SYNTHESIS OF MODEL THE LUENBERGER OBSERVER FOR EXTERNAL CYLINDRICAL GRINDING PROCESS

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Abstract: The problem of diagnosing the actual depth of cut at cylindrical grinding is considered. A mathematical model of the behavior of the grinding wheel and the workpiece during processing is worked out. According to this model it is produced a synthesis the model of the Luenberger observer with the Kalman filter to control the process of external cylindrical grinding. The developed approach is to improve the accuracy of control and the related with them computational *procedures of assessment and management.*

Keywords: grinding, control, model, Luenberger observer, Kalman filter

1. INTRODUCTION

The quality of work pieces depends both on the part blank and processing technology, and technological system (TS) properties. Grinding operations research shows that as a rule at the initial time choosing the right characteristics of a tool, cutting mode, optimal grinding cycle assembling provide set-up parameters of accuracy and quality of the detail surface. However, not only the nominal values of these parameters influence the performance properties of the products but their deviations that significantly increase during the production system operation. The presence of abnormalities is the reason of perturbation actions in technological process caused by instability of TS parameters. In addition instability of detail parameters is defined by exposure in the production process of TS of the changing external factors some of which are unknown and aren't controlled during processing. This problem is particularly acute for finishing operations, where quality parameters of finished products are finally formed and which are the most sensitive to perturbation actions [1].

In the under present-day conditions near 15 ... 20% of finishing operations are carried out by means of external cylindrical grinding. Operations are projected with use of traditional methods not fully accounting the influence of random factors that reduce the stability of manufactured products quality indicators. For quality index stabilization technological modes are assigned on the basis of adverse conditions, for example, the cutting properties renewal of worn grinding wheel is made much earlier than that it requires a real state.

There are used the deterministic models of the technological process, traditional ways of diagnostic and management in forecasting the condition of the TS. With that grinding process is complex stochastic nature, which leads to the spread of products quality index.

2. **PROBLEM STATEMENT**

Thus, for the process of finishing work it is a task of diagnosis as the need to define a number of parameters of the TS in the processing, including those that are not available for direct measurements, what is the purpose of this article. These parameters include, for example, the actual cutting depth, which is largely determined by the laws of material removal and wear of the abrasive tool. Its evaluation task (diagnostics), and therefore a number of other process parameters can be solved using the theory of dynamical Luenberger observers and experience of its application in developments of process control systems of flat grinding [2-4]. According to this experience, besides really existing dynamic object (Fig. 1, the upper part of the scheme), in the scheme of dynamic diagnostic for grinding there are included a dynamic model of the object, the unit of assessments (evaluator), the Kalman filter, the shaping filter, which operates in the mode of the real time.

Figure 1: The scheme of stochastic diagnosis

Interaction between the grinding wheel and work piece is primarily characterized by the parameters of the grinding wheel and work piece form, and their relative position; elastic, damping and other properties of the TS [1]. To develop the grinding wheel and work piece behavior model during machining we are giving the mathematical description of the process that characterizes the interaction between the grinding wheel and work piece.

3. MATHEMATICAL MODEL OF WORKING CONTACT DYNAMICS

As the mathematical model of the grinding wheel can be considered a rotating disk, and circumference in one-dimensional representation. The center of rotation inevitably doesn't coincide with the center of the shape, which determines the imbalance of the wheel, what is usually explained by the appearance of periodically varying forces generated during grinding. In addition, the surface of the wheel doesn't have an ideal geometry. There can be detected well-formed and random deviations of form and waviness with amplitude, frequency and phase changing for the tool life period.

Interaction scheme for the external cylindrical grinding process has the form shown in Fig. 2.

