Calculation of the Wall Shear Stress in the case of an Internal Carotid Artery with stenoses of different sizes

Titus PETRILA[1], Balazs ALBERT*[,2]

*Corresponding author
[1]"Vasile Goldis" Western University, No. 94 Bd. Revolutiei, 310025 Arad, Romania
Member (Assoc.) of Academy of Romanian Scientists,
No. 54 Splaiul Independentei, 050094 Bucharest, Romania
aosrtransilvania@yahoo.com
*[,2]"Babes - Bolyai" University
No. 1 Mihail Kogalniceanu, 400084, Cluj-Napoca, Romania
balazsalb@gmail.com

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Abstract*: In this paper we use a non-Newtonian mathematical model for the blood flow in large vessels – elaborated and presented already by us in a previous paper [1]. We calculate and than compare the values of the wall shear stress, which has a special importance in the possible ruptures of vascular vessels (in the case of a human internal carotid artery with stenosis) in four different cases. The numerical simulations are made using COMSOL Multiphysics 3.3, and the results are compared to some already existing in the literature.*

Key Words*: blood flow; non-Newtonian model; stenosed artery; wall shear stress; rupture of vascular vessels*

1. INTRODUCTION

In this research for blood we accept a non-Newtonian rheological behavior with a variable coefficient of viscosity under the conditions of an unsteady (pulsatile) flow regime, connected with the rhythmic pumping of the blood by the heart. At the same time we admit the incompressibility and homogeneity of the blood while the exterior body forces are neglected. We also take into consideration the viscoelastic behavior of the limiting walls of the vessels, the whole configuration accepting an axial symmetric geometry versus the vertical axis *Oz*. We use a non-Newtonian mathematical model, elaborated in a previous research and presented in [1]. According to this model the motion equations result from the general Cauchy equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = div\mathbf{T},$$

where the stress tensor $\mathbf{T} = -p\mathbf{I} + 2(\mu_s + \mu_{RBC})\mathbf{D}$, where **I** is the unit tensor, the scalar p is the physical pressure, **D** is the rate of strain tensor, μ_s representing the (constant) plasma viscosity while μ_{RBC} is given by the Cross model, i.e.

$$\mu_{RBC} = \frac{\mu_0^*}{1+(k\dot{\gamma})^{1-n}},$$

with $\dot{\gamma}$ being the shear rate, μ_0^* the viscosity coefficient of the blood, k is a time constant and n is the index for a shear thinning behavior.

The evolution equations are joined to some boundary conditions which express the existence of a pressure gradient along the *Oz* axis according to the heart beats and implicitly to the rhythmic blood pushing into the vessel (feature which is important in large vessel). Precisely we have

$$\frac{\partial v}{\partial r} = 0 \text{ and } u = 0 \text{ for } r = 0.$$

At $r = R$, due to the viscoelastic behavior of the vessel's wall, the velocity of the blood must be equal to the displacement velocity of the vessel's wall.

The boundary conditions at "edges" $z = 0$ and $z = L$ of the vessel agree with a physiological pulse velocity given by a periodic time-varying function.

To describe the viscoelastic behavior of the vessel's wall we have used the generalized Maxwell model, which is the most general form of the linear model for viscoelasticity [2].

2. NUMERICAL EXPERIMENTS AND RESULTS

We have analyzed a real case of a human internal carotid artery (ICA), which has a stenosis of 69%. The length of the stenosed arterial segment is 3*cm*, the internal diameter of the blood vessel is 7*mm* and the thickness of the vessel wall is 0.8*mm*. The mass density of the blood has been fixed at $\rho = 1060 kg/m^3$.

For the numerical calculations we have used the Navier-Stokes equations [5],

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u},$$

where the dynamic viscosity, according to the Cross model, is

$$\mu(\dot{\gamma}) = \mu_s + \frac{\mu_0^*}{1 + (k\dot{\gamma})^{1-n}}.$$

The boundary conditions are those mentioned in the previous section. To avoid the transient effect of the initial conditions the results are presented only for the last 5 periods, although the time integration interval is $t \in [0,10s]$. The numerical simulations are made using the programming package COMSOL 3.3. For the constants k, n, and the viscosities μ_s, μ_0^* we have taken the values $k = 1.036$, $n = 0.2$, $\mu_s = 0.00345\, Pa\cdot s$ and $\mu_0^* = 0.0465\, Pa\cdot s$, respectively.

