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An EOQ model with time dependent Weibull deterioration and ramp type demand

Chaitanya Kumar Tripathy[a] and Umakanta Mishra[b*]

[a]*Department of Statistics, Sambalpur University, Jyoti Vihar, Sambalpur, Orissa-768019, India*
[b]*Department of Mathematics, P.K.A. College of Engineering, Chakarkend, Bargarh, Orissa -768028, India*

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A B S T R A C T

This paper presents an order level inventory system with time dependent Weibull deterioration and ramp type demand rate where production and demand are time dependent. The proposed model of this paper considers economic order quantity under two different cases. The implementation of the proposed model is illustrated using some numerical examples. Sensitivity analysis is performed to show the effect of changes in the parameters on the optimum solution.

1. Introduction

The control and maintenance of production inventories of deteriorating items with shortages have attracted much attention in inventory analysis. Deterioration plays an important role in developing inventory models since it is a natural process in many cases. Deterioration is normally identified as decay or damage in goods. Foods, drugs, pharmaceuticals, radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period and thus it plays an important role in analyzing the system.

Shah and Jaiswal (1977), Roychowdhury and Chaudhuri (1983), Dave (1986), Bahari-Kashani (1989), etc studied different types of order-level inventory models for deteriorating items where deterioration rate is considered to be constant. Whitin (1957) considered deterioration of fashion goods at the expiry of prescribed shortage period. Another deteriorating inventory model was developed where deterioration was considered in exponential form (Ghare & Schrader, 1963). Since

* Corresponding author. Tel/Fax: +91 9439412131
E-mail: umakanta.math@gmail.com (U. Mishra)

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then, there have been tremendous works on deteriorating items (Chakrabarti et al., 1998; Covert & Philip, 1973; Mishra, 1975; Goswami & Chaudhuri, 1991, 1992; Fujiwara, 1993; Hariga & Benkherouf, 1994; Wee, 1995; Jalan et al., 1996; Su et al., 1996). To know more work in this line one may consult the review articles written by Nahmias (1982) and Raafat (1991). Traditional inventory problems normally assume that demand is constant and given upfront. However, this simple assumption does not hold in many cases and it can be a function of price, time, etc. Donaldson (1977) is believed to be the first who introduced a linearly time-dependent demand function. There have been tremendous works on time-dependent demand inventory models (McDonald, 1979; Mitra et al., 1984; Ritchie, 1984; Deb & Chaudhuri, 1987; Goyal, 1988; Murdeshwar, 1988; Mandal & Pal, 1998; Panda et al., 2008; Abdul & Murata, 2011). Deng et al. (2007) also presented a review of inventory models for deteriorating items with ramp type demand.

In this paper, we develop economic order quantity (EOQ) models for deteriorating items which are time-dependent and the demand rate is a ramp type function of time. These types of problems are normally observed in the case of new brands of consumer goods. Demand rate for such items usually increases up to certain period and then it almost stabilizes. We assume that the unit production cost and the demand rate to be inversely proportionate. The first model discussed in this paper deals with model where shortage is prohibited and the second one is extended to cover the case of inventory allowing shortage. Two numerical examples are provided to illustrate the solution procedure of our models. Sensitivity analysis is carried out to show the effect of changes in the parameter on the optimum total average cost.

2. Proposed model

2.1. Model 1

To develop the inventory model where shortage is not allowed, the following assumption and notation are used.

a. The lead time is zero.

b. c_1 is the inventory holding cost per unit per unit of time.

c. c_3 is the deterioration cost per unit per unit of time.

d. $R = f(t)$, the demand rate, is assumed to be a ramp type function of time, i.e.

$f(t) = D_0[t-(t-\mu)H(t-\mu)]$, $D_0 > 0$, here $H(t-\mu)$ is a Heaviside's function which may be defined as follows:

$$H(t-\mu)=\begin{cases}1 & if \quad t\geq\mu,\\ 0 & if \quad t<\mu.\end{cases}$$

e. The production rate is $K = \delta f(t)$ where $\delta > 1$ is constant.

f. $\theta(t) = \alpha\beta t^{\beta-1}$ is the deterioration rate; where $0<\alpha<1$, $t \geq 0$ and $\beta > 0$. Generally α is called the scale parameter and β is the shape parameter.

g. C is the total average cost per production cycle.

h. t_1, the production time is greater than μ in no shortage period.

