USE OF LOGARITHMIC FUNCTION FOR DROUGHT SEVERITY ASSESSMENT

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ABSTRACT. – Use of logarithmic function for drought severity assessment. Due to the multiplicative and cumulative effect of various risk events, the use of logarithmic function can provide satisfactory results in the analysis of risks. Based on the monthly values of climatic parameters from fourteen stations in the eastern part of Romania (1961-2006), the author uses the logarithmic function to evaluate and interpret the severity of droughts. With this function, deficient rainfall periods are diagnosed and classified according to three levels (moderate, high and extreme droughts). On these bases, the mathematical modeling allows to assess and evaluate the susceptibility to drought phenomena and the degree of continentalism in the eastern part of Romania.

Keywords: rainfall, logarithm, drought, continentalism.

1. INTRODUCTION

Dryness and drought phenomena evaluation is based on a very broad methodology, allowing a diagnosis more or less precise. Wide range of drought assessment indices and methods can be grouped into several categories, depending on the calculation procedure or the considered type of drought: rainfall indices and criteria, hydric balance and complex indices, diagrams and climograms, indices determined on the basis of satellite images (Stângă, 2009c). In this article, we focus on statistical and mathematical processing of rainfall data with direct reference to the first category of indices. Among these, we mention: the Hellman criterion, Topor rainfall index (Topor, 1964), deciles method (Gibbs and Maher, 1967), drought index Bhalme-Mooley, standardized precipitation index (McKee et al., 1993), effective drought index (Byun şi Wilhite, 1999), Gaussen xerothermic index (Gaussen and Bagnouls, 1953) etc.

Much of these indices and charts serve to a climatic diagnosis, being especially useful in studies of regional climatology. However, with some exceptions, their use in assessing the real significance of the severity of drought is blurring. In some cases, the reason is that these indices evolve linearly between the minimum and maximum values in relation to climatic elements included in the calculation equation. However, the relationship between a climate element and the its risk status may be expressed by a linear function only in very isolated cases. Most times, the effects of risk events are cumulative and/or multiplicative, for which the relationship between these effects and the considered variables are much better described by the exponential or power function (Stângă, 2009a, 2009b).

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2. PRACTICAL USE OF LOGARITHMS

The logarithm of a number to a given base is a power to witch the base must be raised in order to obtain that number ($\log_a x$). A logarithmic function is a function of the form $f(x)=\log_a x$, where a is a positive real number ($a\neq1$). The most important is the natural logarithmic function: $f(x)= \ln x$. A logarithmic growth describes a phenomenon whose intensity can be described as a logarithmic function of some imput. The logarithmic scale is a scale of measurement that use the logarithm of a physical unit instead the unit itself. Data representation on a logarithmic scale can be appropriate when input data cover a large range of values. In addition, the logarithms can be used for standardization and normalization of statistical variables. In statistics, logarithms are frequently used in logistic regression and logit function. Logistic regression is a variation of ordinary regression which is used when the dependent variable is a dichotomous variable (0 and 1) and the independent variables are continuous.

Logarithms, logarithmic functions and logarithmic scales are frequently used to describe different phenomena or probabilities. Thus, the Richter magnitude scale (local magnitude) is a base-10 logarithmic scale obtained by calculating the logarithm of the maximum amplitude of the seismic wave measured on a particular type of seismometer. In chemistry, pH is a negative base-10 logarithm of a molar concentration of dissolved hydronium ions; thus, a high pH indicates a low concentration of hydronium ions, while a low pH indicates a high one. The decibel is a logarithmic unit that indicates the ratio of a physical quantity relative to a specified reference level. The Palermo Technical Impact Hazard Scale is a logarithmic scale used by astronomers to rate the potential hazard of impact of a near-earth object, combining the probability of impact and estimated kinetic yield.

Based on the monthly values of climatic parameters from fourteen stations in the eastern part of Romania (1961-2006), the present paper shows the results of using the logarithmic function to evaluate and interpret the severity of droughts.

3. DROUGHT SEVERITY ASSESSMENT

Generally, when the effect of causal factors is multiplicative, logarithms of measurement data (e. g. monthly or annual values of rainfall or river flow) are characterized by a log-normal distribution (Giurma, 2009). The accuracy of the distribution is more significant if the logarithm is applied to a standardized value (standard deviation, for example) and if the string of data is enough long to be relevant (at least 40 years). Log-normal distribution has zero as lower limit, is unimodal (has only one maximum) and density distribution equation does not depend on the logarithm base, that interfering with a constant. Yet, the analysis of the normal distribution of values in a string of data is not the objective of this paper.