In the case of this real ICA we present some results concerning the wall shear stress, $WSS = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)$, which is believed to have a special importance in the possible ruptures of vascular vessels. We have chosen 4 particular points on the vessel wall of the stenosed ICA (see figure 2) in which the values of the *WSS* are evaluated.

The variation of the wall shear stress (through 5 seconds) – evaluated in those four particular points mentioned in figure1, is presented in figure 2.

It can be clearly seen that the *WSS* reaches very high values in the middle of the stenosis (the red point in figure 1).

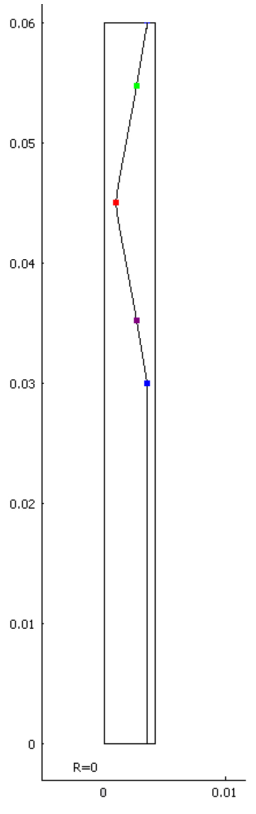

Figure 1. The 4 particular points on the wall of the stenosed ICA (the coordinates on the axes are expressed in *m*)