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Model of this system (Fig. 2) can be constructed on the base of the d'Alembert's principle in the form of differential equations set describing the movement dynamics for centers of the wheel and work piece, and change of the actual cut depth in the external grinding:

$$
\begin{cases}\nm_1\ddot{x}_1 + h_1\dot{x}_1 + c_1x_1 + h_3\dot{t}_f + c_3t_f - h_1\dot{S} - c_1S = 0, \\
m_2\ddot{x}_2 + h_2\dot{x}_2 + c_2x_2 - h_3\dot{t}_f - c_3t_f = 0,\n\end{cases}
$$
\n(1)

where m_1 , m_2 – are reduced masses of the work piece with bench centers and wheel with spindle respectively; h_i – is coefficient of resistance of *i* segment; c_i – is coefficient of rigidity of *i* segment; x_1 and x_2 – are coordinates of wheel and work piece movement center respectively; $S -$ is feed mechanism path per time *t*; $t_f -$ is actual cutting depth.

Figure 2: The equivalent scheme of external grinding machine dynamic system in initial position (a) and in cutting contact (b)

Actual cutting depth (size of the contact zone between work piece and tool on the center line) according to Fig. 2б is defined as

$$
t_f = R + r - L, \qquad (2)
$$

where $R = R_0 + \Delta R$ – is current radius-vector of the wheel surface taking into account its wear and shape deviations ΔR ; $r = r_0 + \Delta r -$ is current radius-vector of the work piece taking into account material removal and shape deviations Δr ; $L = L_0 + \Delta L = L_0 + x_2 - x_1$

– is the current distance between the rotation centers of wheel and work piece.

With a glance of t_f identification (2) and its constituent elements the system (1) will be

$$
\begin{cases}\nm_1\ddot{x}_1 + h_1(\dot{x}_1 - \dot{S}) + h_3(\dot{R} + \dot{r} - \dot{x}_2 + \dot{x}_1) + \\
+ c_1(x_1 - S) + c_3(R_0 + \Delta R + r_0 + \Delta r - L_0 - x_2 + x_1) = 0, \\
m_2\ddot{x}_2 + h_2\dot{x}_2 + c_2x_2 - h_3(\dot{R} + \dot{r} - \dot{x}_2 + \dot{x}_1) - \\
-c_3(R_0 + \Delta R + r_0 + \Delta r - L_0 - x_2 + x_1) = 0.\n\end{cases}
$$
\n(3)

Set of equations (3) in deviations for the initial position of the work piece, Fig. 2a, at the start of its contact with the tool ($L_0 = R_0 + r_0$; $S_0 = 0$; $x_{10} = 0$; $x_{20} = 0$; $t_f = 0$) can be written:

$$
\begin{cases}\nm_1\ddot{x}_1 + h_1\dot{x}_1 + c_1x_1 + h_3(\dot{x}_1 + \dot{R}) + c_3(x_1 + \Delta R) - \\
-h_3(\dot{x}_2 - \dot{r}) - c_3(x_2 - \Delta r) - h_1\dot{S} - c_1S = 0, \\
m_2\ddot{x}_2 + h_2\dot{x}_2 + c_2x_2 + h_3(\dot{x}_2 - \dot{r}) + c_3(x_2 - \Delta r) - \\
-h_3(\dot{x}_1 + \dot{R}) - c_3(x_1 + \Delta R) = 0.\n\end{cases}
$$
\n(4)

As a consequence of the wheel and the work piece rotation variations of geometrical dimensions have periodic or nearly periodic nature, which explains the appearance of internal excitation forces, and which substantially determine the dynamics of the grinding process. For

the further analysis it is reasonable the dynamical system (4) to reduce to the form
\n
$$
\begin{cases}\n\ddot{x}_1 = \frac{1}{m_1} [-(h_1 + h_3)\dot{x}_1 - (c_1 + c_3)x_1 + h_3\dot{x}_2 + c_3x_2] - \\
-\frac{1}{m_1} [h_3(\dot{R} + \dot{r}) + c_3(\Delta R + \Delta r)] + \frac{1}{m_1} [h_1\dot{S} + c_1S], \\
\ddot{x}_2 = \frac{1}{m_2} [-(h_2 + h_3)\dot{x}_2 - (c_2 + c_3)x_2 + h_3\dot{x}_1 + c_3x_1] + \\
+\frac{1}{m_2} [h_3(\dot{R} + \dot{r}) + c_3(\Delta R + \Delta r)].\n\end{cases}
$$
\n(5)

The first components on the right-hand sides of the relations correlation (5) are components with derivatives of the wheel and the work piece center deviations that depend directly on the internal generalized coordinates (geometric and kinematic) of the dynamical system. The second components reflect the influence of wheel and work piece form deviations. The third component of the first equation in the system (5) reflects the impact of penetrating way and grinding wheel feed on a dynamical system.