For $\alpha_1 > 0$, $\gamma > 0$ and $\gamma \neq 2$, the unit production cost $v = \alpha_1 R^{-\gamma}$ is positive. Thus v and R are inversely related which implies that higher demands result in lower per unit production costs and γ remains positive.

We have,

$$\frac{dv}{dR} = -\alpha_1 \gamma R^{-(\gamma+1)} < 0, \frac{d^2 v}{dR^2} = \alpha_1 \gamma(\gamma+1) R^{-(\gamma+2)} > 0.$$

Hence, we observe that the marginal unit cost of production is an increasing function of R. Further and with the increase in demand rate, the unit cost of production decreases with an increasing rate resulting encouragement to the manufacturer to produce more as the demand for the item increases. The nature of the solution of the problem requires restriction $\gamma \neq 2$. At initial time $t = 0$, the production starts with zero level stock. At time t_1, the production stops as the stock attains S level. Market demand and deterioration of items gradually diminishes the inventory level during the time period $t_1 \leq t \leq t_2$ which ultimately falls to zero at time $t = t_2$. At time $t = t_2$ the cycle again repeats.

Let $Q(t)$ be the inventory level at any time t $(0 \leq t \leq t_2)$.

Differential equations governing the instantaneous states of $Q(t)$ during the time interval $0 \leq t \leq t_2$ are as follows,

$$\frac{dQ(t)}{dt} + \theta(t) Q(t) = K - f(t), \qquad 0 \leq t \leq \mu \tag{1}$$

satisfying the initial condition $Q(0) = 0$,

$$\frac{dQ(t)}{dt} + \theta(t) Q(t) = K - f(t), \qquad \mu \leq t \leq t_1 \tag{2}$$

satisfying the condition $Q(t_1) = S$,

$$\frac{dQ(t)}{dt} + \theta(t) Q(t) = -f(t), \qquad t_1 \leq t \leq t_2 \tag{3}$$

satisfying the conditions $Q(t_1) = S$, $Q(t_2) = 0$.

Using $\theta(t) = \alpha\beta t^{\beta-1}$ and ramp type function $f(t)$, Eq. (1) to Eq. (3) we have the following,

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = (\delta - 1) D_0 t, \qquad 0 \leq t \leq \mu \tag{4}$$

satisfying the initial condition $Q(0) = 0$,

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = (\delta - 1) D_0 \mu, \qquad \mu \leq t \leq t_1 \tag{5}$$

satisfying the condition $Q(t_1) = S$,

$$\frac{dQ(t)}{dt}+\alpha\beta t^{\beta-1}Q(t)=-D_0\mu, \qquad t_1\le t\le t_2 \tag{6}$$

satisfying the conditions $Q(t_1)=S$, $Q(t_2)=0$.

Solving the Eqs. (4)-(6) yields,

$$Q(t)=\begin{cases}(\delta-1)D_0\left(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}-\frac{\alpha t^{\beta+2}}{2}\right) & if \quad 0\le t\le\mu\\ (\delta-1)D_0\mu\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}-\alpha t^{\beta+1}-\frac{\mu}{2}-\frac{\alpha\mu^{\beta+1}}{\beta+2}-\frac{\alpha\mu^{\beta+1}}{\beta+1}-\frac{\alpha\mu t^{\beta+1}}{2}\right) & if \quad \mu\le t\le t_1\\ S(1-\alpha t^{\beta}+\alpha t_1^{\ \beta})+D_0\mu\left\{t_1-t+\frac{\alpha}{\beta+1}(t_1^{\ \beta+1}-t^{\beta+1})+\alpha t^{\beta}(t-t_1)\right\} & if \quad t_1\le t\le t_2\end{cases} \tag{7}$$

We neglect the second and higher powers of α throughout the subsequent calculations as $0<\alpha<1$. Since $Q(t_2)=0$, from Eq. (7), we get,

$$S(1-\alpha t_2^{\ \beta}+\alpha t_1^{\ \beta})+D_0\mu\left\{t_1-t_2+\frac{\alpha}{\beta+1}(t_1^{\ \beta+1}-t_2^{\ \beta+1})+\alpha t_2^{\ \beta}(t_2-t_1)\right\}=0.$$