The multiplicative effect of the logarithmic function can be fully exploited to quantify this effect by the construction of certain risk indicators. In a previous study (2009a) we presented a methodology for calculating a rainfall index. Also, in that paper we have discussed a case study on the southern part of the Moldavian Plateau. The working methodology was based on the ratio between rainfall of a particular month (R_i) and the mean rainfall of that month (R_m): *K=Ri/Rm.*

Thus, the positive values correspond to periods when recorded rainfall is higher than mean rainfall, while the negative values correspond to periods of poor rainfall compared with the average. This ratio is very easy to be calculated for any region and any period of time (decade, month, growing season, year), but its relevance strongly depends on the extent of the data string. Although it has only a diagnosis character, this index allows comparisons between different climatic regions or between different time intervals. Subsequently, at that time, we had proceeded to calculate the 10-base logarithm of the ratio Ri/Rm.

For this article, we are testing now the natural logarithm, to obtain more suggestive results by *the rainfall logarithmic index* (RLI). The relation has the form:

$$RLI = \ln(\frac{Ri}{Rm}) = \ln K$$

Getting the K ratio logarithm has as a consequence the fact that subunitary values, with deficit character, become negative, while the values above one remain positive. The zero value of index corresponds to a period in which actual rainfall is equal to the mean rainfall of that interval. *The Rainfall Logarithmic Index (RLI) based on the natural logarithm is more suggestive in the analysis of risk events, due to the exaggeration of the extreme values.* That is why RLI allows to emphasize more clearly the periods of excess and deficient in terms of rainfall.

In the eastern part of Romania, the mean values of the RLI range between -0.20 (Rădăuţi) and -0.33 (Tecuci). In this small amplitude, regional differences are not very well captured. However, both the annual values and the monthly ones reveal an essential feature of temperate continental climate: the poor distribution of precipitations accompanied by the high frecuency of dry periods (Table 1). Although RLI can drop to values below -0.50 for September-October in the southern part of the Moldavian Tableland, it may be noted that negative values are characteristic as well for the wettest months of the year (June, for exemple).

Table 1. ***Rainfall logarithmic index (1961-2006)***

Station	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	AN
Adjud	-0.46	-0.31	-0.28	-0.21	-0.22	-0.17	-0.20	-0.26	-0.43	-0.48	-0.41	-0.37	-0.32
Bacău	-0.25	-0.15	-0.26	-0.15	-0.21	-0.11	-0.15	-0.22	-0.39	-0.39	-0.26	-0.23	-0.23
Bârlad	-0.40	-0.27	-0.37	-0.17	-0.22	-0.12	-0.15	-0.29	-0.53	-0.47	-0.38	-0.33	-0.31
Botoșani	-0.23	-0.27	-0.29	-0.21	-0.19	-0.15	-0.17	-0.20	-0.45	-0.35	-0.24	-0.30	-0.25
Cotnari	-0.23	-0.29	-0.30	-0.16	-0.21	-0.14	-0.15	-0.19	-0.48	-0.42	-0.21	-0.26	-0.25
Focșani	-0.49	-0.33	-0.40	-0.19	-0.21	-0.14	-0.22	-0.32	-0.37	-0.41	-0.50	-0.29	-0.32
Galați	-0.48	-0.28	-0.35	-0.18	-0.23	-0.19	-0.42	-0.22	-0.47	-0.46	-0.36	-0.32	-0.33
Iași	-0.25	-0.23	-0.34	-0.15	-0.27	-0.16	-0.14	-0.27	-0.52	-0.36	-0.24	-0.23	-0.26
Piatra Neamţ	-0.26	-0.20	-0.23	-0.15	-0.16	-0.10	-0.10	-0.18	-0.39	-0.36	-0.22	-0.28	-0.22
Rădăuţi	-0.20	-0.19	-0.24	-0.12	-0.17	-0.11	-0.13	-0.16	-0.35	-0.28	-0.21	-0.26	-0.20
Roman	-0.28	-0.28	-0.33	-0.16	-0.18	-0.11	-0.12	-0.20	-0.49	-0.38	-0.22	-0.28	-0.25
Suceava	-0.24	-0.20	-0.24	-0.16	-0.19	-0.13	-0.21	-0.20	-0.34	-0.28	-0.17	-0.27	-0.22
Tecuci	-0.43	-0.28	-0.36	-0.17	-0.22	-0.16	-0.20	-0.30	-0.46	-0.55	-0.46	-0.34	-0.33
Vaslui	-0.30	-0.25	-0.36	-0.19	-0.31	-0.12	-0.14	-0.36	-0.54	-0.38	-0.33	-0.27	-0.30
AVERAGE	-0.32	-0.25	-0.31	-0.17	-0.21	-0.14	-0.18	-0.24	-0.44	-0.40	-0.30	-0.29	-0.27

