Will gravitational waves confirm Einstein's General Relativity?

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Abstract

Even if Einstein's General Relativity achieved a great success and overcame lots of experimental tests, it also showed some shortcomings and flaws which today advise theorists to ask if it is the definitive theory of gravity. In this proceeding paper it is shown that, if advanced projects on the detection of Gravitational Waves (GWs) will improve their sensitivity, allowing to perform a GWs astronomy, accurate angular and frequency dependent response functions of interferometers for GWs arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, will be the ultimate test for General Relativity. This proceeding paper is also a short review of the Essay which won Honorable Mention at the 2009 Gravity Research Foundation Awards.

Recently, the data analysis of interferometric GWs detectors has been started (for the current status of GWs interferometers see [1]) and the scientific community hopes in a first direct detection of GWs in next years.

Detectors for GWs will be important for a better knowledge of the Universe and either to confirm or rule out the physical consistency of General Relativity or of any other theory of gravitation [2, 3, 4, 5, 6, 7]. In fact, in the context of Extended Theories of Gravity, some differences between General Relativity and the others theories can be pointed out starting by the linearized theory of gravity [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. In this tapestry, detectors for GWs are, in principle, sensitive to a hypothetical *scalar* component of gravitational radiation appearing in extended theories of gravity too, i.e. theories like scalar-tensor gravity [4, 8, 9, 10, 12], bi-metric theory [5], high order theories [2, 3, 6, 7], Brans-Dicke theory [13] and string theory [14].

Reasons of extending General Relativity arise from the fact that, even if Einstein's Theory [15] achieved a great success (see for example the opinion of

Landau who says that General Relativity is, together with Quantum Field Theory, the best scientific theory of all [16]) and overcame lots of experimental tests [15], it also showed some shortcomings and flaws which today advise theorists to ask if it is the definitive theory of gravity [17, 18]. Differently from other field theories like the electromagnetic theory, General Relativity has not been quantized. This fact rules out the possibility of treating gravitation like other quantum theories, and precludes the unification of gravity with other interactions.

On the other hand, one can define *Extended Theories of Gravity* those semi-classical theories where the Lagrangian is modified, in respect to the standard Einstein-Hilbert gravitational Lagrangian, adding high-order terms in the curvature invariants (terms like R^2, $R^{\alpha\beta}R_{\alpha\beta}$, $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, $R\Box R$, $R\Box^k R$) or terms with scalar fields non minimally coupled to geometry (terms like $\phi^2 R$) [17, 18]. In general, it is well known hat terms like those have to be considered in all the approaches to perform the unification between gravity and other interactions. More, from a cosmological point of view, such modifies of General Relativity generate inflationary frameworks which are very important as they solve lots of problems of the Standard Universe Model [19]. We emphasize that we are not telling that General Relativity is wrong. It is well known that, even in the context of Extended Theories, General Relativity remains the most important part of the structure [4, 7, 17, 18]. We are only trying to understand if weak modifies on such a structure could be needed to solve some theoretical and observing problems [17, 18]. We also recall that even Einstein told that General Relativity could not be definitive [28]. In fact, during his famous research on the Unified Field Theory, he tried to realize a theory that he called "Generalized Theory of Gravitation", and he said that mathematical difficulties precluded him to obtain the final equations [28].

In the general context of cosmological evidences, there are also other considerations which suggest an extension of General Relativity. As a matter of fact, the accelerated expansion of the Universe, which is today observed, shows that cosmological dynamic is dominated by the so called Dark Energy, which gives a large negative pressure. This is the standard picture, in which such new ingredient is considered as a source of the *right side* of the field equations. It should be some form of un-clustered non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so called "concordance model" (ΛCDM) which gives, in agreement with the CMBR, LSS and SNeIa data, a good tapestry of the today observed Universe, but presents several shortcomings as the well known "coincidence" and "cosmological constant" problems [20]. An alternative approach is changing the *left side* of the field equations, seeing if observed cosmic dynamics can be achieved extending General Relativity [17, 18, 21, 22]. In this different context, one does not require to find out candidates for Dark Energy and Dark Matter, that, at the present time, have not been found, but only the "observed" ingredients, which are curvature and baryon matter, have to be taken into account. Considering this point of view, we can think that gravity is different at various scales [21] and a room for alternative theories is present. In principle, the most popular Dark Energy and Dark Matter models can be achieved considering $f(R)$ theories of gravity, where R is the Ricci curvature scalar, and/or Scalar-Tensor Gravity [17, 18, 22].