Simplifying and taking the first order approximation over α yields,

$$S=D_0\mu\left\{t_2-t_1+\frac{\alpha}{\beta+1}(t_2^{\ \beta+1}-t_1^{\ \beta+1})+\alpha t_2^{\ \beta}(t_1-t_2)+\alpha t_2^{\ \beta+1}-\alpha t_2^{\ \beta}t_1-\alpha t_1^{\ \beta}t_2+\alpha t_1^{\ \beta+1}\right\}. \tag{8}$$

The total inventory in $0\le t\le t_2$ is as follows,

$$\int_0^{\mu}Q(t)dt+\int_{\mu}^{t_1}Q(t)dt+\int_{t_1}^{t_2}Q(t)dt$$

$$=(\delta-1)D_0\left\{\frac{\mu^3}{6}+\frac{\alpha\mu^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha\mu^{\beta+3}}{2(\beta+3)}\right\}+(\delta-1)D_0\mu\left\{\frac{t_1^{\ 2}}{2}+\frac{\alpha t_1^{\ \beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha t_1^{\ \beta+2}}{\beta+2}-\frac{\mu t_1}{2}\right.$$

$$\left.-\frac{\alpha\mu^{\beta+1}t_1}{\beta+2}-\frac{\alpha\mu^{\beta+1}t_1}{\beta+1}-\frac{\alpha\mu t_1^{\ \beta+1}}{2(\beta+1)}-\frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{2\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha\mu^{\beta+2}}{\beta+1}+\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}\right\}+D_0\mu\left\{\frac{t_2^{\ 2}}{2}\right.$$

$$\left.-\frac{\alpha t_2^{\ \beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_2^{\ \beta+2}}{\beta+2}-t_2t_1-\frac{\alpha t_1t_2^{\ \beta+1}}{\beta+1}+\frac{\alpha t_2t_1^{\ \beta+1}}{\beta+1}+\frac{t_1^{\ 2}}{2}+\frac{\alpha t_1^{\ \beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\ \beta+2}}{\beta+2}\right\} \tag{9}$$

Total number of deteriorated items in $0\le t\le t_2$ is given by

$$=\delta\int_0^{\mu}D_0tdt+\delta\int_{\mu}^{t_1}D_0\mu dt-\int_0^{\mu}D_0tdt-\int_{\mu}^{t_2}D_0\mu dt=\frac{1}{2}D_0\delta\mu(2t_1-\mu)-\frac{1}{2}D_0\mu(2t_2-\mu). \tag{10}$$

The cost of production in $[u,u+du]$ is $Kvdu=\frac{\alpha_1\delta}{R^{\gamma-1}}du$. So the production cost in $0\le t\le t_1$ is

$$\int_0^{\mu}\frac{\alpha_1\delta}{R^{\gamma-1}}du+\int_{\mu}^{t_1}\frac{\alpha_1\delta}{R^{\gamma-1}}du=\frac{\alpha_1\delta D_0^{1-\gamma}}{2-\gamma}[(2-\gamma)\mu^{1-\gamma}t_1+(\gamma-1)\mu^{2-\gamma}],\qquad \gamma\neq 2 \tag{11}$$

Thus the total average cost is as follows,

$$C=\frac{1}{t_2}\Bigg[(\delta-1)D_0c_1\left\{\frac{\mu^3}{6}+\frac{\alpha\mu^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha\mu^{\beta+3}}{2(\beta+3)}\right\}+(\delta-1)c_1D_0\mu\left\{\frac{t_1^{\,2}}{2}+\frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha t_1^{\,\beta+2}}{\beta+2}-\frac{\mu t_1}{2}\right.$$

$$\left.-\frac{\alpha\mu^{\beta+1}t_1}{\beta+2}-\frac{\alpha\mu^{\beta+1}t_1}{\beta+1}-\frac{\alpha\mu t_1^{\,\beta+1}}{2(\beta+1)}-\frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{2\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha\mu^{\beta+2}}{\beta+1}+\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}\right\}+c_1D_0\mu\left\{\frac{t_2^{\,2}}{2}\right.$$

$$\left.-\frac{\alpha t_2^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_2^{\,\beta+2}}{\beta+2}-t_2t_1-\frac{\alpha t_1t_2^{\,\beta+1}}{\beta+1}+\frac{\alpha t_2t_1^{\,\beta+1}}{\beta+1}+\frac{t_1^{\,2}}{2}+\frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\,\beta+2}}{\beta+2}\right\}$$

$$+\frac{1}{2}c_3D_0\delta\mu(2t_1-\mu)-\frac{1}{2}c_3D_0\mu(2t_2-\mu)+\frac{\alpha_1\delta D_0^{1-\gamma}}{2-\gamma}\{(2-\gamma)\mu^{1-\gamma}t_1+(\gamma-1)\mu^{2-\gamma}\}\Bigg] \tag{12}$$

