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THE OPTIMAL CONTROL FOR D.C. POSITION DRIVE SYSTEM

Traian Munteanu, E. Rosu, T. Dumitriu, M. Culea, R. Paduraru, C. Dache

University “Dunarea de Jos” Galati

Abstract: A position drive system with D.C. motor is operated using conventional position, speed and current controls. These controls provide a good dynamic and steady-state behaviour, but they do not take in consideration the conversion’s efficiency. It is very well known that in the transient behaviour, as starting and stopping, the conversion efficiency is diminished to the value about 50 per cent, while in the steady-state it is greater. The main goal of the paper is to develop a new optimal type control law, which minimizes the drawn energy used for the covering of a given position trajectory. The synthesis of the optimal energetic control law is accomplished by comparing the two control methods, conventional and optimal. The experimental results, via the simulation procedure, are also presented.

Keywords: optimal control, energetic criteria, D.C. position drive system

1. INTRODUCTION

The dynamic periods for a position drive system, PDS, as starting and stopping, are very often resorted to. These frequent dynamic periods diminish considerable the conversion efficiency since this is smaller in the dynamic state than in the steady-state. The conventional control using P and/or PI controllers takes into account a good dynamic behaviour with respect to the input and the perturbation of the system, as well as the time for the realization of the given trajectory as being most important. Taking into consideration these requirements in the paper, an optimal control approach oriented to minimize the expenditure energy is developed. Using a model of a D.C. position drive system and adequate performance functional criteria, the non-recursive solution of the Riccati type differential matrix equation, EDMR, is involved as control law for the formulated problem. The solution, via simulation procedure, is applied to the PDS supplied from A.C. – D.C. four quadrants converter.

2. PROBLEM FORMULATION

2.1. *The model of a PDS*

A constant field DC position drive system controlled by armature voltage, Fig.1, is an invariant controllable dynamic system described by the differential equations

$$
\begin{aligned}
\frac{d\varepsilon(t)}{dt} &= \omega(t) \\
\frac{d\omega(t)}{dt} &= -\frac{F_v}{J}\omega(t) + \frac{c}{J}i_A(t) - \frac{1}{J}m_S(t) \qquad (1) \\
\frac{di_A(t)}{dt} &= -\frac{c}{L_A}\omega(t) - \frac{R_A}{L_A}i_A(t) + \frac{1}{L_A}u(t)
\end{aligned}
$$

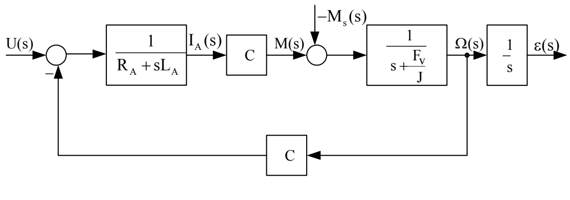

Fig.1 PDS model

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In the space state the differential equations (1) gets the form

$$\overset{\circ}{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{w}(t) \tag{2}$$

where:

The state is given by

$$\mathbf{x}(t) = \begin{bmatrix} \varepsilon(t) & \omega(t) & i_A(t) \end{bmatrix}^T \tag{3}$$

- $\varepsilon(t)$, $\omega(t)$ and $i_A(t)$ are the position, angular speed and armature current;
- $\mathbf{u}(t)$ is the armature voltage as input vector;
- $\mathbf{w}(t)$ is the load torque $m_S(t)$ as the perturbation vector;
- $\mathbf{A}, \mathbf{B}$ and $\mathbf{G}$ are the adequate constant matrices.

2.2 *The quadratic performance criteria*

The control problem consists in finding an admissible armature voltage $u^*(t)$, which transfers the system (2) from the initial state

$$\mathbf{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \tag{4}$$

to a desired state

$$\mathbf{x}_1 = \begin{bmatrix} \varepsilon_1 & 0 & 0 \end{bmatrix}^T \tag{5}$$

in the fixed time t_1, ε_1 being the desired final position, and minimizies the energy consumed.

In order to minimize the drawn energy, the quadratic performance criteria in the form

$$\begin{aligned} J = &\frac{1}{2}\left[\mathbf{x}(t_1) - \mathbf{x}_1\right]^T \mathbf{S}\left[\mathbf{x}(t_1) - \mathbf{x}_1\right] + \\ &+ \frac{1}{2}\int_0^{t_1} \left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right] dt \end{aligned} \tag{6}$$

is associated to system (2), where weighting matrices $\mathbf{S}$ and $\mathbf{Q}$ are 3x3 positive semidefinite matrices and $\mathbf{R}$ is a 1x1 positive definite matrix.