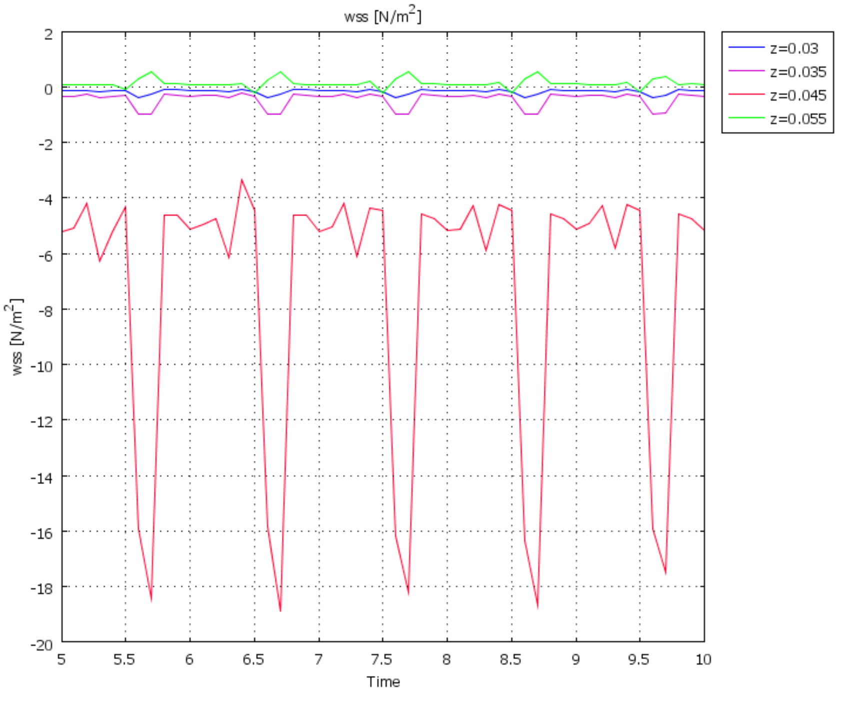

Figure 2. Value of the WSS (through 5 seconds) in the 4 particular points

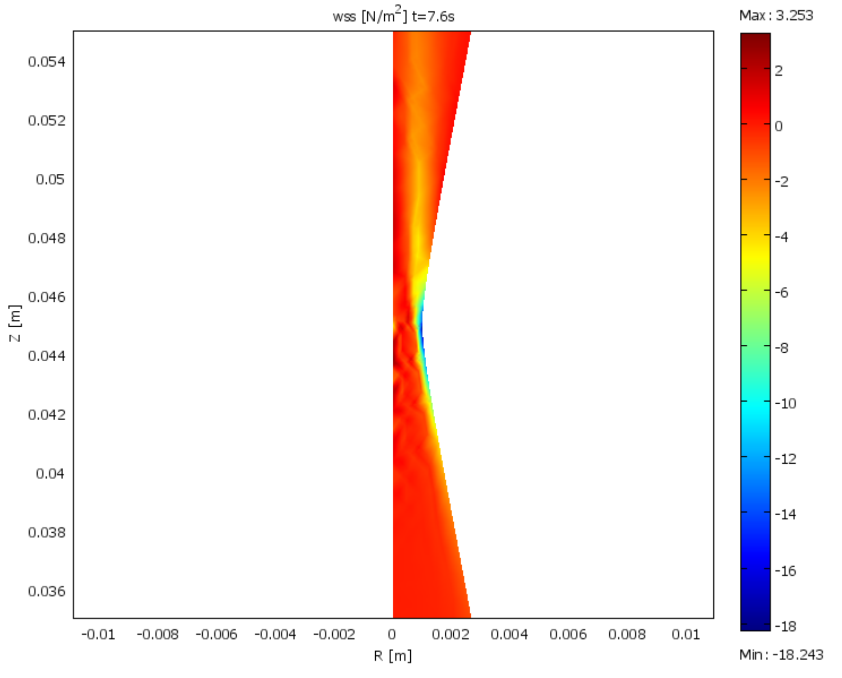

Figure 3. Value of the WSS along the stenosed ICA, at *t*=7.6s

In figures 2 and 3 one can see that the absolute maximum value of the *WSS* is around 18 N/m^2. The "-" sign shows that the wall shear stress acts in opposite direction compared to that of the blood flow. Figure 4 presents the normal values of the *WSS* in some different vascular vessels, according to Papaioannou & Stefanos (2005).

In this figure it can be clearly seen that the normal value of the *WSS* in the case of the arteries is around 1 N/m^2. The comparison of these values with those numerically obtained shows that the value of the *WSS* (numerically obtained) has indeed a significant increase in the stenosed section, as against the normal value.

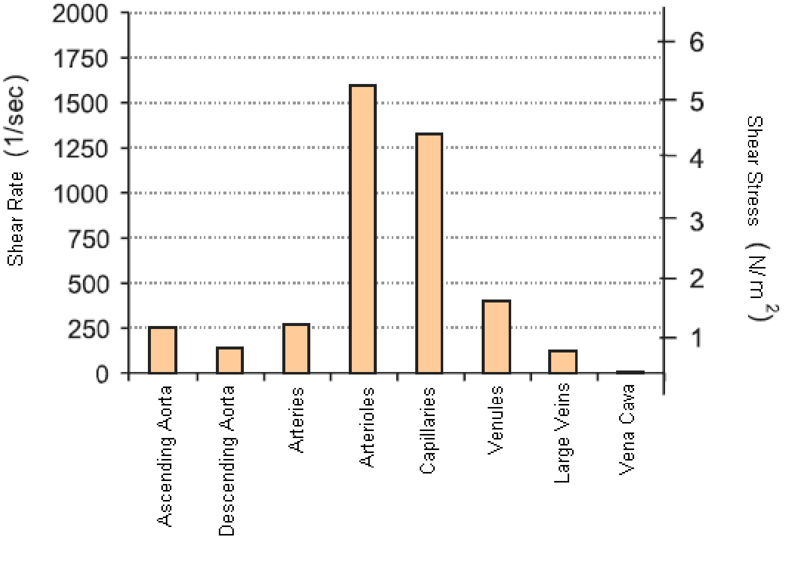

Figure 4. Shear stress values in different blood vessels Papaioannou & Stefanos (2005)

Similarly, in Tu, Yeuh & Liu (2013), page 12, it is stated that in the case of arteries with stenosis the values of the wall shear stress may increase from 10 N/m^2 to 50 N/m^2. A similar result can be found also in the work of Minea (2003), page 169, where – in the case of an ICA with a stenosis of 30 % – the value of the *WSS* is about 20 N/m^2. Trying to make a connection between the values of the wall shear stress (*WSS*) - which is believed to have a special importance in the possible ruptures of vascular vessels - and the degree of the stenosis, we examine numerically the same ICA in four cases: with a stenosis of 30%, 50%, 70% and 90% respectively. The results got for the values of the *WSS* in these four cases are compared. The wall shear stress reaches its highest value in the middle of the stenosis, where the vessel's diameter is the lowest. Thus we have calculated the variation of the *WSS* in the middle of the stenosis in all of the four cases. These results are presented in figures 5-7.