We can find the optimum values of t_1 and t_2 for minimum average cost C from the solutions of the following equations

$$\frac{\partial C}{\partial t_1}=0 \text{ and } \frac{\partial C}{\partial t_2}=0, \tag{13}$$

where

$$\frac{\partial^2 C}{\partial t_1^{\,2}}>0,\quad \frac{\partial^2 C}{\partial t_2^{\,2}}>0,\quad \text{and}\quad \frac{\partial^2 C}{\partial t_1^{\,2}}\frac{\partial^2 C}{\partial t_2^{\,2}}-\frac{\partial^2 C}{\partial t_1\partial t_2}>0.$$

From Eq. (13) we get

$$c_1(\delta-1)D_0\mu\left(t_1+\frac{\alpha t_1^{\,\beta+1}}{\beta+1}-\alpha t_1^{\,\beta+1}-\frac{\mu}{2}-\frac{\alpha\mu^{\beta+1}}{\beta+2}-\frac{\alpha\mu^{\beta+1}}{\beta+1}-\frac{\alpha\mu t_1^{\,\beta}}{2}\right)+c_1D_0\mu\left(\alpha t_1^{\,\beta}t_2+\frac{\alpha t_1^{\,\beta+1}}{\beta+1}\right.$$

$$\left.+\alpha t_1^{\,\beta+1}-\frac{\alpha t_2^{\,\beta+1}}{\beta+1}\right)+D_0\delta\mu c_3+\alpha_1\delta D_0^{1-\gamma}\mu^{1-\gamma}=0 \tag{14}$$

$$-\frac{1}{t_2^{\,2}}\Bigg[(\delta-1)D_0c_1\left\{\frac{\mu^3}{6}+\frac{\alpha\mu^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha\mu^{\beta+3}}{2(\beta+3)}\right\}+(\delta-1)c_1D_0\mu\left\{\frac{t_1^{\,2}}{2}+\frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha t_1^{\,\beta+2}}{\beta+2}-\frac{\mu t_1}{2}\right.$$

$$\left.-\frac{\alpha\mu^{\beta+1}t_1}{\beta+2}-\frac{\alpha\mu^{\beta+1}t_1}{\beta+1}-\frac{\alpha\mu t_1^{\,\beta+1}}{2(\beta+1)}-\frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{2\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha\mu^{\beta+2}}{\beta+1}+\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}\right\}+c_1D_0\mu\left\{\frac{t_1^{\,2}}{2}\right.$$

$$\left.+\frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\,\beta+2}}{\beta+2}\right\}+\frac{1}{2}c_3D_0\delta\mu(2t_1-\mu)+\frac{1}{2}c_3D_0\mu^2+\frac{\alpha_1\delta D_0^{1-\gamma}}{2-\gamma}\{(2-\gamma)\mu^{1-\gamma}t_1+(\gamma-1)\mu^{2-\gamma}\}\Bigg]$$

$$+D_0\mu c_1\left(\frac{1}{2}-\frac{\alpha t_2^{\,\beta}}{\beta+2}+\frac{(\beta+1)\alpha t_2^{\,\beta}}{\beta+2}-\frac{\alpha\beta t_1t_2^{\,\beta-1}}{\beta+1}\right)=0 \tag{15}$$

2.2. Model 2

In this section, we develop a model for deteriorating items when shortage is permitted and completely backlogged and a finite rate of replenishment is assumed for planning horizon. Let c_2 be the shortage cost per unit per unit of time. We start with zero stock at the initial stage. At $t=0$, production starts and continues till $t=t_1$. At this time the stock reaches S level and production is stopped at $t=t_1$. Accumulated inventory during $0 \le t \le t_1$, after meeting the demands during $0 \le t \le t_1$, is available for meeting the demand during $t_1 \le t \le t_2$. The stock is exhausted or reaches zero level at time t_2. Once demand is not satisfied, shortages start to develop and accumulates up to the level P at $t=t_3$. Again, after time t_3 production starts and inventory reaches zero level at time t_4 satisfying the demand during the period $t_3 \le t \le t_4$ along with the backlogged shortages during the period $t_2 \le t \le t_3$. At $t=t_4$, the production cycle completes and new cycle starts. The purpose of the study is to determine the optimum values of C, t_1, t_2, t_3 and t_4 subject to the assumptions stated earlier.

