i*-th and the *j*-th station,

ω_{ij} – base angle between the *i*-th and the *j*-th station,

$\sigma_{\Delta D}$ – measurement error of distance difference.

An average error of geographic longitude determination for the common middle station

$$\sigma_\lambda = 0,5\sigma_{\Delta D} \cos\mathrm{ec}\theta \sqrt{\sin^2 A_{12} \cos\mathrm{ec}^2 \frac{\omega_{23}}{2} + \sin^2 A_{23} \cos\mathrm{ec}^2 \frac{\omega_{12}}{2}} \quad (6)$$

The covariance between geographic coordinates for the common middle station

$$\sigma_{\phi\lambda} = \frac{1}{8}\sigma_{\Delta D}^2 \cos\mathrm{ec}^2\theta \sin(A_1 + A_2)(\cos\mathrm{ec}^2 \frac{\omega_{23}}{2} + \sin(A_2 + A_3)\cos\mathrm{ec}^2 \frac{\omega_{12}}{2}) \quad (7)$$

Also, we can change a GPS-obtained position into a system of two position lines by calculating their elements by using a vector of mean coordinates and elements of its covariance matrix. In this case position lines are regression lines running in the same direction (parallel) (tangent near the actual position) [8]:

a) $$y = \bar{y} + \frac{\sigma_{xy}}{\sigma_x^2}\left(x - \bar{x}\right), \quad (8)$$

b) $$x = \bar{x} + \frac{\sigma_{xy}}{\sigma_y^2}(y - \bar{y}), \Rightarrow y = \bar{y} + \frac{\sigma_y^2}{\sigma_{xy}}(x - \bar{x}), \quad (9)$$

c) $(\bar{x}, \bar{y})$ – centre of gravity of the population (mean position).

In the geographical coordinate system these lines are expressed as follows:

$$\varphi = \varphi_{śr} + \frac{\sigma_{\varphi\Delta l}}{\sigma_{\Delta l}^2}\left(\Delta l - \Delta l_{śr}\right) =$$

$$= \varphi_{śr} + \left(\Delta l - \Delta l_{śr}\right) \cdot \mathrm{tg}\, NR_1, \quad (10)$$

$$\Delta l = \Delta l_{śr} + \frac{\sigma_{\varphi\Delta l}}{\sigma_\varphi^2}\left(\varphi - \varphi_{śr}\right) =$$

$$= \Delta l_{śr} + \left(\varphi - \varphi_{śr}\right) \cdot \mathrm{tg}\, NR_2, \quad (11)$$

$$\frac{\mathrm{tg}\, NR_1}{\mathrm{tg}\, NR_2} = \frac{\sigma_\varphi^2}{\sigma_{\varphi\Delta l}}. \quad (12)$$

Let us illustrate the above considerations of the fusion of ship's position and a cartographic line by the following examples.

EXAMPLE 1.

The first example refers to the fusion of a ship's position from GPS (point) with a linear cartographic object (pier line or depth contour). Such situations often occur when a ship is moored, docked or is close to hydrotechnical objects.

The origin of a local coordinate system $0xy$ is at an established point on the ship (for simplification). In this case the ship is mooring along a pier described by the equation $y = x + 2$ (after a displacement by a vector representing the distance from pier line to assumed coordinate origin) and accuracy $\sigma_{pier} = 1m.$ The running fix, determined by GPS on the ship, had this covariance matrix:

$$\mathbf{R} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

Thus we get $\sigma_{xy} = 0, \sigma_x = \sigma_y = \sqrt{2}$. The GPS position can be considered as a point of intersection of a meridian (vertical line) and parallel (horizontal line).

The matrices $\mathbf{G}$ and $\mathbf{R}$ are as follows:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

while the resultant coordinate vector

LS – $\mathbf{x} = \left[-0,667; 0,667\right]^{\mathrm{T}}$,

WLS – $\mathbf{x} = \left[-0,8; 0,8\right]^{\mathrm{T}}$.

This situation is displayed in Figure 2. We can see that taking into account the accuracy of individual position lines leads to a displacement of *LS* position to the point *WLS* (due to higher accuracy of the cartographic line than that of the GPS-obtained position).