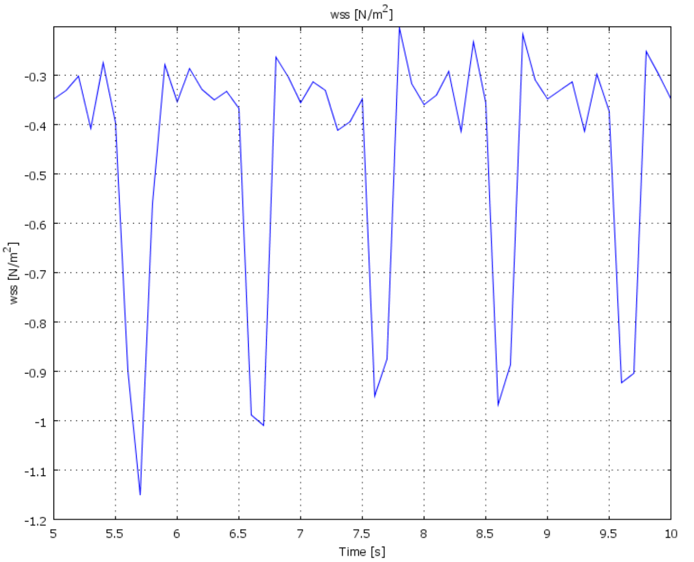

Figure 5. Variation of the WSS (through 5 periods) in the middle of the ICA with a stenosis of 30%

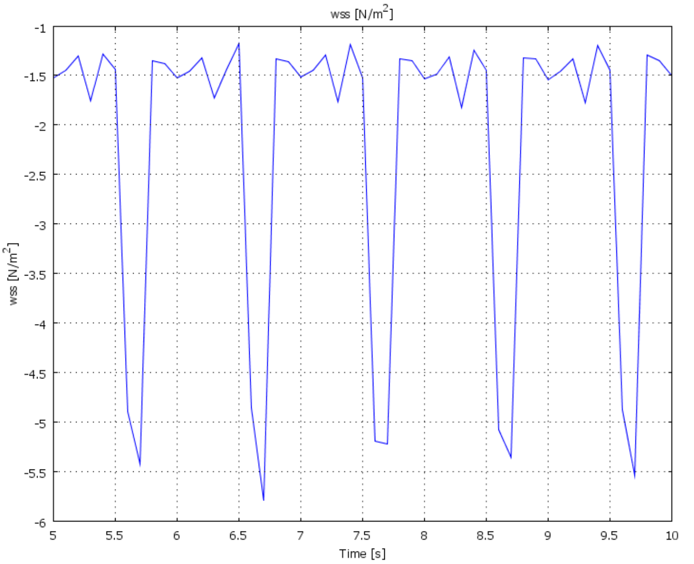

Figure 6. Variation of the WSS (through 5 periods) in the middle of the ICA with a stenosis of 50%