In this proceeding paper we show that, if advanced projects on the detection of GWs will improve their sensitivity, allowing to perform a GWs astronomy [1], accurate angular and frequency dependent response functions of interferometers for GWs arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, will be the ultimate test for General Relativity [2, 3, 4, 5, 6, 7, 10, 11, 12]. The papers which found this essay have been the world's most cited in the official Astroparticle Publication Review of ASPERA during the 2007 with 13 citations [23]. We recall that ASPERA is the network of national government agencies responsible for coordinating and funding national research efforts in Astroparticle Physics, see [30]. This proceeding paper is also a short review of the Essay which won Honorable Mention at the 2009 Gravity Research Foundation Awards [29].

Working with $G = 1$, $c = 1$ and $\hbar = 1$ (natural units), the line element for a GW arising from standard General Relativity and propagating in the z direction is [1, 15, 24, 25]

$$ds^2 = dt^2 - dz^2 - (1 + h_+)dx^2 - (1 - h_+)dy^2 - 2h_\times dxdy, \tag{1}$$

where $h_+(t + z)$ and $h_\times(t + z)$ are the weak perturbations due to the $+$ and the $\times$ polarizations which are expressed in terms of synchronous coordinates in the Transverse Traceless (TT) gauge [15]. In [24, 25] it has been shown that the total frequency and angular dependent response function (i.e. the detector pattern) to the $+$ polarization of an interferometer with arms in the u and v directions in respect to the propagating GW is:

$$\begin{aligned}\tilde{H}^+(\omega) &\equiv \Upsilon_u^+(\omega) - \Upsilon_v^+(\omega)\\ &= \frac{(\cos^2\theta\cos^2\phi - \sin^2\phi)}{2L}\tilde{H}_u(\omega,\theta,\phi) - \frac{(\cos^2\theta\sin^2\phi - \cos^2\phi)}{2L}\tilde{H}_v(\omega,\theta,\phi),\end{aligned} \tag{2}$$

that, in the low frequencies limit ($\omega \to 0$) gives the well known low frequency response function of [26, 27] for the $+$ polarization:

$$\tilde{H}^+(\omega) = \frac{1}{2}(1 + \cos^2\theta)\cos 2\phi + O(\omega)\ . \tag{3}$$

The derivation of eq. (2) has been shown in [24, 25, 29] using the "bouncing photons analysis" that was created in [31]. Actually, this kind of analysis has strongly generalized to angular dependences, scalar waves and massive GWs in [2, 4, 12, 24, 25, 29].

In the same way, the response function for the $\times$ polarization has been obtained as [24, 25, 29]

$$\tilde{H}^\times(\omega) = \frac{-\cos\theta\cos\phi\sin\phi}{L}[\tilde{H}_u(\omega,\theta,\phi) + \tilde{H}_v(\omega,\theta,\phi)], \tag{4}$$

that, in the low frequencies limit ($\omega \to 0$), gives the low frequency response function of [26, 27] for the $\times$ polarization:

$$\tilde{H}^{\times}(\omega) = -\cos\theta \sin 2\phi + O(\omega) \ . \tag{5}$$

The case of massless Scalar-Tensor Gravity has been discussed in [4, 12] with a "bouncing photons analysis" similar to the previous one. In this case, the line-element in the TT gauge can be extended with one more polarization, labelled with $\Phi(t+z)$, i.e.

$$ds^2 = dt^2 - dz^2 - (1 + h_+ + \Phi)dx^2 - (1 - h_+ + \Phi)dy^2 - 2h_\times dxdy. \tag{6}$$