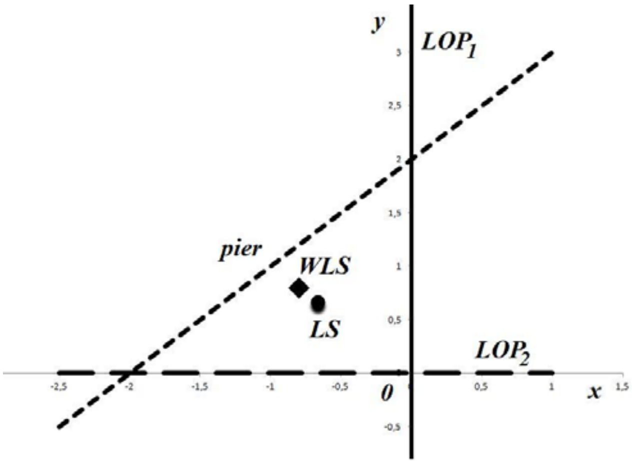

Figure 2. A fusion of a GPS position and pier line.

EXAMPLE 2.

In the other example we perform a fusion of linear objects, e.g. pier line with lines of position of the radionavigational system.

Similarly to Example 1, we adopt $0xy$ at an established point on the ship (for simplification). Now the ship is mooring at a pier described by the equation $y = x + 10$ (after a relevant displacement) and accuracy $\sigma_{pier} = \sqrt{10}\, m$. A running fix was determined on the ship by using a terrestrial hyperbolic system with the following covariance matrix:

$$\mathbf{R} = \begin{bmatrix} 1 & 0,5 \\ 0,5 & 1 \end{bmatrix}.$$

We have then $\sigma_{xy} = 0,5; \sigma_x = \sigma_y = 1$. The matrices $\mathbf{G}$ and $\mathbf{R}$ are as follows:

$$\mathbf{G} = \begin{bmatrix} 6 & 1 \\ -5 & 1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 1 & 0,5 & 0 \\ 0,5 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix},$$

while the resultant coordinate vector

LS – $\mathbf{x} = \left[-0,161; 3,333\right]^{\mathrm{T}}$,

WLS – $\mathbf{x} = \left[-0,115; 7,022\right]^{\mathrm{T}}$.

The situation is illustrated by Figure 3. This time the differences between both estimated positions are larger due to another geometric configuration of position lines and other values of their errors.

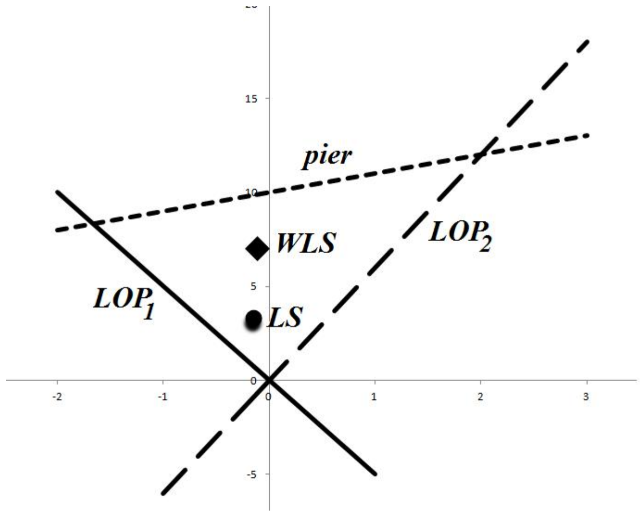

Figure 3. A fusion of positions from a terrestrial radionavigational system with a pier line.

4 CONCLUSIONS

In the above considerations we have shown how additional data, in this case data on cartographic features position and accuracy, can be utilized for estimating ship's position and its covariance matrix. In this way we increase the accuracy and reliability of ship's position being determined. This is of particular importance for ships located in immediate proximity of port and marine facilities and other navigational dangers. Similar situations also occur in aircraft navigation on the airfields. In rail navigation additional requirement is to take into account the spatial position of train or tram tracks, while in truck navigation – position of roads.

Another approach consists in imposing constraints resulting from the Rao-Cramer inequality [7], [8] on the covariance matrix. However, further research into these problems should take into account both deterministic and probabilistic constraints on the coordinate vector, that is the random character of determining coordinates of cartographic objects. The point is to avoid superimposition of two disjoint objects such as a ship and a pier. In the simplest solution the navigator can interfere with the results of calculations. However, such solution is far from satisfactory as it does not take into consideration all possible situations and cannot be automated. It should be borne in mind that cartographic objects are also determined with a definite accuracy.

The presented problem of the fusion of ship position data and data on the location of hydrotechnical objects does not cover all issues of navigational data fusion (spatial data, in more general terms). Problems of integration of measurement data from navigational systems have a long history, starting from a well-known article by R. E. Kalman [5], then his successors, with a period of rapid development of integrated navigational systems in the 1980s and '90s, also in Poland [1]. At present, the focus of data fusion in navigation is shifting towards preliminary processing and fusion of various types of data – measurement, image and text [4]. Also, methods and applications of data and information fusion have been widely extended.

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