4. JOINT MODEL OF CONTACT AND OBSERVATIONS

With designations $y_1 = x_1$, $y_2 = \dot{y}_1 = \dot{x}_1$, $y_3 = x_2$, $y_4 = \dot{y}_3 = \dot{x}_2$ the system (5) can be reduced to the normal Cauchy form [5]

$$
\begin{aligned}\n\dot{y}_1 &= y_2, \\
\dot{y}_2 &= -\frac{1}{m_1} [(\tilde{n}_1 + \tilde{n}_3) y_1 + (h_1 + h_3) y_2 - c_3 y_3 - h_3 y_4] - \\
&\quad -\frac{1}{m_1} [c_3 (\Delta R + \Delta \mathbf{r}) + h_3 (\dot{R} + \dot{r})] + \frac{1}{m_1} [h_1 \dot{S} + c_1 S], \\
\dot{y}_3 &= y_4, \\
\dot{y}_4 &= -\frac{1}{m_2} [(c_2 + c_3) y_3 + (h_2 + h_3) y_4 - c_3 y_1 - h_3 y_2] + \\
&\quad + \frac{1}{m_2} [c_3 (\Delta R + \Delta r) + h_3 (\dot{R} + \dot{r})]\n\end{aligned}
$$

and can be logged via matrix form:

$$
\dot{Y}_0 = A_0 \cdot Y_0 + B_0 \cdot \psi + C_0 \cdot U , \qquad (6)
$$

To solve the problems of process dynamics modeling it is appropriate to write the relation (6) in conjunction with the equation of observation.

In matrix form of state space the system becomes:
 $\dot{Y}_0 = A_0 \cdot Y_0 + B_0 \cdot \Psi + C_0 \cdot U,$

$$
\dot{Y}_0 = A_0 \cdot Y_0 + B_0 \cdot \Psi + C_0 \cdot U,
$$
\n
$$
Z_0 = E_0 \cdot Y_0 + F_0 \cdot V_0,
$$
\n
$$
T_0 = Q_0 \cdot Z_0,
$$
\n(7)

where

where
\n
$$
I_0 = Q_0 \cdot Z_0,
$$
\nwhere
\n
$$
\dot{Y}_0 = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix}, Y_0 = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{c_1 + c_3}{m_1} & -\frac{h_1 + h_3}{m_1} & \frac{c_3}{m_1} & \frac{h_3}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{c_3}{m_2} & \frac{h_3}{m_2} & -\frac{c_2 + c_3}{m_2} & -\frac{h_2 + h_3}{m_2} \end{bmatrix}, B_0 = [B_{01} \ B_{02}],
$$
\n
$$
\hat{A}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, F_0 = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}, V_0 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, T_0 = [t_f], Q_0 = [-1 \ 1],
$$
\n
$$
Y_0 - \text{is vector (column-matrix), which represents the system status vector; } \quad \dot{Y}_0 - \text{is vector of}
$$

system status derivatives; A_0 – is matrix characterizing the dynamic properties of the system; B_0 – is matrix of parameters of work piece and wheel form deviations influence; Ψ – is state vector of work piece and wheel form deviations from nominal parameters; C_0 – is matrix of processing management; $U -$ is vector of control actions connected with cross-feed; $Z_0 -$ is matrix of measuring conditions; A_0 – is matrix of state of measurements; F_0 – is matrix of the meter noise intensity; V_0 – is matrix of independent Gauss white noise of unit intensity meters; T_0 – is matrix of cutting depth; Q_0 – is transformation matrix of aggregate measurements.