Let $Q(t)$ be the inventory level at any time t $(0 \le t \le t_4)$. The flowing differential equations represent the instantaneous states of $Q(t)$ during the time interval $0 \le t \le t_4$.

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = (\delta-1)D_0 t, \qquad 0 \le t \le \mu \tag{16}$$

satisfying the initial condition $Q(0)=0$,

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = (\delta-1)D_0 \mu, \qquad \mu \le t \le t_1 \tag{17}$$

satisfying the condition $Q(t_1)=S$,

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = -D_0 \mu, \qquad t_1 \le t \le t_2 \tag{18}$$

satisfying the conditions $Q(t_1)=S$, $Q(t_2)=0$,

$$\frac{dQ(t)}{dt} = -D_0 \mu, \qquad t_2 \le t \le t_3 \tag{19}$$

satisfying the conditions $Q(t_2)=0$, $Q(t_3)=-P$,

$$\frac{dQ(t)}{dt} = (\delta-1)D_0 \mu, \qquad t_3 \le t \le t_4 \tag{20}$$

satisfying the conditions $Q(t_3)=-P$, $Q(t_4)=0$.

From Eq. (7), the solutions of the Eq. (16) to Eq. (18) can be obtained and the solutions of Eq. (19) and Eq. (20) are as follows,

$$Q(t)=\begin{cases}(\delta-1)D_0\left(\dfrac{t^2}{2}+\dfrac{\alpha t^{\beta+2}}{\beta+2}-\dfrac{\alpha t^{\beta+2}}{2}\right) & if \quad 0\le t\le\mu\\ (\delta-1)D_0\mu\left(t+\dfrac{\alpha t^{\beta+1}}{\beta+1}-\alpha t^{\beta+1}-\dfrac{\mu}{2}-\dfrac{\alpha\mu^{\beta+1}}{\beta+2}-\dfrac{\alpha\mu^{\beta+1}}{\beta+1}-\dfrac{\alpha\mu t^{\beta+1}}{2}\right) & if \quad \mu\le t\le t_1\\ S(1-\alpha t^{\beta}+\alpha t_1^{\beta})+D_0\mu\left\{t_1-t+\dfrac{\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})+\alpha t^{\beta}(t-t_1)\right\} & if \quad t_1\le t\le t_2\\ -D_0\mu(t-t_2) & if \quad t_2\le t\le t_3\\ (\delta-1)D_0\mu(t-t_4) & if \quad t_3\le t\le t_4\end{cases} \tag{21}$$

During the time $t_2 \le t \le t_4$, there is no deterioration as the items produced are sent for meeting the demand, immediately. Hence, total number of deteriorated items during the time $0 \le t \le t_4$ will be the same as the one given in Eq. (10) i.e.

$\frac{1}{2}D_0\delta\mu(2t_1-\mu)-\frac{1}{2}D_0\mu(2t_2-\mu)$.

The total shortage during the time $t_2 \le t \le t_4$ is as follows,

$$\int_{t_2}^{t_3}[-Q(t)]dt+\int_{t_3}^{t_4}-[Q(t)]dt=\frac{1}{2}D_0\mu(t_3-t_2)^2+\frac{1}{2}(\delta-1)D_0\mu(t_4-t_3)^2,$$

and production cost during the time $t_3 \le t \le t_4$ is as follows,

$$\int_{t_3}^{t_4}Kvdu=\alpha_1\delta D_0^{1-\gamma}\mu^{1-\gamma}(t_4-t_3).$$

Hence the cost of production during the time $0 \le t \le t_4$ is computed as,

$$\frac{\alpha_1\delta D_0^{1-\gamma}}{2-\gamma}[(2-\gamma)\mu^{1-\gamma}(t_1+t_4-t_3)+(\gamma-1)\mu^{2-\gamma}]. \qquad \gamma\ne 2$$