The purpose of the first term of the criteria (6) is to guarantee a good dynamic behaviour and a small square error between the final free state $\mathbf{x}(t_1)$ and the desired final state (5). If the matrix $\mathbf{S}$ has the form

$$\mathbf{S} = \begin{bmatrix} s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

the first term of criteria (6), often called the terminal cost, is given by

$$\lambda(t_1) = \frac{1}{2}\left[\mathbf{x}(t_1) - \mathbf{x}_1\right]^T \mathbf{S}\left[\mathbf{x}(t_1) - \mathbf{x}_1\right] = \frac{1}{2}s\left[\varepsilon(t_1) - \varepsilon_1\right]^2 \tag{8}$$

In the same way, by seting

$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \tag{9}$$

the second term of the equation (6) becomes

$$\begin{aligned} &\frac{1}{2}\int_0^{t_1} \left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t)\right] dt = \\ &= \frac{1}{2}\int_0^{t_1} \left[q_1\varepsilon^2(t) + q_2\omega^2(t) + q_3 i_A^2(t)\right] dt \end{aligned} \tag{10}$$

The term most important for consumed energy is

$$\frac{1}{2}\int_0^{t_1} \left[q_3 i_A^2(t)\right] dt = W_{RA} \tag{11}$$

this represents the energy expended in the armature winding, the most important constituent part of the drive dissipation energy.

The term

$$\frac{1}{2}\int_0^{t_1} \left[q_2\omega^2(t)\right] dt \tag{12}$$

represents the accumulated inertial energy. The term minimizes this energy in the aim of preserving the angular speed $\omega(t)$ within the rated limits. In the same way the third term

$$\frac{1}{2}\int_0^{t_1} \left[q_1\varepsilon^2(t)\right] dt \tag{13}$$

will assure a smoothing dynamic for the actual position $\varepsilon(t)$.

The last term

$$\frac{1}{2}\int_0^{t_1} \left[\mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right] dt = \frac{1}{2}\int_0^{t_1} \left[r u^2(t)\right] dt \tag{14}$$

where, $\mathbf{R} = \left[r\right]$ (15)

keeps the control $u(t)$, supply armature voltage, within the admissible limits.

The specified terminal time t_1 required to realize the final state ε_1, corresponds to the actual duration of the dynamic rating obtained using conventional control. Therefore, the optimal control problem is with free end point, specified terminal time and without constraints.

2.3 *The solutions of the problem*

The solution of the problem exists and is unique if the system (1) is controllable, completely observable and the weighting matrices carry out the conditions $\mathbf{Q} \geq \mathbf{0}, \mathbf{S} \geq \mathbf{0}$ and $\mathbf{R} > \mathbf{0}$ [Athans, 1966].

The solution of the optimal problem is given by

$$\mathbf{u}^*(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{y}(t) \tag{16}$$

$\mathbf{x}(t)$ and $\mathbf{y}(t)$, the cost vector, being solutions of the canonical system, Euler-Lagrange,

$$\begin{bmatrix} \overset{\circ}{\mathbf{x}}(t) \\ \overset{\circ}{\mathbf{y}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}} \\ -\mathbf{Q} & -\mathbf{A}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{y}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix} \mathbf{w}(t) \tag{17}$$

with the boundary conditions: the initial state, equation (4), and the transversality condition

$$\mathbf{y}(t_1) = \left[\frac{\partial \lambda(t)}{\partial \mathbf{x}} \right]_{t=t_1} = \mathbf{S}\left[\mathbf{x}(t_1) - \mathbf{x_1} \right] \quad (18)$$

Taken in consideration the transversality condition (18) the cost vector can be written as [Rosu, 1999]

$$\mathbf{y}(t) = \mathbf{P}(t)\mathbf{x}(t) + \mathbf{v}(t) \quad (19)$$

where $\mathbf{P}(t)$ is the solution of the EDMR

$$\begin{aligned} &\overset{\circ}{\mathbf{P}}(t) + \mathbf{P}(t)\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{P}(t) - \\ &-\mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}(t) + \mathbf{Q} = \mathbf{0} \end{aligned} \quad (20)$$

and $\mathbf{v}(t)$ is the solution of the associate differential vector equation, EDVA,

$$\overset{\circ}{\mathbf{v}}(t) + \mathbf{A}^{\mathrm{T}}\mathbf{v}(t) - \mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{v}(t) + \mathbf{P}(t)\mathbf{G}\mathbf{w}(t) = \mathbf{0} \quad (21)$$