Estimated by (7) cutting depth t_f is not free from errors defined by the quality of the measurement process, and noises of measuring devices, and the effects of form random deviation components of work piece and wheel from the nominal range. Therefore, direct acquisition the rate $\frac{dt_j}{dt}$ $\frac{dt_f}{dt}$ of change of the depth of cut by differentiation of the depth t_f of cut defined from (7) is impractical due to the presence of additive noise in the aggregate measurement. To solve the problem of determining the speed of the cutting process it can be constructed the observing system in the form of the Kalman filter [2].

5. CONSTRUCTION OF OBSERVATION MODELS WITH THE KALMAN FILTER

Thus, direct application of the model (7) for constructing the observer is impractical because of the necessity of differentiation of the control signal and the forms deviation that can cause significant difficulties in the application of this approach in the presence of noise in the control channel and computing errors in the differentiation implementation. It is neither a feature of the matrix representation of the model nor description of a dynamic system in deviations, it is already apparent in the original representation (4) of the TS.

To eliminate this disadvantage it is necessary to build a "renewal equation", which in the case of form deviations will have an expression:

$$
X_0 = D_0 \cdot Y_0,
$$
\n(8)

where D_0 – is matrix characterizing the structure of state space parameters estimates and the structure of their linear combinations.

If evaluations of the entire state space are required, therefore D_0 is the unitary matrix. We introduce a modified state vector of the system in the form:

$$
F = Y_0 - B_{02} \cdot \Psi_1,
$$

where Y_0 , B_{02} , Ψ_1 – correspond to the expressions presented in the relationship (7).

From the recording of (7) follows directly that $\dot{\Psi}_1 = \Psi_2$.

From equation (9) it follows:

$$
Y_0 = F + B_{02} \cdot \Psi_1.
$$
\n
$$
(10)
$$

(9)

It is known f. e. [5] that for any jointed in the form matrices $\alpha(t)$ and $\beta(t)$ the ratio *dt* $\frac{\beta(t)}{dt} + \beta(t) \frac{d\alpha(t)}{dt}$ $\frac{d\theta(t)}{dt} = \alpha(t) \frac{d\beta(t)}{dt}$ $\frac{d[\alpha(t) \cdot \beta(t)]}{dt} = \alpha(t) \frac{d\beta(t)}{dt} + \beta(t) \frac{d\alpha(t)}{dt}$ is reasonable.

The derivative \dot{Y}_0 of the state vector (10) which obtained using the equation (10) and the above matrix identity is:

$$
\dot{Y}_0 = \dot{F} + \dot{B}_{02} \cdot \psi_1 + B_{02} \cdot \psi_2.
$$
\n(11)

With the opening of the split form the matrix equation (7) can be rewritten as:

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$$
\dot{Y}_0 = A_0 \cdot Y_0 + B_{01} \cdot \Psi_1 + B_{02} \cdot \Psi_2 + C_0 \cdot U. \tag{12}
$$

Substitution of Y_0 from (9) in the right side of equation (12) produces the result:
 $\dot{Y}_0 = A_0 \cdot [F + B_{02} \cdot \Psi_1] + B_{01} \cdot \Psi_1 + B_{02} \cdot \Psi_2 + C_0 \cdot U$

$$
\dot{Y}_0 = A_0 \cdot [F + B_{02} \cdot \Psi_1] + B_{01} \cdot \Psi_1 + B_{02} \cdot \Psi_2 + C_0 \cdot U \tag{13}
$$

Having compared the left sides of (11) and (13), and after appropriate transformations and grouping can be written the modified equation of state which does not contain Ψ_2 and therefore does not require differentiation of parameters Ψ_1 of forms:
 $\dot{F} = A_0 \cdot F + [A_0 \cdot B_{02} + B_{01} - \dot{B}_{02}] \cdot \Psi_1 + C_0 \cdot U$.

$$
\dot{F} = A_0 \cdot F + [A_0 \cdot B_{02} + B_{01} - B_{02}] \cdot \Psi_1 + C_0 \cdot U. \tag{14}
$$

Comparing equation (14) for the modified state space F and the initial ratio (7) for the state space Y_0 allows us to consider the coefficient in front of the matrix Ψ_1 ,

$$
B_1 = [A_0 \cdot B_{02} + B_{01} - \dot{B}_{02}], \qquad (15)
$$

 \overline{a} as a modified matrix of impact of work piece and wheel form variation.