The total average cost of the system during the time $0 \le t \le t_4$ is as follows,

$$\begin{aligned}C=\frac{1}{t_4}\Bigg[&(\delta-1)D_0c_1\left\{\frac{\mu^3}{6}+\frac{\alpha\mu^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha\mu^{\beta+3}}{2(\beta+3)}\right\}+(\delta-1)c_1D_0\mu\Bigg\{\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{\mu t_1}{2}\\&-\frac{\alpha\mu^{\beta+1}t_1}{\beta+2}-\frac{\alpha\mu^{\beta+1}t_1}{\beta+1}-\frac{\alpha\mu t_1^{\beta+1}}{2(\beta+1)}-\frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{2\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha\mu^{\beta+2}}{\beta+1}+\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}\Bigg\}+c_1D_0\mu\Bigg\{\frac{t_2^2}{2}\\&-\frac{\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_2^{\beta+2}}{\beta+2}-t_2t_1-\frac{\alpha t_1t_2^{\beta+1}}{\beta+1}+\frac{\alpha t_2t_1^{\beta+1}}{\beta+1}+\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\beta+2}}{\beta+2}\Bigg\}\\&+\frac{1}{2}D_0\mu c_2(t_3-t_2)^2+\frac{1}{2}D_0\mu c_2(\delta-1)(t_4-t_3)^2+\frac{1}{2}c_3D_0\delta\mu(2t_1-\mu)-\frac{1}{2}c_3D_0\mu(2t_2-\mu)\\&+\frac{\alpha_1\delta D_0^{1-\gamma}}{2-\gamma}\{(2-\gamma)\mu^{1-\gamma}(t_1+t_4-t_3)+(\gamma-1)\mu^{2-\gamma}\}\Bigg] \qquad , \qquad \gamma\ne 2\end{aligned} \tag{22}$$

The required optimum values of t_1, t_2, t_3 and t_4 which minimize the cost function C can be obtained from the solution of the following equations,

$$\frac{\partial C}{\partial t_1}=0\ ,\frac{\partial C}{\partial t_2}=0\,,\ \frac{\partial C}{\partial t_3}=0 \text{ and } \frac{\partial C}{\partial t_4}=0, \tag{23}$$

subject to the conditions that these values of $t_i\,(i=1,2,3,4)$ satisfy the conditions $D_i>0(i=1,2,3,4)$, where D_i is the Hessian determinant of order i given by

$$D_i=\begin{vmatrix} c_{11} & c_{12}\,\ldots\ldots\ldots c_{1i} \\ c_{21} & c_{22}\,\ldots\ldots\ldots c_{2i} \\ \vdots & \vdots \\ \vdots & \vdots \\ c_{i1} & c_{i2}\,\ldots\ldots\, c_{ii} \end{vmatrix}, \qquad c_{ij}=\frac{\partial^2 C}{\partial t_i \partial t_j} \qquad (i,j=1,2,3,4)$$

From Eq. (23) we get,

$$c_1\left\{(\delta-1)D_0\mu\left(t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}-\alpha t_1^{\beta+1}-\frac{\mu}{2}-\frac{\alpha\mu^{\beta+1}}{\beta+2}-\frac{\alpha\mu^{\beta+1}}{\beta+1}-\frac{\alpha\mu t_1^{\beta}}{2}\right)+D_0\mu\left(\alpha t_1^{\beta}t_2+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\alpha t_1^{\beta+1}-\frac{\alpha t_2^{\beta+1}}{\beta+1}\right)\right\}+\alpha_1\delta D_0^{1-\gamma}\mu^{1-\gamma}=0, \tag{24}$$

$$c_1D_0\mu\left(2t_2-t_2-\frac{\alpha t_2^{\beta+1}}{\beta+1}+\alpha t_2^{\beta+1}-t_1-\alpha t_2^{\beta}t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}\right)-c_3D_0\mu-c_2D_0\mu(t_3-t_2)=0 \tag{25}$$

$$D_0\mu\, c_2(t_3-t_2)-c_2(\delta-1)D_0\mu(t_4-t_3)-\alpha_1\delta D_0^{1-\gamma}\mu^{1-\gamma}=0, \tag{26}$$