The total frequency and angular dependent response function of an interferometer to this "scalar" polarization is [4, 12]

$$\begin{aligned}\tilde{H}^{\Phi}(\omega) = \frac{\sin\theta}{2i\omega L}\{&\cos\phi[1 + \exp(2i\omega L) - 2\exp i\omega L(1 + \sin\theta\cos\phi)] + \\ &- \sin\phi[1 + \exp(2i\omega L) - 2\exp i\omega L(1 + \sin\theta\sin\phi)]\}\,,\end{aligned} \tag{7}$$

that, in the low frequencies limit ($\omega \to 0$), gives the low frequency response function of [9, 14] for the Φ polarization:

$$\tilde{H}^{\Phi}(\omega) = -\sin^2\theta\cos 2\phi + O(\omega). \tag{8}$$

In [2, 3, 4, 7] it has also been shown that, in the framework of GWs, the cases of massive Scalar-Tensor Gravity and $f(R)$ theories are totally equivalent. It is well known that there is a more general conformal equivalence between Scalar-Tensor Gravity and $f(R)$ theories, even if there is a large debate on the possibility that such a conformal equivalence should be also an effective *physical* equivalence [17, 18, 21]. In such cases, because of the presence of a small mass, a longitudinal component is present in the third polarization. This implies that it is impossible to extend the TT gauge to the third mode [2, 3, 4, 6, 7]. But gauge transformations permit to put the line-element due to such a third scalar mode in a conformally flat form [2, 3, 4, 6, 7]:

$$ds^2 = [1 + \Phi(t - v_G z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{9}$$

If the interferometer arm is parallel to the propagating GW, the longitudinal response function, which has been obtained in [2] with the "bouncing photons analysis" and in [7] with a different treatment, associated to such a massive mode is

$$\Upsilon_l(\omega) = \frac{1}{m^4\omega^2 L}(\frac{1}{2}(1+\exp[2i\omega L])m^2\omega^2 L(m^2-2\omega^2)+$$

$$-i\exp[2i\omega L]\omega^2\sqrt{-m^2+\omega^2}(4\omega^2+m^2(-1-iL\omega))+$$

$$+\omega^2\sqrt{-m^2+\omega^2}(-4i\omega^2+m^2(i+\omega L))+$$

$$+\exp[iL(\omega+\sqrt{-m^2+\omega^2})](m^6L+m^4\omega^2L+8i\omega^4\sqrt{-m^2+\omega^2}+$$

$$+m^2(-2L\omega^4-2i\omega^2\sqrt{-m^2+\omega^2}))+2\exp[i\omega L]\omega^3(-3m^2+4\omega^2)\sin[\omega L]), \tag{10}$$

where m in eq. (10) is the small mass of the particle associated to the GW and v_G in eq. (9) is the particle's velocity (i.e. the group velocity as the massive GW has been analysed like a wave-packet [2, 7]). The relation mass-velocity is $m = \sqrt{(1-v_G^2)}\omega$ [2, 7].

Thus, if advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GWs astronomy (we recall that signals from GWs are quite weak) [1], one will only have to look the interferometer response functions to understand if General Relativity is the definitive theory of gravity. In fact, if only the two response functions (2) and (4) will be present, one will conclude that General Relativity is definitive. If the response function (7) will be present too, one will conclude that massless Scalar - Tensor Gravity is the correct theory of gravity. Finally, if a longitudinal response function will be present, i.e. Eq. (10) for a wave propagating parallel to one interferometer arm, or its generalization to angular dependences, one will learn that the correct gravity theory will be massive Scalar - Tensor Gravity which is equivalent to $f(R)$ theories. In any case, the analysed response functions will represent the ultimate test for General Relativity. In fact, General Relativity is the only gravity theory which admits only the two response functions (2) and (4) [4, 7, 17, 18]. Such response functions correspond to the two "canonical" polarizations h_+ and $h_\times$ of standard General Relativity. Thus, if a third polarization will be present, a third response function will be detected by GWs interferometers and this fact will rule out General Relativity like the ultimate theory of gravity.

Resuming, in this proceeding paper we have shown that, by assuming that advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GWs astronomy, accurate angular and frequency dependent response functions of interferometers for gravitational waves arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, will be the ultimate test for General Relativity.

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