Given the specific values presented in table 1, their validation by comparative analysis has been considered appropriate. To this purpose, it was calculated Penman-Monteith reference evapotranspiration ($PM\text{-}ET_0$) based on its correlation with air temperature (T):

$$PM\text{-}ET_0 = 0{,}0048 \cdot T^2 + 0{,}0678 \cdot T + 0{,}4888$$ (Păltineanu ş.a., 2007).

Subsequently, we have proceeded to calculate the evapotranspiration/rainfall ratio (ET/R) and its correlation with RLI. In figure no. 1, is shown the correlation between the mean values of the ratio ET/R and of the RLI for the fourteen meteorological stations in research area. These two variables (ET/R and RLI) are directly proportional and the statistical relevance of the correlation is more than significant (R^2=0.78). The fact that the value "one" of the ratio ET/PP corresponds to a negative value of RLI has already been explained by the poor rainfall distribution, that the proposed index can illustrate.

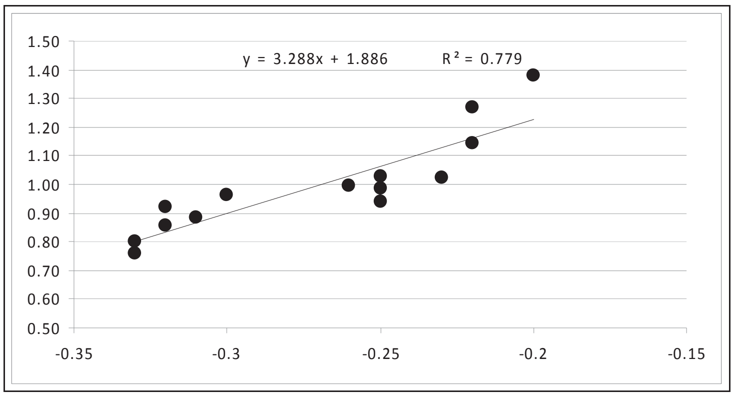

Fig. 1. ***Correlation between ET/R ratio (vertical axis) and RLI (horizontal axis)***

However, the usefulness of this index is much higher in the analysis of case studies to quantify the severity of the different periods of drought. In this situation, index values are not dimmed by mediation and the logarithm properties can be effective exploited. RLI values were grouped into six classes (four of them are considered dry, one is considered normal in terms of rainfall and the last belongs to rainy periods):

1. *Excessive drought* (RLI< -1.00; K < 0.37); *critical drought* (RLI< -2.0; K< 0.135);
2. *Severe drought* (-1.00 ≤ RLI < -0.70; K ≈ 0.37-0.50);
3. *Medium drought* (-0.70 ≤ RLI < -0.40; K ≈ 0.51-0.67);
4. *Dry period* (-0.40 ≤ RLI < -0.10; K ≈ 0.68-0.90);
5. *Normal period* (-0.10 ≤ RLI ≤ +0.10; K ≈ 0.91-1.10);
6. *Rainy periods* (RLI > +0.10; K > 1.10).

On the basis of this index, we find that in the eastern part of Romania, 50% of months have a deficit character, 10-15% of the them are normal in terms of rainfall and for 35-40% months rainfall is higher than average (Table 2).

Table 2. ***Weight of monthly rainfall rating according to RLI classes (1961-2006)***

Type of drought	1	2	3	4	5	6
Adjud	19.93	9.96	9.78	12.68	13.59	34.06
Bacău	11.96	9.42	14.67	15.58	13.77	34.60
Bârlad	18.12	7.25	12.86	13.59	11.05	37.14
Botoșani	15.94	7.97	13.22	15.22	12.14	35.51
Cotnari	16.85	7.79	11.96	14.49	14.49	34.42
Focşani	19.61	8.29	12.71	9.39	14.92	35.08
Galaţi	20.47	7.97	10.14	15.04	12.50	33.88
Iaşi	16.12	8.51	11.23	16.12	13.95	34.06
Piatra Neamţ	12.14	10.51	11.78	16.49	13.04	36.05
Rădăuţi	12.68	7.61	13.22	15.40	15.58	35.51
Roman	15.04	6.34	13.95	17.57	10.87	36.23
Suceava	13.77	8.51	11.05	17.21	13.04	36.41
Vaslui	18.48	7.97	11.59	14.67	11.41	35.87
Average	16.24	8.32	12.17	14.88	13.10	35.29

Taking into account the multiplicative character of the logarithmic function and the other mathematical properties of the logarithms, the graphic representation of data becomes more suggestive to emphasize the frequency and the severity of excessive drought (Fig. 2).