If to take additionally that the damping parameters of the contact zone, the work piece and the grinding wheel masses are not dependent from time during machining, then the matrix elements \dot{B}_{02} are equal to zero and equation (15) takes the form:

$$
B_1 = [A_0 \cdot B_{02} + B_{01}]. \tag{16}
$$

The modified matrix differential equation of the state space is:
 $\dot{F} = A_0 \cdot F + B_1 \cdot \Psi_1 + C_0 \cdot U$.

$$
\dot{F} = A_0 \cdot F + B_1 \cdot \Psi_1 + C_0 \cdot U. \tag{17}
$$

Both the system of differential equations (7) and (17) correspond to coinciding with the accuracy of the symbols Y_0, \dot{Y}_0 and F, \dot{F} the systems of homogeneous linear differential equations $\dot{Y}_0 = A_0 \cdot Y_0$ u $\dot{F} = A_0 \cdot F$. Therefore, the first equation of the system (7) and the correlation (17) are equivalent in the sense of Liapunov [6].

State estimate X_0 on the basis of simulated results of modified system (17) and using transformation (12) is

$$
X_0 = D_0 \cdot F + D_0 \cdot B_{02} \cdot \Psi_1, \qquad (18)
$$

where the matrices X_0 , D_0 , B_{02} , Ψ_1 correspond to the matrix used in (7) and the modified state vector F is defined by (18).

Using the examined approach let's represent matrix product $C_0 \cdot U_0$ of the system (7) as follows:

$$
C_0 \cdot U_0 = C_{00} \cdot S + C_{01} \cdot \dot{S}, \qquad (19)
$$

where $C_{00} = \left[\frac{c_1}{m}\right]^t$, $C_{01} = \left[0 \frac{h_1}{m}\right]$ m_1 , C_{01} – $\begin{bmatrix} 0 & m_1 \end{bmatrix}$ $C_{00} = \left[\frac{c_1}{m} \right]^T$, $C_{01} = \left[0 \frac{h_1}{m} \quad 0 \quad 0 \right]^T$, $\begin{bmatrix} \frac{C_1}{m_1} \end{bmatrix}$, $C_{01} = \begin{bmatrix} 0 & \frac{h_1}{m_1} \end{bmatrix}$ $\left[\frac{c_1}{c_1}\right]^T$ $C = \left[0, \frac{h_1}{h_1}, 0, 0\right]^T$ and $=\left[\frac{c_1}{m_1}\right]^T$, $C_{01} = \left[0 \quad \frac{h_1}{m_1} \quad 0 \quad 0\right]^T$, an and let's reconstruct a model of the system (7) in

a form that does not require differentiation of the control signal. This will also need to convert observation equations of the system in order to restore conditions to be monitored, which allows to:

$$
\dot{F} = A_0 \cdot F + B_1 \cdot \Psi_1 + C_1 \cdot U, \nZ_0 = \mathring{A}_0 F + F_0 \cdot V_0 + D_M \cdot S, \nT_0 = Q_0 \cdot Z_0,
$$
\n(20)

where the matrices A_0 , \hat{A}_0 , Z_0 , F_0 , V_0 , T_0 , Q_0 coincide with the same matrices in correlations