Because of the boundary condition (18) the solutions $\mathbf{P}(t)$ and $\mathbf{v}(t)$ must be calculate recursively and backward in the time, from t_1 to 0. On the other hand the equations (20) and (21) are nonlinear. The schema for integration needs also all the future values of the perturbation vector $\mathbf{w}(t)$, including $\mathbf{w}(t_1)$, which is unrealizable. Obviously, the system (17) cannot be solved.

In [Rosu,1999] and [Rosu,1985] was proposed a non-recursive solution which can be computed at any time t, if the state $\mathbf{x}(t)$ and the perturbation $\mathbf{w}(t)$ are known at the moment of calculus, in the form

$$\begin{aligned} &\mathbf{u}^*(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}(t_1 - t)\mathbf{x}(t) + \\ &+\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{K_1}(t_1 - t)\mathbf{x_1} + \mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{K}_2(t_1 - t)\mathbf{w}(t) \end{aligned}$$
(22)

where:

$$\text{-}\quad \begin{aligned} &\mathbf{P}(t_1 - t) = \left[\mathbf{W}_{21} + \mathbf{W_{22}}\mathbf{e}^{-\mathbf{\Lambda}(t_1 - t)}\mathbf{E}\mathbf{e}^{-\mathbf{\Lambda}(t_1 - t)} \right] \cdot \\ &\cdot \left[\mathbf{W}_{11} + \mathbf{W}_{12}\mathbf{e}^{-\mathbf{\Lambda}(t_1 - t)}\mathbf{E}\mathbf{e}^{-\mathbf{\Lambda}(t_1 - t)} \right]^{-1} \end{aligned} \quad (23)$$

is the non-recursive EDMR solution;

-$\mathbf{K_1}(t_1 - t)$ and $\mathbf{K}_2(t_1 - t)$ are calculate like EDMR solution having the similar forms [2];

$$\text{-}\quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad (24)$$

is the eigenvvalues matrix and $\mathbf{W_{11}}......\mathbf{W_{22}}$ are the parts of the eigenvectors matrix;

$$\text{-}\quad \tau = t_1 - t \quad (25)$$

the remaining time until final moment t_1.

The solution (22) has three components: the first term is the state feedback; the second is the forcing component to achieve the desired final state $\mathbf{x_1}$; the third is the compensating feed-forward of the perturbation $\mathbf{w}(t)$. The solution is a non-recursive and can be computed at any time t and from initial time $t = 0$ to the final time t_1. The solution needs that the state $\mathbf{x}(t)$ and the perturbation $\mathbf{w}(t)$ to be known at the integration time t.

In the above formulation the optimal problem is one without constraints. Obviously there are magnitude limitations for the armature voltage and current and for the angular speed at the rated or maximum admissible values. The introduction of the constraints transforms the problem in nonlinear one, the solution finding becoming difficult, even impossible. The restrictions of the magnitude for the control and the state could be solved by the adequate choosing the weighting matrices Q and R [Rosu,1999].

The structure of the optimal control is presented in Fig.2.

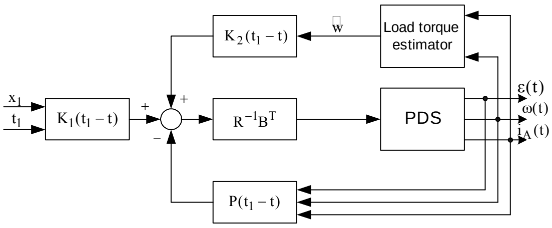


Fig.2 The structure of the optimal control

3. SIMULATION RESULTS

The model (1) and the optimal control law (22) were numerically simulated using Z-transform with zero order hold and Matlab-Simulink software for a PDS with the parameters:

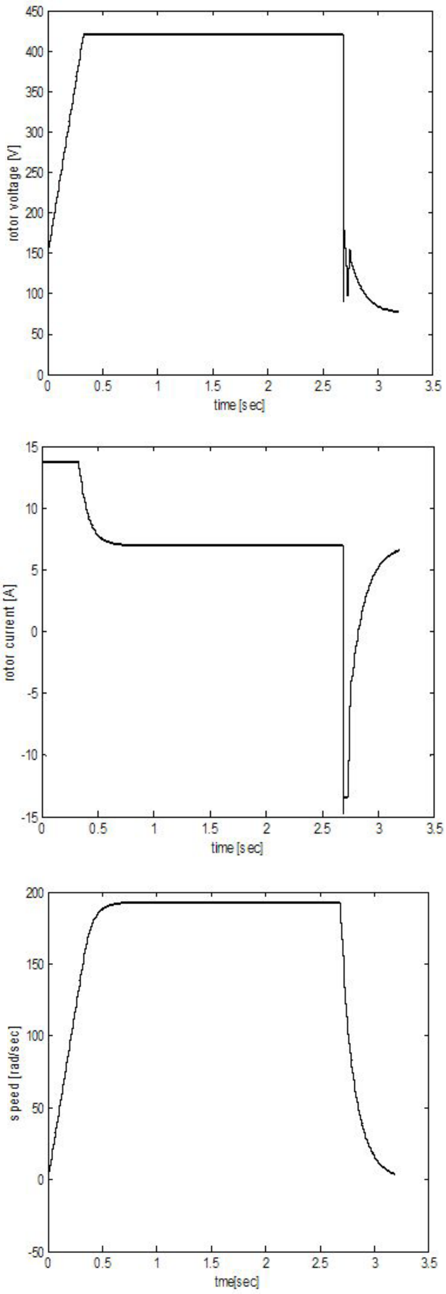
- D.C.motor: $2.2kW, 420V, 6.95A, 192.586 rad/\sec,$ $L_A = 40.5mH, R_A = 10.7\Omega;$
- Maximum load torque: $12.47 Nm$;
- Maximum armature current $I_{AM} = 2I_{AN} = 13.9A$;
- Total inertia: $0.026 Kgm^2$;
- Converter a.c.-d.c. $: 0 \pm 520V, 25A, 4 quadrants, k_D = 42$;
- The viscous friction F_v was neglected.

The conventional control for the PDS model (1) was simulated for a transition from 0 to the final position $\varepsilon_1 = 500 rad$ at the rated load torque $m_S = 12.47 Nm$ and the step position input. In Fig.3 are presented the variations, in order, of the rotor voltage and current, angular speed and position. All the components of the dynamic behaviour are normal and the dynamic is very good, with the final time being 3.2 seconds.

In Fig.4 the variations of the same parameters for the optimal control are presented. The simulated conditions are the same i.e.: final position $\varepsilon_1 = 500 rad$; the final time $t_1 = 3.2\sec$ is physically workable, selected by comparison with conventional control. The weighting matrices are chosen via a procedure presented in [Rosu,1999]. There are many differences between the two control laws. The angular speed variation obtained using optimal control is not the trapezoidal form given by the conventional control. The angular speed has a continuous variation, for both increase and decrease. The step of load torque from $m_{SO} = 2.47Nm$ to rated torque $m_S = 12.47Nm$ at the angular speed $\omega \geq 100 rad/\sec$ does not modify the angular speed increasing because of the compensating feed-forward of the perturbation $\mathbf{w}(t)$, but changes the voltage and current, Fig.5.

The variations of the armature voltage and current are also completely different, but the both are within the admissible limits. There are some differences regarding the position variation, but they are negligible. The final desired position is achieved with a minimum error