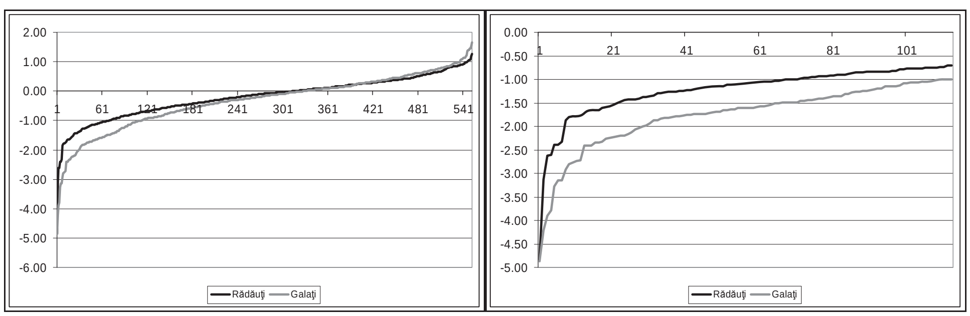

Fig. 2. ***Ascending distribution of RLI (1961-2006) for Galaţi and Rădăuţi (all values to the left and the excessive drought to the right)***

As a specific feature of the Moldavian Plateau climate, it is clear that RLI has a general asymmetry of the left, with the predominance of negative values. For the positive values, the threshold of +2.0 is never touched (the maximum value was +1.72 for January 1966 in Adjud). For the negative ones, however, there are extreme situations (3.4%) when the index value may fall below the critical threshold of -2.0. To this threshold, rainfall does not exceed 13.5% compared with mean value of that period. Below this threshold, drought affect the river flow, the soil water reserves, the groundwater aquifers etc. These are the critical droughts which, overlapping the growing season, completely compromise the agricultural crops and the natural ecosystem. Overall, *the severe and the excessive droughts* account for about 25% of the time; these are the two classes with high and very high risk.

4. RAINFALL ASSIMETRY INDEX

In this context, for regional studies, another index was calculated as the ratio between the sum of negative RLI (deficient rainfall) and the sum of positive RLI (excess rainfall). Changing the sign is to obtain the positive values of the index. This index has been originally named "continentalism index" (Stângă I. C., 2009a) by correspondence with Angot ratio. However, this synthetic index can be also used to describe other types of climate (boreal climate, oceanic climate). That is why, for general use, the expression *"rainfall assimetry index"* seems to be more appropriate. The general formula for calculating the index is:

$$RAI = -\frac{\sum_{RLI \le 0} RLI}{\sum_{RLI \ge 0} RLI}$$

For eastern Romania, the RAI values are comparable with those of the Angot ratio, with roughly the same meaning. Higher values of the index indicate more pronounced continental influences, while lower values (≤ 2.20) show climate without any continental influence. This index can be correlated with the altitude (Fig. 3). Its values move inversely with the altitude, decreasing gradually to the mountains. It should be noted that the threshold value 2.2 correspond to altitudes of 300-400 m. From these altitudes up, the natural setting of climatic elements occurs. Extending the research area, other factors must be taken into account (latitude, position on the continent or in relation with the mountain chains or the wide opening to the plains and plateaus of the Eastern Europe etc.).

This correlation with altitude is not possible for the Angot ratio. The higher altitudes stimulate the convective processes in the warm semester and therefore the value of Angot becomes greater. This could lead to the conclusion that continental features increase with altitude (that is false as can be).

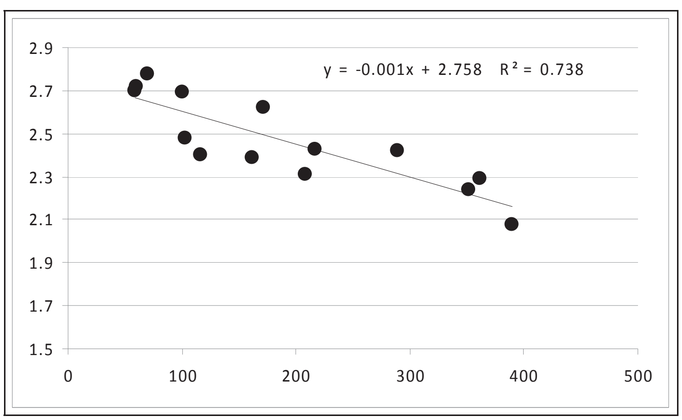

Fig. 3. ***Correlation between the rainfall assimetry index (vertical axis) and the altitude (horizontal axis)***