(7),
$$
F = \begin{bmatrix} F_1 & F_2 & F_3 & F_3 \end{bmatrix}^T, \qquad \dot{F} = \begin{bmatrix} \dot{F_1} & \dot{F_2} & \dot{F_3} & \dot{F_3} \end{bmatrix}^T, C_1 = \begin{bmatrix} 0 & \frac{h_3 h_2}{m_1 m_2} & 0 & \frac{(h_3 + h_2) h_2}{m_2^2} - \frac{c_2}{m_2} \end{bmatrix}^T, D_M = \begin{bmatrix} 0 & \frac{h_2}{m_2} \end{bmatrix}^T.
$$

For each of ΔR_i , Δr_i the shape deviations from nominal their derivatives have random nature and may be characterized by a Gaussian-Markov process with a second-order correlation functions of the form according [2]:

$$
\text{From according [2]:}
$$
\n
$$
K_i(\nu_i, \tau) = \hat{E}_{0i} \cdot \exp(-\alpha_i \nu_i \, |\tau_i|) \cdot \tilde{n}os(\beta_i \nu_i \tau_i), \tag{21}
$$

where V_i – is the wheel or work piece peripheral speed, K_{0i} , α_i , β_i . – parameters of the correlation function which can be determined experimentally.

In the practically important cases the effects stipulated by cross-correlation functions of processes and corresponding energy mutual spectral densities are small concerning the energy range of wheel and work piece spectral characteristics.

Extended forming filter is created using the same method for such random processes allowing to provide the second summand of the first equation of the system (7) by equivalent parameters and variables of shaping filters state equations:

$$
\dot{G} = A_f \cdot G + B_f \cdot W,
$$

\n
$$
R = C_f \cdot G,
$$

\n
$$
H_f = Q_f \cdot R,
$$
\n(22)

where

$$
\dot{G} = \begin{bmatrix} \dot{G}_1 \\ \dot{G}_2 \end{bmatrix}, \qquad G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \qquad W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \qquad R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, \qquad H_f = \begin{bmatrix} \Delta \tilde{R}_i + \Delta \tilde{r}_i \\ \dot{\tilde{R}}_i + \dot{\tilde{r}}_i \end{bmatrix}, A_f = \begin{bmatrix} A_{f1} & A_k \\ A_k & A_{f2} \end{bmatrix},
$$
\n
$$
B_f = \begin{bmatrix} B_{f1} & 0 \\ 0 & B_{f2} \end{bmatrix}, \qquad C_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad Q_f = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \quad \dot{G}_i = \begin{bmatrix} \dot{g}_{1i} \\ \dot{g}_{2i} \end{bmatrix}, \qquad G_i = \begin{bmatrix} g_{1i} \\ g_{2i} \end{bmatrix}, \qquad w_i = \begin{bmatrix} w_{1i} \\ w_{2i} \end{bmatrix},
$$
\n
$$
R_i = \begin{bmatrix} R_{1i} \\ R_{2i} \end{bmatrix}, \qquad A_{fi} = \begin{bmatrix} 0 & 1 \\ \frac{1}{T_{1i}^2} & \frac{T_{2i}}{T_{1i}^2} \end{bmatrix}, \qquad B_{fi} = \begin{bmatrix} k_{fi} \cdot T_{3i} \\ -1 - k_{fi} \cdot T_{3i} \cdot T_{2i} \end{bmatrix}, \qquad T_{Ni} = T_{Ni}(V_i, \alpha_i, \beta_i, N = 1, 2, 3) \qquad [6],
$$

 $\frac{\Delta_{0i} u_i}{2 + R^2}$ 2 $\hat{p} = \sqrt{\frac{2\mathbf{\Lambda}_{0i} \alpha_i}{V_i(\alpha_i^2 + \beta_i^2)}}$ $k_{\hat{n}} = \sqrt{\frac{2K_{0i}\alpha}{r}}$ $=\sqrt{\frac{2K_{0i}\alpha_i}{V_i(\alpha_i^2+\beta_i^2)}}$ $\ddot{}$; $i \in \{\hat{e}, \ddot{a}\}\;$; $g_{1i}, g_{2i}; \dot{g}_{1i}, \dot{g}_{2i}$ - shaping filter auxiliary conditions; w_1, w_2 -

Gaussian white noises of unit intensity which are independent between themselves and noise measurements V_o (7).