$$\begin{aligned}-\frac{1}{t_4^2}\Bigg[&(\delta-1)D_0c_1\left\{\frac{\mu^3}{6}+\frac{\alpha\mu^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha\mu^{\beta+3}}{2(\beta+3)}\right\}+(\delta-1)c_1D_0\mu\left\{\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{\mu t_1}{2}\right.\\ &\left.-\frac{\alpha\mu^{\beta+1}t_1}{\beta+2}-\frac{\alpha\mu^{\beta+1}t_1}{\beta+1}-\frac{\alpha\mu t_1^{\beta+1}}{2(\beta+1)}-\frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{2\alpha\mu^{\beta+2}}{\beta+2}+\frac{\alpha\mu^{\beta+2}}{\beta+1}+\frac{\alpha\mu^{\beta+2}}{2(\beta+1)}\right\}+c_1D_0\mu\left\{\frac{t_2^2}{2}\right.\\ &\left.-\frac{\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_2^{\beta+2}}{\beta+2}-t_2t_1-\frac{\alpha t_1t_2^{\beta+1}}{\beta+1}+\frac{\alpha t_2t_1^{\beta+1}}{\beta+1}+\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\beta+2}}{\beta+2}\right\}\\ &+\frac{1}{2}D_0\mu\, c_2(t_3-t_2)^2+D_0\mu\, c_2(\delta-1)t_3^2+\frac{1}{2}c_3D_0\delta\mu(2t_1-\mu)-\frac{1}{2}c_3D_0\mu(2t_2-\mu)\\ &+\frac{\alpha_1\delta D_0^{1-\gamma}}{2-\gamma}\{(2-\gamma)\mu^{1-\gamma}(t_1-t_3)+(\gamma-1)\mu^{2-\gamma}\}\Bigg]+c_2D_0\mu(\delta-1)(t_4-t_3)+\alpha_1\delta D_0^{1-\gamma}\mu^{1-\gamma}=0.\end{aligned} \tag{27}$$

3. Numerical Example

Example 1. Consider $c_1=4$, $c_3=10$, $D_0=100$, $\mu=12$, $\alpha=0.005$, $\beta=0.4$, $\delta=8$, $\alpha_1=18$ and $\gamma=1.2$ as appropriate units. Using the Mathematica-5.1, we obtain the optimum solution for t_1 and

t_2 of Eq. (14) and Eq. (15) of Model 1, as $t_1^* = 3.90269$, $t_2^* = 125.476$. Using t_1^* and t_2^* in Eq. (12), we get the optimum average cost as $C^* = 348354$.

Example 2. Consider $c_1 = 4$, $c_2 = 6$, $c_3 = 10$, $D_0 = 100$, $\mu = 12$, $\alpha = 0.005$, $\beta = 0.4$, $\delta = 8$, and $\alpha_1 = 18$ and $\gamma = 1.2$ as appropriate units. Using the Mathematica-5.1, we obtain the optimum solution for t_1, t_2, t_3 and t_4 of Eq. (24) to Eq. (27) of Model 2 as $t_1^* = 3.61333$, $t_2^* = 32.5721$, $t_3^* = 48.7701$ and $t_4^* = 51.0835$. Using t_1^*, t_2^*, t_3^* and t_4^* in Eq. (22), we get the optimum average cost as $C^* = 230578$.

4. Sensitivity Analysis

We have performed sensitivity analysis by changing one parameter at a time by 25% and 50%, and keeping the remaining parameters at their original values. Table 1 and Table 2 summarize the results.

Table 1
The summary of the sensitivity analysis when shortage is not permitted

Parameter	% Change	t_1^*	t_2^*	C^*
c_1	+25	4.59006	139.481	465698
	+50	5.06788	152.399	595873
	-25	2.80482	110.01	244564
	-50	0.730086	93.8357	157066
c_3	+25	3.12905	126.696	353085
	+50	2.36735	128.278	358845
	-25	4.68689	124.643	344734
	-50	5.4807	124.216	342291
D_0	+25	3.90296	125.476	435440
	+50	3.90313	125.475	522523
	-25	3.90223	125.478	261271
	-50	3.90123	125.48	174186
μ	+25	6.00256	175.036	409603
	+50	8.38117	230.391	464952
	-25	2.01897	82.7045	171456
	-50	0.31018	48.7833	67226.1
α	+25	4.34322	125.481	356959
	+50	4.1081	125.481	357752
	-25	3.50523	125.45	349443
	-50	3.6992	125.466	348934
β	+25	4.19859	125.659	348735
	+50	4.65648	126.016	349701
	-25	3.70784	125.384	348277
	-50	3.57803	125.338	348142
δ	+25	3.97665	142.236	398470
	+50	4.02168	157.229	443445
	-25	3.76397	106.118	290806
	-50	3.42907	82.35687	221148
α_1	+25	3.90241	125.477	348358
	+50	3.90213	125.478	348362
	-25	3.90297	125.476	348352
	-50	3.90326	125.475	348348
γ	+25	3.90369	125.474	348343
	+50	3.9038	125.474	348342
	-25	3.84433	125.489	348740
	-50	3.82411	125.564	349002

Based on the results of Table 1, the following observations can be made.