Moreover, this index reflects better the natural environment conditions, given that, on the one hand, it is based on the monthly values, and, on the other hand, it has a more complexe calculation method. This one takes into account both the number of deficient months, respectively excess months and the amount rainfall expressed by the deviation from the average. The rainfall assimetry index has the following values: Adjud: 2.69; Bacău: 2.31; Bârlad: 2.62; Botoşani: 2.39; Cotnari: 2.42; Focşani: 2.70; Galaţi: 2.78; Iaşi: 2.48; Piatra Neamţ: 2.29; Rădăuţi: 2.08; Roman: 2.43; Suceava: 2.24; Tecuci: 2.72; Vaslui: 2.40. The interpolation of these values and their correlation with altitude allow mapping of results, emphasizing the some regional differences in eastern part of Romania (Fig. 4). The latitude and the distance from the Carpathian mountains are not included in the equation.

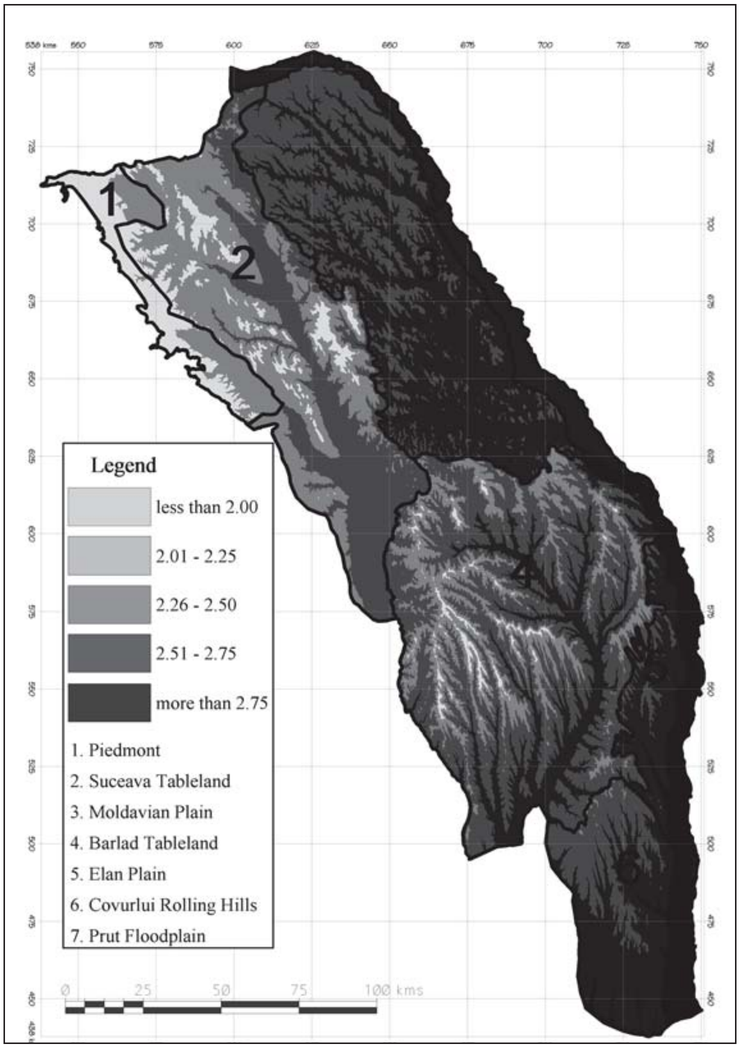

Fig. 4. ***The Rainfall assimetry index in the Moldavian Tableland (1961-2006)***

The proposed RAI is suitable for the analysis of monthly values. The continental influences are felt differently throughout the year, being more pronounced in winter (especially in January) and much lower in the warm season, when, in April-Juny, the Atlantic is strongly felt. This monthly analysis is not an objective of the present article, but some references could be useful. Thus, the *rainfall assimetry index* varies in June between 1.79 (Rădăuţi) and 2.24 (Galaţi), while in October, values range between 2.46 (Rădăuţi) and values above 3.0 in Southern Moldova.

5. CONCLUSIONS

Due to the multiplicative and cumulative effect of various risk events, the use of logarithmic function can provide satisfactory results in the analysis of risks. In this case, the two cantitative indices (RLI and RAI) should be illustrative and should lead to further research on this direction.

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