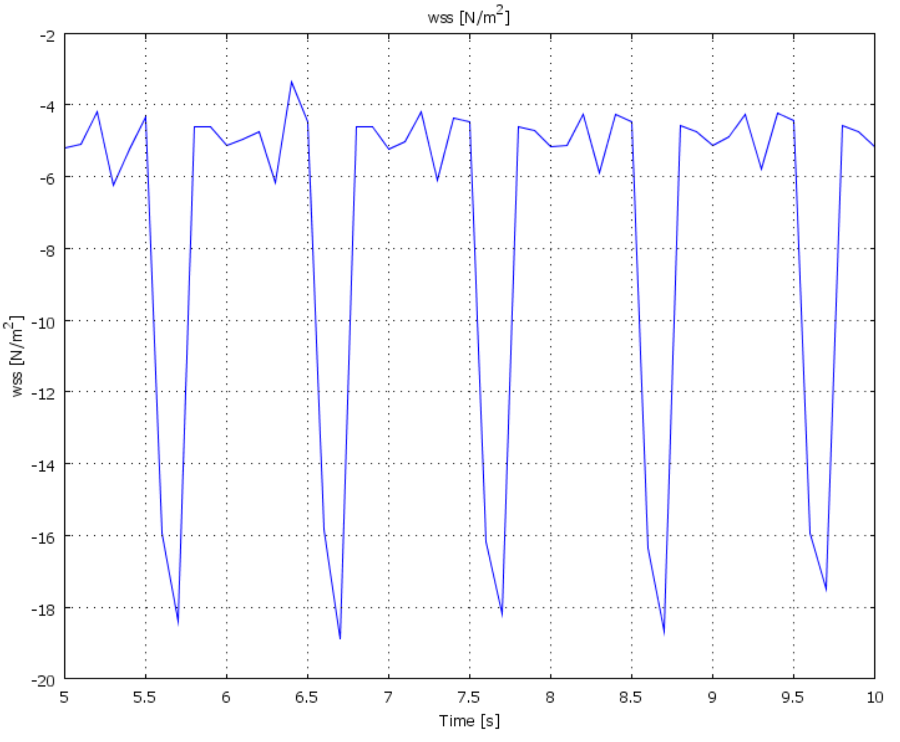

Figure 7. Variation of the WSS (through 5 periods) in the middle of the ICA with a stenosis of 70%

In figures 8-11 the values of the *WSS* along the stenosed ICA are presented, at a certain moment. We present the results for all of the four cases.

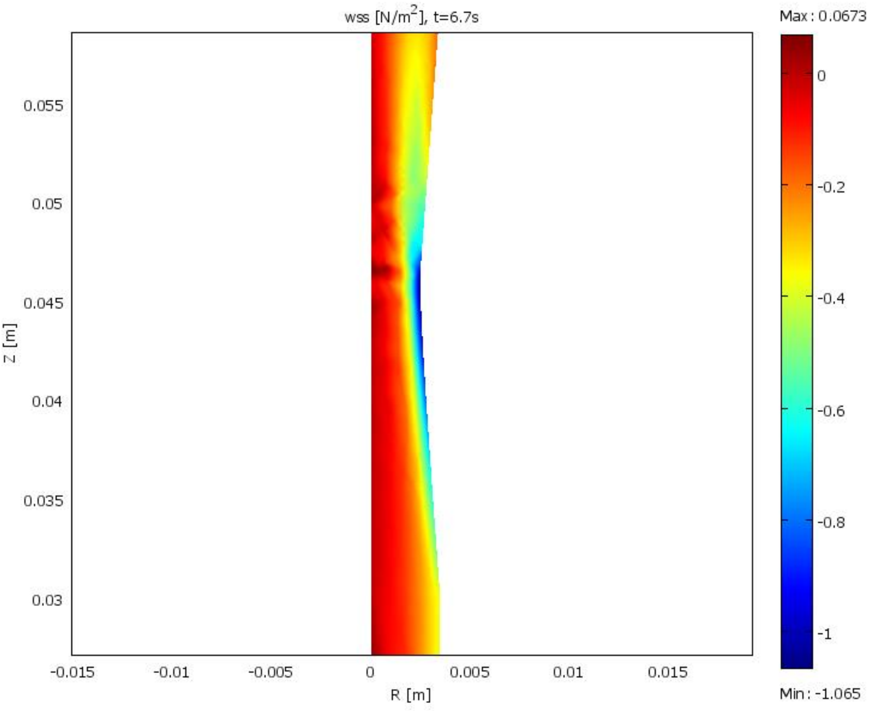

Figure 8. Value of the WSS along the ICA with stenosis of 30% (at t = 6,7*s*)