(i) An increase on the values of the parameters c_1, c_3, D_0, μ, α, β, δ and α_1 will result to an increase on C^*.

(ii) An increase in the values of the parameter γ will result to an in decrease on C^*.

Table 2
The summary of the results when shortage is permitted

Parameter	% Change	t_1^*	t_2^*	t_3^*	t_4^*	C^*
c_1	+25	4.24743	31.9366	51.2598	54.0196	274666
	+50	4.6725	31.2571	53.4892	56.6646	315624
	-25	2.5675	33.0203	45.8238	47.6521	182567
	-50	0.534307	32.8206	41.8616	43.1525	129182
c_2	+25	3.61533	33.637	47.1081	49.032	240066
	+50	3.61684	34.4247	45.9661	47.6144	247101
	-25	3.61053	31.0469	51.4186	54.3279	217035
	-50	3.60634	28.6613	56.3347	60.2866	195949
c_3	+25	2.82575	31.9063	48.1408	50.4594	231058
	+50	2.04748	31.2575	47.51	49.8311	231276
	-25	4.40907	33.2565	49.4004	51.706	229846
	-50	5.21219	33.9619	50.0345	52.3299	228874
D_0	+25	3.6136	32.5716	48.7691	51.0825	288222
	+50	3.61377	32.5713	48.7685	51.082	345866
	-25	3.61285	32.5728	48.7719	51.0851	172934
	-50	3.61184	32.5745	48.7757	51.0886	115289
μ	+25	5.52904	47.9489	70.6475	73.8897	404061
	+50	7.65895	68.0917	97.7001	101.929	632877
	-25	1.85692	20.3445	30.6531	32.1247	110021
	-50	0.2222	10.5044	15.6986	16.439	36937
α	+25	3.89446	34.1653	50.1718	52.4577	237949
	+50	3.74929	33.3127	49.4198	51.7201	239328
	-25	3.36473	31.3454	47.7009	50.0367	232736
	-50	3.48545	31.9216	48.2022	50.5274	231710
β	+25	3.69785	33.5693	49.6514	51.9481	228989
	+50	3.81357	35.285	51.1862	53.4571	226511
	-25	3.54929	31.9404	48.2157	50.5401	231636
	-50	3.50001	31.5206	47.8484	50.1803	232355
δ	+25	3.70254	36.7164	55.3065	57.3714	265247
	+50	3.75928	40.4128	61.1422	63.026	296221
	-25	3.4529	27.7493	41.1869	43.8737	190513
	-50	3.08072	21.702	31.7686	35.1234	141481
α_1	+25	3.61304	32.5725	48.7712	51.0845	230578
	+50	3.61275	32.573	48.7723	51.0854	230578
	-25	3.61631	32.5716	48.7691	51.0825	230578
	-50	3.6139	32.5711	48.768	51.0815	230577
γ	+25	3.61433	32.5705	48.7665	51.0801	230578
	+50	3.61446	32.5702	48.766	51.0796	230578
	-25	3.60486	32.5841	48.7984	51.1093	230566
	-50	3.53385	32.6777	49.027	51.3139	230407

Based on the results of Table 2, the following observations can be made.

4.2.1. Any increase in the values of the parameters c_1 and μ will result to an increase on C^*.

4.2.2. Any increase in the values of the parameters c_2, c_3, D_0, α and δ will result to an increase on the value of C^*.

4.2.3. Any increase in the values of the parameters β will result to a decrease in C^*.

4.2.4. Any increase in the values of the parameters α_1 and γ will result in slight change in C^*.

As we can observe from the results of Table 1 and Table 2, the optimal average cost obtained in no shortage case is more than that of shortage case.

5. Conclusion

In this paper, we have presented a new economic order quantity model when the demand rate is a ramp type function of time. The ramp type demand is generally observed in new brand of consumer goods where demand increases for a certain period and then it stabilizes and becomes almost constant. The proposed model of this paper is considered for two different conditions where shortage is either prohibited for the first case and it is permitted for the second one. The proposed model was analyzed using two numerical examples and they were analyzed when parameters are set to different values.

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