Taking into account (22) extended model of the system (7) takes the form:

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$$
\begin{bmatrix} \dot{F} \\ \dot{G}_f \end{bmatrix} = \begin{bmatrix} A_0 & B_1 \cdot Q_f \\ 0 & A_f \end{bmatrix} \cdot \begin{bmatrix} F \\ G_f \end{bmatrix} + \begin{bmatrix} C_1 \\ 0 \end{bmatrix} \cdot U + \begin{bmatrix} 0 \\ B_f \end{bmatrix} \cdot W,
$$

\n
$$
Z_0 = \dot{A}_0 \cdot Q_0 \begin{bmatrix} F \\ G_f \end{bmatrix} + F_0 \cdot V_0 + D_M \cdot U,
$$

\n
$$
T_0 = Q_0 \cdot Z_0,
$$

\n
$$
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$
\n(23)

where, additionally [6], $G_f = G_f(G)$ *u* $1 \t0 \t0 \t0 \t0 \t0 \t0 \t0$
0 1 0 0 0 0 0 0 (*G*); $Q_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. 0 1 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 $\dot{G}_f = \dot{G}_f(\dot{G}); Q_f$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $=\dot{G}_{f}(\dot{G});$ $Q_{f}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\dot{G}_f = \dot{G}_f(\dot{G}); Q_f =$

The first and second equations of the system (23) should be presented in the form of:

$$
X = A \cdot X + B \cdot U + E \cdot W,
$$

\n
$$
Z = C \cdot X + R \cdot V,
$$
\n(24)

The type and structure of the matrices in correlations (24) is definitely determined by (23).

For the system (24) it is possible to build optimal in mean square the system status estimations in a form of a Kalman filter.

The minimum achievable dispersion of the system status estimations (24) can be estimated by Riccati matrix equation of the form [6, 2]:
 $\dot{\tilde{V}} = A \cdot \tilde{V} + \tilde{V} \cdot A^{T} + E \cdot \Psi_{w} \cdot E^{T} - \tilde{V} \cdot C^{T} \cdot \Psi_{v}^{-1} \cdot C \cdot \tilde{V}$,

$$
\tilde{\mathbf{V}} = \mathbf{A} \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{V}} \cdot \mathbf{A}^{\mathrm{T}} + \mathbf{E} \cdot \mathbf{\Psi}_{\mathrm{w}} \cdot \mathbf{E}^{\mathrm{T}} - \tilde{\mathbf{V}} \cdot \mathbf{C}^{\mathrm{T}} \cdot \mathbf{\Psi}_{\mathrm{v}}^{-1} \cdot \mathbf{C} \cdot \tilde{\mathbf{V}} \tag{25}
$$

which should be solved before to the processing of specific work piece in consequence of the fact that there are no results of observations of a dynamic system therein.

Matrix of the Kalman filter coefficients of amplification is determined by the relationship [6, 2]

$$
K = \tilde{V} \cdot C \cdot \Psi_v^{-1}.
$$
 (26)

Taking into account (25), (26) algorithm of observations filtering can be determined by matrix equations

$$
\hat{Y} = A \cdot \hat{Y} + B \cdot U + K \cdot [Z - B \cdot U - C \cdot \hat{X}],
$$
\n
$$
\begin{bmatrix}\n\hat{i}_f \\
\hat{i}_f\n\end{bmatrix} = \begin{bmatrix}\n-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0\n\end{bmatrix} \cdot \hat{Y}
$$
\n(27)

6. CONCLUSION

The relations (26) and (27) allow us to use the results of coordinate measuring in detail production process for the purpose of constructing of estimations of the processing parameters. These estimates are optimal subject to Gaussian measurement noise and excitement.

The use of this approach is useful in assessments of directly unmeasured parameters. The approach allows to reduce the effects of noises both of the measurements and associated with computational procedures of related assessments. It must be directly used in the implementation of stochastic observation and filtering procedures.

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