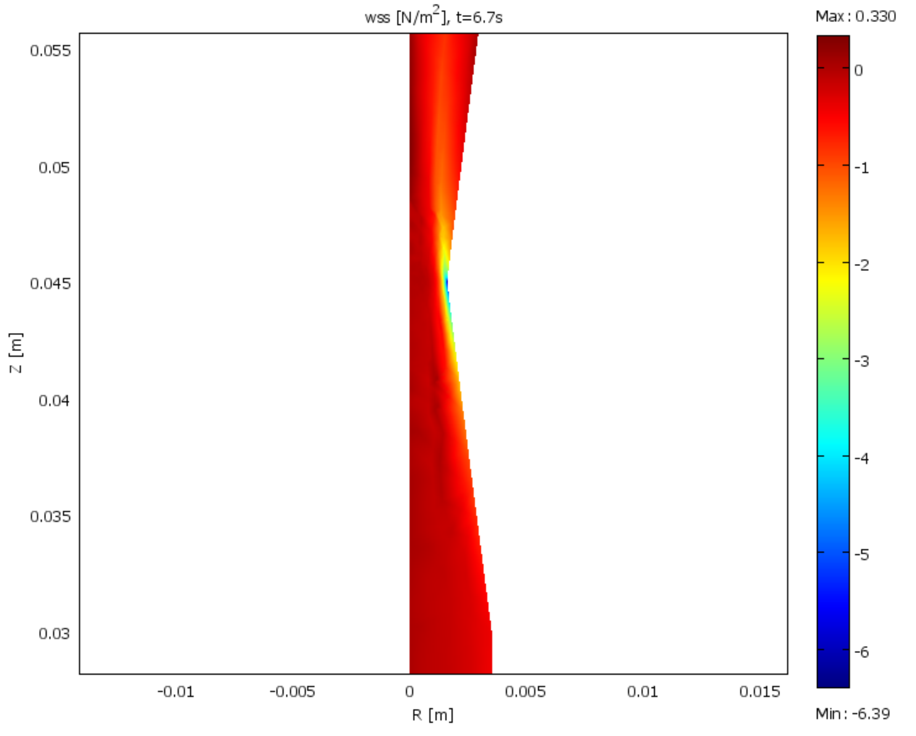

Figure 9. Value of the WSS along the ICA with stenosis of 50% (at t = 6,7*s*)

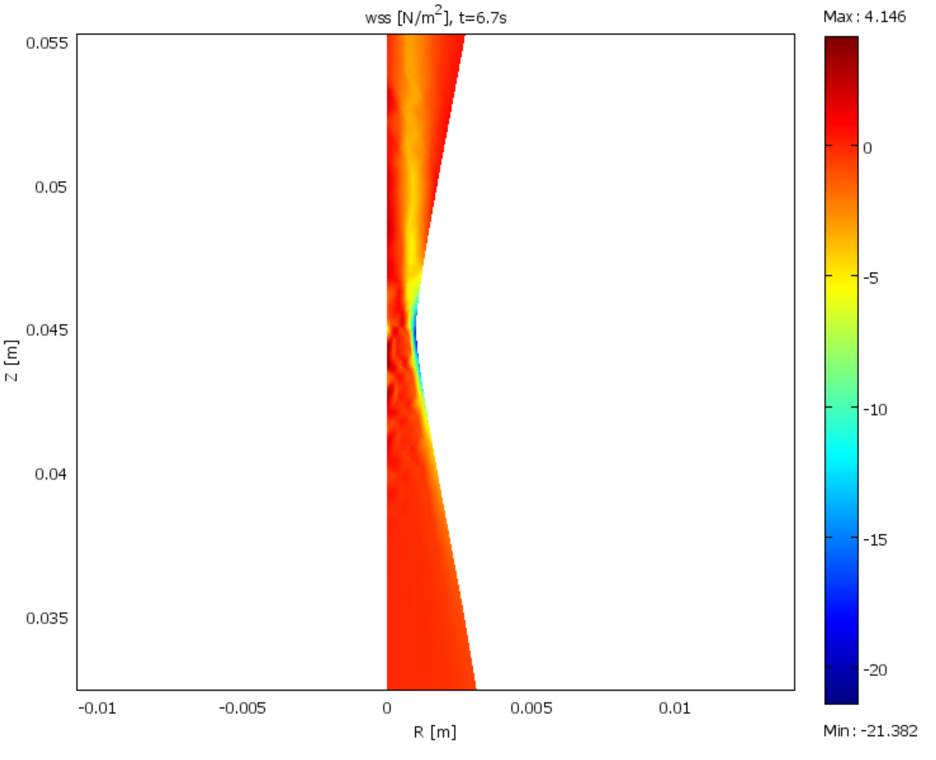

Figure 10. Value of the WSS along the ICA with stenosis of 70% (at t = 6,7*s*)