$$\varepsilon_1 - \varepsilon(t_1) = 500 - 499.6848 = 0.3152 rad \quad (26)$$



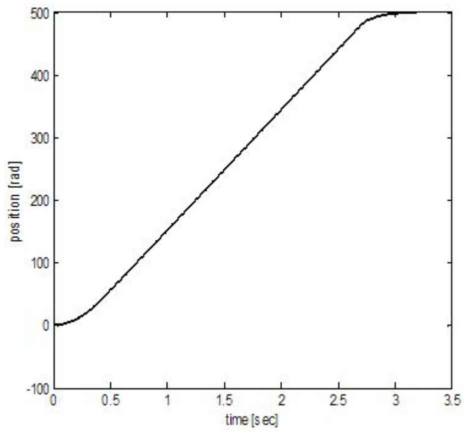
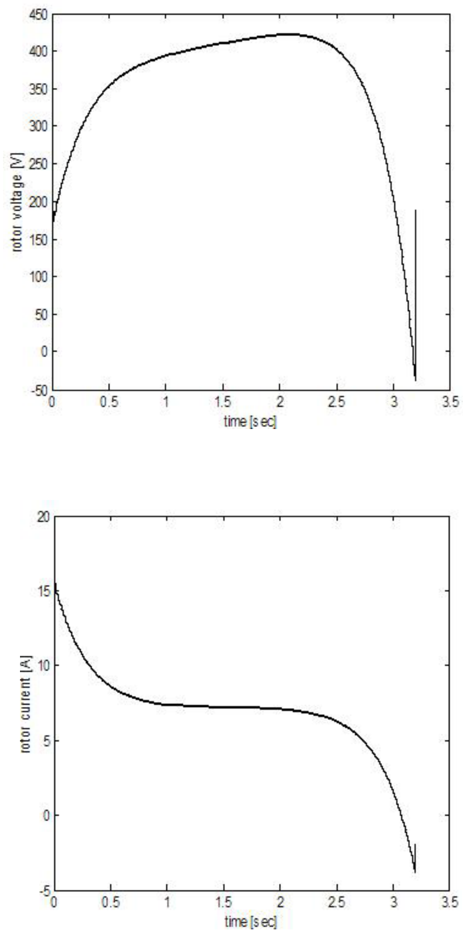


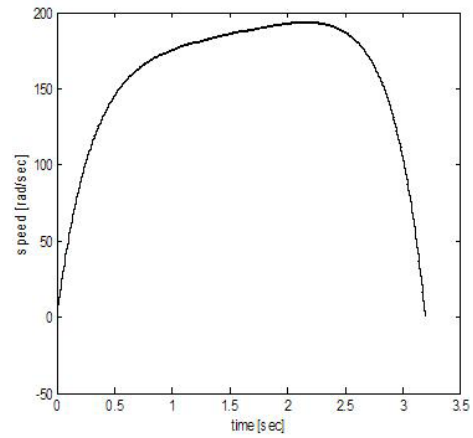
Fig.3. Conventional control, m_S=12.47Nm.

which is 0.063%, a very good response. Also the dynamic behaviour of the drive system is a normal one and physically realisable.

The energy analysis is presented in Table 1 and 2 where:

- W_I is the input, drawn energy, of the system;
- W_O is the output energy, i.e. the mechanically energy for the realisation of the trajectory $0 \rightarrow \varepsilon(t_1)$;





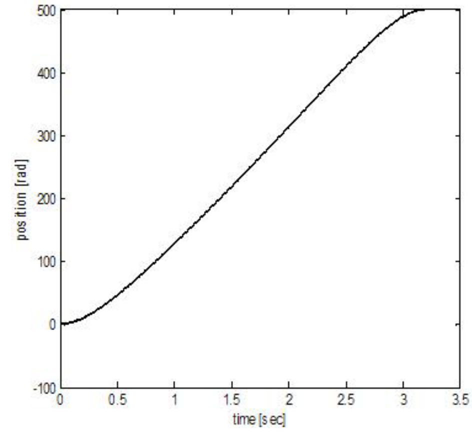
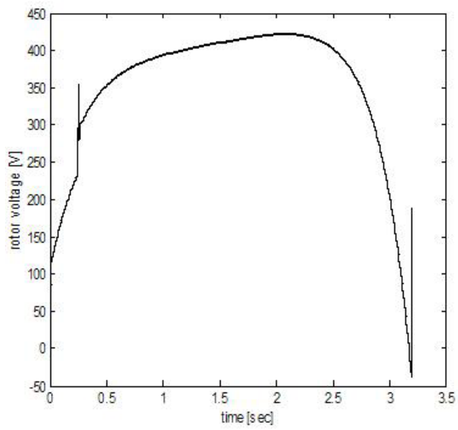
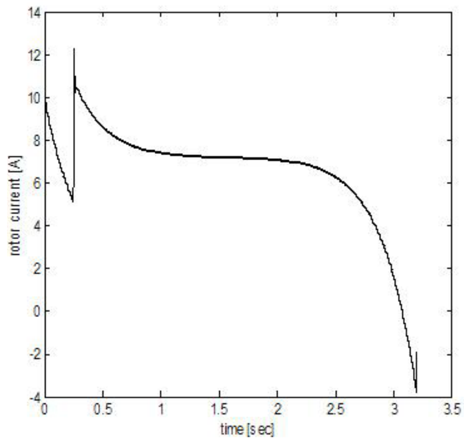


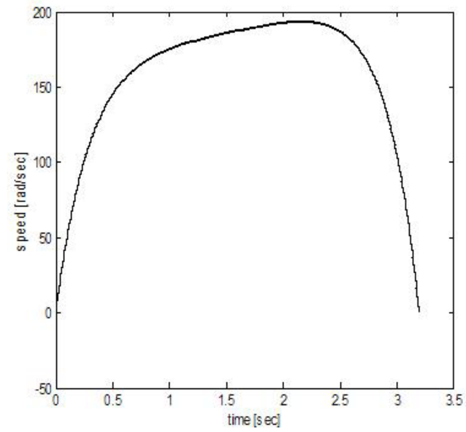
Fig.4. Optimal control, m_S=12.47Nm

- W_{RA} - the copper armature losses energy, the most important constituent part of the drive dissipation energy;
- W_J - the accumulated energy in the inertia masses;
- W_L - the accumulated energy in the armature inductance;



- $\varepsilon(t_1)$ - the final position.





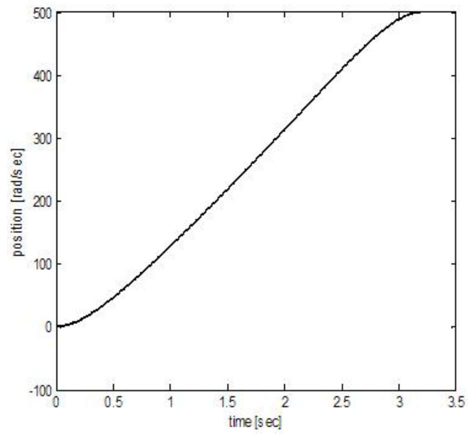


Fig.5. Optimal control, m_{S0}=2.47Nm and m_S =12.47Nm

Tabel 1

Control type	**Load torque, m_S [Nm]**	**W_I [J]**	**W_O [J]**	**W_{RA} [J]**	**W_J [J]**	**W_L [J]**	**$\varepsilon(t_1)$ [rad]**
Conventional	12.47	8423.8	6229.4	2194.0	1.1263	0.4683	499.5471
Optimal	12.47	8173.8	6231.1	1947.1	0.1702	0.2658	499.6848
Absolute difference		-250.0	+1.7	-246.9	-0.956	-0.202	-0.1377
%		-2.96	+0.02	-11.25			

Tabel 2

Control type	**Load torque, m_S [Nm]**	**W_I [J]**	**W_O [J]**	**W_{RA} [J]**	**W_J [J]**	**W_L [J]**	**$\varepsilon(t_1)$ [rad]**
Conventional	2.47 to 12.47	8213.8	6172.5	2040.9	1.17	0.409	499.7409
Optimal	2.47 to 12.47	7738.5	6090.6	1648.8	0.169	0.181	499.6848
Absolute difference		-475.3	-81.9	-392.1	-1.00	-0.23	+0.0561
%		-5.78	-1.33	-19.21			

There are important differences between the two controls. The differences between W_J and W_L are insignificant but correct because the two energies measure the final states of the angular speed and current which are appreciatively the same. The output energies W_O have about the same values since they measure the mechanical work effectuated to realise the displacement $\varepsilon(t_1)$, which is also similar in the two cases. The great difference is for the input energy, from 2.96% to 5.78%, less for optimal control than for conventional. The greater per cent reductions are for the copper armature losses energy W_{RA}, from 11.25% to 19.21%. This result is very important for the motor. The less solicitation of the motor, caused by the armature current form, allows an important overload and a better motor using.

4. CONCLUSION

The proposed optimal energetic control has the following features:

- it is a non-recursive one;
- it provides a significant reduction of the drawn and the copper armature losses energies;
- it assures high dynamic performances, without overshoots, fast compensation of the load torque and a smooth dynamic response;
- it can be implemented using either a conventional automation with some modifications or a new control structure.

In conclusion the optimal control approach proposed in the paper is an energetic one, which increases significantly the conversion efficiency of the PDS.

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