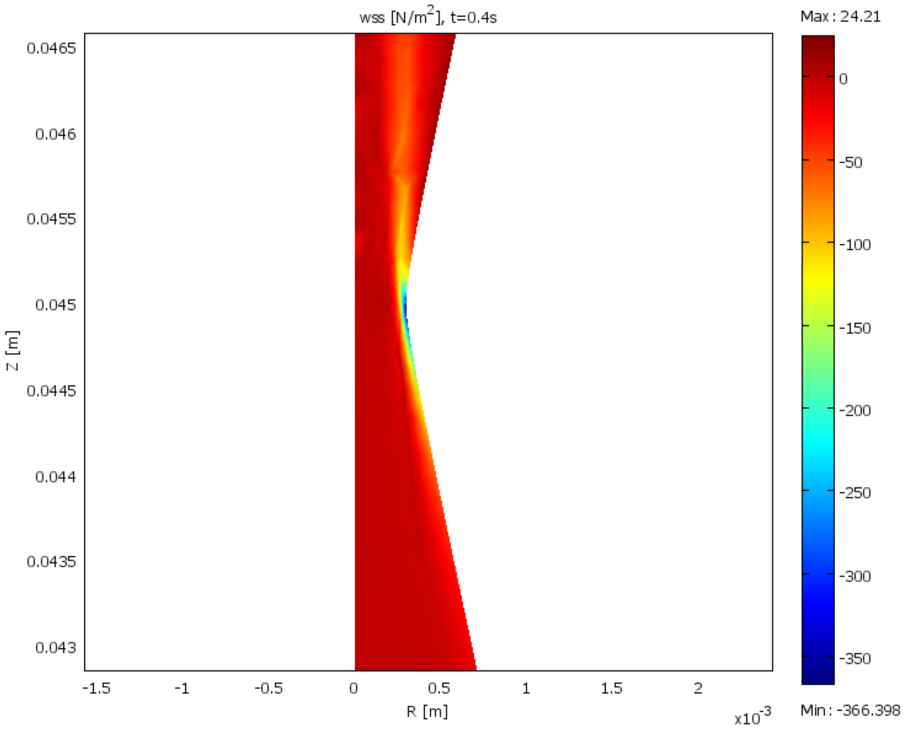

Figure 11. Value of the WSS along the ICA with stenosis of 90% (enlarged picture)

3. CONCLUSIONS

Using a non-Newtonian mathematical model, developed by us in a previous paper, we may calculate the values of the wall shear stress in some particular points of a human internal carotid artery with stenosis. These values numerically obtained are entirely consistent with those already presented in the international literature. This fact validates the accuracy of the used Cross type non-Newtonian mathematical and numerical model for the blood flow together with the viscoelastic behavior of the vessel walls. At the same time our algorithm of calculation of *WSS* provides a useful tool for the assessment of the rupture risk of artery with stenosis.

Furthermore, in the case of the artery with a stenosis of 30 % the highest value of the *WSS* is around $1N/m^2$, a value which does not differ significantly from the value of the *WSS* in a normal artery (without stenosis), according to Papaioannou & Stefanos, (2005). Nevertheless as the degree of the stenosis increases, the maximum value of the wall shear stress increases extraordinary.

In the case of the stenosis of 70% the highest value of the *WSS* is around $20N/m^2$, meanwhile the highest value of the *WSS*, in the case of the stenosis of 90%, passes the value of $350N/m^2$. The higher the value of the *WSS*, the higher the possibility of rupture of the stenosed artery.

REFERENCES

1. B. Albert, V. Vacaras, D. Trif, T. Petrila, Non-Newtonian approach of the blood flow in large viscoelastic vessels with stenosis or aneurysm, *Jokull Journal*, vol. **63**, No. 7, pp. 160-173, 2013.

2. D. Götz, *Three topics in fluid dynamics: Viscoelastic, generalized Newtonian, and compressible fluids*, Verlag Dr. Hut, Muenchen, pp. VII-X, 2012.
3. D. Minea, *Hemodinamica bifurcatiei carotidiene* (in Romanian), Ed. Univ. Transilvania, Brasov, pp. 169, 2003.
4. Th. G. Papaioannou, Ch. Stefanos, Vascular wall shear stress: basic principles and methods, *Hellenic J. Cardiol*., 46: 9-15, 2005.
5. T. Petrila, D. Trif, *Basics of fluid mechanics and introduction to computational fluid dynamics*, Springer U.S.A., 2005.
6. T, Petrila, B. Albert, Calculation of the Wall Shear Stress in the case of a Stenosed Internal Carotid Artery, *Indian Journal of Applied Research*, vol. 3, No. 9, pp. 396-368, 2013.
7. J. Tu, G.-H. Yeoh, Ch. Liu,. *Computational fluid dynamics*, Butterworth-Heinemann, Elsevier, UK, pp. 12, 2013.